Lambda-term problem in contemporary universe

Elena Arbuzova

Dubna State University & Novosibirsk State University

based on joint work with A. D. Dolgov

arXiv:2502.05581, accepted in Eur. Phys. C

Supported by RSF grant 23-42-00066

22th Lomonosov Conference
on Elementary Particle Physics

21 – 27 August 2025, MSU, Moscow, Russia

Outline

- Vacuum energy problem
- Compensation mechanism
- Numerical calculations
- Conclusion

Cosmological constant (Einstein, 1917)

$$S = -\frac{M_{Pl}^2}{16\pi}\int d^4x\sqrt{-g}(R+\Lambda) \equiv -\frac{M_{Pl}^2}{16\pi}\int d^4x\sqrt{-g}R - \varrho_{vac}\int d^4x\sqrt{-g}$$

- Λ – cosmological constant, «Λ-term»
- g – determinant of the metric tensor $g_{\mu\nu}$, $M_{Pl} = 1.22\times 10^{19}$ GeV – Planck mass

The energy-momentum tensor of vacuum:

$$T_{\mu\nu}^{(vac)} = g_{\mu\nu}\varrho_{vac}, \quad \varrho_{vac} = \frac{M_{Pl}^2}{16\pi}\Lambda$$

The Einstein equations with Lambda-term included:

$$\frac{M_{Pl}^2}{8\pi}\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) \equiv \frac{M_{Pl}^2}{8\pi}G_{\mu\nu} = T_{\mu\nu}^{(matt)} + \varrho_{vac}g_{\mu\nu}$$

In homogeneous, isotropic, and 3D-flat space: $ds^2 = dt^2 - a^2(t)d\mathbf{r}^2$

The first Friedman equation:

$$\frac{3H^2M_{Pl}^2}{8\pi} = T_0^0 = \varrho_{matt} + \varrho_{vac}, \quad H = \frac{\dot{a}}{a} \text{ – the Hubble parameter}$$

Vacuum energy problem

Contradiction between theoretical estimates of the magnitude of ϱ_{vac} and observational constraints on its possible value.

It is tempting to identify vacuum energy with cosmological dark energy (DE), since they have the same equation of state: $P = -\varrho$.

- P and ϱ are respectively the pressure and energy densities.

DE makes $\sim 70\%$ of the total cosmological energy density:

$$\varrho_{DE} \sim 1\,\mathrm{keV/cm^3} \approx 10^{-47}\,\mathrm{GeV^4}$$

Theoretical estimates give:

- either infinitely large value
- or, in the case of cancellations of vacuum energies of bosonic and fermionic vacuum fluctuations, the result of the order of the SUSY breaking scale:

$$\varrho^{\mathrm{vac}}_{\mathrm{SUSY}} \sim m^4_{\mathrm{SUSY}} \sim 10^{55}\varrho_{DE}\,, \quad m_{\mathrm{SUSY}} \sim 100\ GeV$$

During cosmological evolution vacuum energy underwent colossal jump during phase transitions from a symmetrical phase to a phase with broken symmetry.

Structure of the QCD-vacuum

Proton mass $m_p \sim 1$ GeV:

- $p = uud$, $m_q \sim 5$ MeV $\Longrightarrow m \approx (15\text{MeV} - E_{bind}) \gtrsim 0.01 m_p$ – ???
  E_{bind} is the binding energy of quarks in a proton.

The missing contribution:

- nontrivial properties of the QCD vacuum $\Longrightarrow$ condensates of quark and gluon fields:

$$\langle \bar{q} q \rangle \neq 0 \,, \quad \langle G_{\mu\nu} G^{\mu\nu} \rangle \neq 0 \,, \quad \varrho_{vac}^{(cond)} \approx 1\text{GeV}^4$$

- Quarks inside a proton destroy the condensates and the proton mass:

$$m_p = 2m_u + m_d - \varrho_{\text{vac}}^{(cond)} l_p^3 \sim 1\text{GeV}, \quad l_p \sim 1/\text{GeV is the proton size}$$

The energy density of the condensate must be negative and by 47 orders of magnitude larger than the observed value $\varrho_{DE} \approx 10^{-47}\,\text{GeV}^4$.

Something else, except for quarks and gluons, "lives" in vacuum and this "something" (new field – ??) compensates ϱ_{vac} by 47 orders of magnitude.

The first model of dynamical reduction of vacuum energy

- Dolgov, 1982 (Nuffield Workshop on the Very Early Universe, proceedings)

$$S_\phi = \int d^4x\sqrt{-g}\mathcal{L} = \int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - U(\phi,R)\right]$$

Equation of motion for the homogeneous field ϕ in FLRW-metric:

$$\ddot{\phi} + 3H\dot{\phi} + \partial U/\partial\phi = 0\,, \quad U = \frac{1}{2}\left(\beta R + m^2\right)\phi^2$$

When $\beta R < 0$ and $|\beta R| > |m^2|$ the EoM in de Sitter space-time has unstable solutions, exponentially rising with time, since $(m_\phi^{(eff)})^2 < 0$ at $R = const$.

With rising ϕ the initial exponential expansion will asymptotically transform into a power-law one:

$$a(t) \sim \exp(H_{vac}t) \implies \phi \sim t \text{ and } a(t) \sim t^\kappa, \text{ where } \kappa = const$$

Reverse reaction of ϕ to the cosmological expansion leads to the transformation of the exponential expansion law into the Friedmann law, despite the presence of nonzero vacuum energy.

Shortcomings of this simple model:

- The energy-momentum tensor of field ϕ is not proportional to $g_{\mu\nu}$:
  $T_{\mu\nu} \neq \Lambda g_{\mu\nu} \Longrightarrow$ the vacuum energy does not vanish, even asymptotically
- The change in the expansion regime is achieved due to the weakening of the gravitational interaction. The gravitational coupling constant decreases with time, first exponentially, and then as $G_N \sim 1/t^2$.
- If such a change in G_N took place in the early universe and later somehow stabilised, then this mechanism could explain the hierarchy of the gravitational and electroweak scales.

Other models:

- Dolgov, Kawasaki, 2003 (arXiv:astro-ph/0307442, astro-ph/0310822)

$$L_{int} = \frac{\partial_\mu \phi \partial^\mu \phi}{2R^2}$$

- Dolgov, Urban, 2008 (arXiv:0801.3090): several different types of the interaction potential between the curvature scalar and a scalar field.

However, in each case, the transition to the canonical cosmology dominated by usual matter was not realised.

Generalisation of the model

$$\mathcal{L}_F = \frac{1}{2}\phi^2\left[\beta R F(R,\phi) + m^2\right]$$

Arbuzova, Dolgov, 2025 (arXiv: 2502.05581 [gr-qc]):

$$\mathcal{L}_f = \frac{1}{2}\phi^2\left[\beta R f(\phi) + m^2\right] \equiv \frac{1}{2}\left[\phi^2 m^2 + \beta R Q(\phi)\right], \quad Q(\phi) = \phi^2 f(\phi)$$

Equation of motion for homogeneous field ϕ:

$$g^{\mu\nu}D_\mu D_\nu\phi + m^2\phi + \frac{1}{2}\beta R\partial_\phi Q = \ddot{\phi} + 3H\dot{\phi} + m^2\phi + \frac{1}{2}\beta R\,\partial_\phi Q = 0$$

- D_μ is the covariant derivative in FLRW-metric and $\partial_\phi Q = \partial Q/\partial\phi$.

The energy–momentum tensor of ϕ is defined as:

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}}$$

Energy-momentum tensor

Correspondingly

$$T_{\mu\nu} = (\partial_\mu\phi)(\partial_\nu\phi) - (1/2)g_{\mu\nu}\left[g^{\alpha\beta}(\partial_\alpha\phi)(\partial_\beta\phi) - m^2\phi^2\right]$$
$$-\beta Q(\phi)\left(R_{\mu\nu} - g_{\mu\nu}R/2\right) + \beta\left(D_\mu D_\nu - g_{\mu\nu}D^2\right)Q(\phi)$$

The covariant derivatives of $Q(\phi)$:

$$D_\mu Q = (\partial_\phi Q)\partial_\mu\phi, \;\; D^2 Q = \partial_\phi^2 Q\partial_\mu\phi\partial^\mu\phi + \partial_\phi Q D^2\phi$$

The trace of the energy-momentum tensor:

$$T_\nu^\nu = -(\partial\phi)^2(3\beta\partial_\phi^2 Q + 1) + 2m^2\phi^2 + \beta QR + 3\beta\left[(\partial_\phi Q)m^2\phi + \beta R(\partial_\phi Q)^2/2\right]$$

For a special case $Q = \phi^2$ the trace is:

$$T_\nu^\nu = -(6\beta+1)(\partial_\mu\phi)(\partial^\mu\phi) + \beta(6\beta+1)R\phi^2 + 2(1+3\beta)m^2\phi^2$$

- This is the well known result. Note, that $T_\nu^\nu = 0$ for $\beta = -1/6$ and $m = 0$.

More Complicated Model

To cure the problems of a simpler model described above we consider:

$$\bar{Q}(\phi, M_0, k) = \phi^2(1 + \sigma\phi^2/M_0^2)^k,$$

- M_0, k are some constant parameters to be fixed below. We take $\sigma = \pm 1$.

The derivatives of $\bar{Q}$ over ϕ are given by:

$$\begin{aligned}
\partial_\phi \bar{Q} &= \frac{2k\sigma\phi^3(1 + \sigma\phi^2/M_0^2)^{k-1}}{M_0^2} + 2\phi(1 + \sigma\phi^2/M_0^2)^k, \\
\partial_\phi^2 \bar{Q} &= \frac{4(k-1)k\sigma^2\phi^4(1 + \sigma\phi^2/M_0^2)^{k-2}}{M_0^4} + \\
&\quad \frac{10k\sigma\phi^2(1 + \sigma\phi^2/M_0^2)^{k-1}}{M_0^2} + 2(1 + \sigma\phi^2/M_0^2)^k
\end{aligned}$$

Curvature Scalar

Taking trace of the Einstein equations

$$\frac{M_{Pl}^2}{8\pi}\left(R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R\right)=\varrho_{vac}g_{\mu\nu}+T_{\mu\nu}^{(matt)}$$

we find the relation:

$$R\left(\beta\bar{Q}+\frac{3\beta^2(\partial_\phi\bar{Q})^2}{2}+\frac{M_{Pl}^2}{8\pi}\right)=$$
$$(\partial\phi)^2(3\beta\partial_\phi^2\bar{Q}+1)-2m^2\phi^2-3\beta m^2\phi(\partial_\phi\bar{Q})-4\varrho_{vac}-\tilde{T}_\nu^\nu$$

$\tilde{T}_\nu^\nu$ is the trace of the energy-momentum tensor of the other kinds of matter.

This equation allows to express R through ϕ and its first derivative.

Dimensionless Variables

Two differential equations that govern cosmological evolution:

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi + \frac{1}{2}\beta R\,\partial_\phi \bar{Q} = 0,$$
$$\dot{H} + 2H^2 = -R/6$$

- Here R is a known function of ϕ and its first derivative.

Dimensionless variables:

$$\tau = tH_0, \quad \varphi = \phi/H_0, \quad h = H/H_0, \; \frac{d}{dt} = H_0\frac{d}{d\tau},$$
$$R = rH_0^2, \; \varrho_{vac} = H_0^4\lambda, \; \bar{Q} = H_0^2 q, \quad M_0 = H_0\mu.$$

- H_0 is an arbitrary normalisation constant.
- We fix $H_0^2 = 8\pi\,\varrho_{vac}^{(in)}/(3M_{Pl}^2)$, where $\varrho_{vac}^{(in)}$ is the original large vacuum energy.

In what follows we denote derivative over τ by prime: $df/d\tau \equiv f'$.

Dimensionless Functions

Dimensionless function q is expressed through the dimensionless field φ:

$$\bar{Q} = \phi^2(1+\sigma\phi^2/M_0^2)^k \Longrightarrow q = \frac{\bar{Q}}{H_0^2} = \varphi^2\left(1+\sigma\varphi^2/\mu^2\right)^k$$

The derivatives of $\bar{Q}$ turn into:

$$\frac{\partial_\phi \bar{Q}}{H_0} \equiv q_1 = \frac{2k\sigma\varphi^3(1+\sigma\varphi^2/\mu^2)^{k-1}}{\mu^2} + 2\varphi^2\left(1+\sigma\varphi^2/\mu^2\right)^k,$$

$$\partial_\phi^2 \bar{Q} \equiv q_2 = \frac{4(k-1)k\sigma^2\varphi^4(1+\sigma\varphi^2/\mu^2)^{k-2}}{\mu^4} + \frac{10k\sigma\varphi^2(1+\sigma\varphi^2/\mu^2)^{k-1}}{\mu^2} + 2\left(1+\sigma\varphi^2/\mu^2\right)^k$$

Dimensionless Equations

Finally we come to following two differential equations

$$h' + 2h^2 = -\frac{r}{6}; \quad \varphi'' + 3h\varphi' + \beta r q_1/2 = 0$$

Dimensionless curvature scalar

$$r = \frac{(3\beta q_2 + 1)(\varphi')^2 - 2(m/H_0)^2\varphi^2 - 3\beta(m/H_0)^2\varphi q_1 - 4\lambda - \tilde{T}/H_0^4}{3\beta^2 q_1^2/2 + M_{Pl}^2/(8\pi H_0^2) + \beta q}$$

- Here $q_1 \equiv \partial_\phi \bar{Q}/H_0$, $q_2 \equiv \partial_\phi^2 \bar{Q}$ are dimensionless derivatives.

We assume:

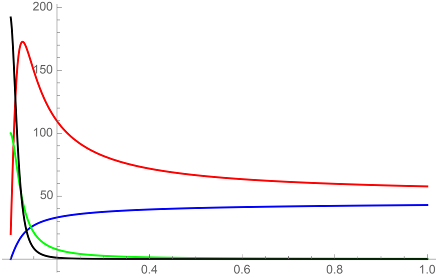
Field ϕ is massless: $m = 0$, the matter is relativistic: $\tilde{T}_\nu^\nu = 0$.

The curvature turns into:

$$r_0 = \frac{(3\beta q_2 + 1)(\varphi')^2 - 4\lambda}{3\beta^2 q_1^2/2 + M_{Pl}^2/(8\pi H_0^2) + \beta q}$$

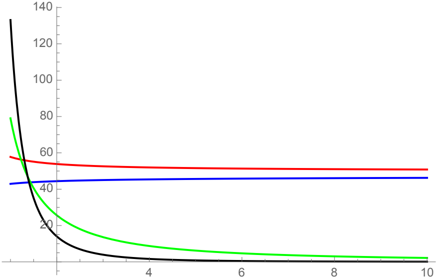
Numerical solutions: $\lambda^{(in)} = 10^4$

Small time τ



Evolution of $10^2\tau h$, $10^3\varphi$, $10^2\varphi'$, $(-r_0)$ as functions of time τ.

Large time τ

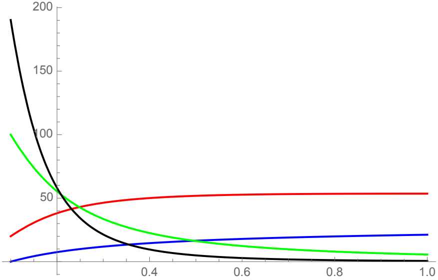


Evolution of $10^2\tau h$, $10^3\varphi$, $3 \cdot 10^4\varphi'$, $(-10^5 r_0)$ as functions of time τ.

The calculations are performed with $\beta = 1$, $\mu = 1$, $k = 3$.
Initial values: $h_{in} = 2$, $\varphi_{in} = 0$, and $\varphi'_{in} = 1$.

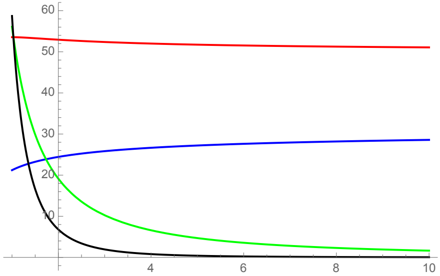
Numerical solutions: $\lambda^{(in)} = 10^2$

Small time τ



Evolution of $10^2\tau h$, $10^2\varphi$, $10^2\varphi'$, $(-10^2 r_0)$ as functions of time τ.

Large time τ



Evolution of $10^2\tau h$, $10^3\varphi$, $10^3\varphi'$, $(-10^4 r_0)$ as functions of time τ.

The calculations are performed with $\beta = 1$, $\mu = 1$, $k = 3$.
Initial values: $h_{in} = 2$, $\varphi_{in} = 0$, and $\varphi'_{in} = 1$.

Compensation of Vacuum Energy

Numerical calculations demonstrate that the curvature, even for very large initial values of $|r_0|$, quickly tends to zero, thereby demonstrating that the vacuum energy is indeed compensated.

Indeed:

- a constant value of curvature correponds to a constant value of the Hubble parameter and therefore to exponentially expanding de Sitter universe.
- The impact of field ϕ results in asimptotical vanishing of R. Thus, the de Sitter expansion turns into power law expansion.

In other words, vacuum energy is completely compensated and we arrive to cosmology dominated by relativistic matter.

Asymptotical Solutions for $r_0 = 0$:

The equations

$$h' + 2h^2 = -\frac{r}{6}; \quad \varphi'' + 3h\varphi' + \beta r q_1/2 = 0$$

with $r_0 = 0$ are trivially solved analytically.

Analytical asymptotic solutions:

$$h(\tau) \to [2(\tau + \tau_0)]^{-1}, \;\; \varphi' \to C\tau^{-3/2}, \;\; \varphi \to const\,,$$

which are in good agreement with the numerical calculations.

- $H(t) \sim 1/(2t)$ is the canonical expression for the Hubble parameter in cosmology dominated by relativistic matter.

Note, that numerical calculations are valid till $\tau \approx 30 - 40$. At larger τ instability of the numerical procedure leads to unreasonable results, but at high τ we have accurate analytic solutions.

Conclusion

The model suggested in this work, efficiently does the job of elimination any original vacuum energy down to zero, and results in realistic cosmology governed by relativistic matter.

According to the relation

$$R = -6(\dot{H} + 2H^2)$$

that is valid in arbitrary homogeneous and isotropic metric, the vanishing of R leads to the equation $\dot{H} + 2H^2 = 0$ and to the Hubble parameter

$$H \equiv \frac{\dot{a}}{a} = \frac{1}{2(t + t_0)}$$

and correspondingly to the scale factor rising as $a(t) \sim t^{1/2}$, which is typical the cosmological model dominated by relativistic matter.

In other words

- We started with the universe with non-zero (and large) vacuum energy. Initially the expansion was the exponential one.

In other words

- We started with the universe with non-zero (and large) vacuum energy. Initially the expansion was the exponential one.
- Later (quite quickly) the expansion turned into a power law form that is typical for cosmology dominated by relativistic matter.

In other words

- We started with the universe with non-zero (and large) vacuum energy. Initially the expansion was the exponential one.
- Later (quite quickly) the expansion turned into a power law form that is typical for cosmology dominated by relativistic matter.
- However, the pre-existing usual matter could be strongly diluted via initial exponential expansion and practically washed out.

In other words

- We started with the universe with non-zero (and large) vacuum energy. Initially the expansion was the exponential one.
- Later (quite quickly) the expansion turned into a power law form that is typical for cosmology dominated by relativistic matter.
- However, the pre-existing usual matter could be strongly diluted via initial exponential expansion and practically washed out.
- On the other hand, in the process of expansion gravitational particle production would be quite efficient that could create enough matter for our normal universe.

Next steps

- We need to include into the model non-relativistic matter and check if the transition to matter dominated cosmology could be successfully achieved.
- Another related problem is a possibility of description of cosmological dark energy in the proposed framework.
- Presumably it can be realised by introduction of more complicated dependence on curvature scalar, R, analogously to the known description of dark energy by modified gravity through $F(R)$ generalisation of GR.

This is supposed to be the matter of future studies.

THANK YOU

for the attention!