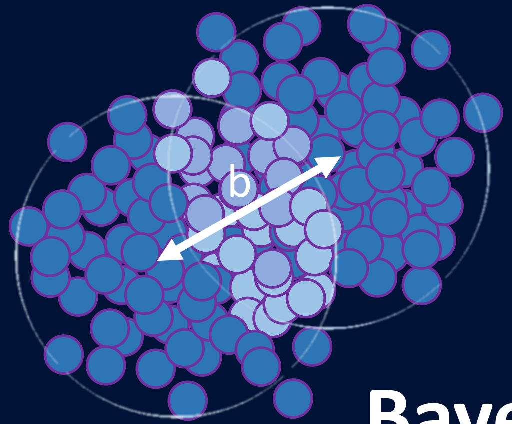

Bayesian approach for centrality determination in nucleus-nucleus collisions at the NICA energy range

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TWENTY-SECOND
LOMONOSOV CONFERENCE ON ELEMENTARY PARTICLE PHYSICS

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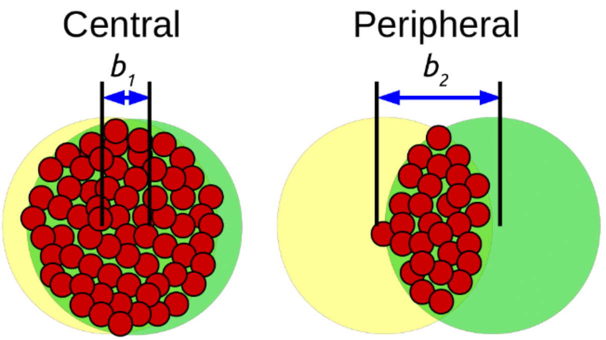
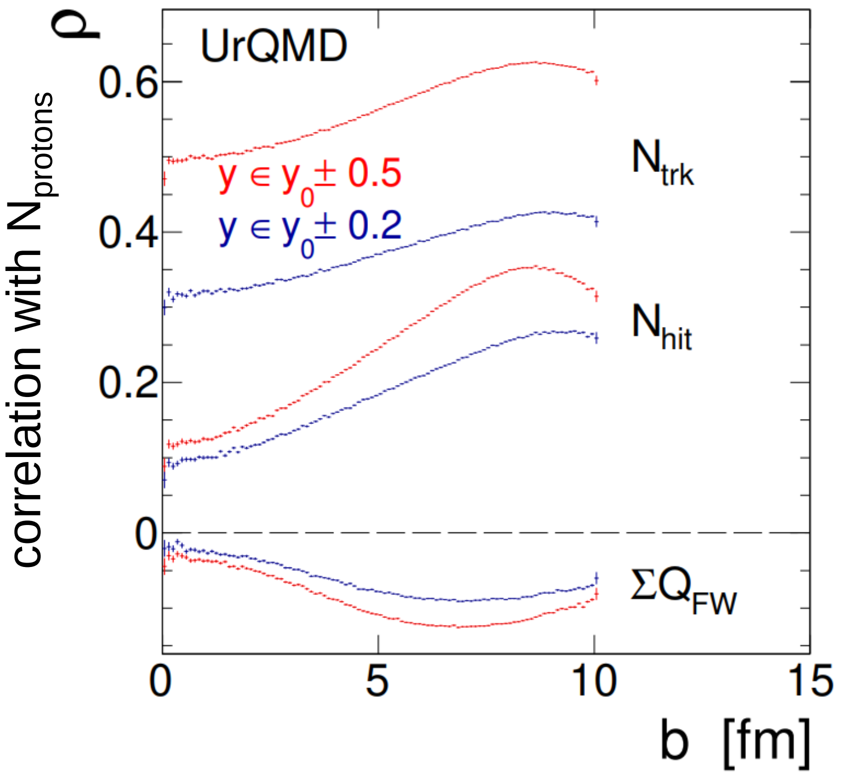
Centrality

- Evolution of matter produced in heavy-ion collisions depend on its initial geometry
- Centrality procedure maps initial geometry parameters with measurable quantities (multiplicity or energy of the spectators)
- **This allows comparison of the future BMAN results with the data from other experiments (STAR BES, NA49/NA61 scans) and theoretical models**

$$c(b) = \frac{\int_0^b \frac{d\sigma}{db'} db'}{\int_0^\infty \frac{d\sigma}{db'} db'} = \frac{1}{\sigma_{A-A}} \int_0^b \frac{d\sigma}{db'} db'$$





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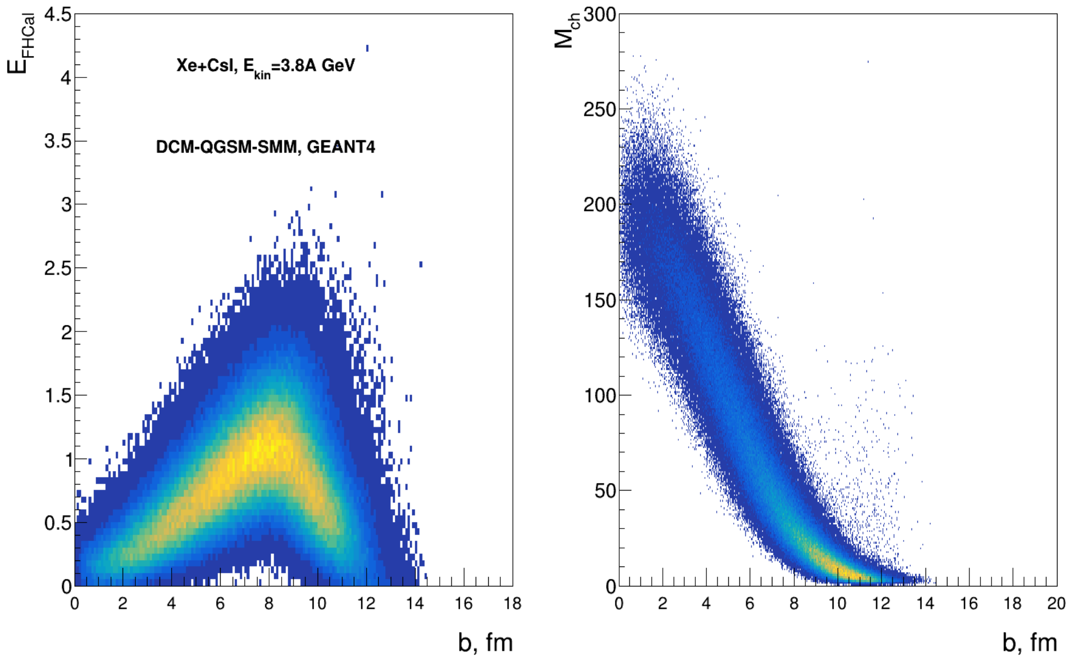
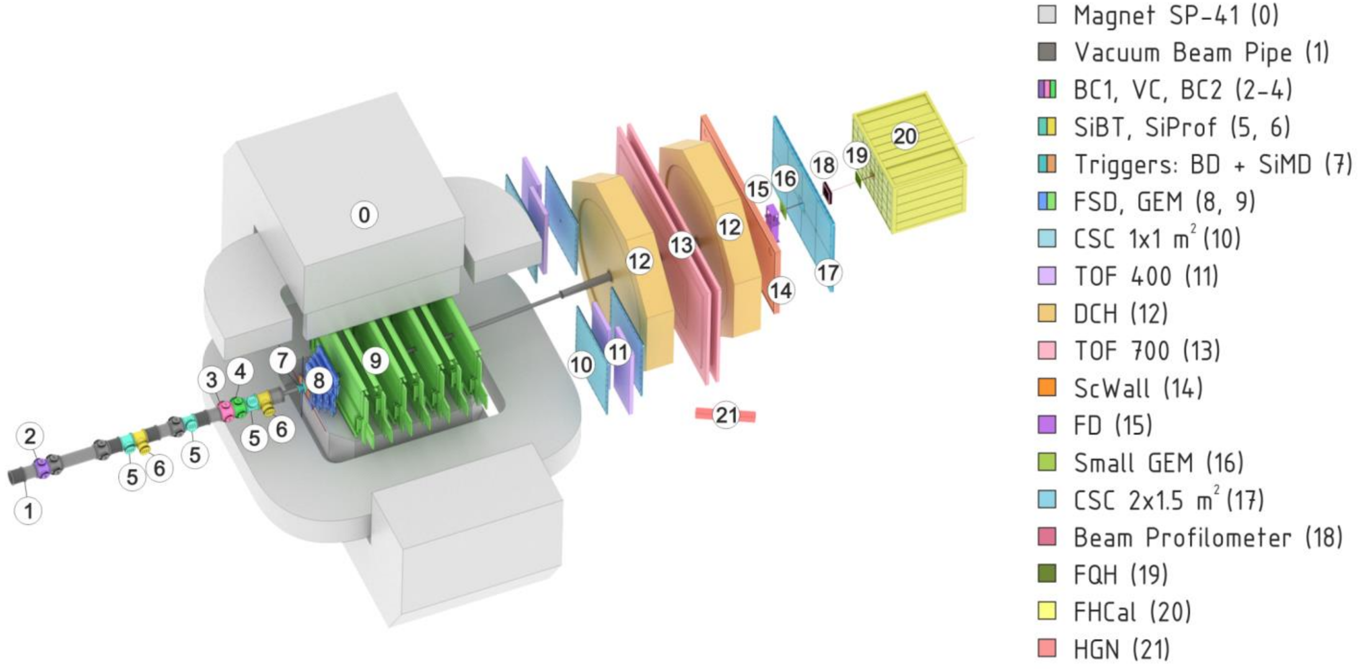
- A number of produced protons is stronger correlated with the number of produced particles (track & RPC+TOF hits) than with the total charge of spectator fragments (FW)
- to suppress self-correlation biases, it is necessary to use spectators fragments for centrality estimation

Centrality determination in BM@N

DCM-QGSM-SMM → GEANT4 → Reconstruction → Centrality determination



Dependence of energy in FHCal and track multiplicity on the impact parameter



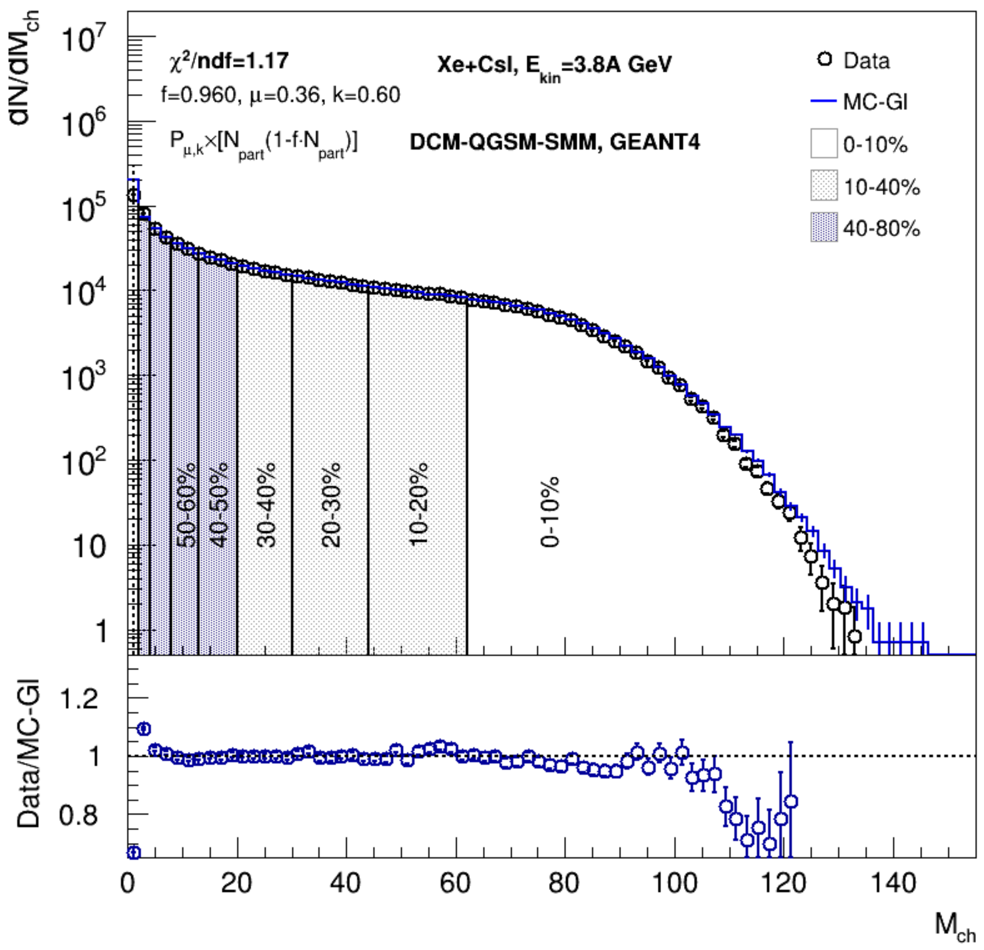
BM@N setup overview

MC-Glauber based centrality framework

MC Glauber data → Evaluate N_a: $N_a = fN_{part}+(1-f)N_{coll}$ → Call $NBD(\mu,k) \times N_a$ → Build multiplicity fitting function

Input multiplicity distribution

Build multiplicity fitting function + Input multiplicity distribution → Evaluate χ^2 → Minimize χ^2 to find f, μ, k



NBD – negative binomial distribution

Parameters of the fit:

- **f –** the fraction of multiplicity from the soft component
- **μ** – mean multiplicity value
- **k –** width of the multiplicity distribution, can be connected to the fluctuations

Implamantation for MPD: https://github.com/FlowNICA/CentralityFramework

The Bayesian inversion method (Γ-fit): 2D fit

- The fluctuation kernel for energy and multiplicity at fixed impact parameter can be describe by 2D Gamma distr.:

$$P(E,M\,|\,c_b) = G_{2D}(E,M,\langle E\rangle,\langle M\rangle,D(E),D(M),R)$$

$\langle E\rangle, D(E)$ – average value and variance of energy

$R(E,M)$ – Pirson correlation coefficient

$$P(E,M) = \int_0^1 P(E,M\,|\,c_b)\,dc_b$$

$$R(E,M) = \frac{\varepsilon_1^2 m_1^2}{\varepsilon_2 m_2} R(E',M')$$

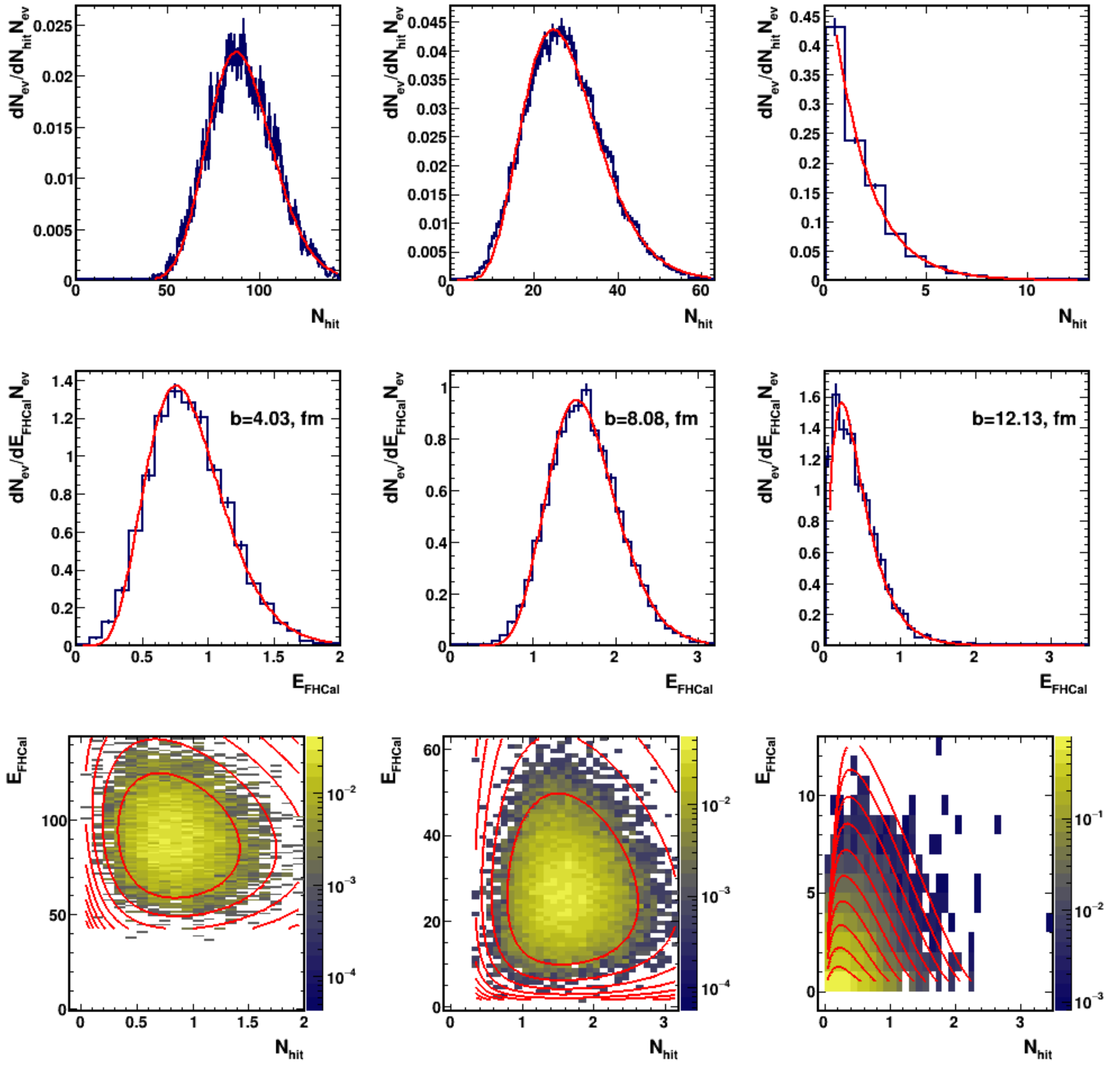
$\varepsilon_1, \varepsilon_2, m_1, m_2$ - fit parameters

$\langle E'(c_b)\rangle$, $D(E'(c_b))$ – average value and var. of energy/mult. from the rec. model data

$$\langle E\rangle = \varepsilon_1 \langle E'(c_b)\rangle, \quad D(E) = \varepsilon_2 D(E'(c_b))$$
$$\langle M\rangle = m_1 \langle M'\rangle, \quad D(M) = m_1^2 D(M') + m_2 \langle M'\rangle$$

$\langle E'(c_b)\rangle, \quad D(E'(c_b))$ - can be approximated by polynomials

The fluctuation of energy and multiplicity at fixed impact parameter



It is possible to find such a rotation angle of the system that $\mathrm{cov}(x, y) = 0$

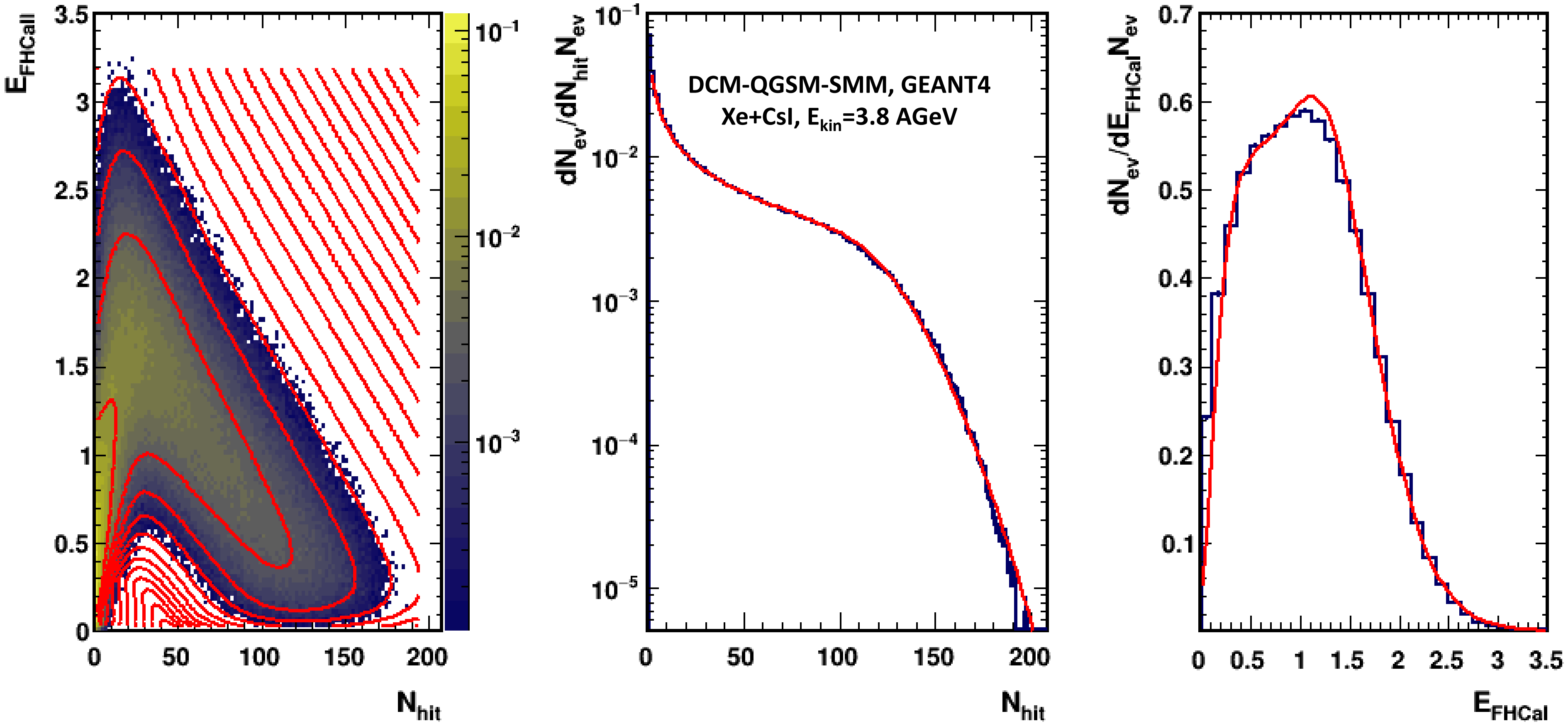
$$\alpha = \arctan\left(\frac{2\sqrt{D(E)D(M)}R(E,M)}{D(E) - D(M)}\right)$$

$$G_{2D} = G(x, \theta_x, k_x) \cdot G(y, \theta_y, k_y)$$

The probability of *b* for fixed range of observables can be find using Bayes' theorem:

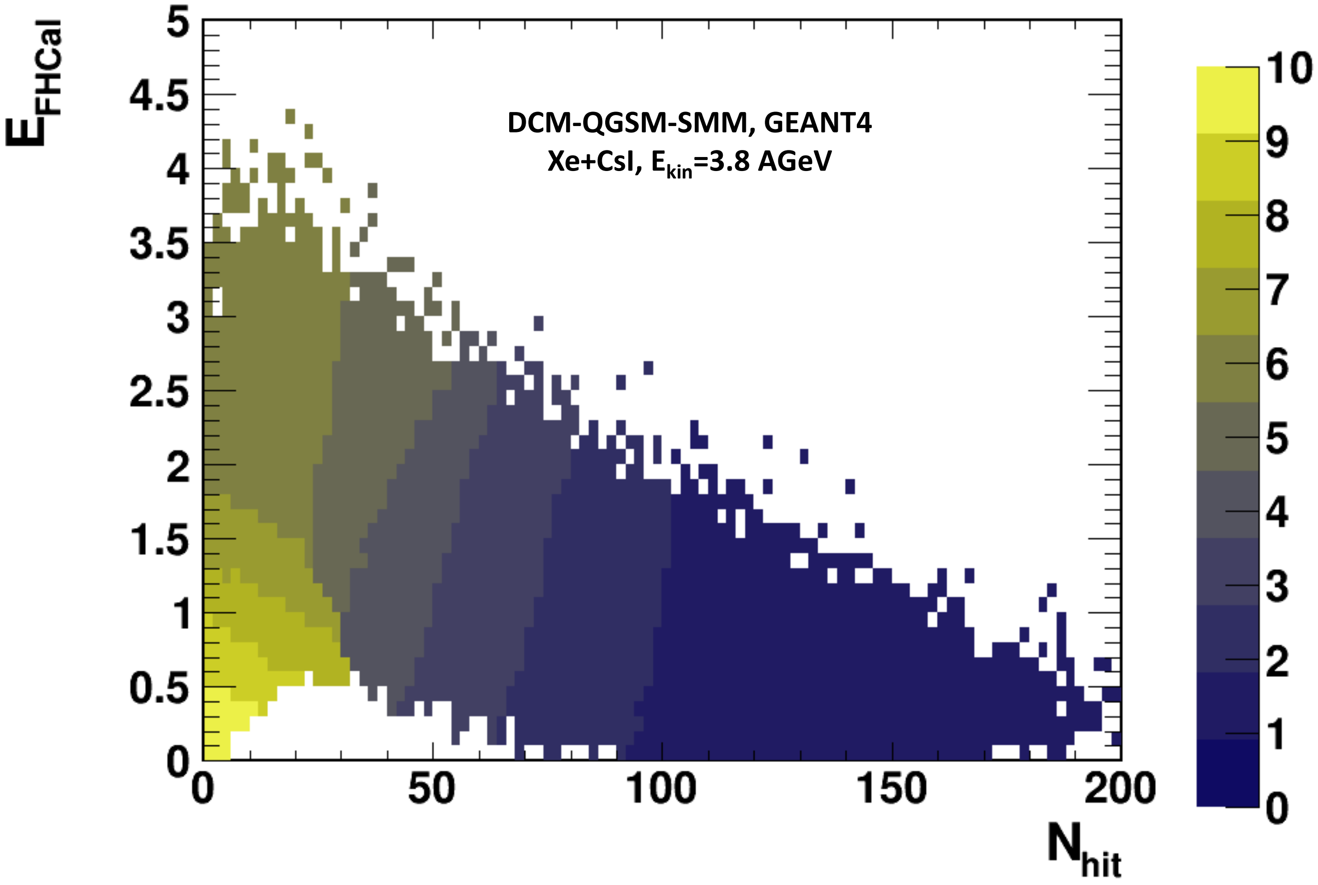
$$P(b \mid N_1, N_2, E_F^1, E_F^2) = P(b) \frac{\int_{N_1}^{N_2} \int_{M_1}^{M_2} P(N, E_F \mid c_b) dE_F dN}{\int_{N_1}^{N_2} \int_{E_F^1}^{E_F^2} \int_0^1 P(N, E_F \mid c_b) dE_F dN dc_b}$$

2D fit results



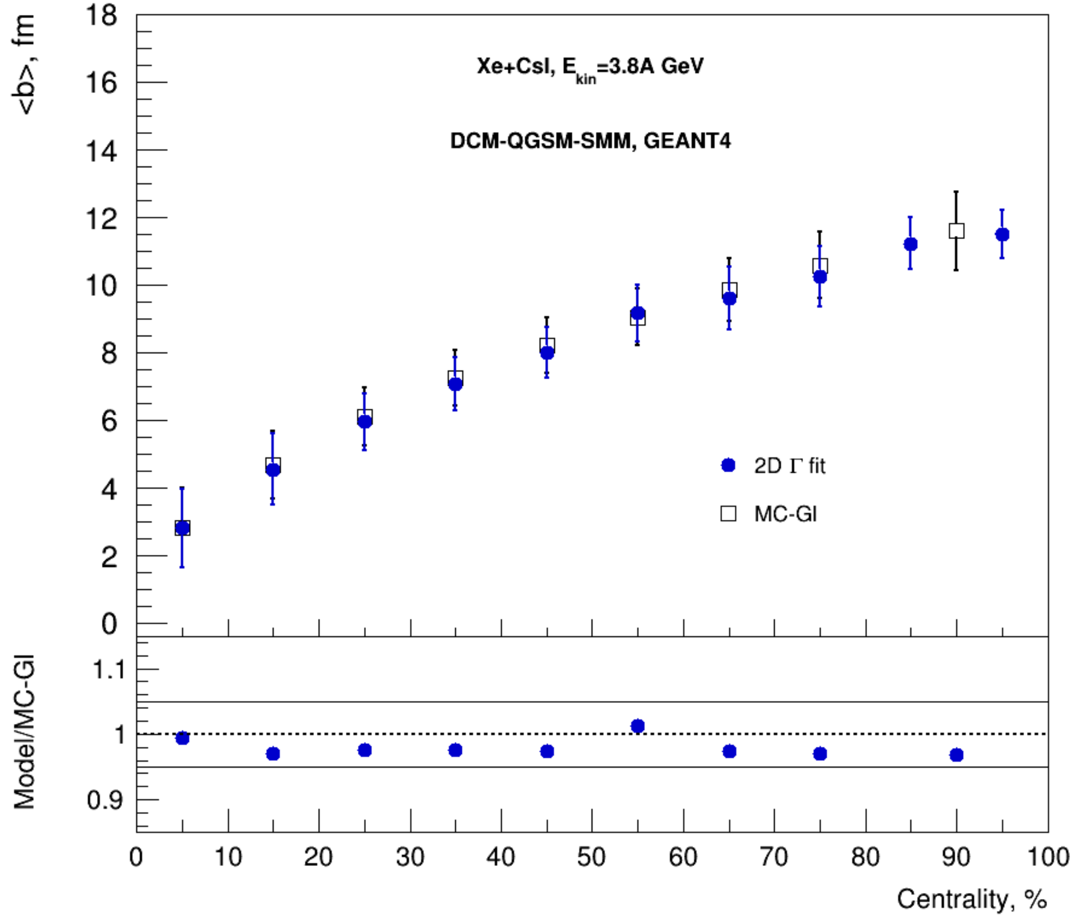
The fit function qualitatively reproduces the multiplicity-energy correlation from FHCal

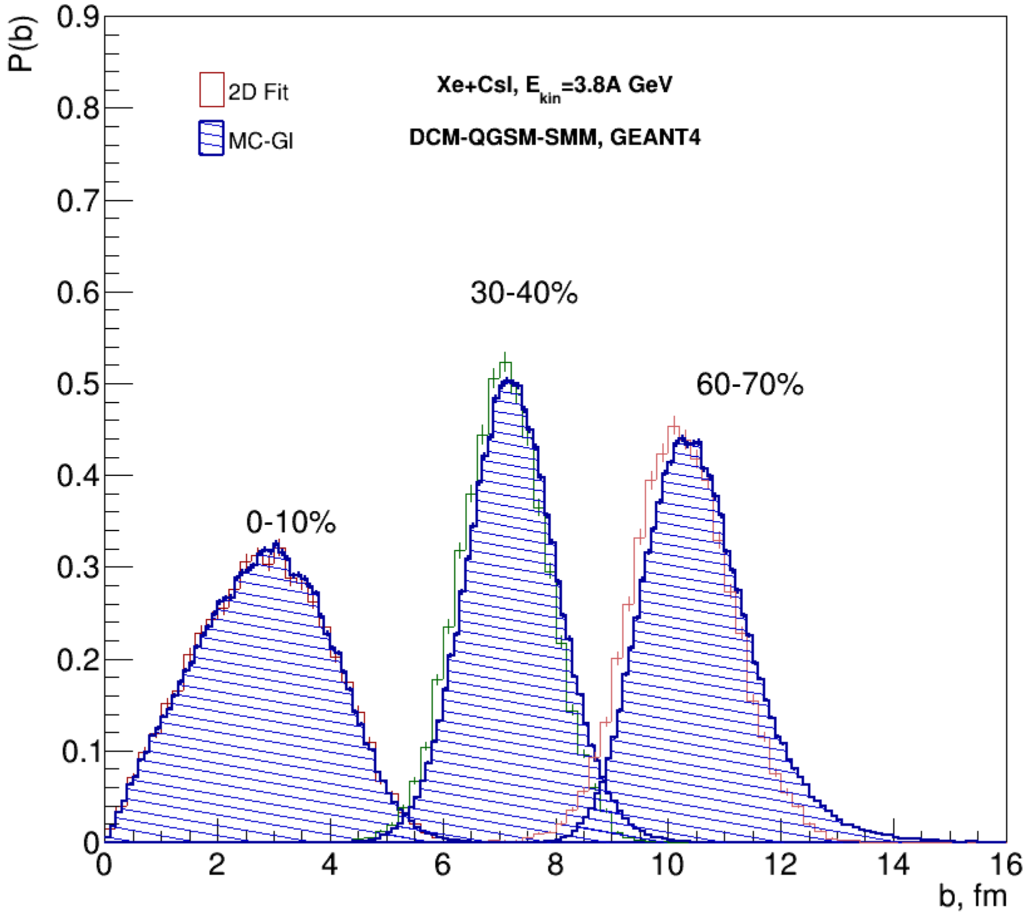
Centrality determination using the forward calorimeter



The K-means method allows to divide a two-dimensional distribution into centrality classes.
In order to correctly apply the class boundaries, it is necessary to match the simulation results with the experiment

Comparison with MC Glauber fit

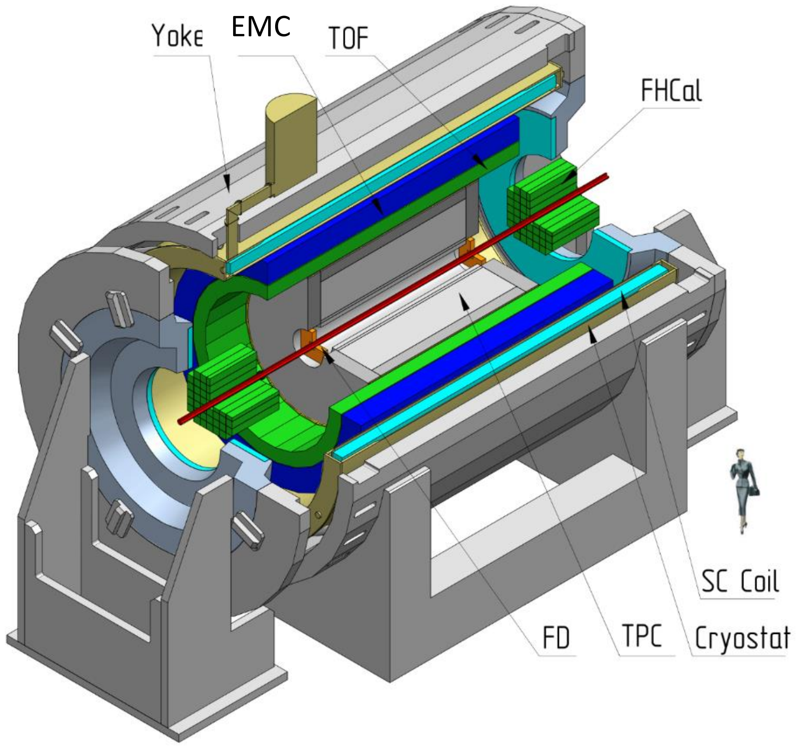




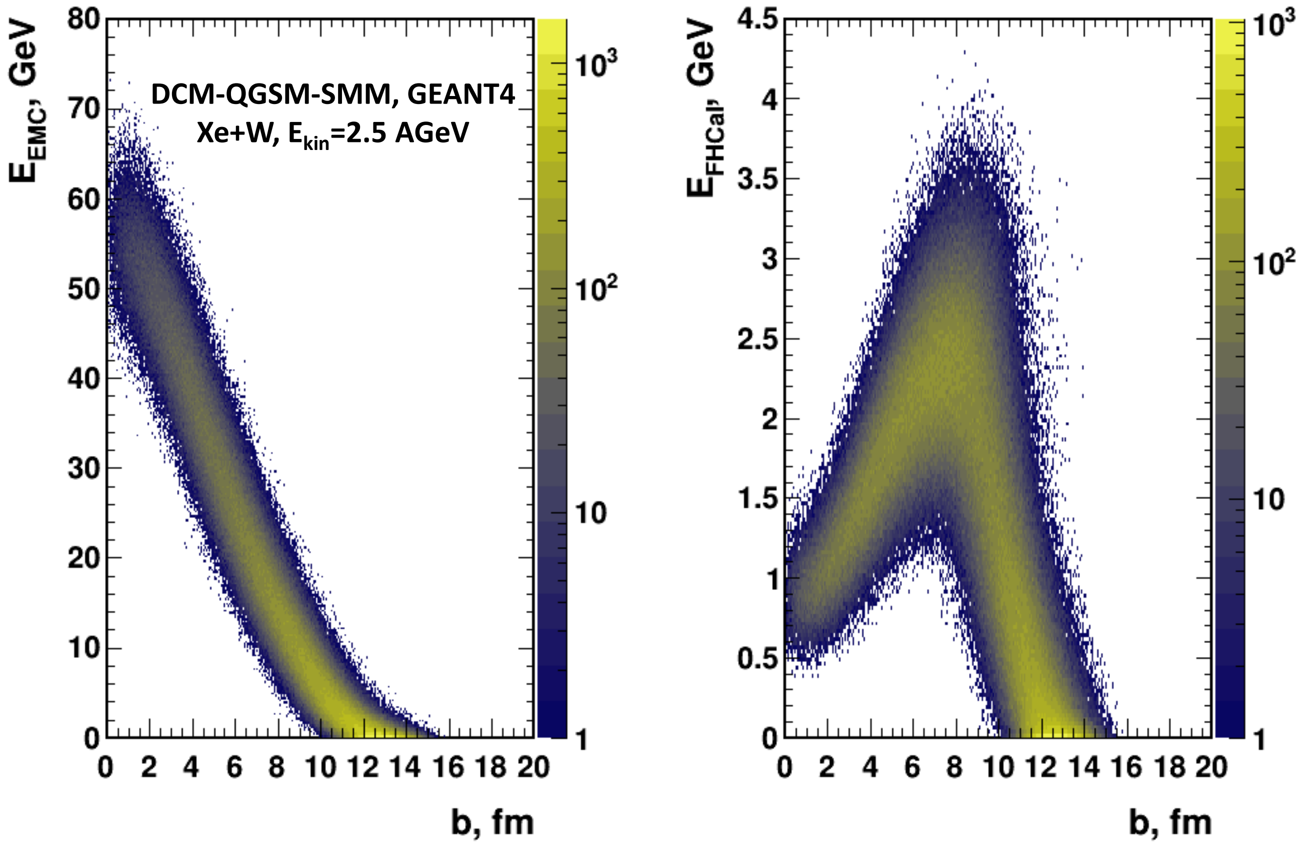
There is agreement within 5%.

MPD Experiment at NICA

DCM-QGSM-SMM → GEANT4 → Reconstruction → Centrality determination

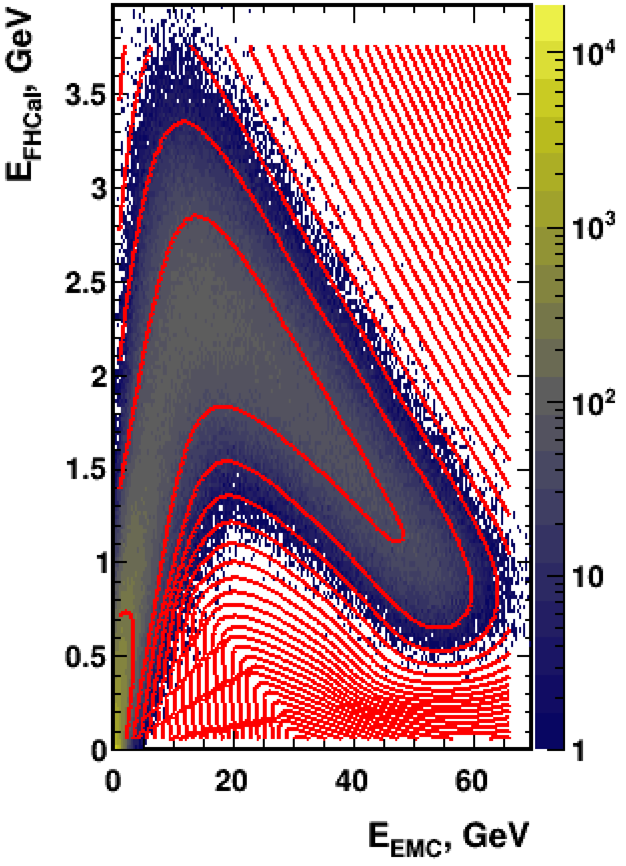


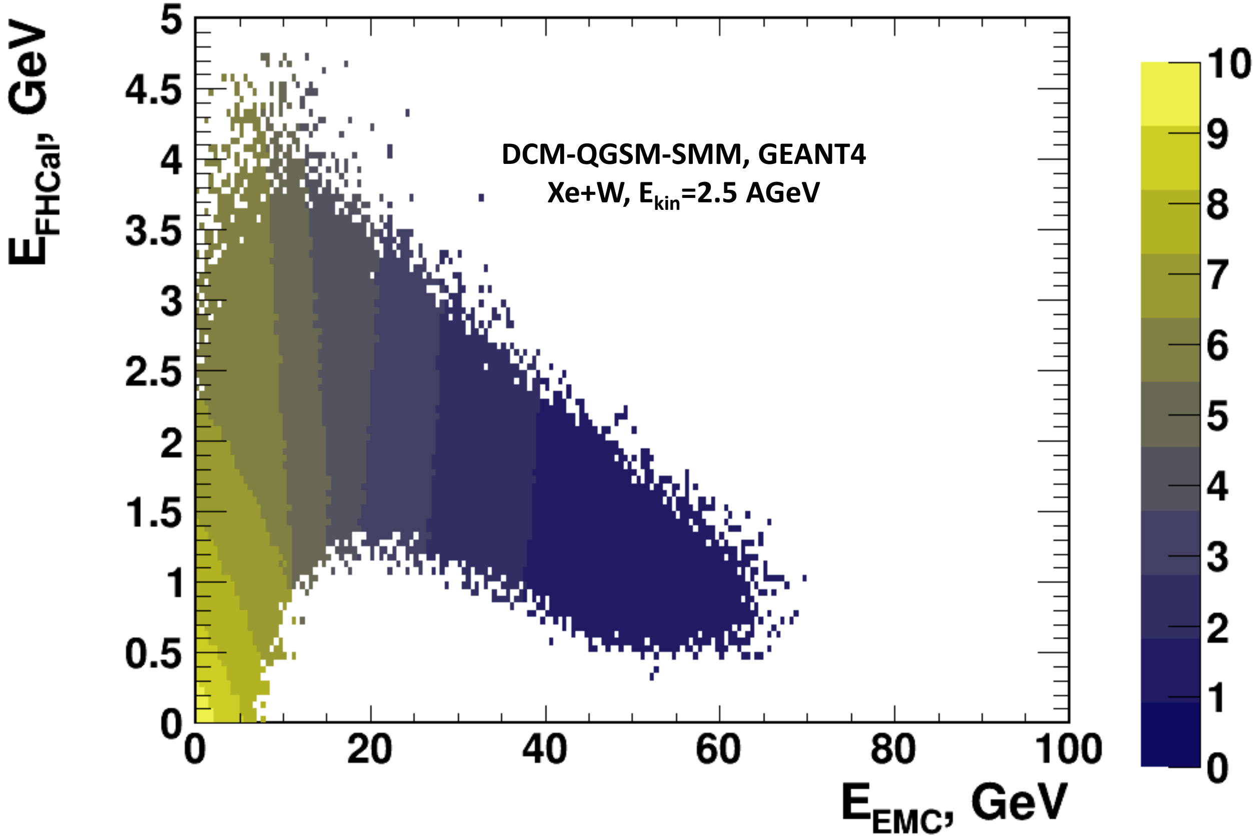
Multi-Purpose Detector (MPD) Stage 1



The analysis was performed using the DCM-QGSM-SMM model.

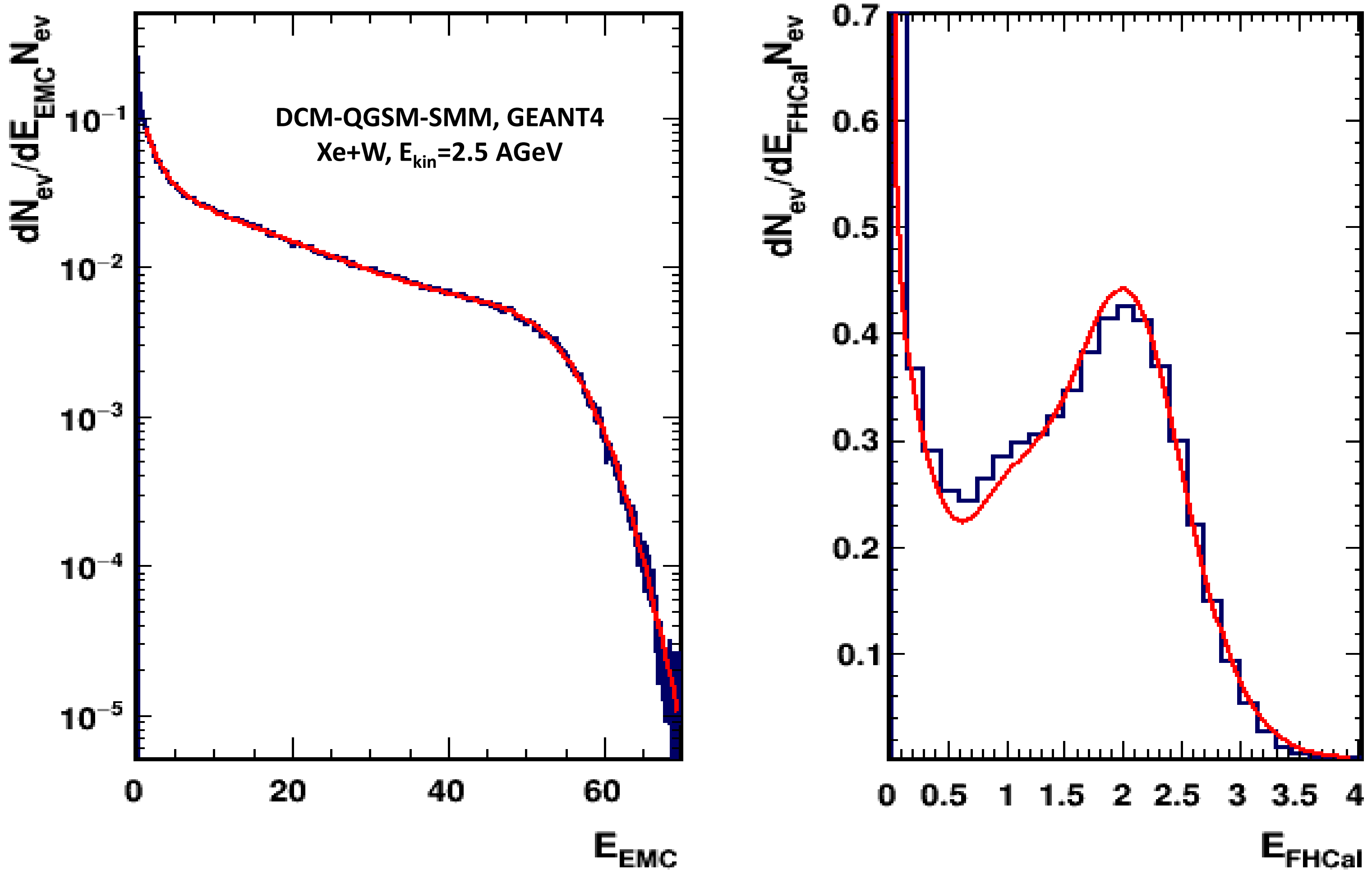
2D Bayesian approach: results





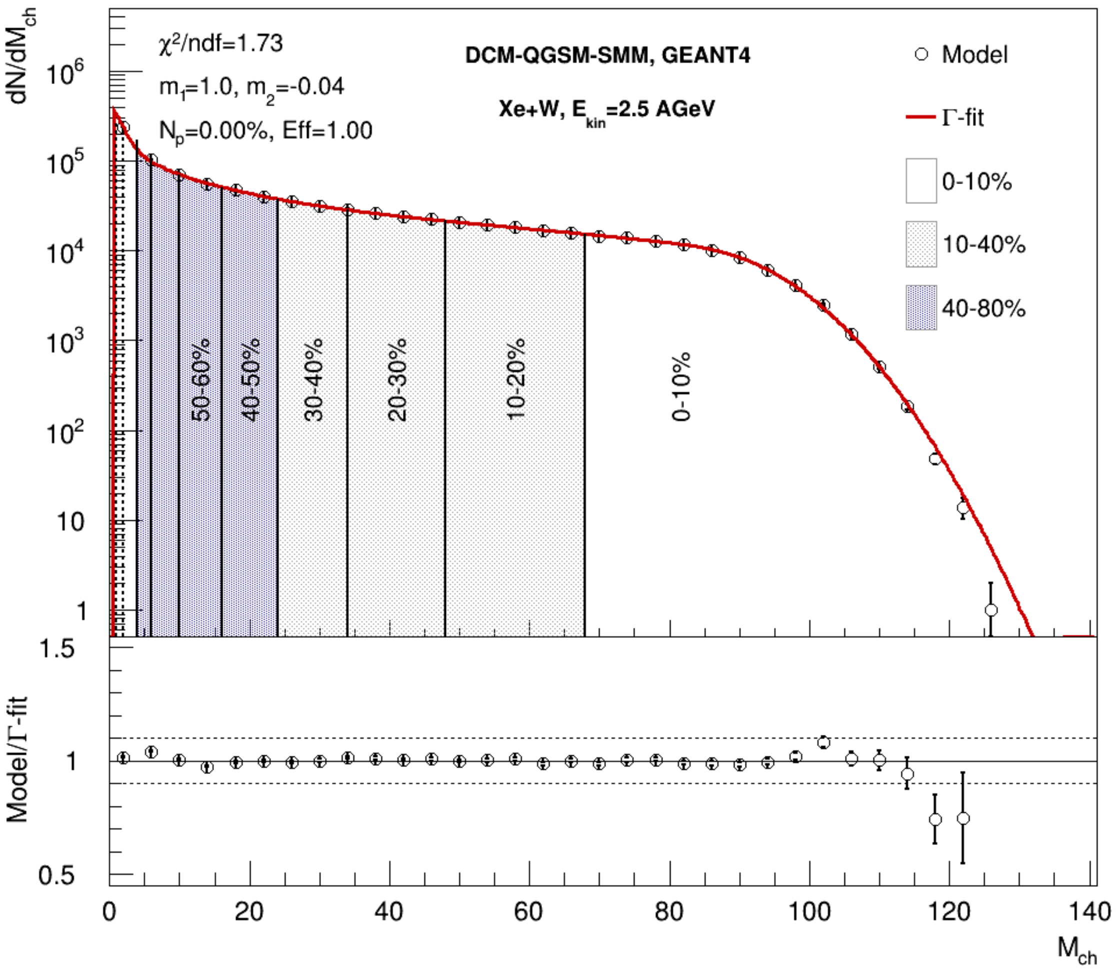
Good agreement between fit and data.

2D Bayesian approach: results

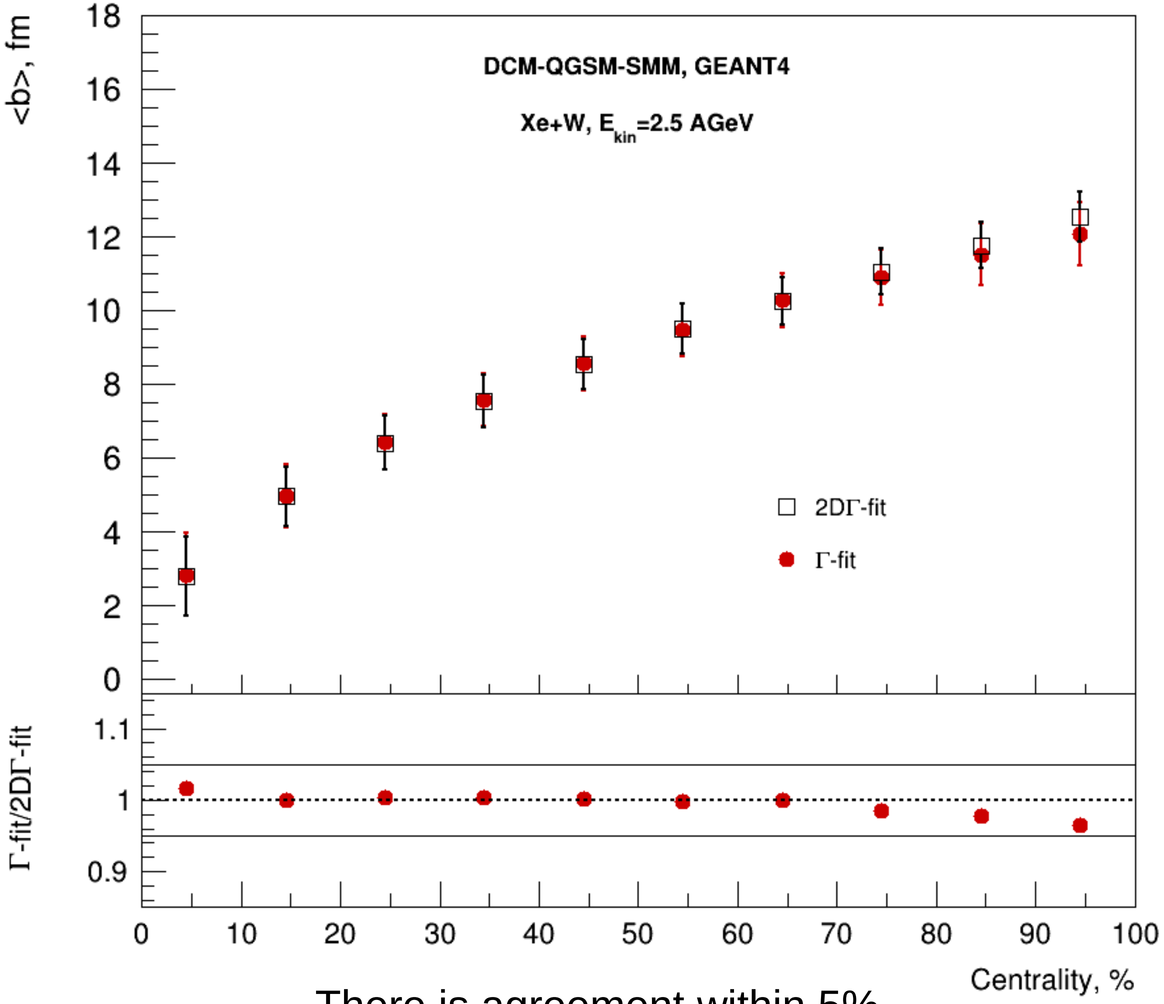


Good agreement between fit and data.

Comparison centrality determination methods



Good agreement with fit



There is agreement within 5%.

Summary and Outlook

- Both the Bayesian inversion and MC – Glauber methods provide consistent results
- The Bayesian inversion method was applied to the BM@N data:
  - Multiplicity-based and 2D approaches using N_{hit} and E_{FHCal} describe simulation data reasonably well.
- The Bayesian inversion method reproduces observables for fixed-target mode at the MPD:
  - Multiplicity-based and 2D approaches using E_{EMC} and E_{FHCal} show results consistent with model data
- In the future, it is planned to study systematics uncertainties using different models (DCM, UrQMD, etc.) and observables (GEM hit multiplicity, etc.)

Thank you for your attention!

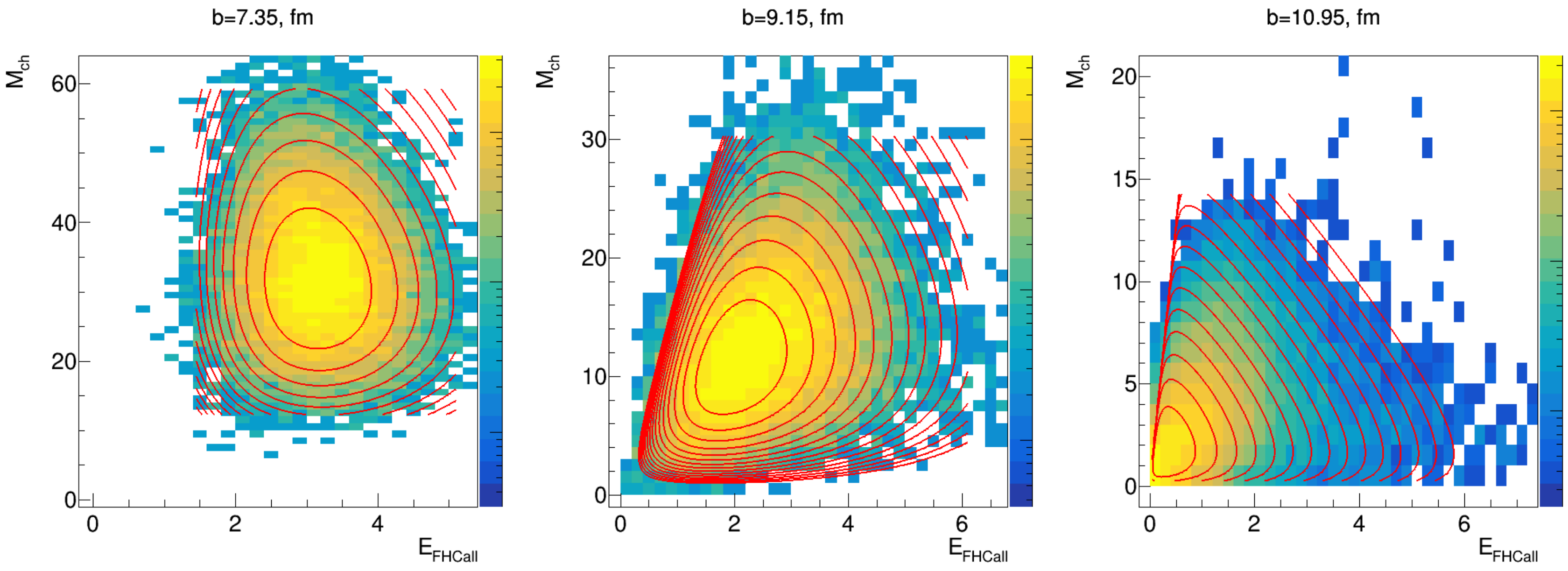
The fluctuation of energy and multiplicity at fixed impact parameter

It is possible to find such a rotation angle of the system that $\mathrm{cov}(x, y) = 0$

$$x = \cos(\alpha)E + \sin(\alpha)M,$$
$$y = -\sin(\alpha)E + \cos(\alpha)M$$

$$\alpha = \arctan\left(\frac{2\sqrt{D(E)D(M)}R(E,M)}{D(E) - D(M)}\right)$$

$$G_{2D}(E_{FH}, M_{ch}, \langle E \rangle, \langle M \rangle, D(E), D(M), R) = G(x, \theta_x, k_x) \cdot G(y, \theta_y, k_y)$$



The distribution of energy and multiplicity at a fixed impact parameter is well described by the gamma distribution

The Bayesian inversion method (Γ-fit): DCM-QSM-SMM based

- The fluctuation kernel for multiplicity at fixed impact parameter is Gamma distr.:

$$P(M \mid c_b) = \frac{1}{\Gamma(k(c_b))\theta^2} M^{k(c_b)-1} e^{-M/\theta}$$

$$c_b = \int_0^b P(b')db'$$ – centrality based on impact parameter

$$\theta = \frac{D(M)}{\langle M \rangle}, \quad k = \frac{\langle M \rangle}{\theta}$$

$\langle M \rangle, D(M)$ – average and variance of Multiplicty

$$P(M) = \int_0^1 P(M \mid c_b)dc_b$$

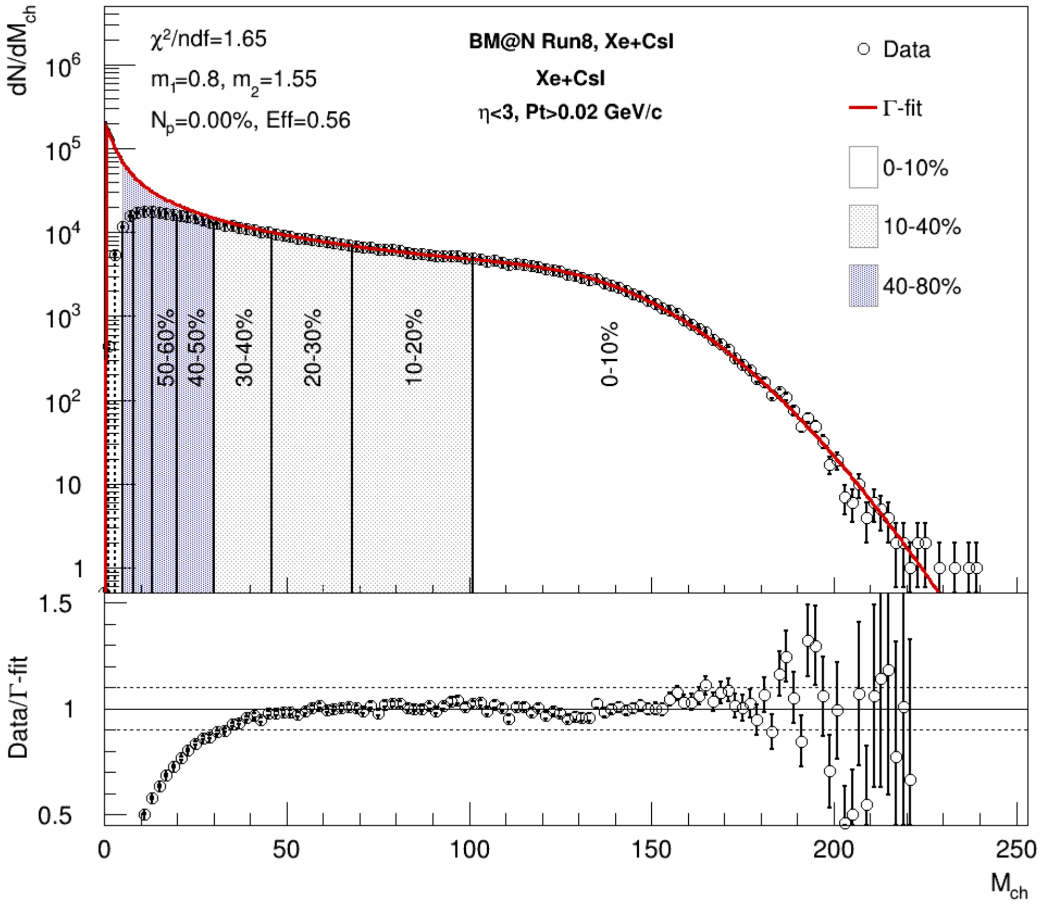
$$\langle M \rangle = m_1 \cdot \langle M' \rangle$$

$$D(M) = m_1^2 \cdot D(M') + m_1 \cdot m_2 \langle M' \rangle$$

$\langle M'(c_b) \rangle$, $D(M'(c_b))$ – average value and var. of energy/mult. from the rec. model data

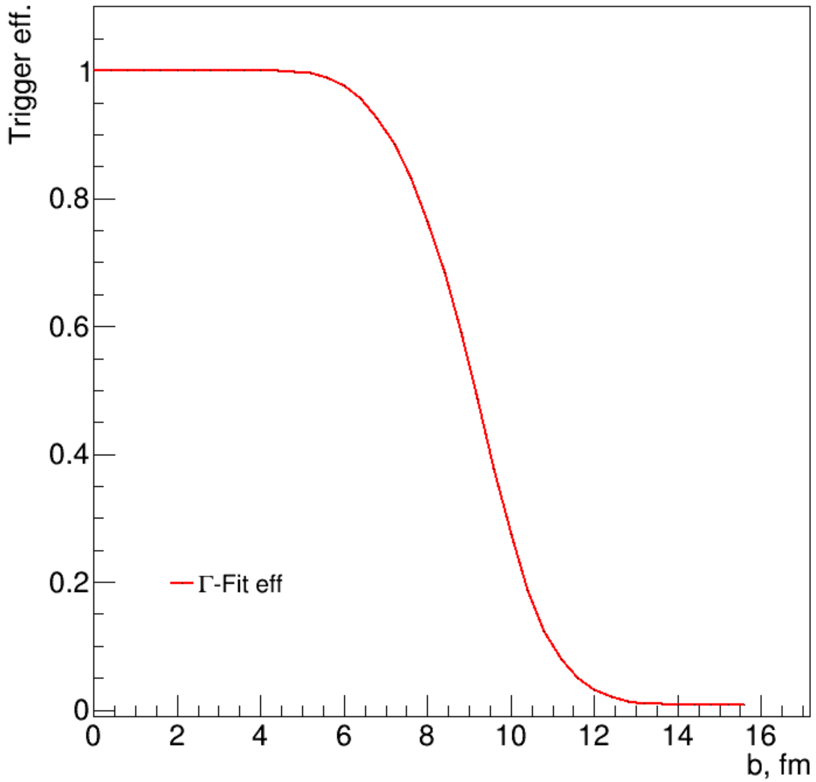
- can be approximated by polynomials or exponential polynomial

Fit results: experimental data



Good agreement with fit

$$P_{eff}(b) = \int_0^{M_{max}} P_{eff}(M)P(M \mid b)dM$$



Convoluted trigger efficiency can be calculated using Bayes' theorem