Dark matter from inflationary quantum fluctuations

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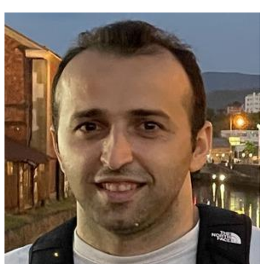

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Introduction

(Image credit: Shutterstock)

Nature and production of dark matter unknown

Most models assume DM particles are produced after inflation.

Could DM originate during inflation?

Scenario

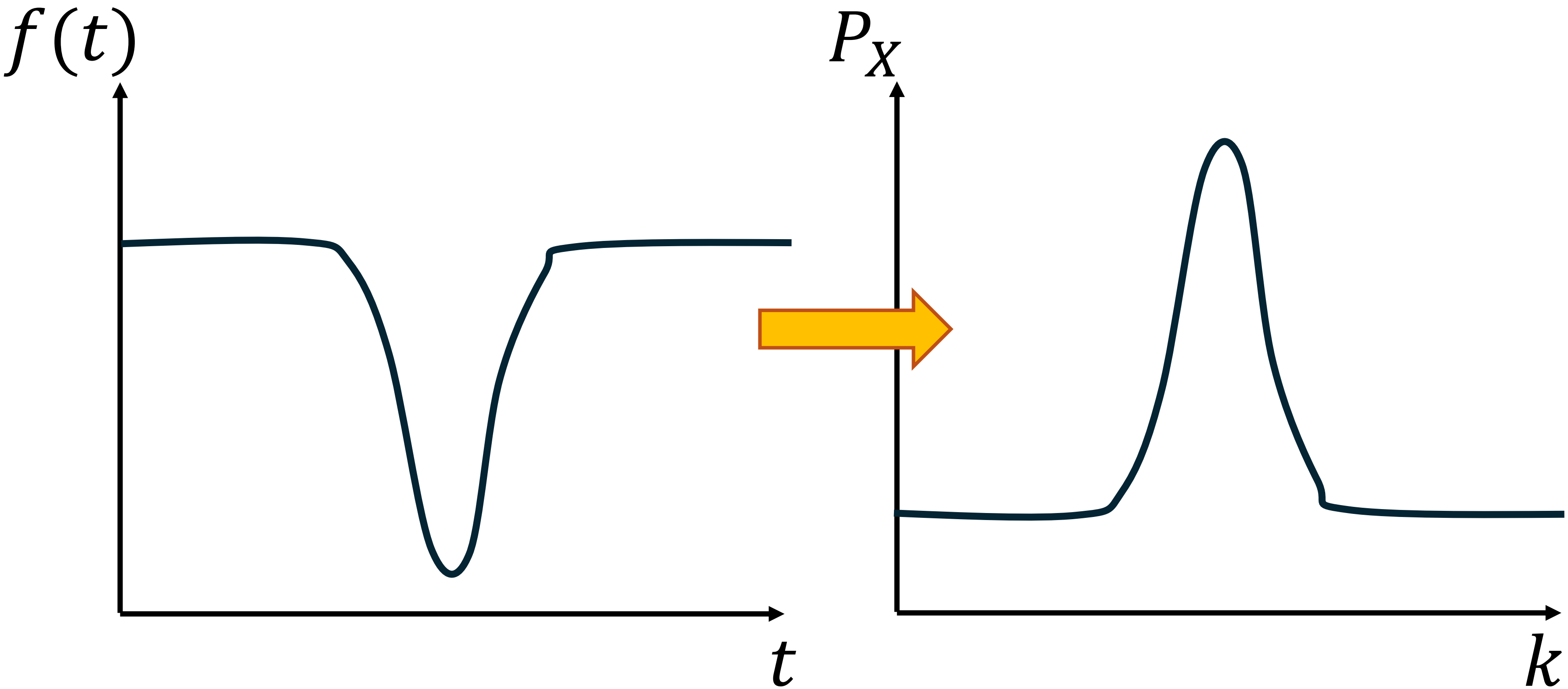
Dark matter arises purely from quantum fluctuations

- Bosonic field (scalar)
- $\langle X \rangle \ll \sqrt{\langle X^2 \rangle}$ (negligible zero mode)
- $\rho_{DM} = \left\langle \frac{1}{2} \dot{X}^2 + \frac{1}{2} (\boldsymbol{\nabla} X)^2 + \frac{m^2}{2} X^2 \right\rangle$

Action for X (spectator field) during inflation

$$S = \frac{1}{2}\int \mathrm{d}^3x\,\mathrm{d}t\,a^3 f^2 \left[\dot{X}^2 - \frac{c_s^2}{a^2}(\partial_i X)^2 - m^2 X^2\right]$$

$f(t) = 1$ in RD



Evolution of X during RD era (no metric perturbation)

Transfer function

$$T(k,\tau) = \frac{X_k(\tau)}{X_{k,i}}$$

$$T''(k,\tau) + 2\frac{a'}{a}T'(k,\tau) + \left(c_s^2k^2 + m^2a^2\right)T(k,\tau) = 0$$

Exact analytic solution

$$T_m(k,\tau) = e^{-\frac{im}{2H}}\,{}_1F_1\left(\frac{3}{4} + \frac{iH_k}{4m}; \frac{3}{2}; \frac{im}{H}\right)$$

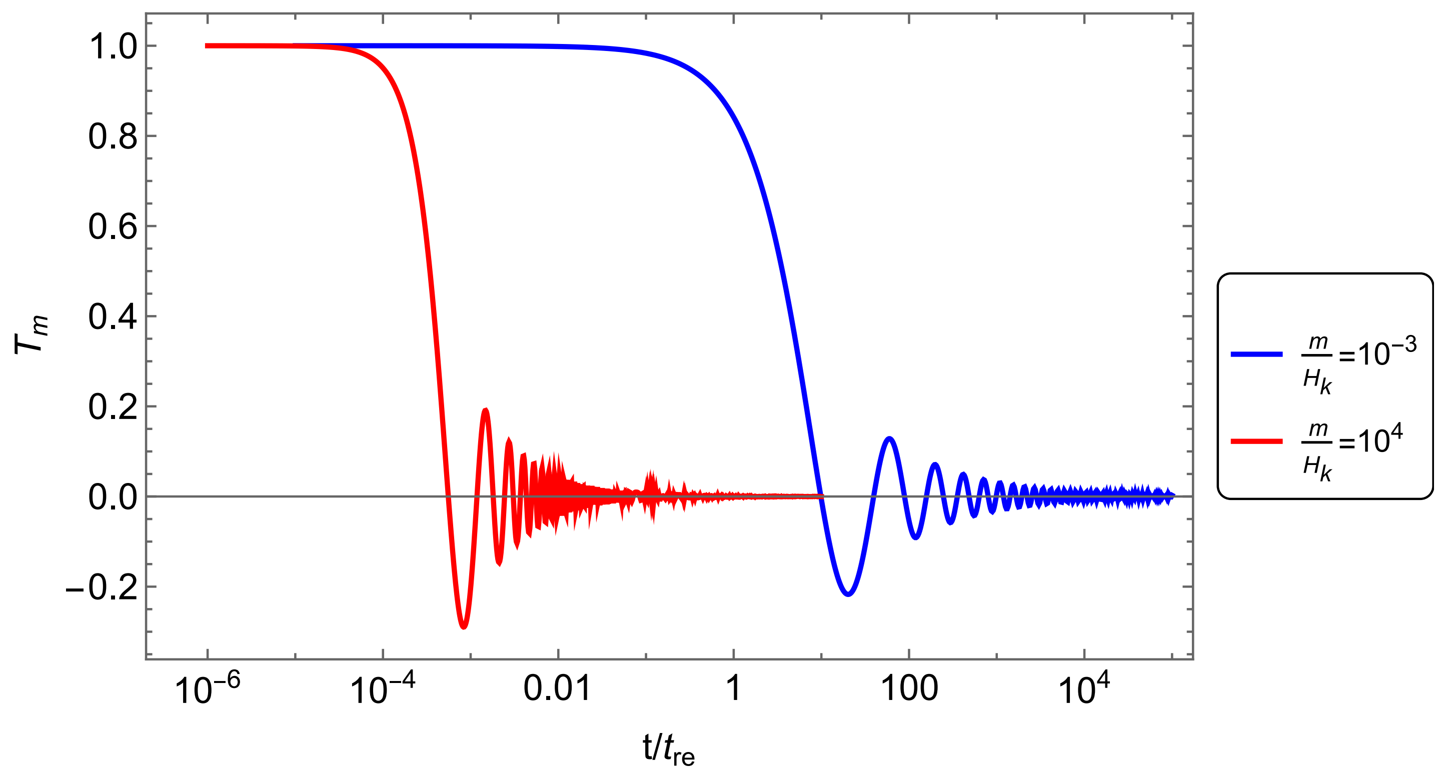
This is not given in the literature

I initially believed the exact solution was already known, but I was mistaken.

Some limiting cases

$$T_m(k,\tau) \approx \frac{1}{x}\sin\left(x + \frac{\mu_k^2}{6}x^3\right)$$

$$T_m(k,\tau) \approx \sqrt{\pi}e^{-\frac{\pi}{8\mu_k}}\mu^{-3/4}\mathrm{Re}\left[\frac{e^{\frac{i\mu}{2}-\frac{3i\pi}{8}}\mu^{\frac{i}{4\mu_k}}}{\Gamma\left(\frac{3}{4}+\frac{i}{4\mu_k}\right)}\left(1-\frac{1}{4\mu\mu_k}+\frac{i}{16\mu\mu_k^2}-\frac{3i}{16\mu}\right)\right]$$



Energy fraction

1. Relativistic regime ($\frac{k}{a} \gg m$)

$$\Omega_X(k,\tau) = \frac{\mathcal{P}_{X,i}(k)}{6M_{\rm Pl}^2} \begin{cases} x^2\left(1+\mu_k^2\, x^2\right) & \propto a^2 \quad \text{For super-Hubble} \\ 1 - \dfrac{\sin(2x)}{x}\left(1+\dfrac{1}{3}\mu_k^2\, x^2\right) + \dfrac{1}{6}\mu_k^2\, x^2\cos(2x) & \propto a^0 \quad \text{For sub-Hubble} \end{cases}$$

$$x = \tau/\tau_{re}$$

2. Non-relativistic regime ($\frac{k}{a} \ll m$)

$$\Omega_X(k,\tau) = \mu^2\left(1+\frac{1}{\mu\mu_k}\right)\frac{\mathcal{P}_{X,i}(k)}{6M_{\rm Pl}^2} \quad \propto a^4 \quad \text{For } m \ll H$$

$$\Omega_X(k,\tau) \approx \frac{4\pi\mu^{1/2}e^{-\frac{\pi}{4\mu_k}}}{\left|\Gamma\left(\frac{3}{4}+\frac{i}{4\mu_k}\right)\right|^2}\frac{\mathcal{P}_{X,i}(k)}{6M_{\rm Pl}^2} \quad \propto a^1 \quad \text{For } m \gg H$$

The transfer function can reproduce all these behaviors.

Variance and skewness

$$\mathcal{P}_{X,i}(k) = \mathcal{A}\left(\frac{H_{\rm inf}}{2\pi}\right)^2 \delta\left[\ln\left(k/k_p\right)\right] \;;$$

Sharply peaked at comoving scale k_p

1. $$\langle \delta_X^2 \rangle = \left\langle \left(\frac{\hat{X}^2 - \sigma_X^2}{\sigma_X^2} \right)^2 \right\rangle = 2$$

$O(1)$ density contrast ($= \sqrt{2}$) from the beginning

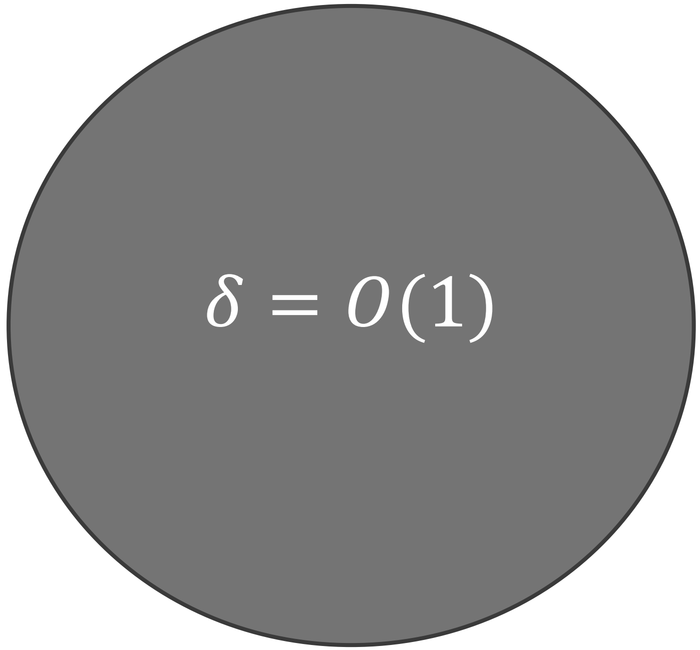
Large isocurvature perturbation

2. $$\langle \delta_X^3 \rangle = 8$$

Highly non-Gaussian

Gravitational collapse

Because of $\delta = O(1)$, DM can form halos even at $z \sim 10^3$

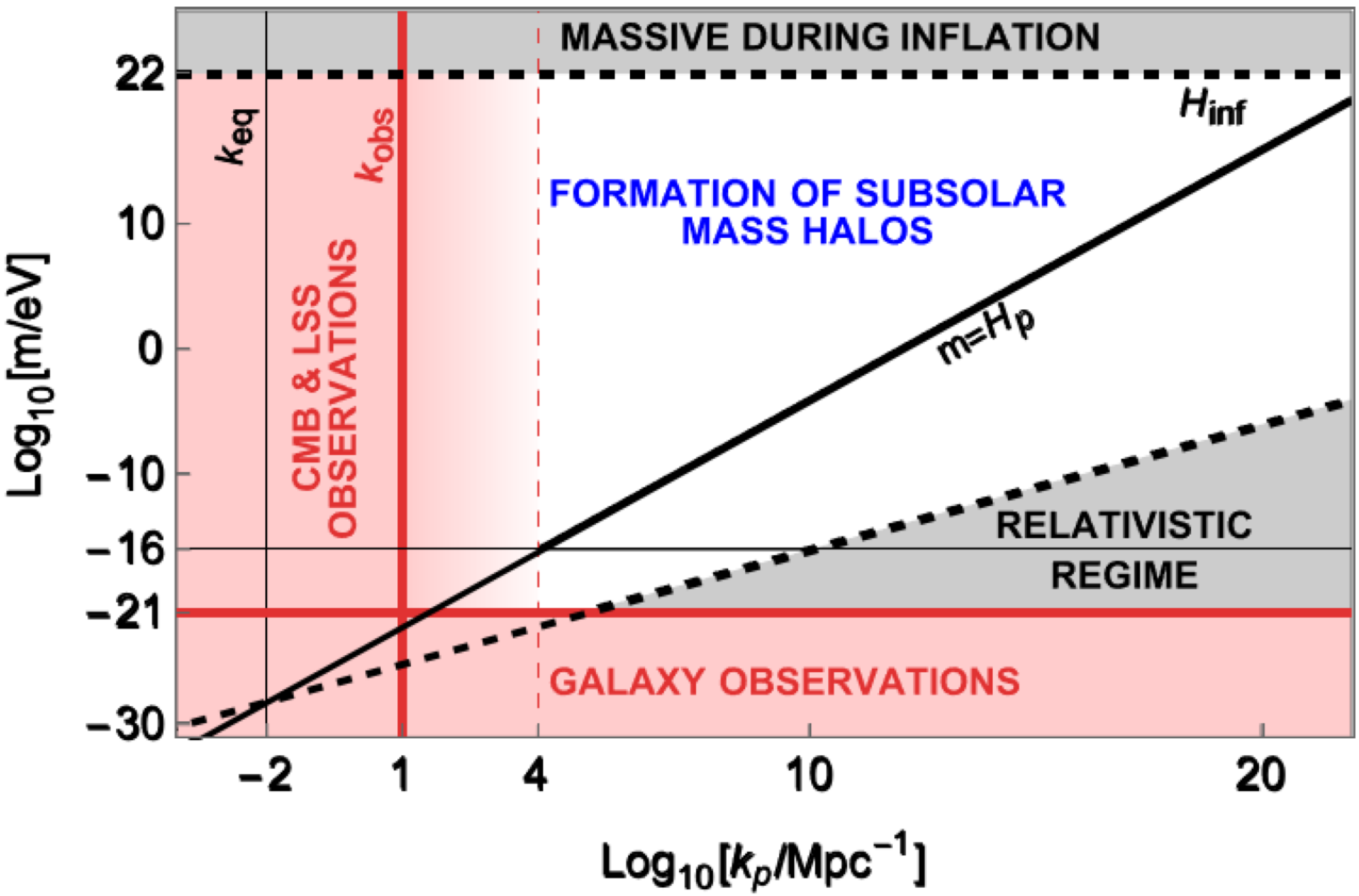


$$t_p \sim \frac{m}{k_p^2}$$

$$t_{ff} \sim \frac{1}{\sqrt{G\rho}}$$

Jeans length

$$\lambda_J \sim \sqrt{Hm}$$



$$M_h \sim \frac{4\pi}{3}\rho_m k_p^{-3} \sim 0.1 M_\odot \left(\frac{k_p}{10\text{kpc}^{-1}}\right)^{-3}$$

Observational implications

Early and abundant formation of subsolar-mass halos

Signals in microlensing?

Formation of early bright objects? (JWST)

Isocurvature perturbations at small scales

Signals in 21cm observations?

Highly non-Gaussian DM perturbation

Deviation from the Press-Schechter prediction based on Gaussian statistics?

Summary

If dark matter arises purely from quantum fluctuations, DM shows distinct features on small scales.

Exact transfer function of a massive field in RD was obtained.