Kinetic growth of axion stars

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TWENTY-SECOND LOMONOSOV
CONFERENCE August, 21-27, 2025
ON ELEMENTARY PARTICLE PHYSICS
MOSCOW STATE UNIVERSITY

A. Dmitriev, DL, A. Panin, I. Tkachev, arXiv:2305.01005
arXiv:2509.XXXXX

QCD axion dark matter

$$m_a \sim 10^{-5} \div 10^{-3}\,\text{эВ}$$

Klaer, Moore '17
Gorghetto et al '21

QCD axions are bosons ϕ

- Solve strong CP problem

  Peccei, Quinn '77

- Tiny interactions:

  $\lambda_{\phi^4} \sim 10^{-50}$, $\lambda_{\phi e^+ e^-} \lesssim 10^{-15}$

  $\Rightarrow$ only gravity!

- Form cold dark matter

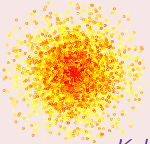
  Preskill et al '83; Abbott, Sikivie '83

  $\rightarrow$ slow: $v_\phi \ll 1$

  $\rightarrow$ overoccupied: $f_{\boldsymbol{p}}(\boldsymbol{x}) \gg 1$

  $=$ wave-like $\phi(t, \boldsymbol{x})$

Axion miniclusters



Kolb, Tkachev '93
Vaquero, Redondo, Stadler '19

- Asteroid masses:

  $M_{tot} \sim 10^{-17} - 10^{-12}\, M_\odot$

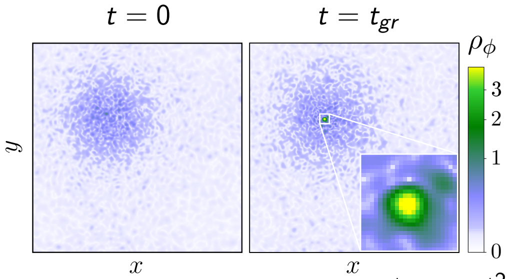
- Gravitationally bound now:

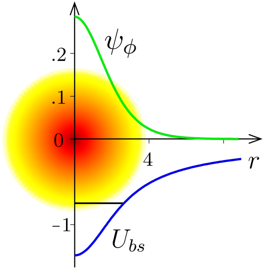
  $R_{mc} \sim$ a.e.

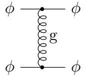
- Include $\sim$ 70% of DM

Bose-Einstein condensation by gravity

DL, Panin, Tkachev '18





- Gravitational scattering: $\sigma_{gr} = \left| \right.$  $\left. \right|^2 \propto \dfrac{(m_\phi G)^2}{v_\phi^4}$
- $\Rightarrow$ **Bose-Einstein condensation:** $t_{gr} \sim (\sigma_{gr} v_\phi n_\phi f_{\mathbf{p}})^{-1} \ll 10^{10}$ **yr** (QCD axions)
- Axion star $=$ condensate at the **lowest level** of U_{bs}
- $\Rightarrow$ Universe is **full** of QCD axion stars!

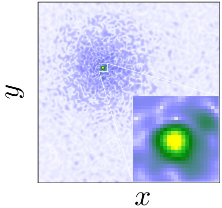
But how do they grow?

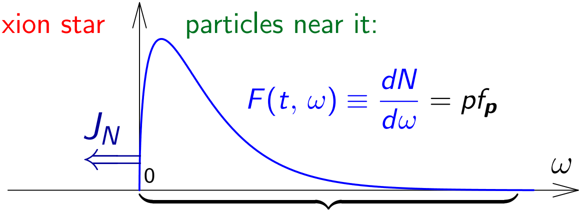
cf. Eggemeier et al '19, Chan et al '22 and see Dmitriev, DL, Tkachev '23

Kinetic equation

e.g., DL, Panin, Tkachev '18

$$\partial_t f_{\mathbf{p}} + \cancel{\mathbf{v}\nabla_x f_{\mathbf{p}}} - \cancel{\nabla_{\mathbf{x}} \bar{U} \nabla_p f_{\mathbf{p}}} = \left(\mathrm{St}\, f_{\mathbf{p}} \propto G^2 f_{\mathbf{p}}^3\right)$$





Approximations

- homogeneous gas: $\nabla_x f_{\mathbf{p}} = 0$
- no mean potential: $\bar{U} = 0$
- collisions: Landau integral

But take into account

- **energy/particle exchange with Bose star**

$$\partial_t F = \underbrace{G^2 \int d\sigma_{gr}\, F^3}_{\text{grav. scattering}}$$

Condensing BC: $J_N\Big|_{\omega=0} \neq 0$

Key observation: scale symmetry!

$$\partial_t F = G^2 \int d\sigma_{gr}\, F^3$$

Symmetry:		**Invariants:**	
$t \to c\,t$ $\omega \to c^{1-2/D}\,\omega$ $F \to c^{1/D}\,F$	c — scaling parameter D — arbitrary	$\omega_s \equiv \beta(t)\,\omega$ $F_s \equiv F/\alpha(t)$	$\alpha \propto t^{-1/D}$ $\beta \propto t^{2/D-1}$

Self-similar Ansatz

= most general & scale-symmetric

$$F = \alpha(t) F_s(\beta(t)\,\omega)$$

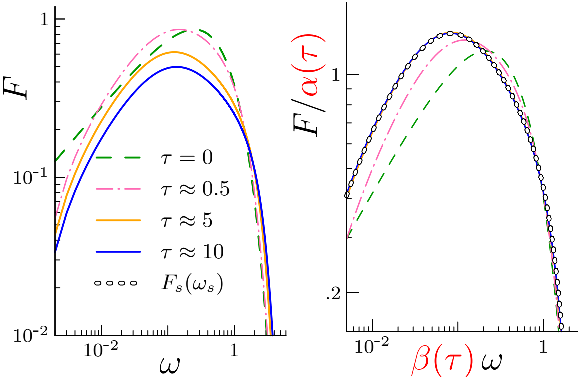
$$\left.\begin{array}{l} E \propto t^{2-5/D} \\ N \propto t^{1-3/D} \end{array}\right\} \Rightarrow E^3/N^5 \propto t$$

Coleman theorem: the Ansatz passes the equation!

$$(2/D-1)\,\omega_s \partial_{\omega_s} F_s - F_s/D = G^2 \int d\sigma_{gr}\, {F_s}^3 \quad \Big| \quad \text{bound/ext conditions} \Leftrightarrow D$$

Numerical evolution

$$\partial_t F = G^2 \int d\sigma_{gr}\, F^3$$



$F(t, \omega)$ is self-similar!

$$F = \alpha F_s(\beta\omega)$$

$$\alpha \propto \tau^{-1/D}, \quad \beta \propto \tau^{2/D-1}$$

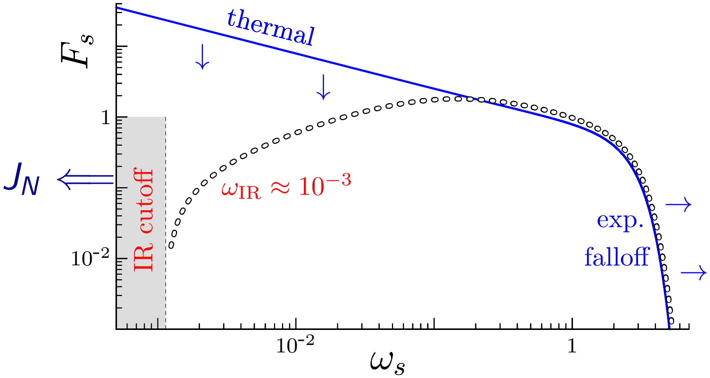
$$(\tau \equiv t/t_{gr})$$

$$D = 2.5$$

boundary conditions $\Rightarrow$ $E = \text{const}$ $\Rightarrow$ $D = 2.5$

Self-similar solutions are attractors!

Self-similar profiles



$$\left.\begin{array}{l} E \propto t^{2-5/D} \\ N \propto t^{1-3/D} \end{array}\right\} \Rightarrow E^3/N^5 \propto t$$

Adiabatic self-similarity = approximate self-similarity

$$\partial_t F = G^2 \int d\sigma_{gr}\, F^3 + J_{\text{ext}}(t,\,\omega) \longleftarrow$$

external source

symmetry breaking!

Approx. self-similar!

$$F = \alpha F_s(\beta\omega)$$

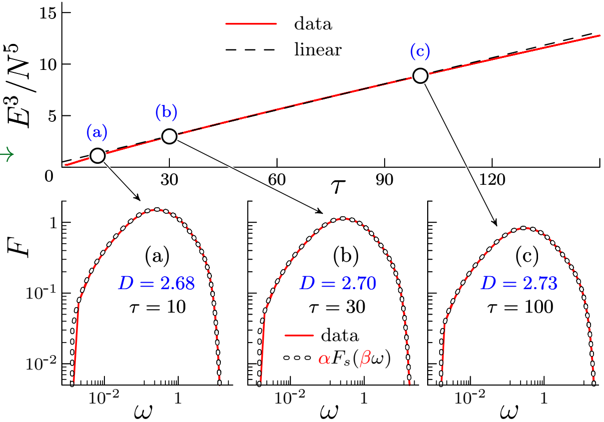
$\alpha \propto t^{-1/D}$, $\beta \propto t^{2/D-1}$ $\rightarrow$

But $D = D(t)$

slowly!

Small parameter:

$$t\partial_t \ln D \ll 1$$

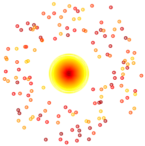


$$(2/D - 1)\,\omega_s \partial_{\omega_s} F_s - F_s/D = G^2 \int d\sigma_{gr}\, {F_s}^3 + J_{\text{ext},s}$$

All self-similar solutions are <u>kinetic attractors!</u>

Growth of axion star

Dmitriev, DL, Panin, Tkachev '23



- Adiabatic self-similarity:

  $D = D(t)$, but slowly $\Rightarrow$ $\boxed{E^3/M^5 \approx (t - t_i)/t_*}$

  self-similar eq. of state

- Heuristic constants: $t_i,\ t_* = \text{const}$

- Energy & mass conservation:

$$\underset{\text{gas}}{E} + \underset{\text{star}}{E_{bs}} = \text{const}, \qquad \underset{\text{gas}}{M} + \underset{\text{star}}{M_{bs}} = \text{const}$$

- Axion star: $E_{bs} = -\gamma M_{bs}^3$

  γ $\downarrow$ known constant

$$\frac{(1 + x^3/\epsilon^2)^3}{(1 - x)^5} = \frac{t - t_i}{t_{gr} - t_i}$$

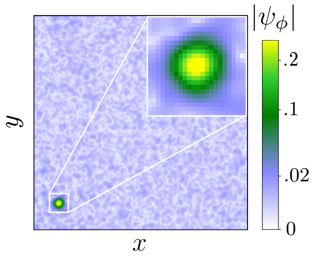
- $x(t) \equiv M_{bs}/M_{tot}$
- $\epsilon^2 = E_{tot}/\gamma M_{tot}^3$
- t_i — find from the fit

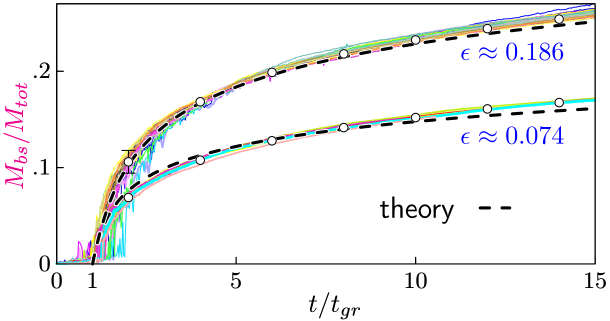
Simple algebraic law!

Comparing with exact simulations

Dmitriev, DL, Panin, Tkachev '23

$\boxed{f \gg 1} \Rightarrow$ random waves $\psi_\phi(t, x)$



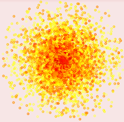


Coincides with all known simulations!

cf. Levkov et al '18, Eggemeier et al '19, Chan et al '22, Schive et al '14, Dmitriev et al '23

What does it mean for cosmology?

Miniclusters



$M \sim 10^{-(17\div 12)}\, M_\odot$

$\Phi = \delta\rho_\phi/\bar{\rho}_\phi\big|_{\mathrm{RD}} = 0 \div 10^3$

Hogan, Rees '88; Kolb, Tkachev '93

QCD axion $\boxed{m_a \sim 10^{-4}\,\mathrm{eV}}$

- Relaxation time:

  $t_{gr} \sim 5 \cdot 10^8\,\mathrm{yr}/\Phi^4 \gtrsim \mathrm{hr}$

- Time to condense 10% of minicluster:

  $t_{10} \sim \dfrac{7 \cdot 10^{18}\,\mathrm{yr}}{\Phi^8} \cdot \underbrace{(10\%)^9}_{x} \underbrace{< 10^{10}\,\mathrm{yr}}_{\text{if } \boxed{\Phi \gtrsim 1}}$

- $\Rightarrow$ $\boxed{\gtrsim 10\ \%\ \text{of dark matter} = \text{axion stars?}}$

- Heavy axion stars: $M_{bs} \sim 10^{-17} - 10^{-12}\, M_\odot$
  $R_{bs} \sim 100\,\mathrm{km}$

- Observational signatures: Bose-novae, extra radio background, parametric radioexplosions, . . .

We need distribution of miniclusters!

cf. Maseizik, Sigl '24; Gorghetto, Hardy, Villadoro, '24

Conclusions

Kinetic theory

- Gravitational kinetics is controlled by self-similar attractors!
  = non-thermal fixed points of kinetic evolution

  in short-range kinetics: Svistunov '91; Semikoz, Tkachev '95; Berges, Sexty '12

- Adiabatic self-similarity: $\underbrace{D = D(t)}$
  problems with broken scale symmetry!

  Applications in ordinary (short-range) kinetics? *cf. Micha, Tkachev '03*

Axion stars

- Adiabatic self-similarity $\Rightarrow$ analytic growth law $M_{bs}(t)$
  $\Rightarrow$ distribution of axion stars in the Universe!
- But:
  - Generalization to non-uniform gas in minicluster?
  - Scattering integral for condensation onto Bose star?
  - Distribution of axion miniclusters?

Thank you for attention!

The talk is supported by the grant RSF 22-12-00215