Effects of inhomogeneities on the propagation of gravitational waves from binaries of compact objects

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*A. Ali and ASM, JCAP **01**, 054 (2017)*

*S. S. Pandey, A. Sarkar, A. Ali and ASM, JCAP **06**, 021 (2022)*

*A. Halder, S. S. Pandey, ASM, JCAP **08**, 064 (2023)*

S. S. Pandey, A. Halder, ASM, Phys. Rev. D 110, 043531 (2024)

Outlook:

- Observations tell us that the present Universe is inhomogeneous up to scales (< 500 h^{-1} Mpc) [*Features:* Spatial volume is dominated by voids; peculiar structures at very large scales] ***[Sloan Digital Sky Surveys; Giant arc ~ 1 Gpc]***

- Cosmology is very well described by spatially homogeneous and isotropic FLRW model

  (modulo recent tensions: Hubble, S-8.. ?)

- Observational concordance comes with a price: more that 90% of the energy budget of the present universe comes in forms that have never been directly observed (DM & DE); DE not even theoretically understood

- Scope for alternative thinking without modifying GR or extending SM; application of GR needs to be more precisely specified on large scales

- Backreaction from inhomogeneities could modify the evolution of the Universe; Gravitational wave propagation ⇒ compact object parameters in GW astronomy

Propagation of Gravitational Waves from binaries

- Several detection events of compact binary mergers since LIGO and VIRGO

- Observed GW parameters are crucial for inferring source parameters, viz., mass, merger rate

- Multi-messenger astronomy opening up new observational window to physics of BH formation & many aspects of early universe physics

- Present observation of GWs comes from sources that are well within (much smaller than) the scale of observed global homogeneity

- *Backreaction induced changes in observed GW parameters – corresponding modification in inferred source parameters*

Problem of course-graining or averaging

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$$

$$G_{\mu\nu} = < T_{\mu\nu} >$$

Einstein's equations: ***nonlinear***

$$< G_{\mu\nu}(g_{\mu\nu}) > = < T_{\mu\nu} > \neq G_{\mu\nu}(< g_{\mu\nu} >)$$

Einstein tensor constructed from average metric tensor will not be same in general as the average of the Einstein tensor of the actual metrics

Different approaches of averaging

Macroscopic gravity: *(Zalaletdinov , GRG '92;'93)*

$$\left\langle g^{\mu\lambda}\right\rangle\left\langle R_{\lambda\nu}\right\rangle-\frac{1}{2}\delta^{\mu}_{\nu}\left\langle g^{\lambda\rho}\right\rangle\left\langle R_{\lambda\rho}\right\rangle+C^{\mu}_{\nu}=\kappa\left\langle T^{\mu}_{\nu}\right\rangle$$

(additional mathematical structure for covariant averaging scheme)

Perturbative schemes: *(Clarkson et al, RPP '11; Kolb, CQG '11)*

$$g_{\mu\nu}=\bar{g}_{\mu\nu}+\delta g_{\mu\nu} \qquad \overline{G}^{\mu}_{\nu}+\delta G^{\mu}_{\nu}=\kappa\left\langle T^{\mu}_{\nu}\right\rangle$$

Spatial averages : ***(Buchert, GRG '00; '01)****

Lightcone averages: *(Gasperini et al., JCAP '09;'11)*

Bottom-up approach [discrete cosmological models]: (Tavakol , PRD'12; JCAP'13)

Using the Einstein equations:

$$3\frac{\ddot{a}_D}{a_D} = -4\pi G\langle\rho\rangle_D + Q_D + \Lambda$$

Q: Backreaction due to averaging

$$3H_D^2 = 8\pi G\langle\rho\rangle_D - \frac{1}{2}\langle R\rangle_D - \frac{1}{2}Q_D + \Lambda$$

$$0 = \partial_t\langle\rho\rangle_D + 3H_D\langle\rho\rangle_D$$

where the average of the scalar quantities on the domain D is

$$\langle f\rangle_D(t) = \frac{\int_D f(t, X^1, X^2, X^3)\, d\mu_g}{\int_D d\mu_g}$$

Integrability condition:

$$\frac{1}{a_D^6}\partial_t\left(a_D^6 Q_D\right) + \frac{1}{a_D^2}\partial_t\left(a_D^2\langle R_D\rangle\right) = 0$$

ρ = local matter density

R = Ricci-scalar

$H_D = \frac{\dot{a}_D}{a_D}$ = domain dependent Hubble rate

The kinematical backreaction QD
is defined as

$$Q_D = \frac{2}{3}\left(\langle\theta^2\rangle_D - \langle\theta\rangle_D^2\right) - 2\sigma_D^2$$

where θ is the local expansion rate,

$\sigma^2 = 1/2\sigma_{ij}\sigma^{ij}$ is the squared rate of shear

Acceleration equation for the global domain D:

$$\frac{\ddot{a}_D}{a_D} = \sum_{\ell} \lambda_\ell \frac{\ddot{a}_\ell(t)}{a_\ell(t)} + \sum_{\ell \neq m} \lambda_\ell \lambda_m \left(H_\ell - H_m\right)^2$$

2-scale interaction-free model (Weigand & Buchert, PRD '10)***:***

M – those parts that have initial overdensity ("Wall")

E – those parts that have initial underdensity ("Void")

$$D = M \cup E \qquad H_D = \lambda_M H_M + \lambda_E H_E$$

Void fraction: $\lambda_E = {|E|}/{|D|}$ Wall fraction: $\lambda_M = {|M|}/{|D|}$

$$\lambda_M + \lambda_E = 1$$

Acceleration equation:

$$\frac{\ddot{a}_D}{} = \lambda_M \frac{\ddot{a}_M}{} + \lambda_E \frac{\ddot{a}_E}{} + 2\lambda_M \lambda_E (H_M - H_E)^2$$

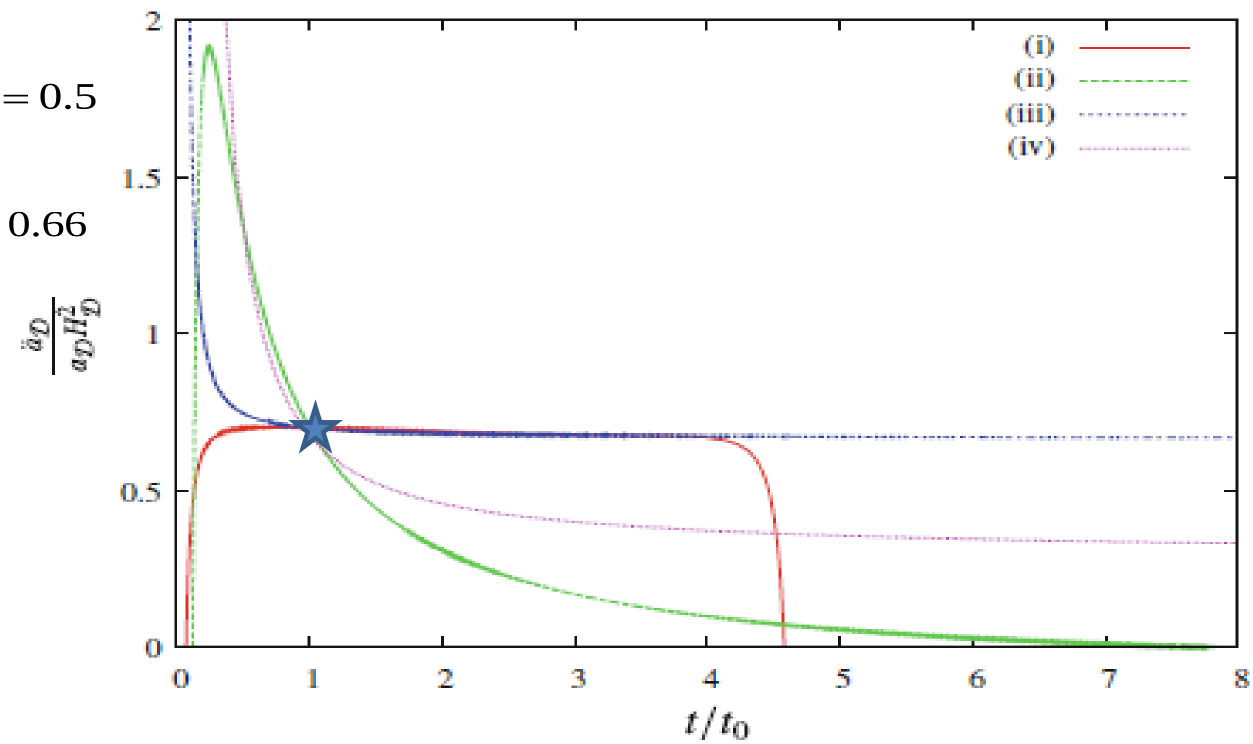
Future evolution assuming present acceleration

$$a_M \propto c_M t^{\beta} \qquad\qquad a_E \propto c_E t^{\alpha}$$

Present wall fraction, $\lambda_{M_0} = 0.09$ *[Weigand & Buchert, PRD '10]*

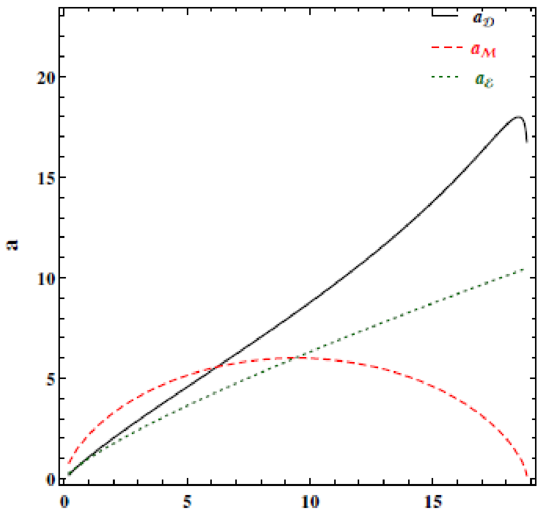
(i) $\alpha = 0.995, \beta = 0.5$

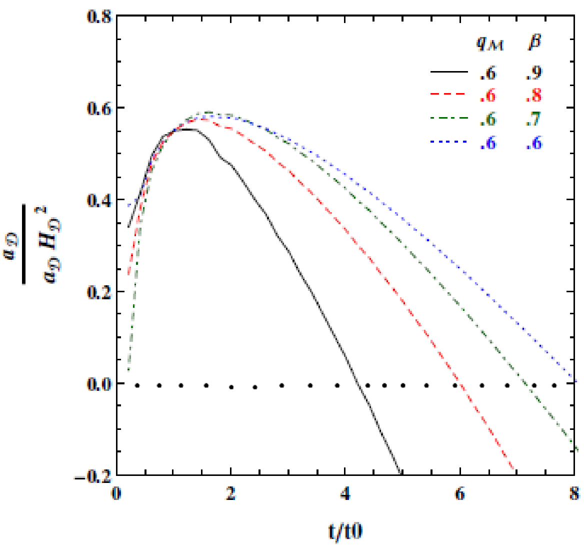
(ii) $\alpha = 1.02, \beta = 0.66$



N.Bose & ASM,
GRG (2013)

Future evolution of the global domain *[A. Ali, ASM JCAP '17]*





$$q = 0.6, \beta = 0.7$$

$$a_E = c_E t^\beta$$

$$a_M = \frac{q}{2q-1}(1-\cos\theta)$$

$$t = \frac{q}{2q-1}(\theta - \sin\theta)$$

$$H_D = \lambda_M H_M + \lambda_E H_E$$

As time evolves, H_E *falls off more rapidly compared to* H_M

Even though the wall occupies a tiny fraction of the total volume, the decrease of λ_M *is more than compensated by the comparative evolution of* H_E *and* H_M

Decelerating future evolution !

Analogous scalar field cosmology *[A. Ali, ASM, JCAP '17]*

Effective perfect fluid E-M tensor in the Backreaction formalism:

$$\rho_{eff}^{D} = \langle \rho \rangle_{D} - \frac{1}{16\pi G} Q_{D} - \frac{1}{16\pi G} \langle R \rangle_{D}$$

$$P_{eff}^{D} = -\frac{1}{16\pi G} Q_{D} + \frac{1}{16\pi G} \langle R \rangle_{D}$$

Buchert equations recast in standard Friedman form:

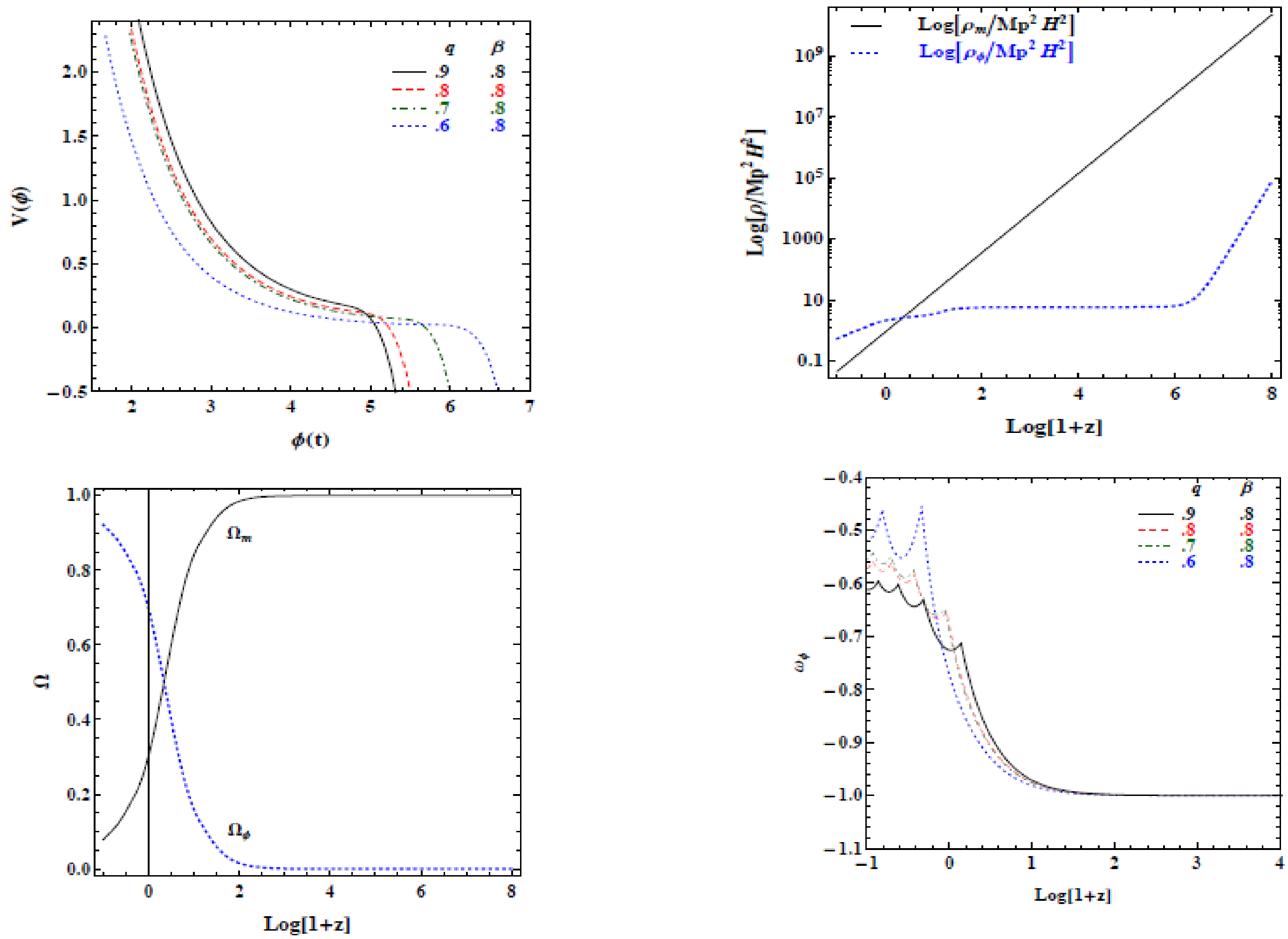
$$3\frac{\dot{a}_{D}}{a_{D}} = -4\pi G\left(\rho_{eff}^{D} + 3P_{eff}^{D}\right) + \Lambda$$

$$3H_{D}^{2} = 8\pi G \rho_{eff}^{D} + \Lambda$$

$$\dot{\rho}_{eff}^{D} + 3H_{D}\left(\rho_{eff}^{D} + P_{eff}^{D}\right) = 0$$

corresponding to energy density and pressure of effective global scalar field at scales much larger than the scale of inhomogeneities

Scalar field dynamics



Thawing quintessence *[Recent DESI BAO Observations !]*

Observational Impact of Inhomogeneities:

Multi-messenger Astronomy

Backreaction model

(2-scale void-wall) *[Pandey, Ali, Sarkar, ASM '22]*

$$t = t_0 \left(\frac{\phi - \sin\,\phi}{\phi_0 - \sin\,\phi_0} \right) ,$$

$$a_o = \frac{f o^{1/3}}{2} (1 - \cos\,\phi) \, ,$$

$$a_u = \frac{f u^{1/3} (\phi_0 - \sin\,\phi_0)}{\pi t_0} t^{\beta}$$

$$a_{\mathcal{D}} = \left(\frac{a_u^3 + a_o^3}{a_{u,0}^3 + a_{o,0}^3} \right)^{1/3}$$

$$H_{\mathcal{D}} = H_u \frac{a_u^3}{a_u^3 + a_o^3} + H_o \frac{a_o^3}{a_u^3 + a_o^3}$$

$$\mathcal{Q}_{\mathcal{D}} = \mathcal{Q}_o + \mathcal{Q}_u + 6 f o (1 - f o)(H_o - H_u)^2$$

$$\mathcal{Q}_o = 0 \qquad \mathcal{Q}_u = 0$$

Light propagation in backreaction model

Angular diameter distance

$$D_A = \frac{c}{1+z_1}\int_0^{z_1}\frac{dz}{H(z)}$$

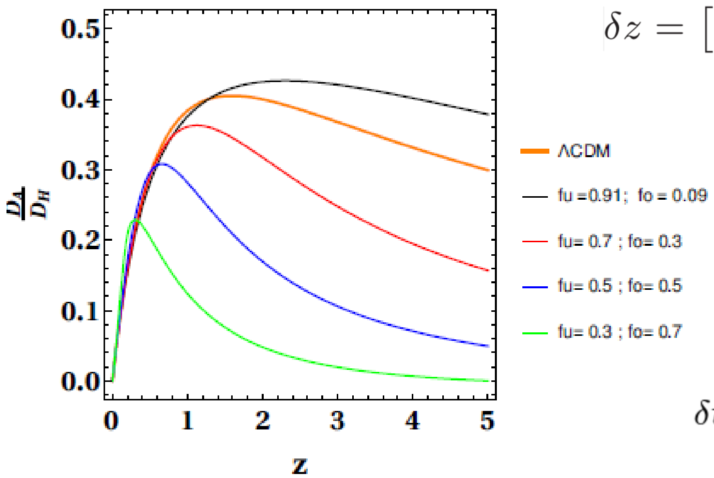
Covariant scheme:

[Rasanen '09]

$$1+z = \frac{1}{a_{\mathcal{D}}},$$

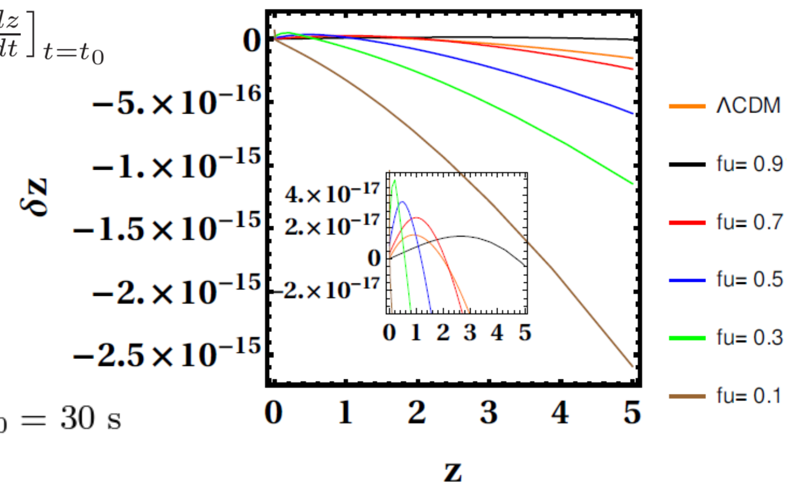
$$H_{\mathcal{D}}\frac{d}{dz}\left((1+z)^2 H_{\mathcal{D}}\frac{dD_A}{dz}\right) = -\frac{4\pi G}{c^4}\langle\rho_{\mathcal{D}}\rangle D_A$$

Change in observed redshift during the time interval of observation: Red-shift drift



$$\delta z = \left[\frac{dz}{dt}\right]_{t=t_0}$$

$$\delta t_0 = 30 \text{ s}$$



Angular diameter versus red-shift

Departure from ΛCDM *in terms model parameters*

Gravitational wave amplitude

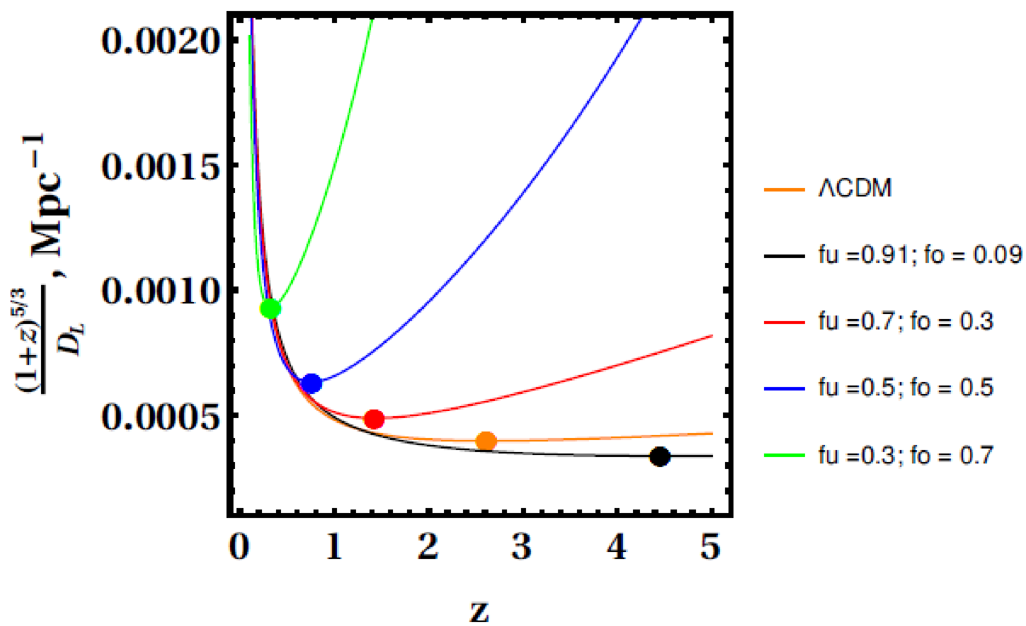
Binaries in early inspiral stage
(Keplerian approximation)

$$h_\times = \frac{G^{5/3}(1+z)^{5/3}}{D_L c^4} \frac{m_1 m_2}{(m_1+m_2)^{1/3}} (-4\omega^{2/3}) \sin 2\omega t$$

$$D_L = (1+z)D$$

Red-shift dependent part of GW amplitude

$(1+z)^{5/3}/D_L$



Deviations from ΛCDM *get amplified at higher z*

Change in gravitational wave observables

** Amount of change depend upon effect of inhomogeneities in the path of propagation – model parameters* fu, fo, and β

Red-shift minima (model dependence)

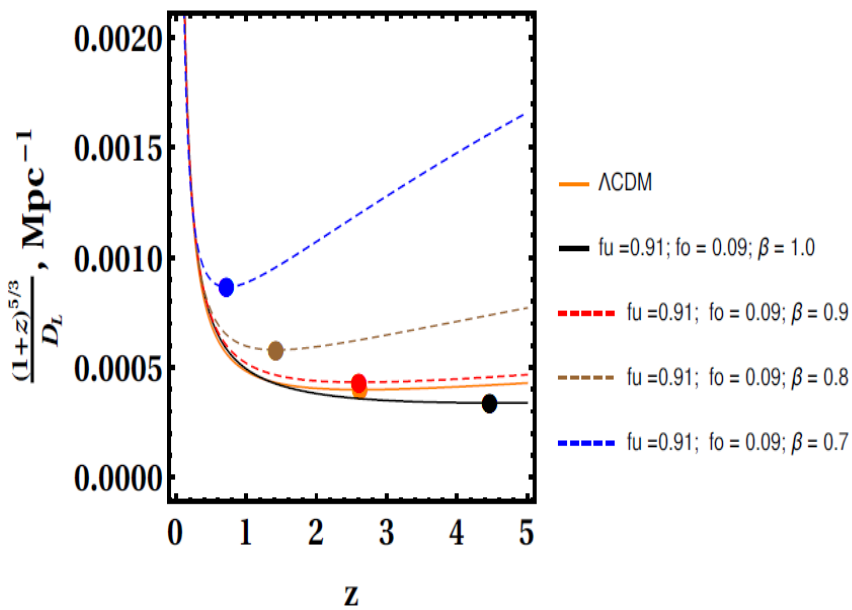
$(1+z)^{5/3}/D_L$ has a minimum at $z_{\min}$

S. S. Pandey, A. Sarkar, A. Ali, ASM, JCAP 06, 021 (2022)

$$(1+z_{\min})\left[\frac{d}{dz} ln[D_L]\right]_{z=z_{\min}} = \frac{5}{3}$$

Independent of binary characteristics, cosmological model, GW detector

*[Rosado et al., PRL **116**, 101102 (2016)]*



Minima Varies !

PHYSICAL REVIEW D **110,** 043531 (2024)

Analyzing the 21-cm signal brightness temperature in the Universe with inhomogeneities

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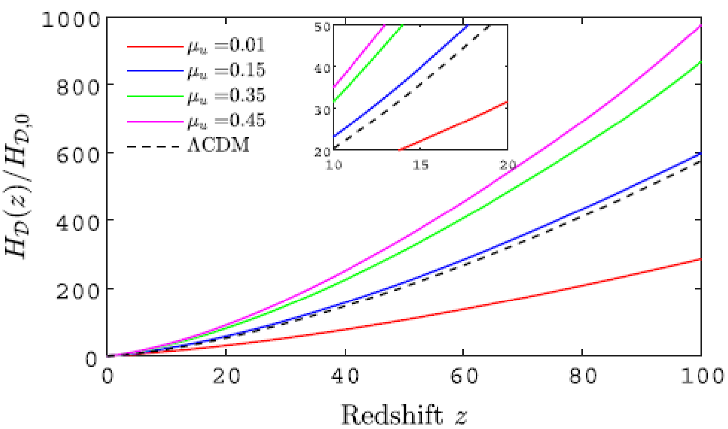
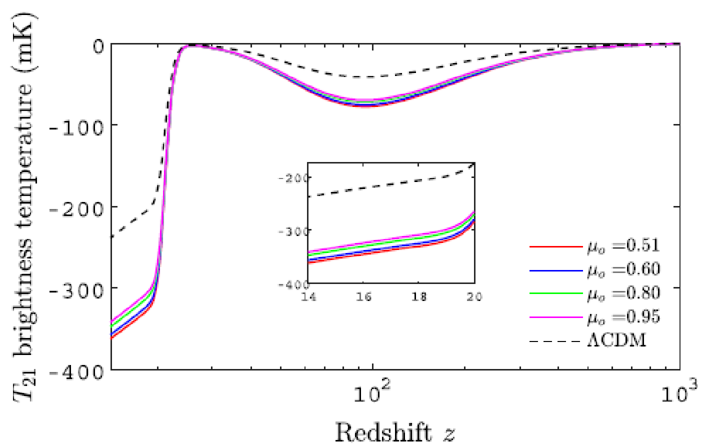




Multidomain model *[A. Halder, S. S. Pandey, ASM, JCAP **08**, 064 (2023)]*

Overdense regions

$$a_{o_i} = \frac{q_{o_i}}{2q_{o_i}-1}(1-\cos\phi)$$
$$t_i = \frac{q_{o_i}}{2q_{o_i}-1}(\phi-\sin\phi)$$

Underdense regions

$$a_{u_i} = c_{u_i} t^{\beta_i}$$

Global acceleration:

$$\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} = \left(\sum_i -\lambda_{o_i} q_{o_i} H_{o_i}^2\right) + \left(\sum_j \lambda_{u_j} \frac{\beta(\beta-1)}{t^2}\right) + \left(\sum_k \sum_l \lambda_k \lambda_l \left(H_l - H_k\right)^2\right)$$

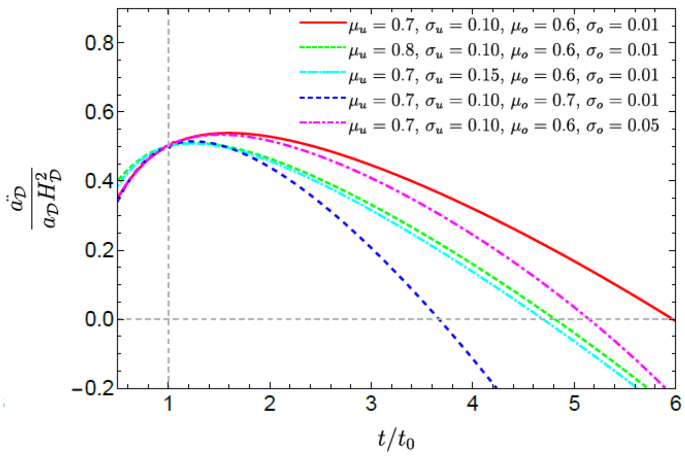
Gaussian profile for volume fractions

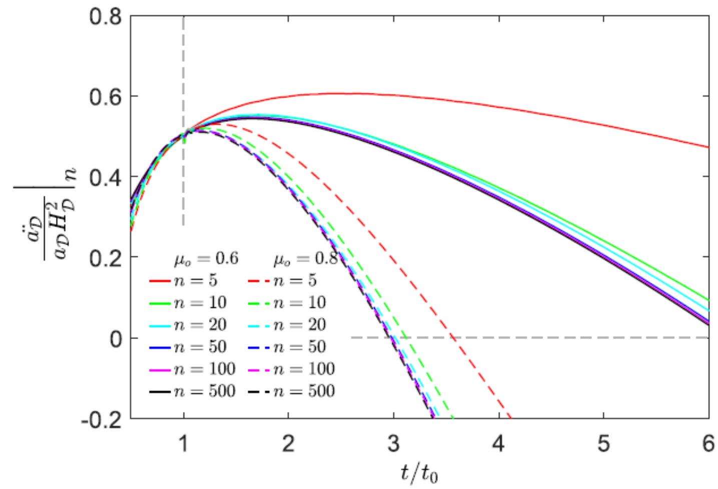
$$\lambda_{u_i,0} = \frac{N_u}{\sigma_u\sqrt{2\pi}} e^{-(\beta_i-\mu_u)^2/2\sigma_u^2}$$

$$\lambda_{o_i,0} = \frac{N_o}{\sigma_o\sqrt{2\pi}} e^{-(q_{o_i}-\mu_o)^2/2\sigma_o^2}$$

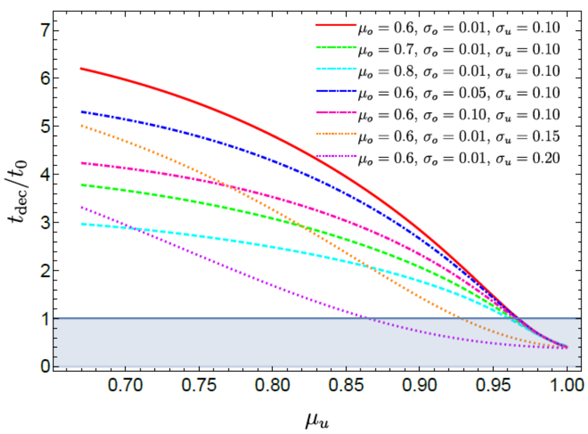
Evolution in multidomain model *(No Big-rip !)*

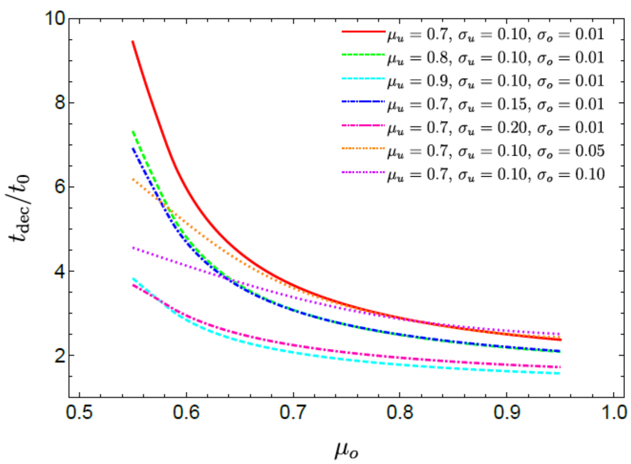
Global acceleration





Onset of future decelerating phase $t_{\rm dec}$

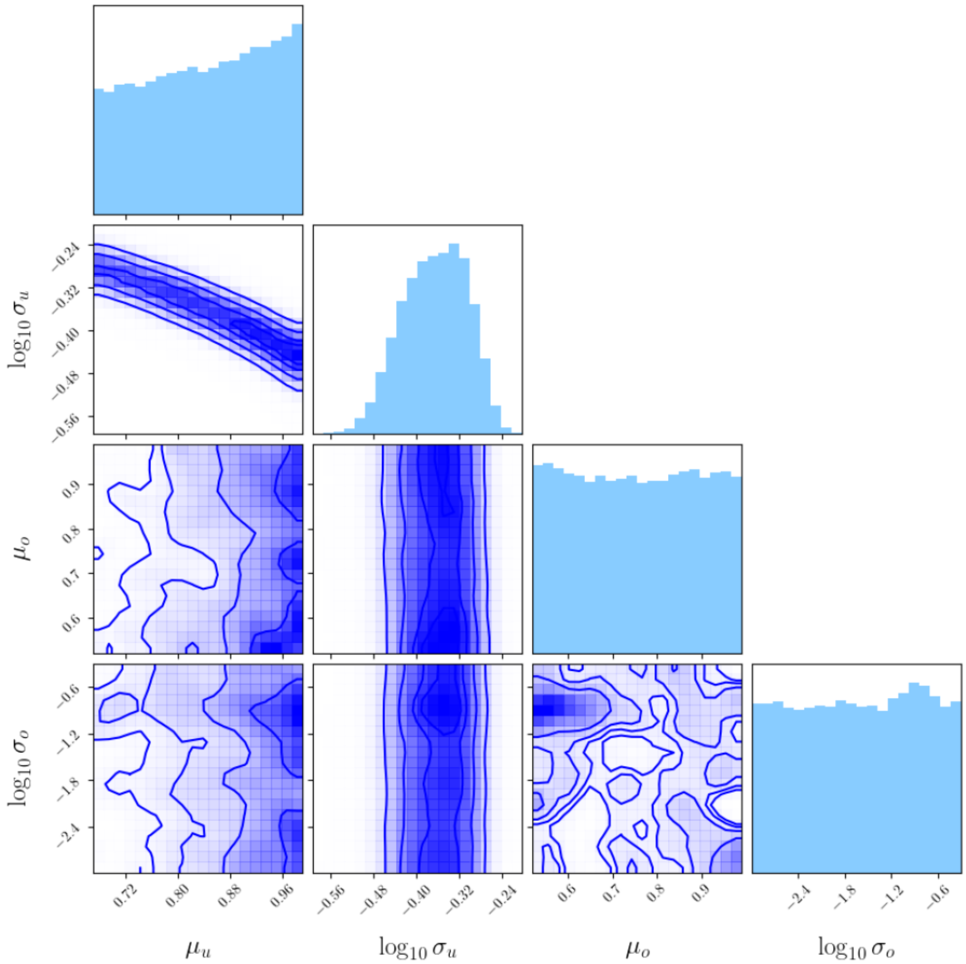




Observational constraints *[multi-scale Gaussian model]*

[A. Halder… ASM, JCAP '23]

(Bayesian analysis using Union 2.1 Supernova I a data)



Effect of inhomogeneities: gravitational wave propagation (Summary)

[A. Ali, ASM, JCAP 01, 054 (2017); S. S. Pandey, A. Sarkar, A. Ali, ASM, JCAP 06, 021 (2022); A. Halder, S. S. Pandey, ASM, JCAP 08, 064 (2023); S. S. P., A. H., ASM, PRD 110. 043531 (2024)]

- Effect of backreaction due to inhomogeneities on the future evolution of the accelerating universe *(Spatial averaging in the **Buchert framework**)*

- The global homogeneity scale (or cosmic event horizon) impacts the role of inhomogeneities on the evolution, causing the acceleration to slow down significantly with time.

- Backreaction could be responsible for a decelerated era in the future. ***(Avoidance of big rip !)*** *Possible within a small region of parameter space*

- Analogous scalar field cosmology: Form of potential fixed by backreaction model; Observational constraints from data analysis

- Dip in 21-cm signal (explanation for EDGES result !)

- Modification of binary GW parameters due to backreaction from inhomogeneities. Clear signature of red-shift drift. ***Significance in Multimessenger Astronomy & EU Physics***

- Effect may be tested in more realistic models, e.g., models with no ansatz for subdomains, & other schemes of backreaction