Multiscalar-metric gravity: merging gravity, dark energy and dark matter

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Abstract:

The status of a modification of General Relativity (GR) – Spontaneously Broken Relativity (SBR) – for merging gravity, dark energy (DE) and dark matter (DM) is presented. The modification is principally grounded on a multiscalar-metric concept of spacetime endowed with two dynamical structures: a basic metric and a set of the reversible multiscalar fields. The latter ones serve geometrically as distinct dynamical coordinates among arbitrary observer's ones and physically as a kind of gravitational Higgs fields producing the possible spontaneous symmetry breaking (SSB). The effective field theory (EFT) of the extended gravity based on the multiscalar-metric spacetime – the metagravity – is discussed and of some of its particular realisations as SBR are explicated. Generally, SBR physically results in appearance of massive tensor and scalar gravitons, with their implications shortly discussed. Some of the emerging consequences and problems are briefly discussed. Possible prospects for SBR for future deeper unification of gravity with matter in the SSB context are shortly indicated.

Status and prospects

- Motivation
  - Cosmological and astrophysical evidence for Dark Energy (DE) and Dark Matter (DM).
  - Numerous proposals for DM candidates, absence of convincing candidate.
  - The cosmological constant (CC) issue and Hubble tension.
- Feasibility of substantial modification of general relativity (GR). Multiscalar-metric spacetime and gravity concept: tagging spacetime with additional intrinsic scalar fields.
- Effective field theory (EFT) of multiscalar-metric gravity (metagravity).
- Spontaneosly broken relativity (SBR):
  - General relativity. *Dark hole:* a GR-like black hole with a scalar graviton halo – signature of the multiscalar-metric gravity. Asymptotically flat rotation curves.
  - Weyl-transverse relativity: CC screening.
- Problems and prospects of the proposed spacetime and gravity modification.

Motivation: DM and DE problems

- The standard ΛCDM model – homogenous, isotropic spatially flat Friedmann–Robertson–Walker Universe dominated (at present) by Dark Energy and Dark Matter: 5% SM matter, 25% (cold) DM (0.5-1.6% 3 SM ν), 70% DE.
  - $\Omega_{tot} = \rho/\rho_c \simeq 1$ from CMB data $\Rightarrow$ spatially flat Universe.
  - SN lumi. distance vs z $\Rightarrow$ accelerated expansion of Universe $\Rightarrow$ $\Omega_\Lambda \simeq 0.7$.
  - **Strong motivation for cold DM:** CMB anisotropy vs present day baryonic density fluctuations $\Rightarrow \Omega_m \simeq 0.3$
- DM is observed in gravitationally bound structures from dwarf galaxies to galaxy clusters: v_r dispersion, rotation curves, gravitational lensing vs visible density.
  - From largest structures: $\langle \rho_{DM} \rangle \simeq 0.264\rho_c$, consistent with ΛCDM.
  - DM in galaxies: *asymptotically flat rotation curves*, the baryonic Tully–Fisher relation, $v_{rot\,\infty}^{3\div4} \propto M_{bar}$, for $M_{bar} = 10^6 \div 10^{12} M_\odot$ *(DM / SM matter coupling?)*.
- No natural DM candidate, no preferred BSM model providing it, no signal in direct DM searches: accelerator production, galactic WIMP detection . . . Small scale ($\ll 1$ Mpc) cold DM dynamical problems: • *Missing satellites:* few $M > 10^7 M_\odot$ subhaloes in the MW, much less than expected from simulations. • *The cusp-core problem:* no central DM density peak $\rho(r) \propto 1/r^{0.8\div1.4}$ • *Small DM haloes* of the largest MW satellites . . .
- The vacuum energy (CC) issues:
  - Huge CC from EFT vs tiny observational CC • Hubble tension (a hint to dynamical DE?)

Multiscalar-metric spacetime concept

/ Yu.F. Pirogov, EPJ C 76 (2016) 215 /

For description of gravity in GR the spacetime is supposed to be a metric manifold.

- To merge gravity with DE and DM, a modified concept of spacetime manifold with arbitrary observer's coordinates x^μ is considered, endowed with a two-component structure: basic metric $g_{\mu\nu}(x)$, $\mu, \nu = 0, 1 \ldots d$ (d is the spacetime dimension) supplemented by the set of multi-scalars $Z^a(x)$, $a = 0, 1 \ldots d$ transforming under piecewise auxiliary Lorentz group. / For definiteness, we assume $d = 4$ in what follows. / The multi-scalars define the set of distinct coordinates on the manifold, $z^\alpha(x) \equiv \delta^\alpha_a Z^a(x)$, in a piecewise manner. For homogeneity, Z^a are supposed to enter only through derivatives, more particularly, through the auxiliary metric $\zeta_{\mu\nu} \equiv \partial_\mu Z^a \partial_\nu Z^b \eta_{ab}$. In arbitrary coordinates, $g_{\mu\nu}$ and $\zeta_{\mu\nu}$ are used for construction of EFT merging gravity and dark components.

The modified spacetime concept is in fact the essence of the whole approach. The future development should confirm (or, conceivably, otherwise) the adopted concept.

Effective field theory (EFT) of multiscalar-metric gravity (*metagravity*) */ Yu.F. Pirogov, EPJ C 76 (2016) 215, IJMP Conf. Ser. 47 (2018) 1860101 /*

Gravity is described by an EFT built in observer's arbitrary kinematic coordinates x^α with a basic dynamical metric $g_{\mu\nu}(x^\alpha)$ and an auxiliary metric $\zeta_{\mu\nu}$. We introduce an effective scalar field (serving as a DM messenger):

$$\sigma = \log(\sqrt{-g}/\sqrt{-\zeta}),$$

and tensor fields, the effective metric (i.e. the one defining the observables) $\bar{g}_{\mu\nu}$, and the metric/quasi-metric correlator $\bar{æ}^{\mu}_{\nu}$ (a kind of dynamical DE):

$$\bar{g}_{\mu\nu} \equiv e^{\bar{w}(\sigma)} g_{\mu\nu},\ \bar{æ}^{\mu}_{\nu} \equiv \bar{g}^{\mu\lambda}\zeta_{\lambda\nu} \quad (\bar{g}^{\mu\nu} \equiv \bar{g}^{-1\,\mu\nu}),$$

where $\bar{w}(\sigma)$ is the Weyl form-factor. The two marginal cases are of special interest: at $\bar{w} = 0$ the effective metric presents the GR metric $\bar{g}_{\mu\nu} \equiv g_{\mu\nu}$, and at $\bar{w} = -\sigma/2$ the metric of Weyl-transverse gravity, $\bar{g}_{\mu\nu} \equiv (g_{\mu\nu}/(-g)^{1/4})(-\zeta)^{1/4}$. The effective action of the multiscalar-metric gravity (*metagravity* in what follows) is $S[g_{\mu\nu}, Z^a] = \int \bar{\mathcal{L}}(\bar{g}_{\mu\nu}, \bar{æ}^{\mu}_{\nu}, \sigma)\sqrt{-\bar{g}}\, d^4x$. The metagravity is self-sufficient, but in reality it should be supplemented by ordinary matter and DM fields. For definiteness, $\bar{æ}^{\mu}_{\nu}$ and σ do not directly interact with the ordinary matter and are coupled only to tensor gravity and DM. →

Effective field theory (EFT) of multiscalar-metric gravity (metagravity) (cont'd)

... $\rightarrow$ The Lagrangian: $\bar{\mathcal{L}} = \bar{\mathcal{L}}_G + \bar{\mathcal{L}}_{DS} + \bar{\mathcal{L}}_M$.

- Tensor gravity:
$$\bar{\mathcal{L}}_G = \bar{\mathcal{L}}_G(\bar{R})$$
$\bar{R} = \bar{g}^{\mu\nu} R_{\mu\nu}(\bar{g}_{\alpha\beta})$, where $R_{\mu\nu}$ is Riemann curvature tensor expressed via the effective metric.
- Dark substances – the scalar graviton (SG) and dark energy (DE):
$$\bar{\mathcal{L}}_{DS} = \bar{\mathcal{L}}_{SG}(\sigma, \partial_\mu\sigma, \bar{g}^{\mu\nu}, \bar{R}^{\mu\nu}, \bar{æ}^{\alpha}_{\beta}) - \bar{V}_{DE}(\bar{æ}^{\mu}_{\nu}, \sigma)$$
- Conventional matter (CM) and possible additional DM (sterile ν's?):
$$\bar{\mathcal{L}}_M = \bar{\mathcal{L}}_{CM}(\bar{g}_{\mu\nu}, \phi_{CM}) + \bar{\mathcal{L}}_{DM}(\bar{g}_{\mu\nu}, \partial_\mu\sigma, \phi_{DM}, \phi_{CM})$$
$$\bar{\mathcal{L}}_{DM} = \bar{\mathcal{L}}(g_{\mu\nu}, \phi_{DM}, \phi_{CM}) - \partial_\mu\sigma J^{\mu}_{DM}(\phi_{DM})$$

σ is assumed to interact with possible additional DM but doesn't interact directly with CM. In this sense, σ is a DM messenger.
The EFT *a priori* admits multiple realisations. Somewhat restricted but sufficiently general versions are considered in what follows.

Spontaneously broken relativity (SBR)

$$\bar{\mathcal{L}}_G = -\frac{1}{2} M_{\rm Pl}^2 \, \bar{R}(\bar{g}_{\mu\nu})$$

For simplicity, we split $\bar{\mathcal{L}}_{DS}$ into scalar graviton and pure DE terms:

$$\bar{\mathcal{L}}_{DS} = \bar{\mathcal{L}}_{SG} + \bar{\mathcal{L}}_{DE}$$

$$\bar{\mathcal{L}}_{SG} = \frac{M_{\rm Pl}^2}{2} \left\{ \left[\bar{g}^{\mu\nu} F_g(\sigma, \bar{æ}^{\alpha}_{\beta}, \bar{R}_{\mu\nu}) + \bar{R}^{\mu\nu} F_R(\sigma, \bar{æ}^{\alpha}_{\beta}, \bar{R}_{\mu\nu}) \right] \partial_\mu \sigma \partial_\nu \sigma - v_\sigma(\sigma) \right\}$$

The form-factors F_g, F_R are chosen as

$$F_g = \Upsilon^2 + \kappa_s(\bar{æ}^{\alpha}_{\beta}) + a(\bar{R}), \;\; F_R = b(\bar{R}) \, .$$

Υ characterizes the constant part of the coupling between scalar and tensor gravity. Possible dependence of the coupling on DE and the scalar curvature are accounted by $\kappa_s(æ^{\alpha}_{\beta})$ and $a(\bar{R})$ form-factors. A possible non-minimal coupling of σ to tensor gravity is parameterized by the term $\propto b(\bar{R})\bar{R}^{\mu\nu}$. A dependence of F_g on σ is neglected under the supposed shift symmetry $\sigma \to \sigma + const$.

- The dark energy term: $\bar{\mathcal{L}}_{DE} = -\bar{V}_{DE} \equiv -\bar{V}_{æ}(\bar{æ}^{\mu}_{\nu}) \equiv -M_{\rm Pl}^2 \bar{v}_{æ}(\bar{æ}^{\mu}_{\nu}) =$
$= -M_{\rm Pl}^2 \left(\bar{\Lambda} + G_1 \bar{æ}^{\mu}_{\mu} + G_2 (\bar{æ}^{\mu}_{\mu})^2 + H_2 \bar{æ}^{\mu}_{\nu} \bar{æ}^{\nu}_{\mu} + \dots \right)$

Such a decomposition is motivated by the requirement that the shift symmetry $\sigma \to \sigma + const$ is violated only by the potential $v_\sigma(\sigma)$.

Field equations

$$\frac{\delta S}{\delta g^{\lambda\rho}} = \frac{\delta S}{\delta \bar{g}^{\mu\nu}}\frac{\delta \bar{g}^{\mu\nu}}{\delta g^{\lambda\rho}} + \frac{\delta S}{\delta \sigma}\frac{\delta \sigma}{\delta g^{\lambda\rho}}$$

$\Rightarrow$ Einstein's equations (traceless terms are explicitly separated):

$$M_{\rm Pl}^2\left(\bar{R}_{\lambda\rho} - \frac{1}{4}\bar{g}_{\lambda\rho}\bar{R}\right) = \bar{T}_{\lambda\rho} - \frac{1}{4}\bar{g}_{\lambda\rho}\bar{T} + \\ + \bar{g}_{\lambda\rho}\left[\frac{1}{4}(1+2\bar{w}')\left(M_{\rm Pl}^2\bar{R} + \bar{T}\right) - \frac{1}{\sqrt{-\bar{g}}}\frac{\delta\sqrt{-\bar{g}}\bar{\mathcal{L}}}{\delta\sigma}\right] \quad (1)$$

$$\bar{T}_{\lambda\rho} = \frac{2}{\sqrt{-\bar{g}}}\frac{\delta\sqrt{-\bar{g}}\bar{\mathcal{L}}}{\delta \bar{g}^{\lambda\rho}}\,, \quad \bar{\mathcal{L}} = \bar{\mathcal{L}}_{SG} + \bar{\mathcal{L}}_{CM} + \bar{\mathcal{L}}_{DM} + \bar{\mathcal{L}}_{DE}$$

see backup

$$\frac{\delta S}{\delta Z^a} = 0 \quad \Rightarrow \quad \nabla_\lambda\left\{\eta_{ab}\partial_\rho Z^b\left[{\zeta^{-1}}^{\lambda\rho}\left(\frac{\bar{w}'}{2}\left(M_{\rm Pl}^2\bar{R} + \bar{T}\right) - \frac{1}{\sqrt{-\bar{g}}}\frac{\delta\sqrt{-\bar{g}}\bar{\mathcal{L}}}{\delta\sigma}\right) \\ - M_{\rm Pl}^2\left(\frac{\partial\bar{v}_{æ}}{\partial æ^\mu_\rho}\bar{g}^{\mu\lambda} + \frac{\partial\bar{v}_{æ}}{\partial æ^\mu_\lambda}\bar{g}^{\mu\rho}\right)\right]\right\} = 0 \quad (2)$$

Field equations

Eq. (2) can be integrated right away:

$$e^{2\bar{w}+\sigma}\left[\frac{\bar{w}'}{2}\left(M_{\rm Pl}^2\bar{R}+\bar{T}\right)-\frac{1}{\sqrt{-\bar{g}}}\frac{\delta\sqrt{-\bar{g}}\bar{\mathcal{L}}}{\delta\sigma}-\frac{M_{\rm Pl}^2}{2}\frac{\partial\bar{v}_{\ae}}{\partial \ae^\lambda_\rho}\ae^\lambda_\rho-\frac{1}{4}\eta_{ab}\partial_\lambda Z^a f^{b\lambda}\right]=M_{\rm Pl}^2\lambda_s\,, \tag{3}$$

where $M_{\rm Pl}^2\lambda_s$ is an integration constant and $f^{b\lambda}$ ($\nabla_\lambda f^{b\lambda}=0$) is an arbitrary covariantly divergenceless bi-vector transforming as a vector both relative to the multiscalar and observer's coordinates. The term including $f^{b\lambda}$ may influence an asymptotic behavior of the solution. From (3) and (1) one obtains the equation for the scalar graviton field (put here $\bar{\mathcal{L}}_{SG}$, $\bar{\mathcal{L}}_{DM}$):

$$\frac{M_{\rm Pl}^2}{\sqrt{-\bar{g}}}\partial_\mu\left[\sqrt{-\bar{g}}\left(\bar{g}^{\mu\nu}\left(\Upsilon^2+\kappa_s(\ae^\alpha_\beta)+a(\bar{R})\right)+b(\bar{R})\,\bar{R}^{\mu\nu}\right)\partial_\nu\sigma\right]+\frac{\partial V_\sigma^{\rm eff}}{\partial\sigma}=$$

$$=\nabla_\mu J^\mu_{DM}+(1+2\bar{w}')M_{\rm Pl}^2\left[\frac{1}{2}\frac{\partial\bar{v}_{\ae}}{\partial\ae^\lambda_\rho}\ae^\lambda_\rho+\frac{1}{4}\eta_{ab}\partial_\lambda Z^a f^{b\lambda}\right], \tag{4}$$

$$V_\sigma^{\rm eff}(\sigma)=M_{\rm Pl}^2\left[v_\sigma(\sigma)+\lambda_s e^{-2\bar{w}-\sigma}\right].$$

Note: at $\bar{w}\neq-\sigma/2$, $M_{\rm Pl}^2\lambda_s\neq0$ makes σ self-interacting even if $V_\sigma(\sigma)\equiv0$. In this context, the Weyl-transverse case $\bar{w}=-\sigma/2$ is exceptional: the DE and auxiliary $f^{b\lambda}$ contributions in (4) also disappear which singles out the Weyl-transverse case among the rest of the realisations.

Field equations: $\bar{w}' \neq -1/2$ case

- Einstein's equation (1) is represented in a conventional form as

$$M_{\rm Pl}^2\left(\bar{R}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\bar{R}\right) = \bar{T}^{\rm eff}_{\mu\nu} \tag{5}$$

The effective energy–momentum tensor:

$$\bar{T}^{\rm eff}_{\mu\nu} = \bar{T}_{\mu\nu} + \bar{g}_{\mu\nu}M_{\rm Pl}^2\left[\frac{1}{2}\frac{\partial v_{æ}}{\partial æ^{\lambda}_{\rho}}æ^{\lambda}_{\rho} + \lambda_s e^{-2\bar{w}-\sigma} + \frac{1}{4}\eta_{ab}\partial_\lambda Z^a f^{b\lambda}\right] \tag{6}$$

$$\bar{T}_{\mu\nu} = \frac{2}{\sqrt{-\bar{g}}}\frac{\delta\sqrt{-\bar{g}}\bar{\mathcal{L}}}{\delta\bar{g}^{\mu\nu}}, \quad \bar{\mathcal{L}} = \bar{\mathcal{L}}_{SG} + \bar{\mathcal{L}}_{CM} + \bar{\mathcal{L}}_{DM} + \bar{\mathcal{L}}_{DE}$$

see backup

σ:

$$\frac{M_{\rm Pl}^2}{\sqrt{-\bar{g}}}\partial_\mu\left[\sqrt{-\bar{g}}\left(\bar{g}^{\mu\nu}\left(\Upsilon^2 + \kappa_s(æ^{\alpha}_{\beta}) + a(\bar{R})\right) + b(\bar{R})\,\bar{R}^{\mu\nu}\right)\partial_\nu\sigma\right] + \frac{\partial V^{\rm eff}_{\sigma}}{\partial\sigma} =$$

$$= \nabla_\mu J^\mu_{DM} + (1+2\bar{w}')M_{\rm Pl}^2\left[\frac{1}{2}\frac{\partial\bar{v}_{æ}}{\partial æ^{\lambda}_{\rho}}æ^{\lambda}_{\rho} + \frac{1}{4}\eta_{ab}\partial_\lambda Z^a f^{b\lambda}\right]$$

$$V^{\rm eff}_{\sigma}(\sigma) = M_{\rm Pl}^2\left[v_\sigma(\sigma) + \lambda_s e^{-2\bar{w}-\sigma}\right]$$

Special case: spontaneously broken Weyl-transverse relativity (SBWTR, $\bar{w} = -\sigma/2$)

- Einstein's equation (1) takes the form:

$$M_{\rm Pl}^2\left(\bar{R}_{\lambda\rho} - \frac{1}{2}\bar{g}_{\lambda\rho}\bar{R}\right) = \bar{T}_{\lambda\rho} - \frac{1}{4}\bar{g}_{\lambda\rho}\left(M_{\rm Pl}^2\bar{R} + \bar{T}\right), \qquad (7)$$

$$\bar{T}_{\mu\nu} = \frac{2}{\sqrt{-\bar{g}}}\frac{\delta\sqrt{-\bar{g}}\bar{\mathcal{L}}}{\delta\bar{g}^{\mu\nu}}, \quad \bar{\mathcal{L}} = \bar{\mathcal{L}}_{SG} + \bar{\mathcal{L}}_{CM} + \bar{\mathcal{L}}_{DM} + \bar{\mathcal{L}}_{DE}$$

see backup

σ: $\frac{M_{\rm Pl}^2}{\sqrt{-\bar{g}}}\partial_\mu\left[\sqrt{-\bar{g}}\left(\bar{g}^{\mu\nu}\left(\Upsilon^2 + \kappa_s(æ^\alpha_\beta) + a(\bar{R})\right) + b(\bar{R})\,\bar{R}^{\mu\nu}\right)\partial_\nu\sigma\right] + \frac{\partial V_\sigma}{\partial\sigma} = \nabla_\mu J^\mu_{DM}$

▶ $\bar{g}_{\mu\nu} \equiv e^{-\sigma/2}g_{\mu\nu} = \left(g_{\mu\nu}/(-g)^{1/4}\right)(-\zeta)^{1/4} \Rightarrow \sqrt{-\bar{g}} \equiv \sqrt{-\zeta} \Rightarrow$ the Lagrangian CC term $\sqrt{-\bar{g}}M_{\rm Pl}^2\bar{\Lambda} \equiv \sqrt{-\zeta}M_{\rm Pl}^2\bar{\Lambda}$ does not vary with $g_{\mu\nu} \Rightarrow$ **The Lagrangian CC is irrelevant.** Impose a covariant conservation of $\bar{T}_{\lambda\rho}$ $\Rightarrow$ $\partial_\mu\left(M_{\rm Pl}^2\bar{R} + \bar{T}\right) = 0 \Rightarrow$ An effective CC reappears in the r.h.s. of (7) as an arbitrary integration constant unrelated to the Lagrangian CC.

↑ A possible path to explain CC issues? */ Yu.F. Pirogov, Grav. Cosmol. 28 (2022) 3, 263/*

▶ The massive tensor graviton appears via SSB mechanism (no ghosts, smooth $m_g \to 0$ transition), see the realisation in backup.

Conventional case: spontaneously broken GR (SBGR) ($\bar{w} = 0$, $\bar{g}_{\mu\nu} \equiv g_{\mu\nu}$)

- Einstein's equation (1) takes a GR-like form:

$$M_{\rm Pl}^2 \left(\bar{R}_{\lambda\rho} - \frac{1}{2}\bar{g}_{\lambda\rho}\bar{R} \right) = \bar{T}_{\lambda\rho} - \bar{g}_{\lambda\rho}\frac{1}{\sqrt{-\bar{g}}}\frac{\delta\sqrt{-\bar{g}}\bar{\mathcal{L}}}{\delta\sigma} \tag{8}$$

$$\bar{T}_{\mu\nu} = \frac{2}{\sqrt{-\bar{g}}}\frac{\delta\sqrt{-\bar{g}}\bar{\mathcal{L}}}{\delta\bar{g}^{\mu\nu}}, \quad \bar{\mathcal{L}} = \bar{\mathcal{L}}_{SG} + \bar{\mathcal{L}}_{CM} + \bar{\mathcal{L}}_{DM} + \bar{\mathcal{L}}_{DE}$$

see backup

σ:

$$\frac{M_{\rm Pl}^2}{\sqrt{-\bar{g}}}\partial_\mu \left[\sqrt{-\bar{g}}\left(\bar{g}^{\mu\nu}\left(\Upsilon^2 + \kappa_s(æ^\alpha_\beta) + a(\bar{R})\right) + b(\bar{R})\,\bar{R}^{\mu\nu}\right)\partial_\nu\sigma\right] + \frac{\partial V_\sigma^{\rm eff}}{\partial\sigma} =$$

$$= \nabla_\mu J^\mu_{DM} + M_{\rm Pl}^2\left[\frac{1}{2}\frac{\partial\bar{v}_æ}{\partial æ^\lambda_\rho}æ^\lambda_\rho + \frac{1}{4}\eta_{ab}\partial_\lambda Z^a f^{b\lambda}\right]$$

$$V_\sigma^{\rm eff}(\sigma) = M_{\rm Pl}^2\left[V_\sigma(\sigma) + \lambda_s e^{-\sigma}\right]$$

To illustrate the discussed theory, consider a local stationary spherically symmetric solution $\rightarrow$ ...

Spontaneously broken GR: scalar graviton dark holes

/Yu.F. Pirogov, Eur.Phys.J.C 72 (2012) 2017/

To start with, search for a static spherically symmetric vacuum solution merging a GR-like BH and a scalar graviton halo.

$$ds^2 \equiv \bar{g}_{\mu\nu}\, dx^\mu dx^\nu = A(r)dt^2 - C(r)r^2(\sin^2\theta\, d\phi^2 + d\theta^2) - A^{-1}(r)dr^2$$

For simplicity, neglect $\bar{V}_{DE}$ (and set $\bar{\Lambda} = 0$), put $V_\sigma = 0$, choose $\lambda_s < 0$ in (3), restricted case $f^{b^\lambda} = 0$. An exact solution (a generalization of Schwarzschild):

/ Yu.F. Pirogov, O.V. Zenin, Phys. At. Nucl. 88 (2025) 2, 356/

$$\begin{aligned} A(r) &= (1 - r_g/r)\,(r/r_h)^{\frac{4\Upsilon^2}{1+2\Upsilon^2}} \\ C(r) &= (r/r_c)^{\frac{4\Upsilon^2}{1+2\Upsilon^2}} \\ \sigma(r) &= \frac{2}{1+2\Upsilon^2}\log(r/r_h) \end{aligned}$$

- r_g is the Schwarzschild radius of the central BH.
- The scalar profile parameter:

$$r_h = \Upsilon\left[-\frac{2}{1+2\Upsilon^2}\,\frac{1}{\lambda_s}\right]^{1/2}$$

- r_c is fixed by the gauge-invariant condition:

$$\frac{d(\text{measured circle length})}{d(\text{measured radius})} \to 2\pi\,,\ r \to \infty$$

$$\Rightarrow r_c = r_h\left[1 + 2\Upsilon^2\right]^{\frac{1}{2\Upsilon^2}}\ \ (r_c \simeq r_h e \text{ at } \Upsilon^2 \ll 1)$$

- Non-flat asymptotic at $r \to \infty$! An anomalous radial acceleration term $\propto 1/r$.

For a particle at rest:

$$\frac{d^2r}{ds^2} = \left[-\frac{r_g}{2r^2} - \frac{2\Upsilon^2}{1+2\Upsilon^2}\left(1 - \frac{r_g}{r}\right)\frac{\mathbf{1}}{\mathbf{r}}\right]\cdot\left(\frac{r}{r_h}\right)^{\frac{4\Upsilon^2}{1+2\Upsilon^2}} \quad \rightarrow \quad \ldots$$

Spontaneously broken GR: scalar graviton dark holes

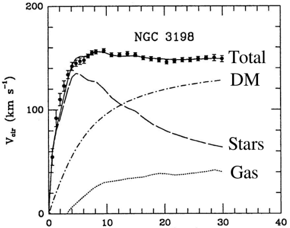
. . . $\longrightarrow$ An apparent rotation velocity for a circular orbit with the radius r:

$$v_{rot}(r) = \left[\left(1 + 2\Upsilon^2\right) \frac{1}{2} \frac{r_g}{r - r_g} + \mathbf{2\Upsilon^2} \right]^{1/2}$$

At r $\to \infty$ $\mathbf{v_{rot}} \to \sqrt{2}\Upsilon$ $\longleftarrow$ an asymptotically flat rotation curve.

Can one model DM haloes manifested in galaxies?

A typical asymptotic v_{rot} in large spiral galaxies is $\sim$ 300 km/s ($\sim$ 30 km/s in DM dominated dwarfs), thus $\Upsilon \sim 10^{-4} \div 10^{-3}$ $\Rightarrow M_s = \Upsilon M_{\rm Pl} \sim 10^{14 \div 15}$ GeV *(close to GUT scale?)* A caveat: large deviations from GR-like behaviour – not a problem at $r < r_h$ /arXiv:2504.05011/, still need a cutoff at $r \gg r_h$. $\longrightarrow$ Take into account the neglected DE term $M_{\rm Pl}^2 v_{æ}(æ^{\alpha}_{\beta})$ $\longrightarrow$ asymptotically flat solutions can be looked for.



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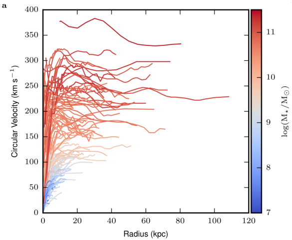

F. Lelli (2022)

Problems and prospects

- Prove that SBR is theoretically quite consistent for merging of gravity with DE and DM.
- Its phenomenological adequacy requires a more thorough study of the following problems:
  1. A more detailed study of finite vacuum solutions (dark holes) in the SBR framework: static spherically symmetric – generalization of GR Schwarzschild solution, and rotating as a generalization of Kerr solution.
  2. Gravitational waves: dispersion, extra polarisations, scalar waves?
  3. Necessity and conceivable DM sources of the scalar graviton. Modification of vacuum solutions.
  4. Study global solutions taking into account DE and DM sources of the scalar graviton.
  5. Connection with the conventional (SM) matter.

► In conclusion, SBR presents a perspective route for future deeper unification of gravity with matter in the common context of SSB.

References

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Backup

Scalar graviton, DE and matter contributions to $\bar{T}_{\mu\nu}$

$$\bar{T}_{\mu\nu}^{(SC,DE,DM,CM)} = \frac{2}{\sqrt{-\bar{g}}}\frac{\delta\sqrt{-\bar{g}}\bar{\mathcal{L}}_{(SG,DE,DM,CM)}}{\delta\bar{g}^{\mu\nu}}:$$

$$\begin{aligned}\frac{1}{\sqrt{-\bar{g}}}\frac{\delta\sqrt{-\bar{g}}\bar{\mathcal{L}}_{SG}}{\delta\bar{g}^{\mu\nu}} = \frac{M_{\rm Pl}^2}{2}\Big\{&\partial_\mu\sigma\partial_\nu\sigma\left(\Upsilon^2+\kappa_s(æ^\alpha_\beta)+a(\bar{R})\right)\\
&-\bar{g}_{\mu\nu}\left[\frac{1}{2}\partial_\lambda\sigma\partial_\rho\sigma\left(\bar{g}^{\lambda\rho}\left(\Upsilon^2+\kappa_s(æ^\alpha_\beta)+a(\bar{R})\right)+b(\bar{R})\bar{R}^{\lambda\rho}\right)\right]\\
&+\partial_\lambda\sigma\partial_\rho\sigma\left[\bar{g}^{\lambda\rho}\zeta_{\mu\alpha}\frac{\partial\kappa_s}{\partial æ^\nu_\alpha}+\bar{R}_{\mu\nu}\left(\bar{g}^{\lambda\rho}\frac{\partial a}{\partial\bar{R}}+\bar{R}^{\lambda\rho}\frac{\partial b}{\partial\bar{R}}\right)\right]+b\left(\partial_\alpha\sigma\partial_\mu\sigma\bar{g}^{\alpha\rho}\bar{R}_{\rho\nu}+\partial_\nu\sigma\partial_\alpha\sigma\bar{g}^{\alpha\rho}\bar{R}_{\rho\mu}\right)\\
&+\frac{1}{2}\nabla_\rho\nabla_\alpha\left(\partial_\gamma\sigma\partial_\beta\sigma b\right)\bar{g}^{\alpha\beta}\bar{g}^{\rho\gamma}\bar{g}_{\mu\nu}+\frac{1}{2}\nabla_\rho\nabla_\alpha\left(\partial_\mu\sigma\partial_\nu\sigma b\right)\bar{g}^{\rho\alpha}-\nabla_\rho\nabla_\mu\left(\partial_\nu\sigma\partial_\alpha\sigma b\right)\bar{g}^{\rho\alpha}\\
&+\left[\bar{g}_{\mu\nu}\bar{g}^{\rho\lambda}\nabla_\rho\nabla_\lambda-\nabla_\nu\nabla_\mu\right]\left(\partial_\alpha\sigma\partial_\beta\sigma\left(\bar{g}^{\alpha\beta}\frac{\partial a}{\partial\bar{R}}+\bar{R}^{\alpha\beta}\frac{\partial b}{\partial\bar{R}}\right)\right)\Big\}+\frac{1}{2}\bar{g}_{\mu\nu}V_\sigma(\sigma)\end{aligned}$$

$$\begin{aligned}\frac{1}{\sqrt{-\bar{g}}}\frac{\delta\sqrt{-\bar{g}}\bar{\mathcal{L}}_{DE}}{\delta\bar{g}^{\mu\nu}} &= -\frac{\partial\bar{V}_æ}{\partial æ^\mu_\beta}\zeta_{\nu\beta}+\frac{1}{2}\bar{g}_{\mu\nu}\bar{V}_æ\\
\frac{1}{\sqrt{-\bar{g}}}\frac{\delta\sqrt{-\bar{g}}\bar{\mathcal{L}}_{DM}}{\delta\bar{g}^{\mu\nu}} &= \frac{1}{\sqrt{-\bar{g}}}\frac{\delta\sqrt{-\bar{g}}\bar{\mathcal{L}}(\bar{g}_{\lambda\rho},\phi_{DM},\phi_{CM})}{\delta\bar{g}^{\mu\nu}}\\
&\quad-\partial_\lambda\sigma J_{DM\rho}-\frac{1}{2}\bar{g}_{\lambda\rho}(-\partial_\mu\sigma J^\mu_{DM})-\partial_\mu\sigma\frac{\delta J_{DM\nu}}{\delta\bar{g}^{\lambda\rho}}\bar{g}^{\mu\nu}\,.\end{aligned}$$

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Neglect matter and v_σ. Split the DE tensor field into its trace and the traceless part:

$$\bar{æ} \equiv \bar{æ}^\mu_\nu \delta^\nu{}\mu \,, \; \bar{\tilde{æ}}^\mu_\nu \equiv \bar{æ}^\mu_\nu - \frac{1}{4}\delta^\mu_\nu \bar{æ}$$

Consider the even quartic DE potential:

$$\bar{V}_{æ} = \frac{1}{2} M^2_{\rm Pl} \bar{v}_2 \left(\bar{\tilde{æ}}^\mu_\nu \bar{\tilde{æ}}^\nu_\mu + \bar{\beta}_4 \left((\frac{1}{4}\bar{æ})^2 - 1 \right)^2 \right) \,, \; \bar{v}_2 > 0 \;\; \bar{\beta}_4 > 0.$$

$\frac{d\bar{V}_{æ}}{d\bar{æ}} = 0$ at $\bar{æ} = 0$ (trivial unbroken phase, higher $\bar{V}_{æ}$) and $\bar{æ} = 4$ (broken flat phase, lower $\bar{V}_{æ}$). In flat background, choose proper coordinates $x^\alpha_* = \delta^\alpha_a Z^a_*$, so that $g_{\alpha\beta} = \bar{g}_{\alpha\beta} = \zeta_{\alpha\beta} = \eta_{\alpha\beta}$, $\bar{æ}^\alpha_\beta = \delta^\alpha_\beta, \sigma = 0$. Small perturbations around the flat background (indices are lowered with the background metric, $\partial_\alpha = \partial/\partial x^\alpha_*$, $\chi^\alpha = \delta^\alpha_a \chi^a$):

$$Z^a = Z^a_* + \chi^a \,, \; g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta} \,, \; \bar{g}_{\alpha\beta}(x_*) = \eta_{\alpha\beta} + \bar{h}_{\alpha\beta}(x_*) \,, \; \bar{æ}^\alpha_\beta(x_*) = \delta^\alpha_\beta - \bar{f}^\alpha_\beta(x_*) \,,$$

$$\bar{h}_{\alpha\beta} \equiv h_{\alpha\beta} + \bar{w}'/2(h - 2\partial\chi)\eta_{\alpha\beta} \,, \; \eta_{\alpha\gamma}\bar{f}^\gamma_\beta(x_*) = \bar{h}_{\alpha\beta}(x_*) - \partial_\alpha\chi_\beta(x_*) - \partial_\beta\chi_\alpha(x_*)$$

$\bar{w}' = -1/2 \Rightarrow \bar{f} = 0$.

In a vicinity of the flat background: $\bar{V}_{æ} = \frac{1}{2} M^2_{\rm Pl} \bar{v}_2 \bar{f}^\alpha_\beta \bar{f}^\beta_\alpha + \frac{1}{4}(\bar{\beta}_4 - 1)\bar{f}^2 + \ldots$ $\rightarrow$ ...

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... →
$$\bar{T}_{æ\alpha\beta} = 2M_{\rm Pl}^2 \bar{v}_2 \bar{f}_{\alpha\beta}$$

In unitary gauge $\chi^\alpha = 0$, $\bar{f}_{\alpha\beta} = \bar{h}_{\alpha\beta}$ ⇒ substitute the tensor graviton field $\bar{h}_{\alpha\beta}$ by the DE field $\bar{f}_{\alpha\beta}$ in $\bar{R}_{\alpha\beta}$. In linear approximation (LA):

$$\bar{R}_{\alpha\beta} = \frac{1}{2}\left(\partial_\alpha\partial_\gamma \bar{f}^\gamma_\beta + \partial_\beta\partial_\gamma \bar{f}^\gamma_\alpha - \partial^2 \bar{f}_{\alpha\beta}\right), \ \bar{R} = \partial_\gamma\partial_\delta \bar{f}^{\gamma\delta} \tag{9}$$

Eq. (7) in LA:

$$\partial_\alpha\partial_\gamma \bar{f}^\gamma_\beta + \partial_\beta\partial_\gamma \bar{f}^\gamma_\alpha - \partial^2 \bar{f}_{\alpha\beta} - \frac{1}{2}\partial_\gamma\partial_\delta \bar{f}^{\gamma\delta}\eta_{\alpha\beta} = 4\bar{v}_2 \bar{f}_{\alpha\beta}\,. \tag{10}$$

From Eqs. (**??**), and the multiscalar FE (2):

$$\partial_\alpha\left(4\bar{v}_2 \bar{f}^\alpha_\beta - \frac{1}{2}\partial_\gamma\partial_\delta \bar{f}^{\gamma\delta}\delta^\alpha_\beta\right) = 0$$ ← actually follows from (10)

Require a covariant conservation of the omitted non-DE $\bar{T}_{\alpha\beta}$ terms in the r.h.s. of (10) ⇒ from the reduced Bianchi identity $\nabla^\alpha\left(\bar{R}_{\alpha\beta} - 1/2\bar{g}_{\alpha\beta}\bar{R}\right) = 0$ one has in LA $\partial_\alpha \bar{f}^{\alpha\beta} = 0$. ⇒ tensor FE (10) reduces to

$$\left(\partial^2 + m_g^2\right)\bar{f}_{\alpha\beta} = 0\,,\ m_g^2 = 4\bar{v}_2 \tag{11}$$

⇒ Due to $\partial_\alpha \bar{f}^{\alpha\beta} = 0$ and $\bar{f} = 0$ Eq. (11) describes exactly 5 independent degrees of freedom without ghosts.