Non-singular cosmologies in Horndeski theories and their stability

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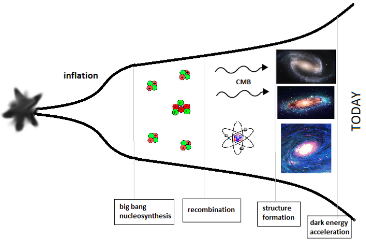
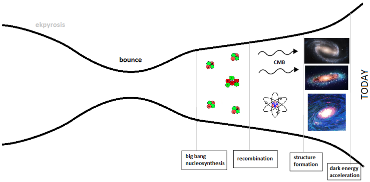
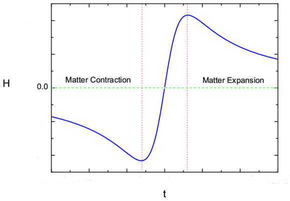
Motivation



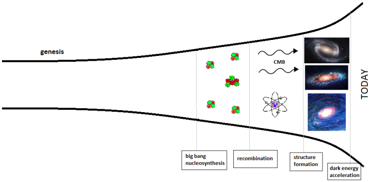
- Standard Big Bang cosmology has its characteristic set of problems at early times including an initial singularity problem.
- Inflation can successfully solve the majority of issues, but it **is geodesically incomplete in the past**.
- Alternative scenarios propose non-singular solutions:
  - Bouncing models (contraction → bounce → expansion)
  - Genesis models (expansion from Minkowski space)

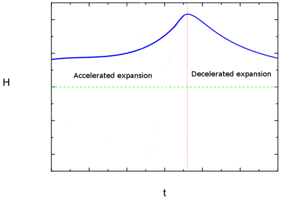
- Both scenarios require the Hubble parameter H to grow:
  - a Universe with a bounce ($\dot{H} > 0$ during the bouncing stage)





  - a Universe starting off with Genesis ($\dot{H} > 0$ at the onset of expansion)





Non-singular cosmologies and NEC violation

- Both scenarios require **violation of the Null Energy Condition** (NEC):

$$T_{\mu\nu}k^{\mu}k^{\nu} > 0 \quad (g_{\mu\nu}k^{\mu}k^{\nu} = 0)$$

NEC for a homogeneous stationary fluid: $\quad p + \rho > 0$

- NEC ensures that the Hubble parameter never grows

$$\dot{H} = -4\pi G(p+\rho) + \frac{\kappa}{a^2} < 0$$

and the energy density in standard cosmologies always decreases

$$\frac{d\rho}{dt} = -3H(\rho + p) < 0.$$

- NEC is satisfied in GR + conventional matter
- Ways to violate NEC:
  - add exotic matter
  - modify GR $\longrightarrow$ scalar-tensor theories like Horndeski theories

Horndeski theory

Horndeski (1974), Nicolis, Rattazzi, Trincherini (2009), Deffayet, Gao, Steer, Zahariade (2011)

$$S = \int \mathrm{d}^4 x \sqrt{-g}\,(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5)\,,$$

$$\mathcal{L}_2 = F(\pi, X),$$

$$\mathcal{L}_3 = K(\pi, X)\Box\pi,$$

$$\mathcal{L}_4 = -G_4(\pi, X)R + 2G_{4X}(\pi, X)\left[(\Box\pi)^2 - \pi_{;\mu\nu}\pi^{;\mu\nu}\right],$$

$$\mathcal{L}_5 = G_5(\pi, X)G^{\mu\nu}\pi_{;\mu\nu} + \frac{1}{3}G_{5X}\left[(\Box\pi)^3 - 3\Box\pi\pi_{;\mu\nu}\pi^{;\mu\nu} + 2\pi_{;\mu\nu}\pi^{;\mu\rho}\pi_{;\rho}{}^{\nu}\right].$$

π is a scalar field, $X = \pi_{,\mu}\pi^{,\mu}$, $\pi_{,\mu} = \partial_\mu\pi$, $\pi_{;\mu\nu} = \nabla_\nu\nabla_\mu\pi$, $\Box\pi = g^{\mu\nu}\nabla_\nu\nabla_\mu\pi$, $G_{iX} = \partial G_i/\partial X$.

- Equations of motion are 2nd order (2+1 DOFs and no Ostrograsky ghost)
- Generality: any STT with 2nd order EOMs, belong to the Horndeski group.
- Special cases: Brans-Dicke theory, $f(R)$-gravity, k-essence, kinetic gravity braiding, Fab Four, any inflation on a scalar and many more...
- Healthy NEC violation $\longrightarrow$ suitable framework for non-singular cosmologies

Stability issues

- Another subtlety with non-singular cosmologies: stability at the perturbative level.

$$ds^2 = (1+2\alpha)dt^2 - \partial_i\beta\, dtdx^i - a^2(1+2\zeta\delta_{ij} + h_{ij}^T)dx^i dx^j$$

Quadratic action for tensor h_{ij}^T and scalar ζ DOFs

$$S^{(2)}_{h+\zeta} = \int \mathrm{d}t\mathrm{d}^3x\; a^3\left[\frac{\mathcal{G}_\mathcal{T}}{8}\left(\dot{h}_{ij}^T\right)^2 - \frac{\mathcal{F}_\mathcal{T}}{8a^2}\left(\partial_k h_{ij}^T\right)^2 + \mathcal{G}_\mathcal{S}\dot{\zeta}^2 - \mathcal{F}_\mathcal{S}\frac{(\nabla\zeta)^2}{a^2}\right]$$

$$\mathcal{G}_\mathcal{S} = \frac{\Sigma\mathcal{G}_\mathcal{T}^2}{\Theta^2} + 3\mathcal{G}_\mathcal{T}, \qquad \mathcal{F}_\mathcal{S} = \frac{1}{a}\frac{\mathrm{d}\xi}{\mathrm{d}t} - \mathcal{F}_\mathcal{T}, \qquad \xi = \frac{a\mathcal{G}_\mathcal{T}^2}{\Theta}$$

Stability and (sub)luminality conditions

$$\mathcal{G}_\mathcal{T}, \mathcal{F}_\mathcal{T} > \epsilon > 0, \quad \mathcal{G}_\mathcal{S}, \mathcal{F}_\mathcal{S} > \epsilon > 0, \quad \mathcal{F}_\mathcal{T} \leq \mathcal{G}_\mathcal{T}, \quad \mathcal{F}_\mathcal{S} \leq \mathcal{G}_\mathcal{S}$$

- ***Complete*** stability for $\forall t$? $\longrightarrow$ *No-go theorem*

No-go theorem

Libanov, Mironov, Rubakov (2016)

Kobayshi (2016)

No-go theorem: there are no cosmological solutions with $a > 0$ in Horndeski theory which are free from gradient instabilities for $t \in (-\infty, +\infty)$.

$$\mathcal{F}_\mathcal{S} = \frac{1}{a}\frac{\mathrm{d}\xi}{\mathrm{d}t} - \mathcal{F}_\mathcal{T} \qquad \longrightarrow \qquad \xi(t_2) - \xi(t_1) = \int_{t_1}^{t_2} a(t)\left(\mathcal{F}_\mathcal{T} + \mathcal{F}_\mathcal{S}\right)\mathrm{d}t$$

- To avoid gradient instabilities, there must be a moment t_0 when $\xi(t)$ changes sign, i.e., $\xi(t_0) = 0$.
- With $a \neq 0$ making $\xi = \dfrac{a{\mathcal{G}_\mathcal{T}}^2}{\Theta}$ cross zero requires either
  - $\mathcal{G}_\mathcal{T} \to 0$, which corresponds to a strong coupling regime, or
  - $\Theta \to \infty$, which corresponds to a singularity in the Lagrangian

Hence, ξ cannot cross zero in a healthy way $\longrightarrow$ *non-singular cosmologies in Horndeski theory are always plagued with gradient instabilities .*

Ways to Evade the No-Go Theorem

Basic strategy: *resolve the contradiction between stability and healthy $\xi(t_0) = 0$.*

- Protect ξ from crossing zero with rapid decay of $\mathcal{G}_{\mathcal{S},\mathcal{T}}, \mathcal{F}_{\mathcal{S},\mathcal{T}} \to 0$ in

$$\xi(t_2) - \xi(t_1) = \int_{t_1}^{t_2} a(t)\,(\mathcal{F}_\mathcal{T} + \mathcal{F}_\mathcal{S})\,\mathrm{d}t \quad \to \quad \text{converges}$$

  Solution: stability does not require ξ to cross zero.
  Note: Strong coupling problem? Not necessarily .

  Ageeva, Evseev, Melichev, Rubakov (2018), Ageeva, Petrov, Rubakov (2021)

- Make $\xi = \dfrac{a{\mathcal{G}_\mathcal{T}}^2}{\Theta}$ cross zero by adjusting Θ:
  - Vanishing $\mathcal{G}_\mathcal{T}(t_*) = \Theta(t_*) = 0$: fine-tuning and strong coupling at t_*
  - $\Theta \equiv 0$ for $\forall t \to$ constraints on the Lagrangian $\to$ 2 DOFs like in GR

  Mironov, Shtennikova (2023)

- Go beyond Horndeski: $\xi = \dfrac{a{\mathcal{G}_\mathcal{T}}^2}{\Theta} \quad \to \quad \xi = \dfrac{a\mathcal{G}_\mathcal{T}\,(\mathcal{G}_\mathcal{T} + \mathcal{D}\dot{\pi})}{\Theta}$

  (for a review see e.g. 2409.16108)

- Cuscuton theory: $\mathcal{G}_\mathcal{S} \equiv 0 \to$ non-dynamical scalar mode

Cuscuton theory

- Cuscuton theory: an example of "minimally modified gravity" theories (involves only 2 DOFs)
- Cuscuton was formulated as a subclass of k-essence:

$$S = S_{\mathrm{EH}} + \int d^4x \sqrt{-g}\, P(X, \pi), \qquad \left(X \equiv -\tfrac{1}{2}\partial_\mu \pi\, \partial^\mu \pi\right)$$

$$\delta_\pi S = 0 \quad \xrightarrow{\text{FRLW}} \quad (P_{,X} + 2XP_{,XX})\ddot{\pi} + 3HP_{,X}\dot{\pi} + P_{,X\pi}\dot{\pi}^2 - P_{,\pi} = 0$$

Afshordi et al. (2007)

- By definition: cuscuton is scalar field π that is
  (1) non-dynamical at the background level

$$P_{,X} + 2XP_{,XX} = 0 \qquad \longrightarrow \qquad P(X,\pi) = c_1(\pi)\sqrt{|X|} + c_2(\pi)$$

  (2) has no propagating scalar DOF at the pertubation level

$$S^{(2)}_\zeta = \int \mathrm{d}t\mathrm{d}^3x\, a^3 \left[\mathcal{G}_\mathcal{S}\dot{\zeta}^2 - \mathcal{F}_\mathcal{S}\frac{(\nabla\zeta)^2}{a^2}\right]$$

$$\mathcal{G}_\mathcal{S} = \frac{X}{H^2}(P_{,X} + 2XP_{,XX}), \qquad \mathcal{F}_\mathcal{S} = -\frac{\dot{H}}{H^2}M_{Pl}^2$$

Cuscuton theory and non-singular cosmologies

- Original cuscuton: $P(X,\pi) = \pm\mu^2\sqrt{2X} - V(\pi)$

$$\boxed{\pm\text{sign}(\dot\pi)\cdot 3H\mu^2 = -V_\pi}$$

- Accompanying Einstein eqs:

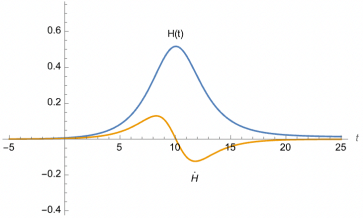
$$H^2 = \frac{8\pi}{3M_{Pl}^2}V(\pi), \qquad \dot H = \mp\frac{4\pi}{M_{Pl}^2}(\mu^2\sqrt{2X})$$

- NEC-violating phase with $\dot H > 0$ corresponds to "$-$" sign in $P(X,\pi)$
- Let us construct a complete Genesis solution:
  - Original Genesis: $H \sim \frac{1}{(-t)^3}, \qquad a \sim 1 + \frac{1}{(-t)^2}$ as $t \to -\infty$

    Creminelli et al. (2010)
  - Kination at late times: $H \sim \frac{1}{3t}$, as $t \to +\infty$

Genesis driven by the cuscuton field: a toy-model

$$H^2 = \frac{8\pi}{3M_{Pl}^2} V(\pi)$$
$$\dot{H} = \frac{4\pi}{M_{Pl}^2}(\mu^2\sqrt{2X})$$
$$3H\mu^2 = V_\pi$$



- Reconstruction technique: for a chosen $H(t)$ we find suitable $V(\pi)$
- Suitable Hubble parameter:

$$H(t) = \left[\left(4\,\frac{\Lambda^3}{f^3}\cdot\frac{t^2\,(1-\tanh(t/\tau))}{2\,(1+\alpha/3)} + 3\cdot\frac{1+\tanh(t/\tau)}{2}\right)\sqrt{2\tau^2+t^2}\right]^{-1}$$

$$H(t) \sim \frac{1}{(-t)^3} \quad (t\to-\infty), \qquad\qquad H(t) \sim \frac{1}{3t} \quad (t\to+\infty)$$

Mironov, Rubakov, VV (2019)

- $H(t)$ immediately defines $V(t)$ and $\pi(t)$
- The scenario is completelu stable by design (no other matter is present)

More realistic Genesis scenario and potential problems

The following steps towards a more realistic Genesis model:

- The late time kination can be driven by an additional massless scalar
- Additional matter components source scalar perturbations
  - Possible issues with stability (invoking ghosts or gradient instabilities)
  - Superluminal propagation of the scalar perturbation mode (kinetic mixing effect)
- Power spectrum of scalar perturbations that agrees with observational data (+ non-Gaussianities analysis)
- Extended cuscuton theory may provide a more natural and rich framework

Conclusion

- Scalar-tensor theories with modified gravity provide a vast playground for various non-singular cosmologies
- Possibly the main issue with non-singular cosmologies is to ensure their stability at the linearized level
- Minimally modified gravity theories seem to be specifically suitable for realistic, completely stable non-singular cosmologies

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- Minimally modified gravity theories seem to be specifically suitable for realistic, completely stable non-singular cosmologies

Thank you for your attention!