Luminality and non-decay of Gravitational Waves in Modified Gravity

S. Mironov, A. Shtennikova, **M. Valencia-Villegas**

Institute for Theoretical and Mathematical Physics, MSU,

Institute for Nuclear Research of the Russian Academy of Sciences

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Outline

1. Why modified gravity?

2. Theoretical options after GW170817:

   - Historically standard approach: Constrain the Scalar couplings
   - In this talk: "**Double down**"

     add Scalar—Photon couplings

3. Gravitational Wave decay constraints:

   A novel theory to bypass them.

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1. Introduction:

Why "Modified Gravity"?

Our reference theory, Einstein's gravity (GR)

- generally suffers singularities, if the Null Energy Condition (NEC) holds (Penrose, Hawking theorems),
- ΛCDM (Cosmological Constant Problem, Weinberg (1989)),
- challenges ahead?
  (DESI BAO data, *e.g.* A.G. Adame *et al.*, DESI 2024 VI. JCAP 02 (2025))

$\vdots$

1. A paradigm: Horndeski theory

Scalar modification of GR that

- keeps second order equations (No Ghosts),
- does not satisfy the NEC in general,

Unique answer: **Horndeski theory** (1974)

Extensively used for

- early and late time cosmology
- compact objects and other modified gravity solutions

There are generalizations: Beyond Horndeski (Gleyzes-Langlois-Piazza-Vernizzi theory) and DHOST (Ben Achour, Crisostomi, Koyama, Langlois, Noui Tasinato (2016) JHEP 12, 100 [arXiv:1608.08135])

1. What is Horndeski theory?

- Take a real scalar field π besides the metric field,
- On top of GR take 4 general Scalar Potentials $G_i(\pi, X)$, $i = 2, 3, 4, 5$, where $X = \nabla_\mu \pi \nabla^\mu \pi$,
- Most general combinations of R and $(\nabla^2\pi)^p$ with $p \leq 3$ and G_i coefficients, with **second order equations of motion**.

$$\mathcal{L}_\pi = \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5,$$
$$\mathcal{L}_2 = G_2(\pi, X),$$
$$\mathcal{L}_3 = G_3(\pi, X)\Box\pi,$$
$$\mathcal{L}_4 = G_4(\pi, X)R - 2G_{4X}(\pi, X)\left[(\Box\pi)^2 - \pi_{;mn}\pi^{;mn}\right],$$
$$\mathcal{L}_5 = G_5(\pi, X)G^{mn}\pi_{;mn} + \frac{G_{5X}}{3}\left[(\Box\pi)^3 - 3\Box\pi\pi_{;mn}\pi^{;mn} + 2\pi_{;mn}\pi^{;ml}\pi_{;l}{}^{n}\right],$$

1. Review: The Speed of Perturbations

- Take the Perturbed metric

$$\mathrm{d}s^2 = (\eta_{\mu\nu} + \delta g_{\mu\nu})\mathrm{d}x^\mu \mathrm{d}x^\nu$$

with FLRW background

$$\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\mathrm{d}t^2 + a^2\,\delta_{ij}\mathrm{d}x^i \mathrm{d}x^j$$

and metric perturbation,

$$\delta g = a^2\,\left(2\,S_i \mathrm{d}t\,\mathrm{d}x^i + (\partial_i F_j + \partial_j F_i + 2\,h_{ij})\,\mathrm{d}x^i\,\mathrm{d}x^j\right)\,,$$

- The Photon is the perturbation $A_i(\eta, \vec{x})$
- Scalar sector remains untouched

1. Review: The Speed of Perturbations

- Quadratic action for Graviton,

$$\mathcal{S}_{Graviton} = \frac{1}{2}\int \mathrm{d}t\, \mathrm{d}^3x\, a^3 \left(\mathcal{G}_\tau\, \dot{h}_{ij}^2 - \frac{\mathcal{F}_\tau}{a^2}\,(\partial_k h_{ij})^2\right)\,, \qquad (2)$$

  thus, the Speed of Gravitational Waves is,

$$c_g^2 = \frac{\mathcal{F}_\tau}{\mathcal{G}_\tau}\,,$$

- Quadratic action for the Photon,

$$\mathcal{S}_{Photon} = \frac{1}{4}\int \mathrm{d}t\, \mathrm{d}^3x\, a \left(\mathcal{G}_A\, \dot{A}_i^2 - \frac{\mathcal{F}_A}{a^2}\,(\partial_k A_i)^2\right)\,, \qquad (3)$$

  thus, the Speed of the Photon is,

$$c^2 = \frac{\mathcal{F}_A}{\mathcal{G}_A}\,.$$

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2. (nearly) Luminal GWs: observation vs. theory

- **The event** GW170817 and the electromagnetic counterpart GRB170817A set the speed of GWs (c_g) very close to the speed of light c

$$\left|\frac{c_g}{c} - 1\right| \leq 5 \times 10^{-16}\,, \tag{4}$$

in this talk, I *assume* exactly Luminal GWs, $c_g \equiv c$

- Now, what's with **the theory**? in Horndeski theory on a cosmological FLRW background, the speed of GWs is

$$c_g^2 = \frac{2G_4 - 2\ddot{\pi}XG_{5,X} + XG_{5,\pi}}{2G_4 - 4XG_{4,X} - 2\dot{\pi}XHG_{5,X} - XG_{5,\pi}} \neq \mathbf{1} = \mathbf{c^2}\,, \tag{5}$$

2. Luminal GWs. Option (B)

(B) Historically standard approach: Constrain the Scalar couplings by

assuming minimal Photon such that $c_g = c = 1$.

- Consequence of (B): necessarily $G_{4X} = G_5 \equiv 0$

$$c_g^2 = \frac{2G_4 - 2\ddot{\pi}XG_{5,X} + XG_{5,\pi}}{2G_4 - 4XG_{4,X} - 2\dot{\pi}XHG_{5,X} - XG_{5,\pi}} = \mathbf{1} = \mathbf{c^2}\,, \qquad (6)$$

  - We are left with Brans-Dicke type theory $G_4 = G_4(\pi)$.
  - We lose the Fab-Four, in the context of Cosmological constant problem (Charmousis et. al., 2011)

Is there another option?

Yes!

Luminal GWs. Option (A)

- (A) Supplement Horndeski theory with the Scalar—Photon couplings that satisfy

$$\frac{c_g(t)}{c(t)} = 1\,. \tag{7}$$

Note that here we will not assume $c(t) \equiv 1$.

Basic idea is: *Gravity couples universally to all matter.*

What if

the *scalar modification of gravity* at cosmological scales also shares this *universal coupling* property? In particular, DE Scalar — Photon

How to obtain
Dark Energy—Photon couplings
such that we see

$$\frac{c_g(t)}{c(t)} = 1 \ ?$$

KK is a possibility

(A) Horndeski Theory + DE —Photon couplings

- Our proposal reads

$$\mathcal{L} = \text{Horndeski Theory} + \mathcal{L}_{4A} + \mathcal{L}_{5A} \tag{8}$$

where the **Dark Energy—Photon couplings** are

$$\mathcal{L}_{4A} = -\frac{1}{4} G_4\, F^2 + G_{4,X}\, F_\mu{}^\sigma\, F_{\nu\sigma}\, \pi^{;\mu} \pi^{;\nu} \tag{9}$$

$$\mathcal{L}_{5A} \quad = \quad G_5(\pi) \left(\frac{1}{8} F^{\mu\nu} F_\mu{}^\rho (-4\, \pi_{;\nu\rho} + g_{\nu\rho} \Box \pi) + \frac{1}{2} F_{\mu\nu} \nabla_\sigma F^{\nu\sigma}\, \pi^{;\mu} \right)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

S. Mironov, A. Shtennikova and **M. V-V** (2024), Phys.Lett.B 858, 139058 [arXiv:2405.02281]

S. Mironov, A. Shtennikova and **M. V-V** (2025), Phys. Rev. D 111, L101501 [arXiv:2412.13460]

Partial Conclusion:

- A different viewpoint: in this talk the speed tests suggest the way Dark Energy couples to the Photon such that $c_g(t)/c(t) = 1$, which is what was actually measured in GW170817 and GRB170817,
- We recover non-minimal couplings (Fab Four),
- We are brought to a $U(1)$ vector-scalar Galileon.
- How does the theory look? $\cdots$

Partial Conclusion:

- Add $\mathcal{L}_{4A}$ and $\mathcal{L}_{5A}$ to the Horndeski Lagrangian

$$\mathcal{L}_{=}\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 + \mathcal{L}_{4A} + \mathcal{L}_{5A},$$
$$\mathcal{L}_2 = G_2(\pi, X),$$
$$\mathcal{L}_3 = G_3(\pi, X)\Box\pi,$$
$$\mathcal{L}_4 = G_4(\pi, X)R - 2G_{4X}(\pi, X)\left[(\Box\pi)^2 - \pi_{;mn}\pi^{;mn}\right],$$
$$\mathcal{L}_5 = G_5(\pi)\, G^{mn}\pi_{;mn}\ ,$$
$$\mathcal{L}_{4A} = -\frac{1}{4}G_4(\pi, X)\, F^2 + G_{4,X}\, F_\mu{}^\sigma F_{\nu\sigma}\nabla^\mu\pi\nabla^\nu\pi$$
$$\mathcal{L}_{5A} = G_5\ \left(\frac{1}{8}F^{\mu\nu}F_\mu{}^\rho(-4\nabla_\nu\nabla_\rho\pi + g_{\nu\rho}\Box\pi) + \frac{1}{2}F_{\mu\nu}\nabla_\sigma F^{\nu\sigma}\nabla^\mu\pi\right)$$

with this theory, **automatically**

$$\frac{c_g(t)}{c(t)} = 1$$

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3. GWs $\rightarrow$ DE?

- In general, with G_{4X}, $G_5 \neq 0$,

  GWs $\rightarrow$ DE decay channels **hss** and **hhs**

- Problem holds in Beyond Horndeski theory $\mathcal{L}_{BH}$ (GLPV) and in any disformally related theory

$$\mathcal{L}_{BH} = \text{Horndeski Theory} + \mathcal{L}_{F4} \tag{11}$$

$$\mathcal{L}_{F4} = F_4(\pi, X)\, \epsilon^{\mu\nu\rho\sigma}\, \epsilon^{\mu'\nu'\rho'}{}_{\sigma}\, \pi_{;\nu\nu'}\, \pi_{;\rho\rho'}\, \pi_{;\mu}\, \pi_{;\mu'}$$

- Usual conclusion without DE — Photon couplings:
  Need! $F_4 = G_5 = G_{4X} = 0$

  Creminelli, Lewandowski, Tambalo Vernizzi (2018) JCAP 12, 025 [arXiv:1809.03484]

- Conclusion changes with DE — Photon couplings!

3. DE — Photon couplings: GWs $\nrightarrow$ DE

- GW decay is suppressed if

$$F_4\Big(4G_4 + X(2G_{4,X} + 3G_{5,\pi})\Big) + X\, F_{4,X}\Big(2G_4 + XG_{5,\pi}\Big) = 0$$

$$+4G_{4,X}^2 + 4G_4G_{4,XX} + G_{5,\pi}\Big(4G_{4,X} + 2XG_{4,XX} + G_{5,\pi}\Big) \tag{12}$$

Creminelli, Lewandowski, Tambalo Vernizzi (2018) JCAP 12, 025 [arXiv:1809.03484]

- Without DE — Photon couplings $F_4 = \frac{-2G_{4,X}}{X}$ and $G_5 = 0$ by $c_g = c = 1$.

- With DE — Photon couplings F_4 is free: use this Freedom and satisfy Eqn. (12)!!

$$F_4(\pi, X) = \frac{1}{2X^2}\Big(2G_4 - X(4G_{4,X} + G_{5,\pi}) + \frac{4J_4(\pi)}{2G_4 + XG_{5,\pi}}\Big), \tag{13}$$

3. SUMMARY

$$\mathcal{L} = \mathcal{L}_{\text{Horndeski}} + \mathcal{L}_{\text{Beyond Horndeski}}(F_4(\pi, X)) + \mathcal{L}_{\text{BH}_{4A}} + \mathcal{L}_{5A}\,, \qquad (14)$$

$$\mathcal{L}_{\text{BH}_{4A}} = -\frac{G_4}{4}\,F^2 + \frac{2G_{4,X} + X\,F_4(\pi, X)}{2}(F_{\mu\nu}\pi^{\mu})^2\,,$$

$$\mathcal{L}_{5A} = G_5(\pi)\,\left(\frac{1}{8}F^{\mu\nu}F_{\mu}{}^{\rho}(-4\nabla_\nu\nabla_\rho\pi + g_{\nu\rho}\Box\pi) + \frac{1}{2}F_{\mu\nu}\nabla_\sigma F^{\nu\sigma}\nabla^{\mu}\pi\right)$$

Automatically luminal and with suppressed GW decay (Notice, this theory is NOT disformally related to Maxwell electrodynamics).

We are left with 3 free potentials of π, X (namely, G_2, G_3, $G_4(\pi, X)$) and 2 free potentials of π (namely, G_5, $J_4(\pi)$).

Conclusions

- We obtained an extension of Beyond Horndeski theory Eqn (14) that
  - automatically propagates luminal Gravitational waves, $\frac{c_g(t)}{c(t)} = 1$
  - GWs do not decay into Dark Energy,
  - has nonminimal couplings of the Scalar of Dark Energy to both the Graviton and the Photon
- This is the largest Beyond Horndeski theory (and DHOST) that enjoys this property. F_5 and other DHOST quad+qubic remain ruled out.
- orthogonal tests to Modifications of Gravity? Pandora's Box??

  Ferreira, Wolf, Read, (2025). The Spectre of Underdetermination in Modern Cosmology

2405.02281

and, 2412.13460

Thanks for your attention!

Support slides:

(A) Horndeski Theory + Dark Energy—Photon couplings

- One possibility: First, take 5D Horndeski theory, just as a tool,
- compactify with the Ansatz (16) and the Cylinder condition

$$ {}^{(5)}g_{B\,C} = \begin{pmatrix} g_{\mu\,\nu} + A_\mu\, A_\nu & A_\mu \\ A_\nu & 1 \end{pmatrix} , \tag{16} $$

- 4D fields A_μ and g are but components of ${}^{(5)}g$
    - **their speeds are *bound* to be the same ...**
    - ... But, caveat, we ignore the Dilaton $\longrightarrow$ compute speeds and check

Quadratic Action

- Quadratic action about FLRW, With h_{ij} Graviton and $A_i(t,\vec{x})$ Photon perturbation:

  $A_i(t,\vec{x})$ decouples

- Quadratic action for Graviton and Photon read,

$$\mathcal{S}_{Graviton} = \frac{1}{2}\int \mathrm{d}t\,\mathrm{d}^3x\,a^3\,\left(\mathcal{G}_\tau\,\dot{h}_{ij}^2 - \frac{\mathcal{F}_\tau}{a^2}\,(\partial_k h_{ij})^2\right)\,, \tag{17}$$

$$\mathcal{S}_{Photon} = \frac{1}{4}\int \mathrm{d}t\,\mathrm{d}^3x\,a\,\left(\mathcal{G}_A\,\dot{A}_i^2 - \frac{\mathcal{F}_A}{a^2}\,(\partial_k A_i)^2\right)\,, \tag{18}$$

- Simple relation $\mathcal{S}_{Graviton}$ to $\mathcal{S}_{Photon}$,

$$\mathcal{G}_A = \mathcal{G}_\tau + \Delta_l \tag{19}$$

$$\mathcal{F}_A = \mathcal{F}_\tau\,. \tag{20}$$

Quadratic Action

- Speed of GWs and Light, respectively, on the cosmological medium,

$$c_g^2 = \frac{\mathcal{F}_\tau}{\mathcal{G}_\tau}, \qquad c^2 = \frac{\mathcal{F}_A}{\mathcal{G}_A}$$

with $\mathcal{F}_A = \mathcal{F}_\tau, \qquad \mathcal{G}_A = \mathcal{G}_\tau + \Delta_l$

$$\frac{c_g^2}{c^2} = \frac{\mathcal{F}_\tau}{\mathcal{G}_\tau}\frac{\mathcal{G}_A}{\mathcal{F}_A} = \frac{\mathcal{G}_A}{\mathcal{G}_\tau} = 1 + \frac{\Delta_l}{\mathcal{G}_\tau}\,, \tag{21}$$

with $\Delta_l = -2\dot{\pi} X H G_{5,X}$

- We get Luminal GWs with $G_{5,X} = 0$:

nonminimal couplings $G_4(\pi, X)$, $G_5(\pi)$ preserve

$$\frac{c_g^2(t)}{c^2(t)} = 1\,. \tag{22}$$

With Dilaton

- Where does $G_{5X} = 0$ come from?

on FLRW

$$\frac{c_g^2}{c^2} = 1 - \frac{2\dot{\pi}\, X\, (H - l)\, G_{5,X}}{\mathcal{G}_\tau}\,, \tag{23}$$

$l = 0$ in our case

(Cross check: with Dilaton (ϕ), $l = \frac{\dot{\phi}}{\phi}$. 5D hom/ isotropy means $l = H$).

Disformally related? Not quite

- $\mathcal{L}_{4A}$ is disformally equivalent to Maxwell, (Babichev, Charmousis, Muntz, Padilla and Saltas (2024))

$$(T:\, g_{\mu\nu} \to \tilde{g}_{\mu\nu} = A(\pi, X)\, g_{\mu\nu} + B(\pi, X)\pi_{;\mu}\pi_{;\nu})$$

$$-\frac{1}{4}F^2 \xrightarrow{T} \mathcal{L}_{4A} = -\frac{1}{4}G_4\, F^2 + G_{4,X}\, F_\mu{}^\sigma F_{\nu\sigma}\, \pi^{;\mu}\pi^{;\nu}$$

But, notice that $\mathcal{L}_{5A}$ is NOT disformally equivalent to Maxwell,

$$-\frac{1}{4}F^2 \overset{T}{\nrightarrow} \mathcal{L}_{5A} = G_5\, \left(\frac{1}{8}F^{\mu\nu}F_\mu{}^\rho(-4\,\pi_{;\nu\rho} + g_{\nu\rho}\Box\pi) + \cdots\right)$$

this will be essential in what follows.

*S. Mironov, A. Shtennikova and **M. V-V** (2025), Phys. Rev. D 111, L101501 [arXiv:2412.13460]*