Cosmological Constant Suppression in Non-Stationary Scalar Covariant State

Asya Aynbund [1], Valery V. Kiselev [1,2]

aynbund.asya@phystech.edu, kiselev.vv@phystech.edu

[1] Moscow Institute of Physics and Technology (MIPT)
[2] Institute of High Energy Physics (IHEP- NRC KI)

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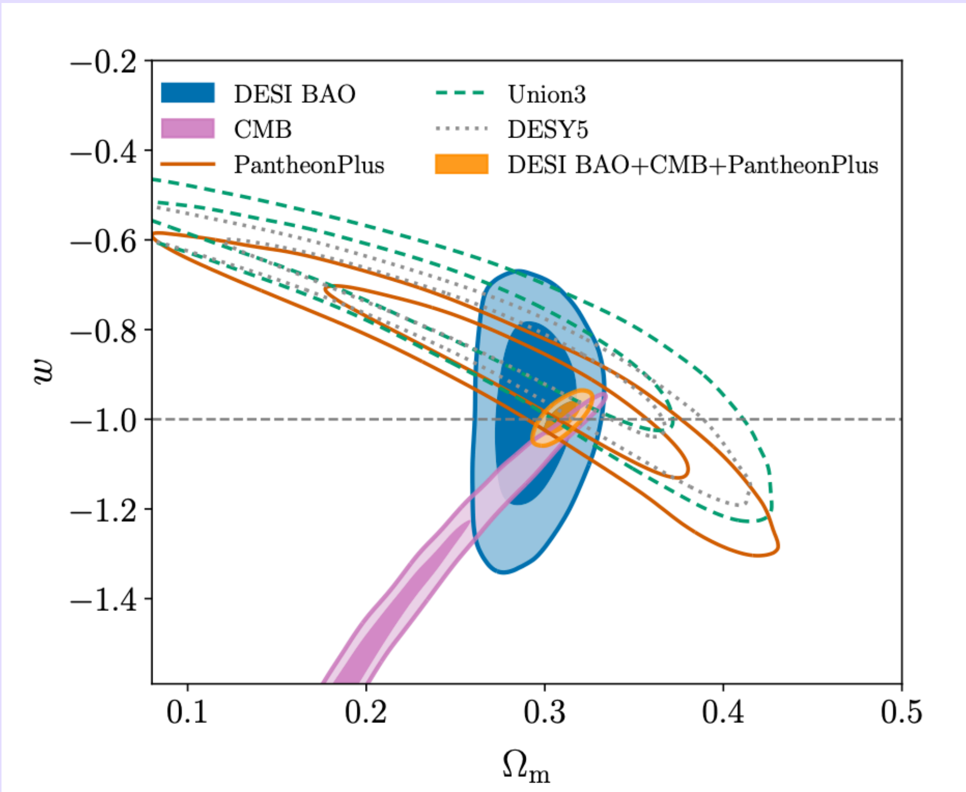
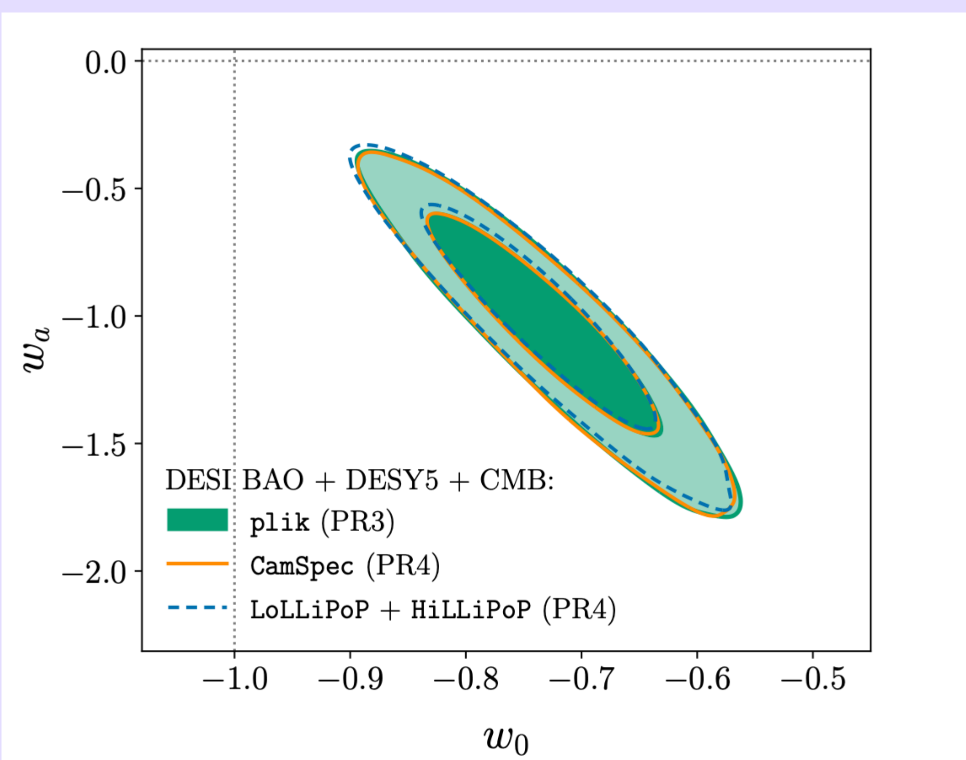
Introduction

- Cosmological constant problem: according to the data — 10^{-3} eV, based on estimates in theoretical models — 10^{18} GeV;
- No theoretical models with vacuum energy density suppression $\Rightarrow$ Zeldovich: the cosmological constant $\leftrightarrow$ density of vacuum energy generated by ZPM;
- 4D isotopic model in contrast to 3D isotropic





State parameter: $w = \dfrac{p}{\rho} \approx -1$

- $\rho \sim \Lambda^4$
- ΛCDM:

$$\Omega_m = 0.3095 \pm 0.0069, \quad w = -0.997 \pm 0.025$$

- $w_0 w_a$CDM: $w = w_0 + (1-a) w_a$

$$w_0 = -0.827 \pm 0.063, \quad w_a = -0.75^{+0.29}_{-0.25}$$

Scale $\Lambda_{DE} = 10^{-3}\text{eV}$

- Model assumptions for considering:
  - Nothing (or emptiness): classical limit

$$T^{\text{emp.}}_{\mu\nu}\,\big|_{\text{cl}} = 0 \quad \leftrightarrow \quad T^{\text{cl.}}_{\mu\nu} = \text{tr}\left(\hat{\rho}\hat{T}_{\mu\nu}\right)$$

  - Crucial: quantum fluctuations
  - Non-stationary universe $\Rightarrow$ no pure vacuum
  - Possible invariant vacuum:

$$\text{tr}\left(\hat{\rho}\hat{T}_{\mu\nu}\right)_{\text{inv.}} = W_{\text{vac}} \cdot \langle \text{vac} \,|\, \hat{T}_{\mu\nu} \,|\, \text{vac} \rangle + \text{tr}_{\text{non-vac}}\left(\hat{\rho}\hat{T}_{\mu\nu}\right) = W_{\text{vac}} \cdot \rho^{\text{bare}}_{\text{vac}} g_{\mu\nu} + T_{\mu\nu}\left[\Phi_{\text{cl.}}\right]$$

$$\Lambda \sim 10^{16}\text{GeV}, \rho^{\text{bare}}_{\text{vac}} \sim \Lambda^4 \qquad W_{\text{vac}} \ll 1$$

$$n_{\text{eff}} \sim \frac{\tilde{m}_{\text{Pl}}}{\Lambda} \quad W_{\text{vac}} = \text{e}^{-n_{\text{eff}}} \quad n_{\text{eff}} \sim 250$$

- Two fundamental scales in inflation: Planck mass $\tilde{m}_{\text{Pl}}$ and inflationary plateau height V_C

Dark energy and n_{eff}

- Non-separable $|\text{vac}\rangle \Rightarrow$ non-invariant $T_{\mu\nu}$:

$$\rho_{\text{DE}} = W_{\text{DE}} \cdot \langle \text{vac} \,|\, \hat{T}_{00} \,|\, \text{vac} \rangle \cdot g^{00} = W_{\text{DE}} \cdot \rho_{\text{vac}}^{\text{bare}}, \quad p_{\text{DE}} = w_{\text{DE}} \cdot \rho_{\text{DE}}$$

- Using the conservation law in the expanding Universe:

$$\dot{\rho}_{\text{DE}} + 3H\left(\rho_{\text{DE}} + p_{\text{DE}}\right) = 0 \Rightarrow \dot{W}_{\text{DE}} \cdot \rho_{\text{vac}}^{\text{bare}} + 3H\left(1 + w_{\text{DE}}\right) W_{\text{DE}} \cdot \rho_{\text{vac}}^{\text{bare}} = 0$$

- Under $H \equiv \dot{a}/a$ and introducing the e-folding in terms of scale factor $a(t)$ — $N \equiv \ln a(t)$:

$$w_{\text{DE}} = -1 - \frac{1}{3}\left(\ln W_{\text{DE}}\right)'$$

- In vicinity of $N = 0$, the effective number of quanta

$$n_{\text{eff}}(N) \approx n_{\text{eff}} + n' \cdot N + \frac{1}{2} n'' \cdot N^2$$

- While

$$W_{\text{DE}} = \mathrm{e}^{-n_{\text{eff}}(N)} \Rightarrow w_{\text{DE}} \approx -1 + \frac{1}{3}\left(n' - n'' \cdot (1 - a)\right)$$

- Empirical data correspond to approximate constraints

$$0.3 < n' < 0.75, \quad 1.7 < n'' < 3$$

Action with cut-off

- Action of real scalar field $\phi(x)$:

$$S = \int \mathrm{d}^4 x \frac{1}{2} \left(\phi(x) \left(-\partial_\mu \partial^\mu \phi(x) \right) - m^2 \phi^2(x) \right)$$

- Applying:
  - ✓ Fourier back-and-forth
  - ✓ Wick rotation $p_0 = \mathrm{i}p_4$ with $p^2 = -p_E^2$ and back: $\mathrm{i}p_E = k$
  - ✓ 4D isotropic solutions with a cut-off alike Λ
- we arrive to:

$$S_{4\mathrm{D}} = \frac{1}{4\pi} \frac{\left\langle p_{\mathrm{E}}^3 \right\rangle_\Lambda}{\Lambda^6} \int \mathrm{d}\tau \frac{1}{2} \phi(\tau) \left(-\frac{\mathrm{d}^2}{\mathrm{d}\tau^2} - m^2 \right) \phi(\tau)$$

- New Cut-off Scale Λ: $\frac{1}{4\pi} \frac{\left\langle p_{\mathrm{E}}^3 \right\rangle_\Lambda}{\Lambda^6} \mapsto \frac{1}{\Lambda^3}$
- 4D Isotropic Cut-off Implementation:

$$S_{4\mathrm{D}} = \frac{1}{\Lambda^3} \int \mathrm{d}\tau \frac{1}{2} \left(\dot{\phi}^2 - m^2 \phi^2 \right)$$

Non-stationary coherent state: interaction with gravity

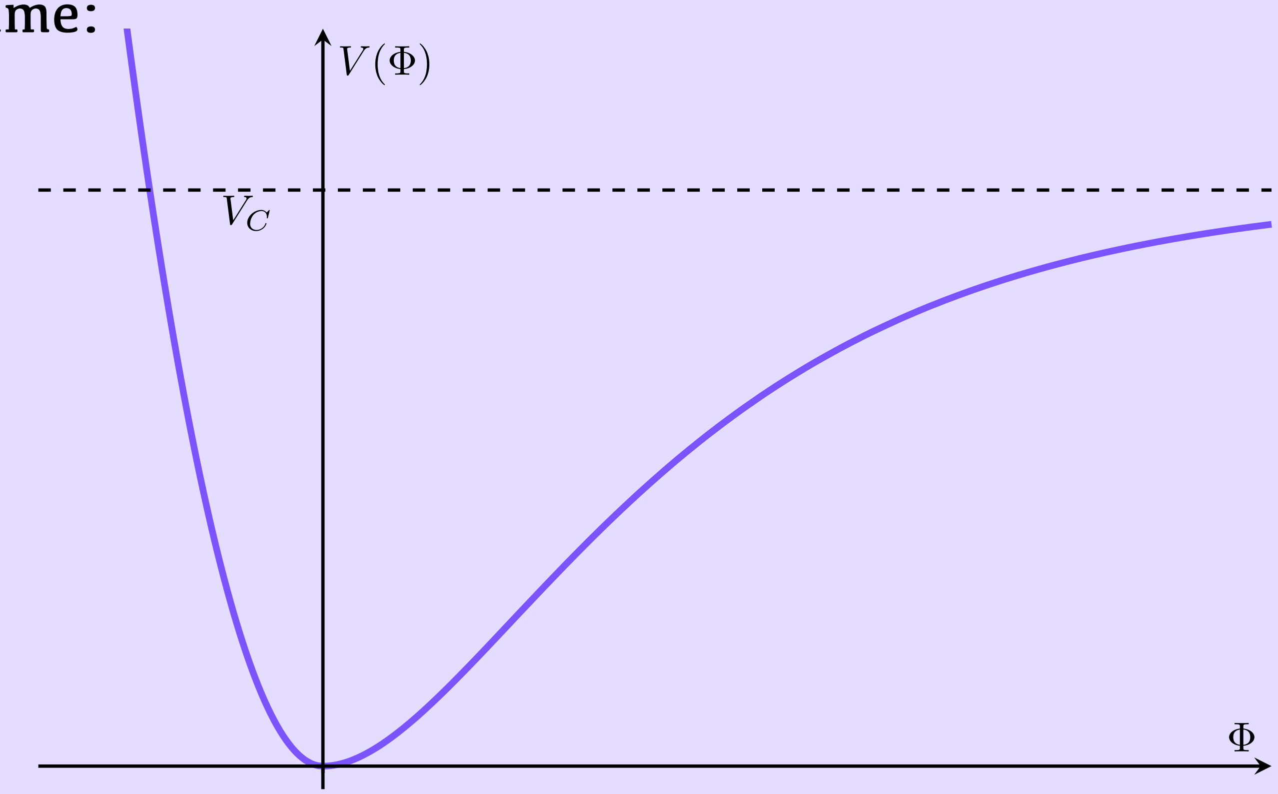
- The action in the Jordan frame:

$$S_{\text{int}} + S_{\text{EH}} = -\frac{1}{2}\tilde{m}_{\text{Pl}}^2 \int \mathrm{d}^4x \sqrt{-g} R \cdot \left(1 + \frac{\varphi}{\Lambda_{\text{int}}}\right)$$

- The conformal transformation to the Einstein frame:

$$\Omega(\varphi) = 1 + \frac{\varphi}{\Lambda_{\text{int}}}$$

$$g_{\mu\nu} \mapsto \frac{1}{\Omega(\varphi)} g_{\mu\nu}$$



- The canonical form under the field substitution:

$$\Phi = \tilde{m}_{\text{Pl}} \sqrt{3/2} \ln \left(1 + \frac{\varphi}{\Lambda_{\text{int}}}\right)$$

- At the potential of free scalar field in the Jordan frame the potential of Φ equals:

$$V_E = \frac{1}{2} m^2 \Lambda_{\text{int}}^2 \left(1 - \exp\left(-\frac{\Phi}{\tilde{m}_{\text{Pl}}} \cdot \sqrt{2/3}\right)\right)^2$$

- It has the plateau and inflaton mass at the bottom:

$$V_C = \frac{1}{2} m^2 \Lambda_{\text{int}}^2 \quad m_{\text{inf}}^2 = \frac{2}{3} m^2 \cdot \frac{\Lambda_{\text{int}}^2}{\bar{m}_{\text{Pl}}}$$

Numerical estimates

- The contribution of vacuum state to the average stress-energy tensor is suppressed as:

$$\rho_{\text{vac}} \sim (10^{-3}\ \text{eV})^4, \qquad \Lambda \sim 10^{16}\ \text{GeV}\,.$$

- Then $\langle n \rangle \sim 250 \Rightarrow$ the energy of initial coherent state in the reference volume:

$$\langle E \rangle \sim \langle n \rangle m \sim \tilde{m}_{\text{Pl}}\,.$$

- Two energy scales the reduced Planckean mass and the scale of inflation plateau.

Conclusion

- We have shown the bare value of ZP can be suppressed under the following conditions (+covariant formalism):
  - the non-stationary coherent state of scalar field in the expanding universe
  - the finite volume
  - natural estimates for the cut-off scale
- The model involves two scales of energy
- Detailed results are provided in arXiv preprint 2411.16181, 2501.05274 (both to be submitted)

Thank you for your attention!

Scalar field and spatial-temporal covariance

- Suppose true spatial-temporal structure (Wick rotation, no poles):

$$\langle \text{vac} \,|\, T_{\mu\nu}(x) \,|\, \text{vac} \rangle = \int \frac{\mathrm{d}^4 p_E}{(2\pi)^4} \frac{1}{p_E^2 + m^2} \left(p_\mu^E p_\nu^E - \frac{1}{2} g_{\mu\nu}^E (p_E^2 + m^2) \right)$$

- Replacement:

$$p_\mu^E p_\nu^E \mapsto \frac{1}{4} g_{\mu\nu}^E p_E^2 \,.$$

- Therefore:

$$\langle \text{vac} \,|\, T_{\mu\nu}(x) \,|\, \text{vac} \rangle = \frac{1}{4} g_{\mu\nu} \int \frac{\mathrm{d}^4 p_E}{(2\pi)^4} \left(1 + \frac{m^2}{p_E^2 + m^2} \right) \Rightarrow w_{\text{vac}} = -1$$

- True structure of spatial-temporal quantum fluctuations in the vacuum supposes the covariant four-dimensional formulation.

Finite volume and spatial structure

- The scalar field in a finite volume:

$$S = V_{[3]} \int \mathrm{d}t \, \frac{1}{2} \left((\partial_t \phi(t))^2 - m^2 \phi^2(t) \right)$$

- Using oscillator coordinate, mass, frequency and scale of energy:

$$\langle 0 | m^2 \phi^2 | 0 \rangle = \langle 0 | (\partial_t \phi)^2 | 0 \rangle = \frac{1}{2} \frac{m}{V_{[3]}},$$

- So, the energy density equals:

$$\langle 0 | T_{00} | 0 \rangle = \frac{1}{2} \langle 0 | (\partial_t \phi)^2 + m^2 \phi^2 | 0 \rangle = \frac{1}{2} \frac{m}{V_{[3]}},$$

- while the pressure is given by:

$$\mathtt{p} = \frac{1}{2} \langle 0 | (\partial_t \phi)^2 - m^2 \phi^2 | 0 \rangle = 0.$$

- Does not correspond to the vacuum: the parameter fits the **dust**, i.e. particles with zeroth pressure

Covariant model

- Action for the field in the **finite** volume in the **covariant 4d** form:

$$S_\Lambda = \frac{1}{\Lambda^3}\int \mathrm{d}\tau \frac{1}{2}\Big((\partial_\tau\phi)^2\big\langle\partial_\mu\tau\partial_\nu\tau\big\rangle g^{\mu\nu} - m^2\phi^2\Big),$$

- the covariant ST-structure in the model is defined by:

$$\big\langle\partial_\mu\tau\partial_\nu\tau\big\rangle = \frac{1}{4}g_{\mu\nu}, \quad \langle(\partial_\lambda\tau)^2\rangle = 1$$

- Analogously for the state with no quanta of oscillator:

$$\langle \text{vac}\,|\,T^\Lambda_{\mu\nu}\,|\,\text{vac}\rangle = g_{\mu\nu}\langle\text{vac}\,|\,\big(-\tfrac{1}{4}(\partial_\tau\phi)^2+\tfrac{1}{2}m^2\phi^2\big)\,|\,\text{vac}\rangle = g_{\mu\nu}\frac{1}{8}\,m\Lambda^3\,.$$

- and the bare value of energy density:

$$\rho^{\text{bare}}_{\text{vac}} = \langle\text{vac}\,|\,T^\Lambda_{00}\,|\,\text{vac}\rangle = \frac{1}{8}\,m\Lambda^3 > 0.$$

Non-stationary coherent state

- The contribution of vacuum state to the average stress-energy tensor is suppressed as:

$$\langle T^{\Lambda}_{\mu\nu}\rangle_{\text{vac}} = \langle \textsf{vac}\,|\,T^{\Lambda}_{\mu\nu}\,|\,\textsf{vac}\rangle \cdot \mathrm{e}^{-\langle n\rangle} = \rho^{\text{bare}}_{\text{vac}} \cdot \mathrm{e}^{-\langle n\rangle} = \frac{1}{8}\, m\Lambda^3 \cdot \mathrm{e}^{-\langle n\rangle},$$

- the energy of coherent state in the final volume:

$$\langle E\rangle = \omega_{\text{osc}}\big(\langle n\rangle + \tfrac{1}{2}\big) = m\big(\langle n\rangle + \tfrac{1}{2}\big),$$

- the same quantity in terms of energy density within a finite volume:

$$\langle E\rangle = V_{[3]}\,\rho^{\text{bare}}_{\text{vac}}\big(\langle n\rangle + \tfrac{1}{2}\big) \cdot 2 = V_{[3]}\,\frac{1}{4}\, m\Lambda^3(\langle n\rangle + \tfrac{1}{2}\big),$$

- Therefore, the reference volume:

$$V_{[3]} = \frac{4}{\Lambda^3}$$