Universal Constraints on Black Hole Hairs: From No-Short-Hair Theorems to Observational Signatures

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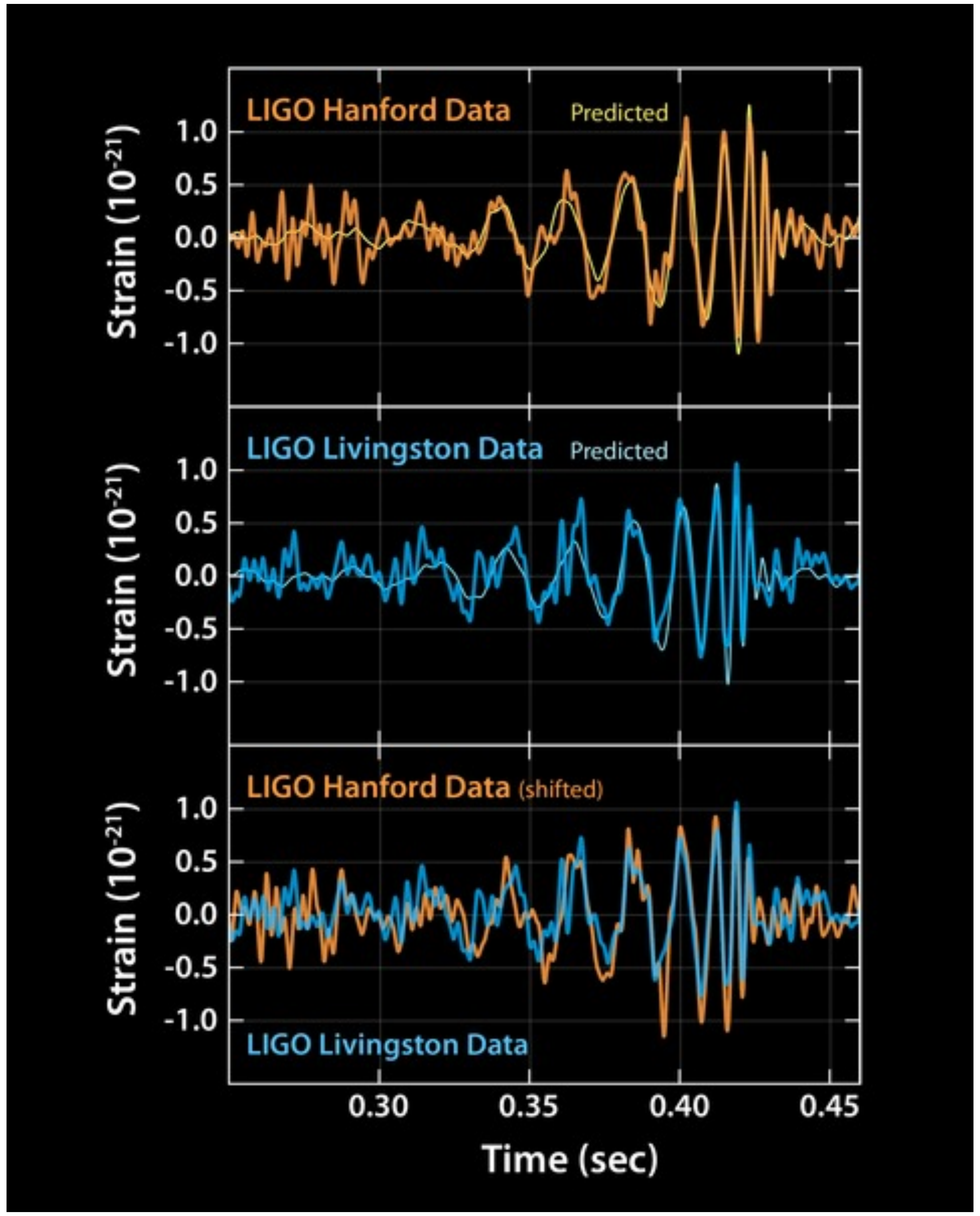
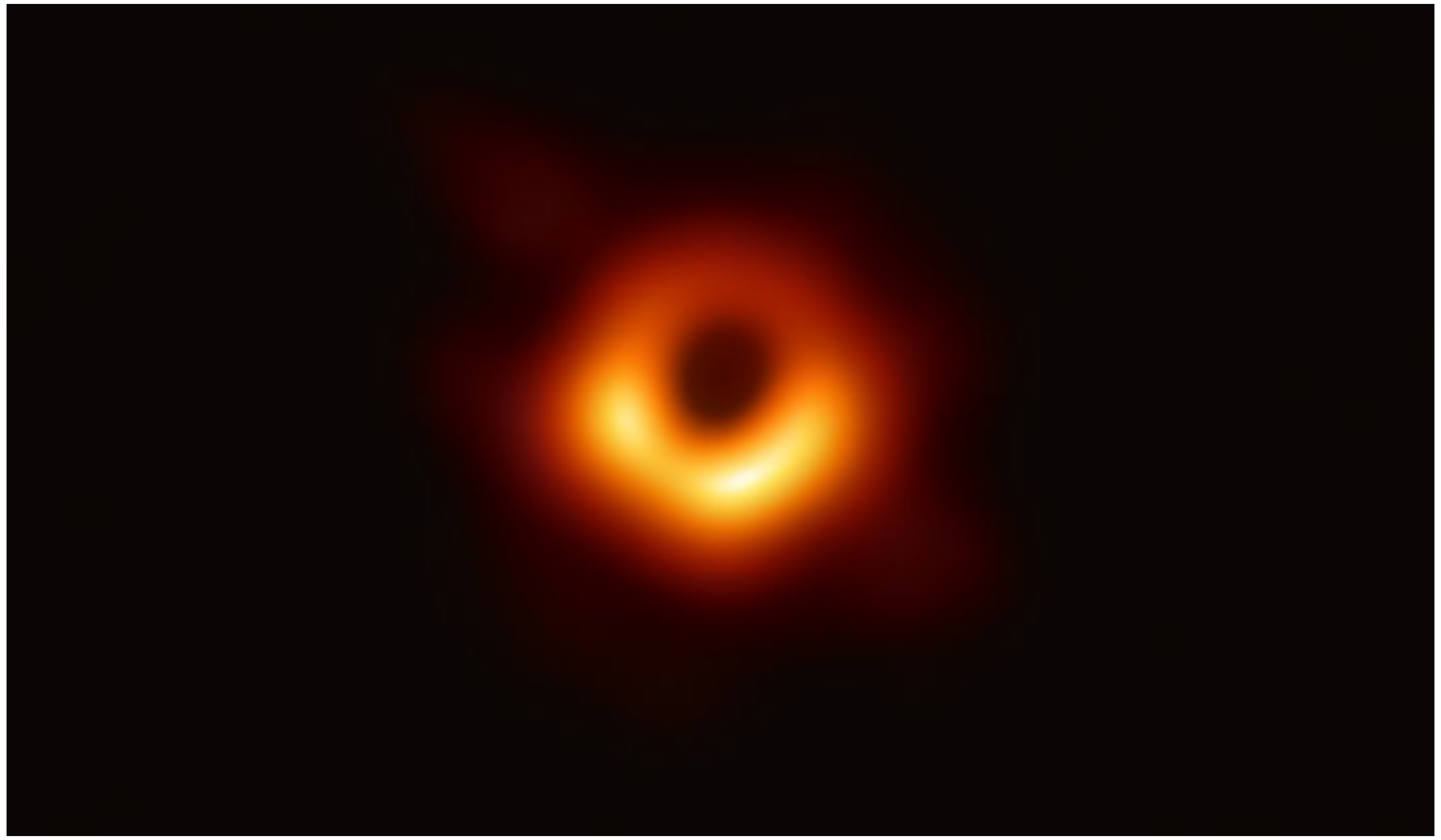
10th Anniversary of GW150914

- **GR+Kerr paradigm** has shown extraordinary observational success….

GW150914 (LIGO-Virgo, 2015)

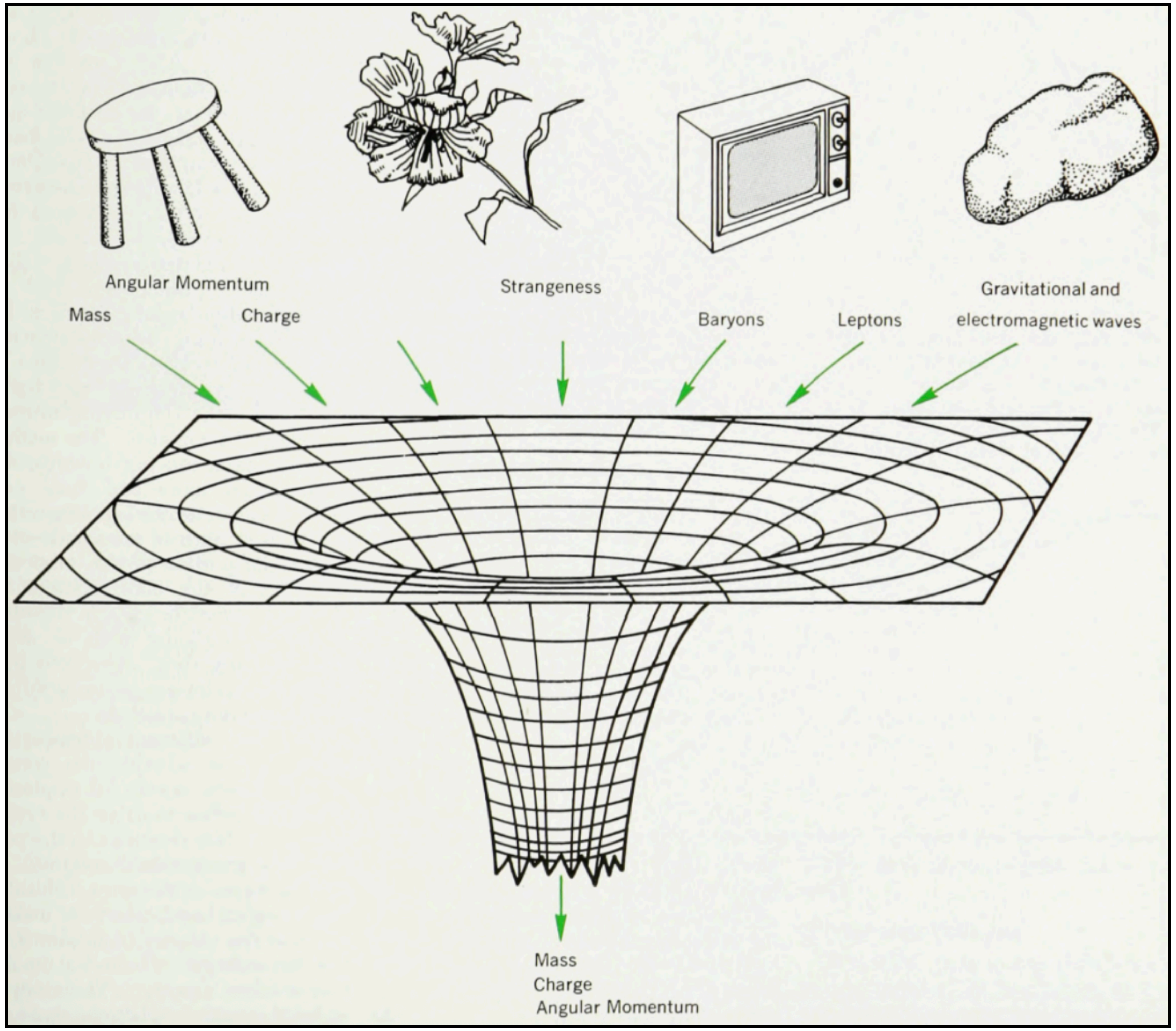


M87 shadow (EHT, 2017)



- **<u>No-hair conjecture:</u>** The final product of a gravitational collapse in GR is a Kerr(-Newman) BH specified by mass, spin, (and EM charges) alone.

[Ruffini+Wheeler, Phys. Today. 24, 1- 30 (1971)]



No-hair Tests

- This conjecture is later supported by **Bekenstein's no-scalar-hair theorem** for canonical scalar field (extended to vector, spin-2 fields too).

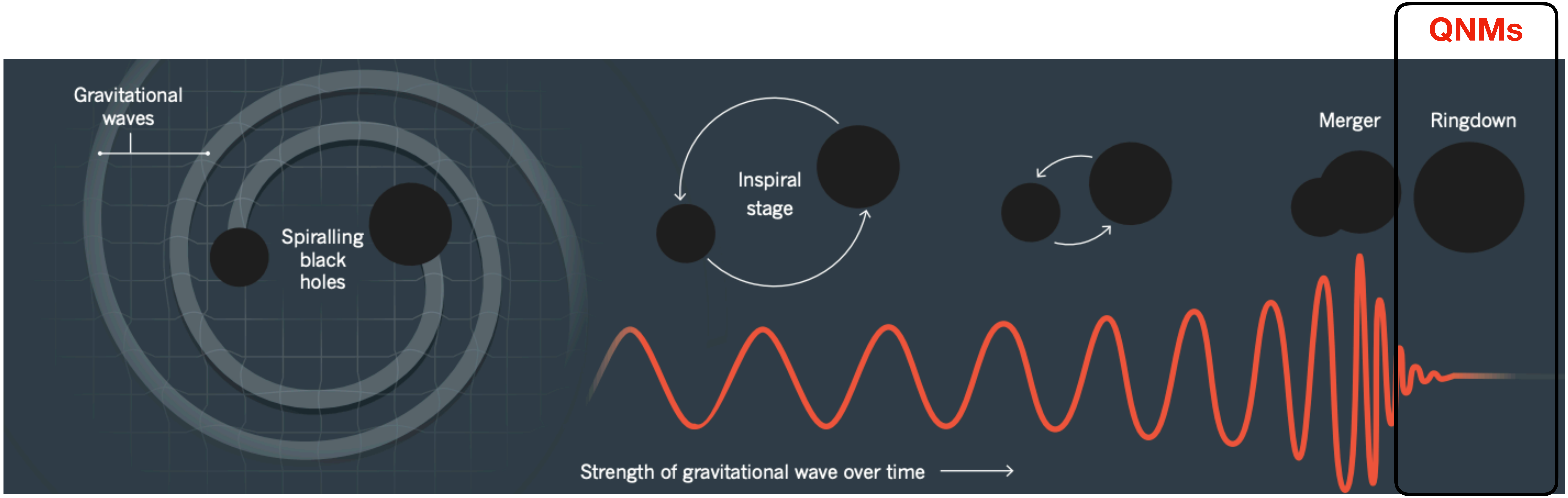
  [Bekenstein, PRD 5, 1239 (1972); PRD 5, 2403 (1972)]

- These results lend a strong support to Kerr paradigm: **Astrophysical BHs are adequately described by the Kerr metric**.

- It is only with such remarkably consistent framework, we have the luxury to constraint potential deviations from it using observations.

- In particular, several GW tests has strongly constraining potential **"hairy modifications"** (any new attributes of BH beyond mass and spin).

Three Phases of Binary Mergers



[Figure: M. Zastrow, Nature 537, S198–S199 (2016)]

No-hair Test with GW150914

[Isi+, PRL 123, 111102 (2019)]

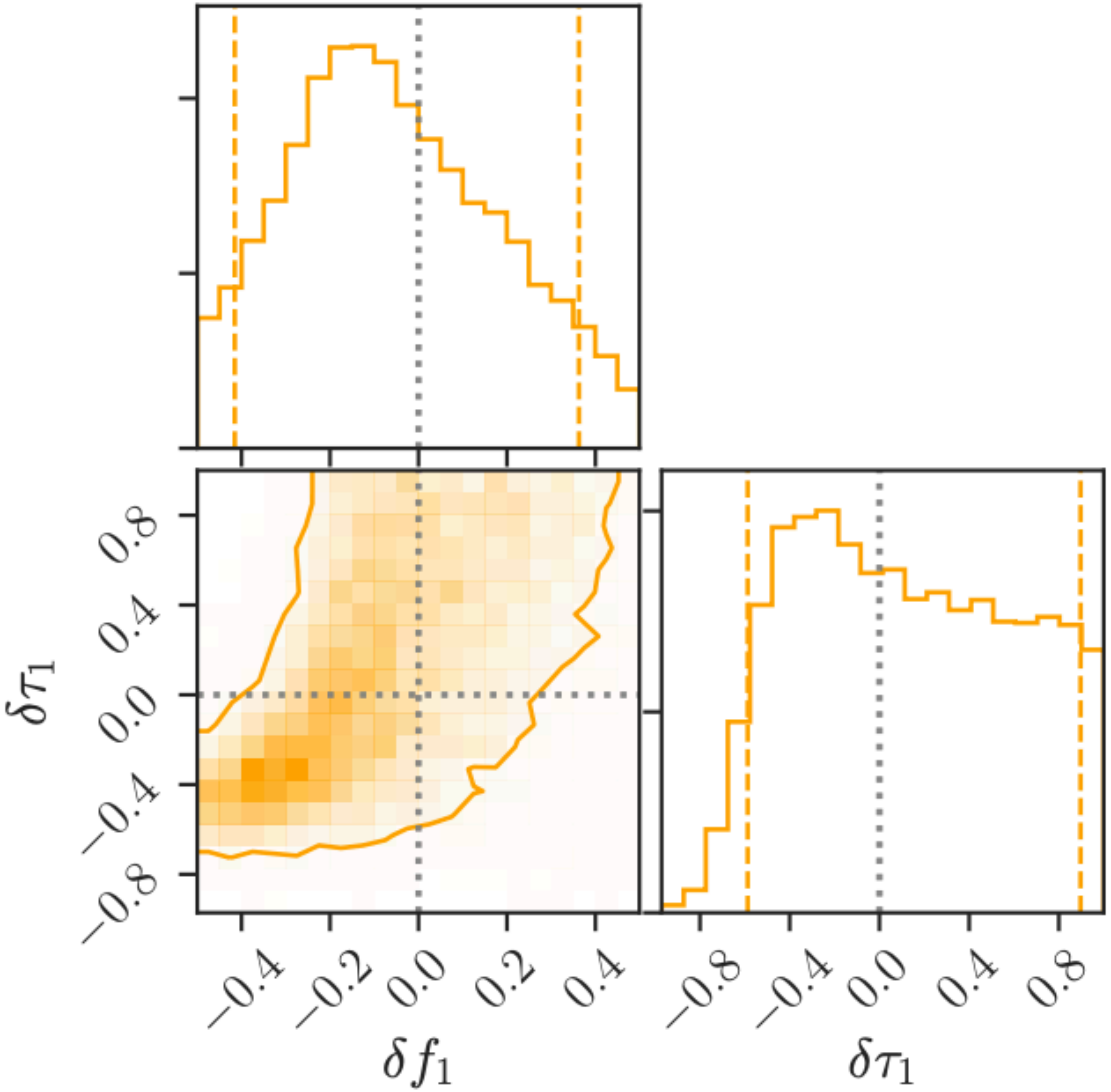
- Any deviations δf_1 and $\delta \tau_1$ from zero signify deviations from no-hair theorem: New waveform

$$\underbrace{A_0\, e^{i\omega_0 t}\, e^{-t/\tau_0}\, S_0(\theta,\phi)}_{\text{fundamental mode}} + \underbrace{A_1\, e^{-i\omega_1' t}\, e^{-t/\tau_1'}\, S_1(\theta,\phi)}_{\text{1st overtone}}$$

where $\omega_1' = 2\,\pi f_1^{GR}(1+\delta f_1)$ and

$$\tau_1' = \tau_1^{GR}(1+\delta\tau_1)\,.$$

- Hairy modifications are not fully ruled out but constrained!



Hairy BH Solutions

- Many explicit counter-examples have been found too, like **coloured BHs in Einstein-Yang-Mills theory**, BHs with **dilatonic and axionic hairs,** and many more.

  [Bizon, PRL 64, 2844-2847 (1990); Kanti+, PRD 54, 5049- 5058 (1996); Campbell+, PLB 263, 364-370 (1991)]

- Even scalar-hairy BH solutions exist in and beyond GR with non-canonical scalar couplings. [for a review, see Herdeiro+Radu, arXiv: 1504.08209 [gr-qc]]

- Following **Coleman et. al. (1992)**, hairs can be categorized into two major groups -

  [arXiv: hep-th/9201059]

  (i) <u>Global hairs:</u> Satisfy global conservation laws (e.g., mass, spin, EM charge)

  (ii) <u>Non-global hairs:</u> Not associated with any global charge (this talk).

Non-Global Hairs

- The effect of a non-global hair Φ (not necessarily a scalar field) is prescribed by the corresponding stress-energy tensor $T^{\mu}_{\nu}[\Phi]$.

- The non-existence of global charge implies hair decays too fast: $\rho \equiv -T^{0}_{0}$ $\sim r^{-n}$ asymptotically!

- Then, it could happen that the hair is too **“short”** (i.e., has little/no effect beyond a radius r_{hair}) to have any measurable effect on observables accessed far from the BH.

Questions

- Such short-hairy BHs (for which the hair is confined only to the near-horizon region) will mimic Kerr BHs and can lead to ambiguous verification of the no-hair property.

- Therefore, following questions are of central observational importance:

  Q1. Can such short-hairy BHs exist?

  $\Longrightarrow$ No, all existing hairs must extend beyond a certain radius away from the horizon.

  Q2. If not, what is the minimum extension of a hair?

  $\Longrightarrow$ For non-rotating case, $r_{hair} > r_\gamma^1$, i.e., the innermost light ring (LR). For rotating case, the minimum size is given by a more complicated relation.

Non-existence of Short-hairs: Static Case

- General static and spherically symmetric metric in $D \geq 4$ dimensions:

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{k(r)} + h(r)\,d\Omega^2_{D-2}\,.$$

- Asymptotically flatness: $f(r) \to 1$, $k(r) \to 1$, and $h(r) \sim r^2$.

- Represents a non-extremal BH if $f(r_H) = k(r_H) = 0$, where the event horizon r_H is a single root. **[Vishveshwara, Jour. of Math. Phys. 9,1319–1322 (1968)]**

- If this metric is sourced by some hairs, then the corresponding T^μ_ν has only 3 independent components $\{T^0_0 \equiv -\rho(r), T^r_r \equiv p(r), T^{\theta_1}_{\theta_1} \equiv p_T(r)\}$.

Non-existence of Short-hairs: Static Case

- No field equations will be assumed! But conservation law $\nabla_\mu T^\mu_\nu = 0$ must hold.

- Radial component can be rewritten as: [RG+, PRD 108 (2023) 4, 4]

$$\boxed{\hat{P}'(r) = \frac{h^{D/2-1}(r)}{2f(r)}\,[p(r)+\rho(r)]\,\Delta(r) + \frac{h^{D/2-1}(r)}{2}\,h'(r)\,T(r)\,,}$$

where $\hat{P} = h^{D/2}p$, $\Delta = \underbrace{f(r)\,h'(r)}_{R(r)} - \underbrace{h(r)\,f'(r)}_{L(r)}$, and $T = T^\mu_\mu$ is the trace.

- The **4-dimensional case in GR** was first studied by **Nunez et. al. (1996)** and later by **S. Hod (2011)**. [Nunez+, PRL 76 (1996) 571-574; Hod, PRD 84 (2011) 124030]

Non-existence of Short-hairs: Static Case

- Energy conditions on matter (T^{μ}_{ν}): [Nunez+, PRL 76 (1996) 571-574; Hod, PRD 84 (2011) 124030]

  >> Weak energy condition, $p+\rho \geq 0$ and $\rho \geq 0$

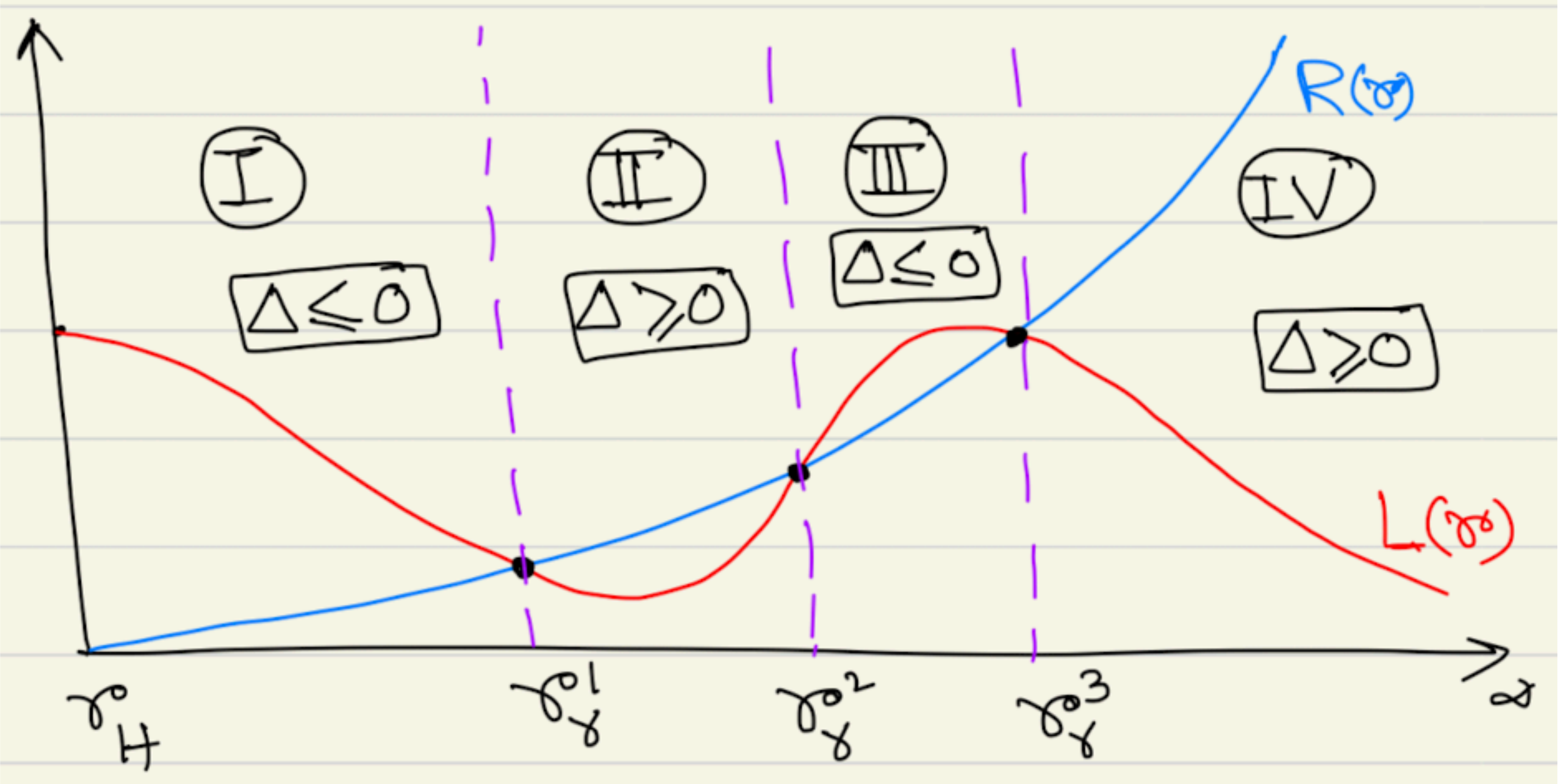
  >> Non-positive trace condition, $T \leq 0$

  >> Non-global hair, $\hat{P} \to 0$ as $r \to \infty$.

- We have $$\hat{P}'(r) = \frac{h^{D/2-1}}{2f} \underbrace{(p+\rho)}_{\geq 0} \underbrace{\Delta(r)}_{?} + \frac{h^{D/2-1}}{2} \underbrace{h'(r)}_{>0} \underbrace{T(r)}_{\leq 0} ,$$

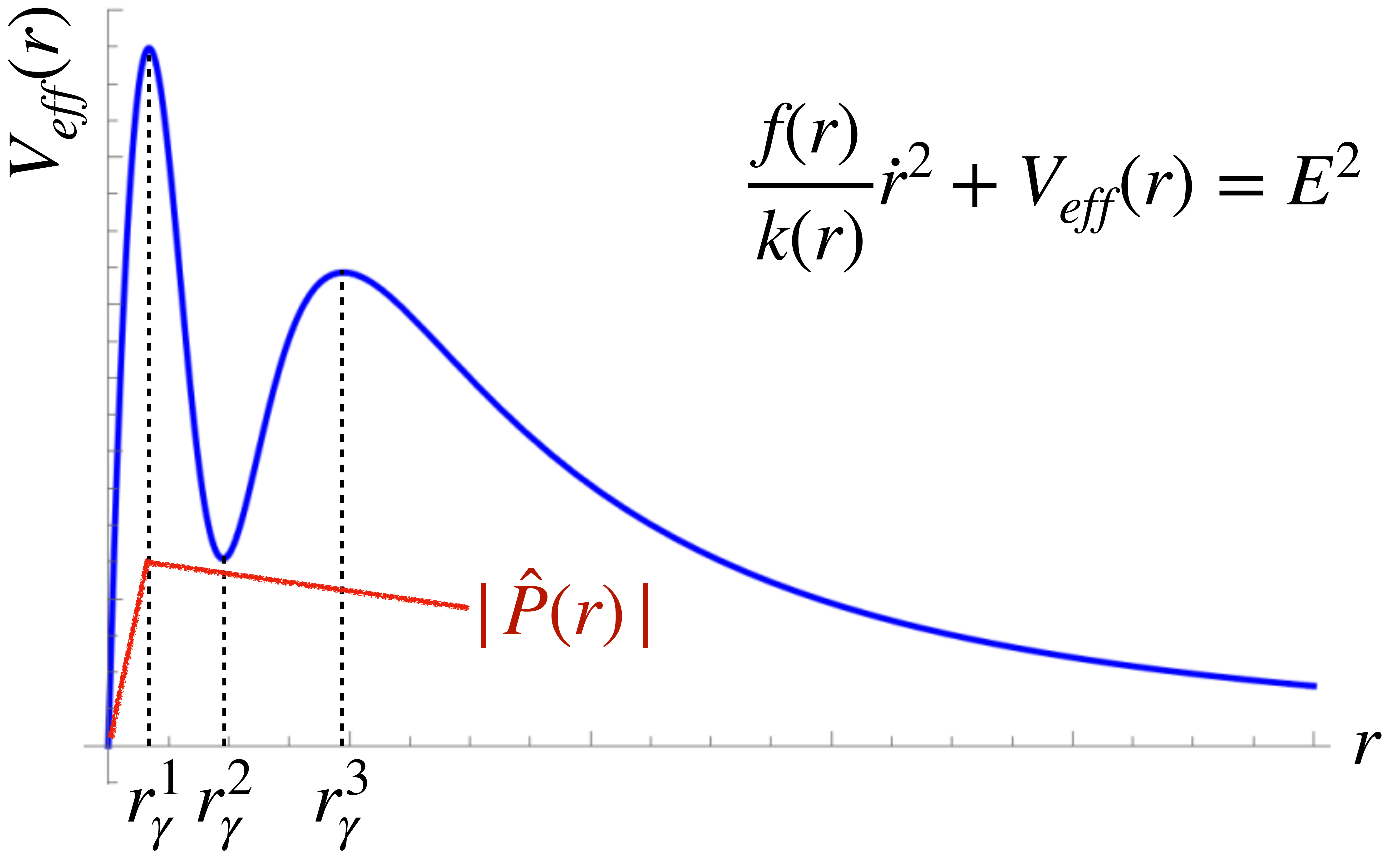
- Regularity at horizon $\Longrightarrow p+\rho\,|_H = 0 \Longrightarrow \hat{P}(r_H) = h^{D/2}(r_H)\,p(r_H) \leq 0$ and $\hat{P}'(r_H) < 0.$

Non-existence of Short-hairs: Static Case



$$\Delta = \underbrace{f(r)\,h'(r)}_{R(r)} - \underbrace{h(r)\,f'(r)}_{L(r)} \implies \hat{P}'(r_H \le r \le r_\gamma^1) \le 0.$$

A Toy Geodesic Potential with Multiple LRs



Hairosphere must extend beyond the innermost LR (r_γ^1)

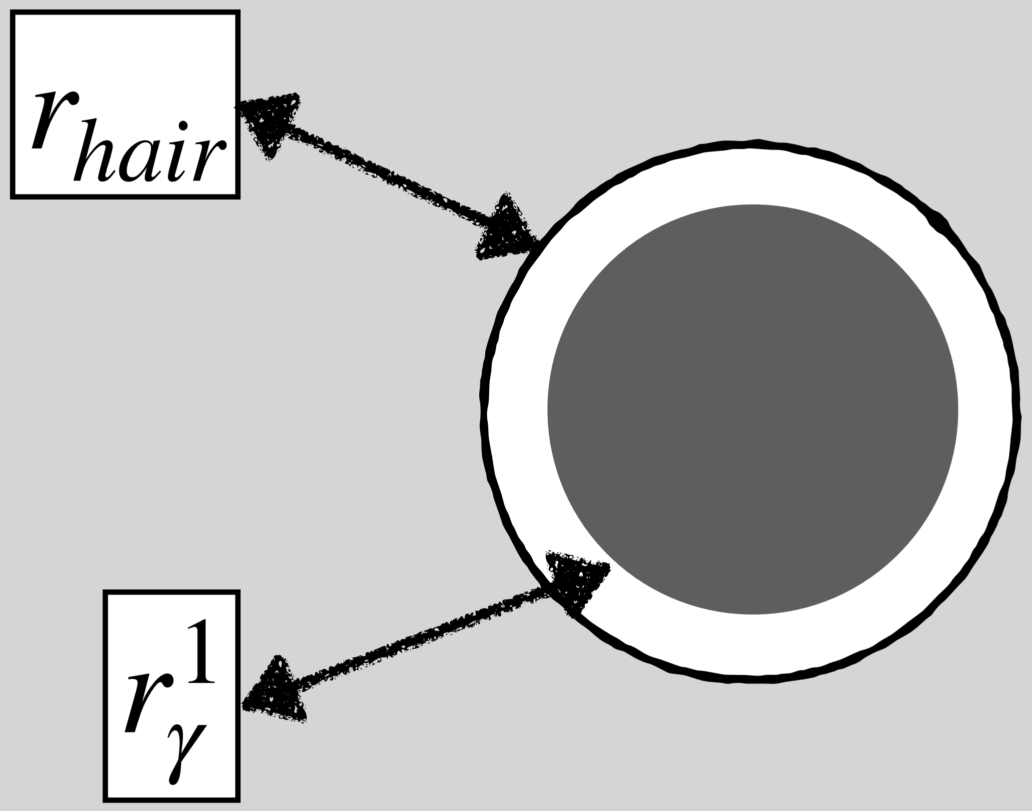
Summary

- Static BH
- Spherically sym.
- Asymp. flatness

- WEC
- $T \leq 0$
- $\hat{P} \to 0, r \to \infty$

Theory Agnostic

No short hair



A Few Generalizations

- Static BH beyond spherical symmetry: $r_{hair}(\theta) > r^1_\gamma(\theta)$ for some ranges in θ.
  [Acharya+Sarkar, PRD 109 (2024) 6, 064084]

- Some results have been obtained also for static asymptotically dS and AdS BHs. [Cai+, PRD 58 (1998) 024002; Ishibashi+, CQG 41 (2024) 8, 085010]

- We have recently generalized the theorem for rotating and asymptotically flat BHs with some extra assumptions, like Klein-Gordon separability etc $\Longrightarrow$ no short hair property holds at least at the poles. [RG+Singha, PRD 111 (2025) 4, 4]

- We have also shown a few extensions for exotic compact objects.
  [RG+, PRD 108 (2023) 4, 4; PRD 111 (2025) 4, 4]

Theoretical and Observational Consequences

- No-short hair theorems provide the first unified understanding of profiles of hair around BHs.
- If no evidence of hairs are found till the innermost LR of BHs, then there is no hair.
- For BHs with a single LR, present observations have already started to probe the near-LR regions using both shadow and QNM observations.
- **Thus, no-short hair theorem strengthen the observational verification of the no-hair result, by ruling out those hairs that could have evaded observations.**

Theoretical and Observational Consequences

$$\hat{P}'(r) = \frac{h^{D/2-1}(r)}{2f(r)}\,[p(r)+\rho(r)]\,\Delta(r) + \frac{h^{D/2-1}(r)}{2}\,h'(r)\,T(r)\,,$$

- Among all **BHs in GR** satisfying the aforementioned energy conditions, the vacuum solution (Schwarzschild-Tangherlini BH) has the largest inner LR for the same ADM mass, i.e., $r_\gamma^1 \leq r_\gamma^{ST}$.
- If the spacetime has a single LR, then we have the inequality for the shadow radius: $r_{sh} \leq \sqrt{\dfrac{D-1}{D-3}}\, r_\gamma^{ST}$, i.e., ST BH cast the largest shadow. **[Lü+Lyu, PRD 101 (2020) 4, 044059]**
- This can potentially be used as a test of GR and other matter assumptions via EHT observations.
- A similar bound can also be found on eikonal QNMs. **[S. Hod, S. Chakraborty, ...]**

Future Directions and Conclusions

- Can we test the no-short hair theorem directly?

- Yes, construct a short-hairy BH metric by violating some assumptions, e.g., consider $T > 0$.

- Such a BH was constructed by **Brown and Husaine** with

$$-g_{tt} = \frac{1}{g_{rr}} = 1 - \frac{2M}{r} + \frac{Q^{2k}}{r^{2k}} \quad \text{where } k > 1.$$

[Int. J. Mod. Phys. D 6 (1997) 563-573]

- QNMs **(arXiv: 1404.7149)** and shadow **(arXiv: 2209.08202)** signatures of these metric have been studied theoretically. Bayesian inference on $\{Q, k\}$.

- It would be interesting to study hairy properties of other objects, like wormholes etc.

THANK YOU

	T	$T_t^t - T_r^r$	$T_t^t(r \to \infty) \sim$
Skyrme	$-\frac{1}{16\pi}(f^2\,\mu\,F'^2 + \frac{\sin^4 F}{e^2\,r^4})$	$-\frac{\mu}{16\pi}(f^2 + \frac{2\sin^2 F}{e^2\,r^2})F'^2$	$\frac{1}{r^6}$
YM	0	$-\frac{\mu w'^2}{2\pi f^2 r^2}$	$\frac{1}{r^6}$
YMD + V	$-\frac{1}{4\pi}(\mu\,\phi'^2 + 4V)$	$-\frac{\mu}{4\pi}(\phi'^2 + 2\frac{w'^2}{f^2\,r^2}e^{2\gamma\phi})$	$\frac{1}{r^6}$ or $\frac{1}{r^{10}}$
Y M H	$-\frac{1}{4\pi}[\mu\,\phi'^2 + 4V + \frac{f^2\,\phi^2}{2\,r^2}\,(1+w)^2]$	$-\frac{\mu}{4\pi}(\phi'^2 + 2\frac{w'^2}{f^2\,r^2})$	$\frac{e^{-2m\,r}}{r^2}$
NAProca	$-\frac{m^2}{8\pi\,r^2}\,(1+w)^2$	$-\frac{\mu w'^2}{2\pi f^2\,r^2}$	$\frac{e^{-2m\,r}}{r^2}$

[Nunez+, PRL 76 (1996) 571-574]