TWENTY-SECOND LOMONOSOV CONFERENCE ON ELEMENTARY PARTICLE PHYSICS

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MOSCOW STATE UNIVERSITY

Self-similarity of cumulative high-p_T hadron production in heavy ion collisions at high energies

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- Motivation
- High-p_T hadron production in p+p
- Cumulative production in p+A at U70, FNAL
- Spectra of charged hadrons in Au+Au in non- and cumulative range at STAR in BES-I
- Cumulative production in collider and fixed target mode at RHIC and NICA: perspectives
- Conclusion

Motivation & Goals

Search for new symmetries in Nature

Systematic analysis of inclusive cross sections of particle production in p+p, p+A and A+A collisions to search for general features of constituent structure, interaction and fragmentation over a wide scale range.

z-Scaling as a tool in high energy physics

Development of z-scaling approach for description of cumulative hadron production in inclusive p+A and A+A collisions and verification of self-similarity principle.

The approach can be used to search for and study

- Symmetry of constituent interactions at small scales
- Origin of flavor, spin,…
- Similarity and difference of u,d,s,c,b,t quark fragmentation
- Strangeness as a probe to search for new physics
- Phase transitions in p+p, p+A and A+A systems
- …………..

Fundamental principles & symmetries

"Fundamental symmetry principles dictate the basic laws of physics, control the structure of matter and define the fundamental forces in nature."

Leon M. Lederman

ν_μ

1988
M. Schwartz
J. Steinberger

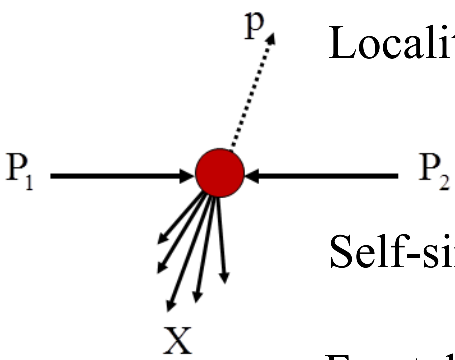
"…for every conservation law there must exist a continuous symmetry...."

Emmy Nöether

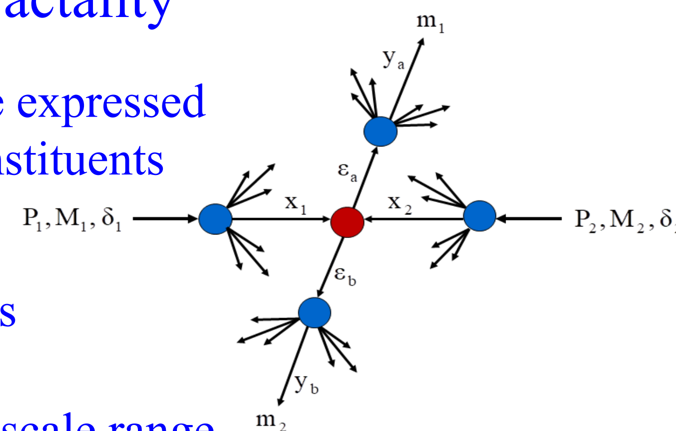
Self-similarity is a property of physical phenomena and principle to construct theories

- Self-similarity is the symmetry of repeatability of structures and processes with change in a scale.
- The self-similarity principle states that structures and processes repeats with change of a scale.
- Self-similarity is the unifying concept for theories of fractals and chaos.
- Phenomenon that is self-similar looks the same or behaves the same when viewed at different magnifications.

Principles: locality, self-similarity, fractality



Locality: collisions of hadrons and nuclei are expressed via binary interactions of their constituents (partons, quarks and gluons,...).



Self-similarity: interactions of the constituents are mutually similar.

Fractality: self-similarity is valid over a wide scale range.

- The principles are reflected as regularities in measurable observables and can be usually expressed as scalings in a suitable representation of data.
- z-Scaling of differential cross sections of inclusive particle production in p+p, p+A and A+A is used as a tool to search for and study of principles and symmetries that reflect properties of interactions at constituent level.
- z-Scaling is based on the principles of *locality, self-similarity, fractality*.

Self-similarity in inclusive reactions & z-scaling

The assumption of self-similarity of hadron interactions at a constituent level transforms to the requirement of universal description of inclusive spectra by a scaling function $\Psi(z)$ that depends on a self-similarity parameter z.

Hypothesis of z-scaling :

$s^{1/2}$, p_T, θ_{cms} — Inclusive particle distributions can be described in terms of constituent sub-processes and parameters characterizing general properties of the system. — x_1, x_2, y_a, y_b; $\delta_1, \delta_2, \varepsilon_a, \varepsilon_b, c$

$Ed^3\sigma/dp^3$ — Scaled cross section $\Psi(z)$ of inclusive particle production depends in a self-similar way on a single scaling variable z. — $\Psi(z)$

The self-similarity parameter z is a dimensionless quantity, expressed through the dimensional values P_1, P_2, p, M_1, M_2, m_1, m_2, characterizing the process of inclusive particle production.

Procedure to construct function $\Psi(z)$ based on maximum fractal entropy was suggested.

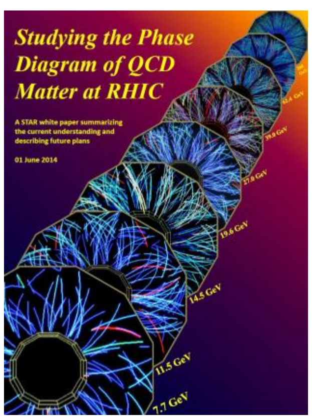
Int. J. Mod. Phys. A 32 (2017) 1750029.
Physics 5(2) (2023) 537.

Scaling, Universality vs. basic principles & symmetries

"Scaling" and "Universality" are concepts developed to understanding critical phenomena. Scaling means that systems near the critical points exhibiting self-similar properties are invariant under transformation of a scale. According to universality, quite different systems behave in a remarkably similar fashion near the respective critical points. Critical exponents are defined only by symmetry of interactions and dimension of the space.

Harry E. Stanley, Grigory I. Barenblatt,…

Beam Energy Scan program at RHIC to search for and study phase transition and critical phenomena in nuclear matter

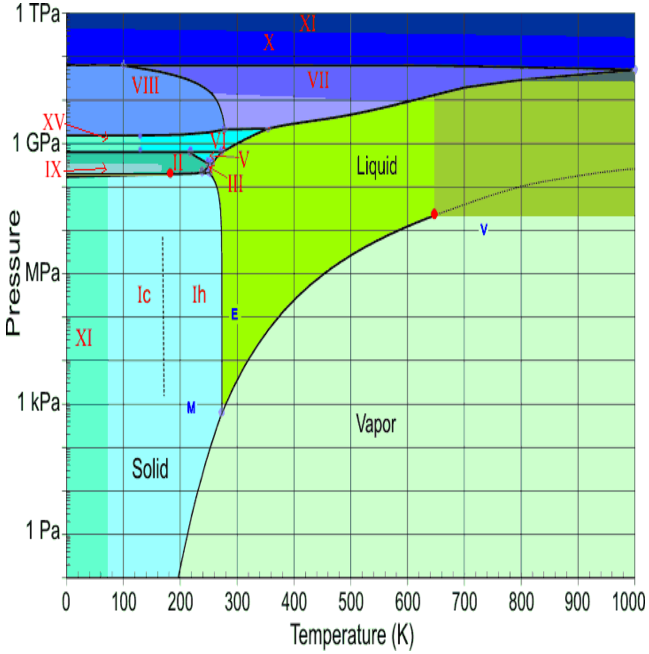


- The idea is to vary the collision energy and look for the signatures of QCD phase boundary and QCD critical point i.e. to span the phase diagram from the top RHIC energy (lower μ_B) to the lowest possible energy (higher μ_B).
- To look for the phase boundary, we would study the established signatures of QGP at 200 GeV as a function of beam energy. Turn-off of these signatures at particular energy would suggest the crossing of phase boundary.
- Near a critical point, there would be enhanced fluctuations in multiplicity distributions of conserved quantities (net-charge, net-baryon).

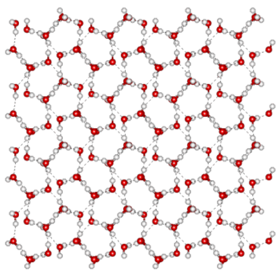
STAR collaboration

Phase diagram of H_2O and Nuclear Matter

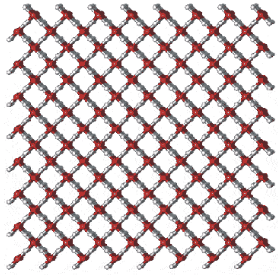
The phase diagram of water is established



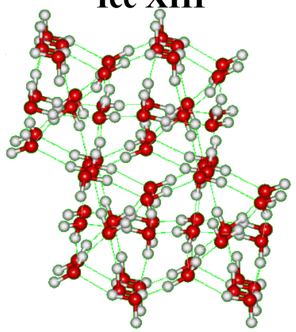
Ice III



Ice X

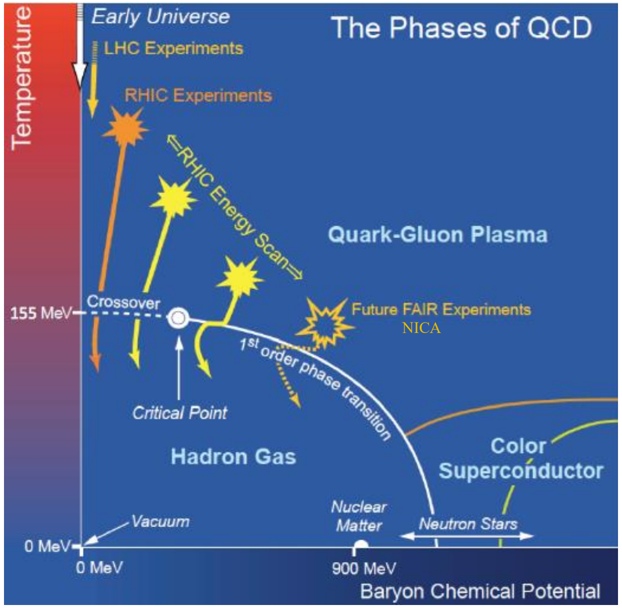


Ice XIII



- Phases (ice I-XV, liquid, vapor)
- Phase boundaries
- Phase transitions
- Triple Point (16)
- Critical Point (1)

The phase diagram of strongly interacting nuclear matter is under study

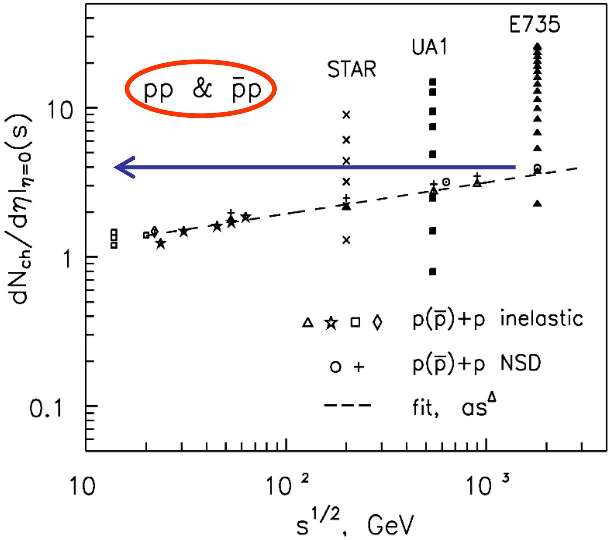


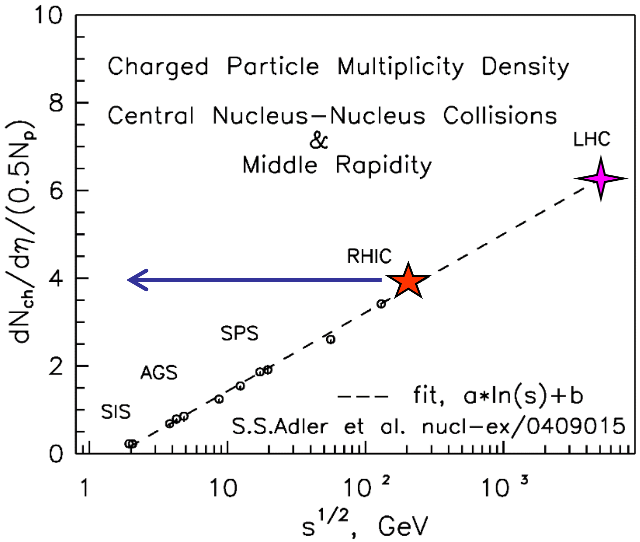
- Phases - ?
- Phase boundaries -?
- Phase transitions - ?
- Triple Point - ?
- Critical Point - ?

Characteristics of produced medium in pp/$\bar{p}$p & AA

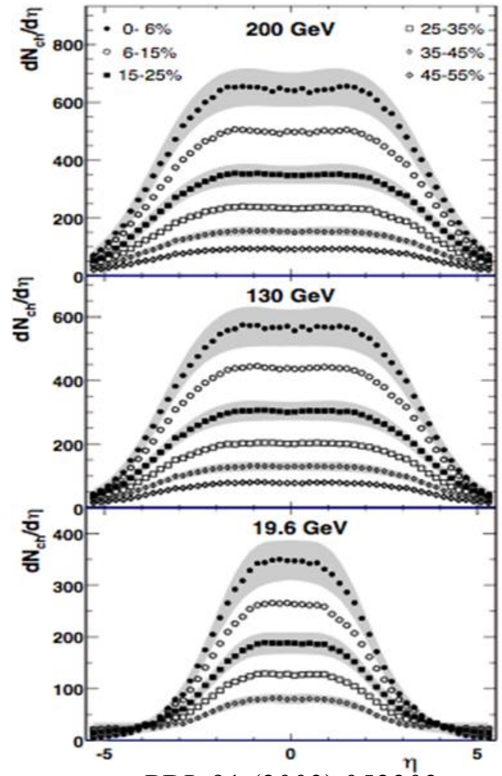
- Particle multiplicity N_{ch}
- Multiplicity density $dN_{ch}/d\eta$
- Mean transverse momentum $<p_T>$
- Energy density $\varepsilon_{Bj} = 1/(\pi R^2\tau)\, dE_T/dy$

Phys.At.Nucl.70 (2007)1294





RHIC & PHOBOS



PRL 91 (2003) 052303

- Multiplicity density $dN_{ch}/d\eta$ in pp & $\bar{p}$p collisions is much larger than $dN_{ch}/d\eta/(0.5N_p)$ in central AA collisions at AGS, Sp$\bar{p}$S and RHIC.
- Is medium produced in pp at high $dN_{ch}/d\eta$ similar one than in AA ?
- Are there common properties of hadron production in pp & AA ?

Self-similarity of negative hadron production in p+p collisions

p+p is of interest by itself:

- verification and search for new features
- search for a phase transition with different probes

p+p interaction is a reference for p+A and A+A physics

V.V. Abramov et al., Sov. J. Nucl. Phys. 31 (1980) 484.
V.V. Abramov et al., JETP Lett. 33 (1981) 289.
J.W. Cronin et. al., Phys. Rev. D11 (1975) 3105.
D. Antreasyan, J.W Cronin et al., Phys. Rev. D 19 (1979) 764.
D.E. Jaffe et al., Phys. Rev. D 40 (1989) 2777.

I.Zborovský & MT, Int. J. Mod. Phys. (2015) 1560103.

Self-similarity of h^- production in p+p

Self-similarity parameter

$$z = z_0 \Omega^{-1}$$

$$z_0 = \frac{s_\perp^{1/2}}{(dN_{ch}/d\eta|_0)^c m_N}$$

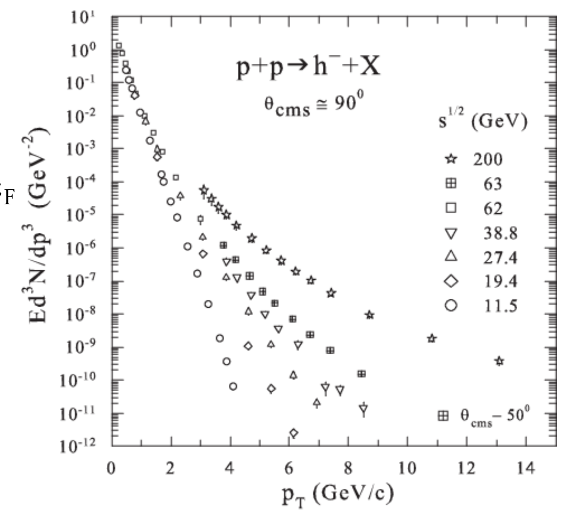
$$\Omega = (1-x_1)^\delta (1-x_2)^\delta (1-y_a)^{\varepsilon_F} (1-y_b)^{\varepsilon_F}$$

- $dN_{ch}/d\eta|_0$ - multiplicity density
- c - “specific heat” of bulk matter
- δ - proton fractal dimension
- ε_F - fragmentation fractal dimension

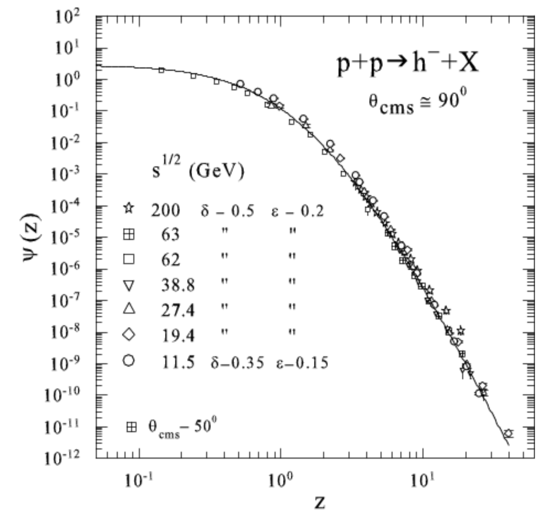
Scaling function

$$\Psi(z) = \frac{\pi}{(dN/d\eta)\cdot\sigma_{inel}} \cdot J^{-1} \cdot E\frac{d^3\sigma}{dp^3}$$

Strong sensitivity of data point to √s at high p_T



“Collapse” of data point onto a single curve



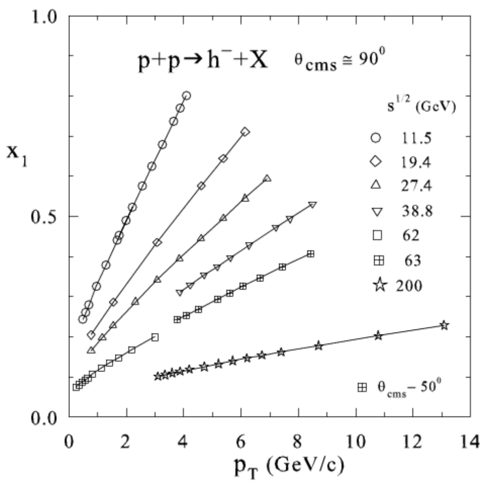
- Energy independence of Ψ(z)
- Centrality independence of Ψ(z)
- Power law at high z
- Saturation at low z

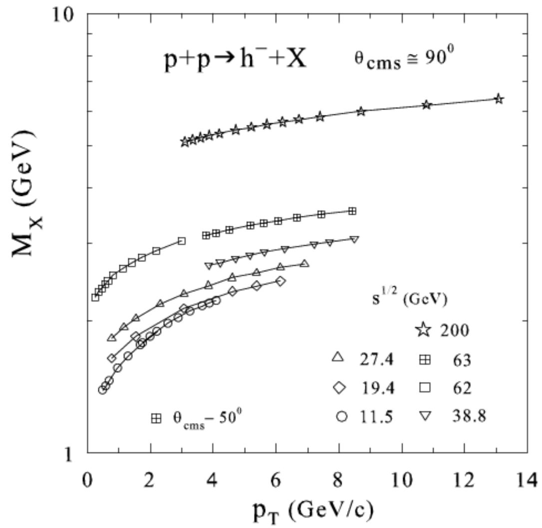
- Universality: the same shape of Ψ(z) vs. √s, p_T
- Asymptotic behavior of Ψ(z) at high z – power law.

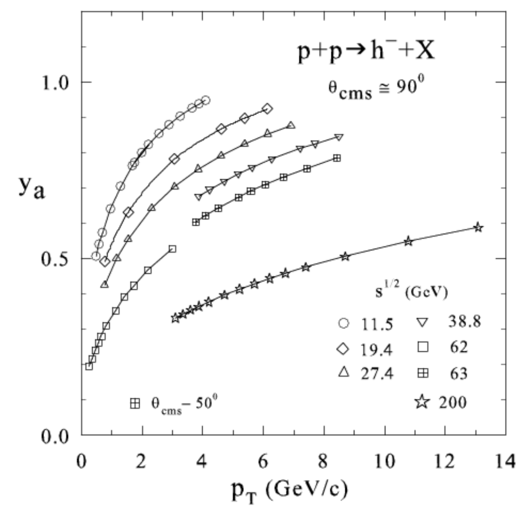
Constituent sub-process of h^- production in p+p

Constituent level of particle production in terms of

Momentum fraction x_1 — **Recoil mass M_X** — **Energy loss $\Delta E/E=(1-y_a)$**







x_1

- increases with p_T
- decreases with $\sqrt{s}$

$$(x_1 P_1 + x_2 P_2 - p/y_a)^2 = M_X^2$$

M_X

- increases with p_T
- increases with $\sqrt{s}$

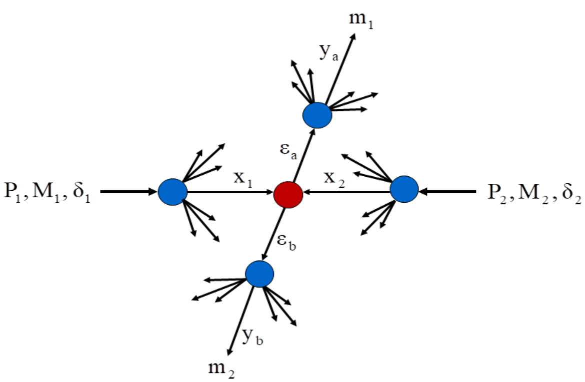
$$M_X = x_1 M_1 + x_2 M_2 + m_2/y_b$$

$\Delta E/E$

- decreases with p_T
- increases with $\sqrt{s}$

- p+p is a reference for p+A and A+A
- high x_1 and p_T – physics nearby kinematic boundary

Energy loss in p+p



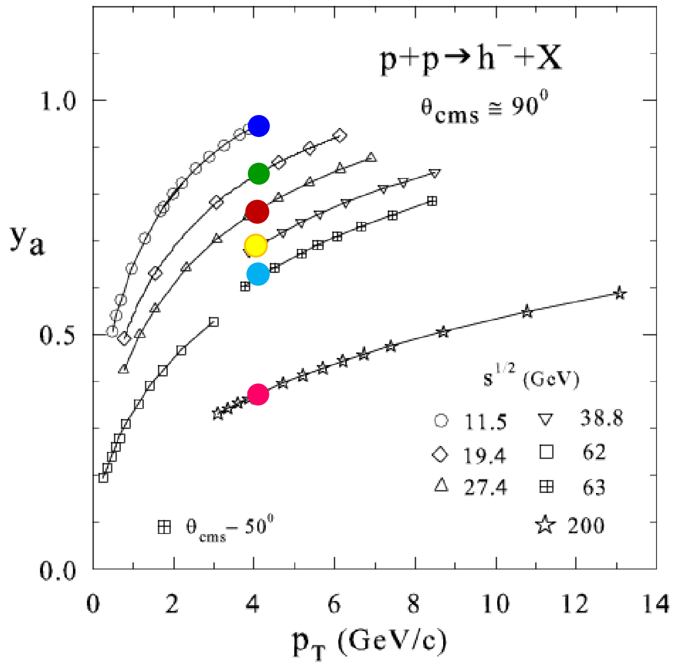
$(x_1P_1+x_2P_2-q)^2 = M_X^2$

$q=p/y_a$

p – momentum of produced hadron

q – momentum of scattered constituent

Energy loss $\Delta E/E=(1-y_a)$



$\Delta E/E$

- decreases with p_T
- increases with $\sqrt{s}$

p_T= 4 GeV/c

5 %
energy loss
$q \approx 4.2$ GeV/c

17 %
energy loss
$q \approx 4.8$ GeV/c

25 %
energy loss
$q \approx 5.3$ GeV/c

32 %
energy loss
$q \approx 5.9$ GeV/c

38 %
energy loss
$q \approx 6.5$ GeV/c

68 %
energy loss
$q \approx 12.5$ GeV/c

Conclusions I

- Self-similarity of particle structure, constituent interactions and fragmentation process of hadron production in p+p collisions was found.
- Properties of the scaling function were described.
- Model parameters – structural and fragmentation dimensions and specific heat, were determined from data analysis.

Properties of $\Psi(z)$ in pp collisions

- Energy independence of $\Psi(z)$ ($s^{1/2} > 20$ GeV)
- Angular independence of $\Psi(z)$ ($\theta_{cms}=3^0$-90^0)
- Multiplicity independence of $\Psi(z)$ ($dN_{ch}/d\eta=1.5$-26)
- Saturation of $\Psi(z)$ at low z ($z < 0.1$)
- Power law, $\Psi(z) \sim z^{-\beta}$, at high z ($z > 4$)
- Flavor independence of $\Psi(z)$ (π,K,φ,Λ,..,D,J/ψ,B,Υ,…, top)

These properties reflect self-similarity, locality, and fractality
of hadron interactions at a constituent level.
It concerns the structure of the colliding objects,
constituent interactions and fragmentation process.

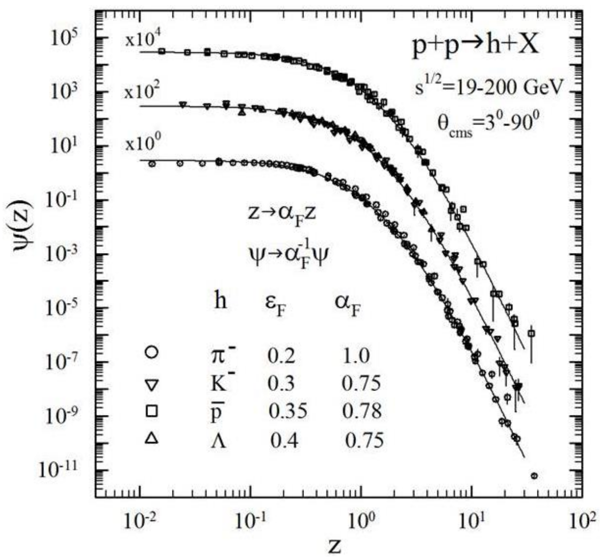
Self-similarity of strangeness production in p+p

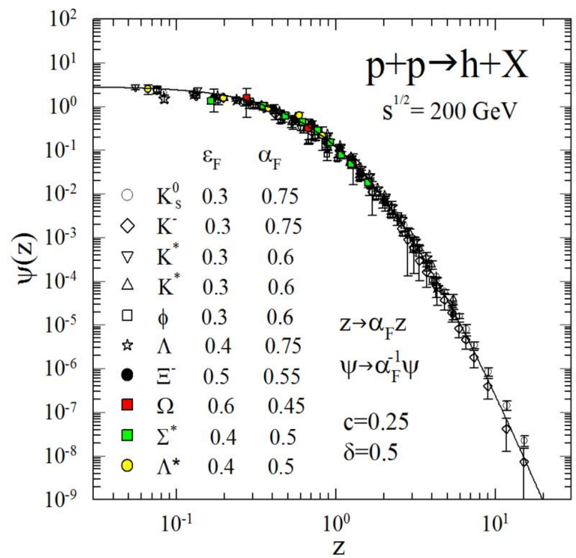
Universality: flavor independence of the scaling function

K^0_S, K^-, K^*, ϕ, Λ, Ξ, Ω, Σ^*, Λ^*

"Collapse" of data points onto a single curve

M.T.& I.Zborovský
Int. J. Mod. Phys.
A24,1417(2009)
A32,1750029 (2017)
Phys. Part. Nucl.
51, 141 (2020)





STAR:
PRL 92 (2004) 092301
PRL 97 (2006) 132301
PLB 612 (2005) 181
PRC 71 (2005) 064902
PRC 75 (2007) 064901
PRL 108 (2012) 072302

PHENIX:
PRC 75 (2007) 051902
PRD 83 (2011) 052004
PRC 90 (2014) 054905

Solid line for π^- meson is a reference frame

$\varepsilon_\pi = 0.2, \quad \alpha_\pi = 1$

- Energy independence
- Angular independence
- Flavor independence
- Saturation for $z < 0.1$

- Power law $\Psi(z) \sim z^{-\beta}$ at large z
- ε_F, α_F independent of p_T, $s^{1/2}$

Self-similarity of cumulative hadron production in p+A collisions

p+A is of interest by itself:

- verification of scaling and search for new phenomena
- search for a phase transition with different probes

p+A interaction is a reference for A+A physics

I. Zborovský, MT, Phys. Rev. D75 (2007) 094008.
I. Zborovský, MT et. al., Int . J. Mod. Phys. A16 (2001) 1281.
A. Aparin, MT, Nucl. Phys. B 245 (2013) 149-152.
A. Aparin, MT, Phys. Part. Nucl. Lett., 11 (2014) 91-100;
11 (2014) 381-390; 11 (2014) 391-403.

Cumulative particle, process, region,…

A.M.Baldin & V.S.Stavinsky (1971,1973)

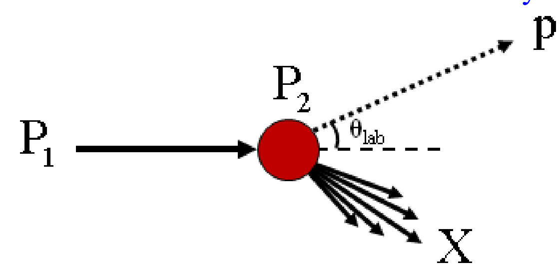
A.M.Baldin

V.S.Stavinsky

The cumulative particle is a particle produced in the region forbidden for free nucleon kinematics:

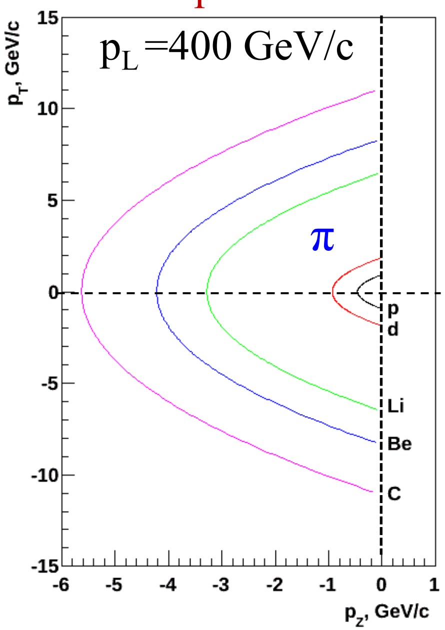
$$P_1 + P_2 \rightarrow p + X$$

$$(P_1 + P_2 - p)^2 = M_X^2 \qquad \min M_X \Rightarrow \quad p^A_{max} > p^p_{max}$$



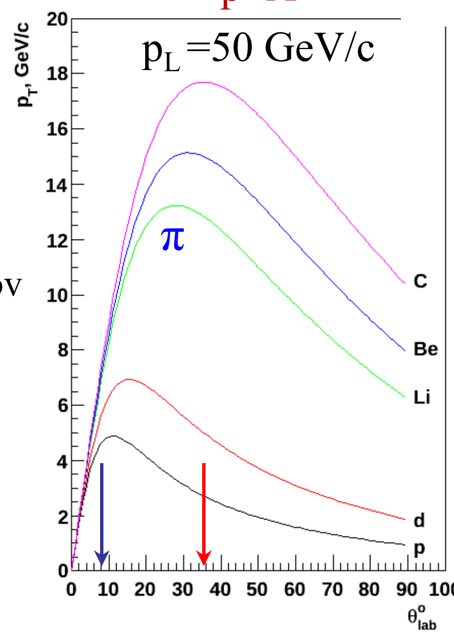
p+A

G.A.Leksin (1980)



p+A

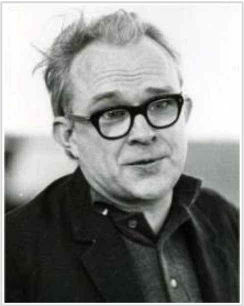
V.V.Ammosov (2013)



Conservation laws:

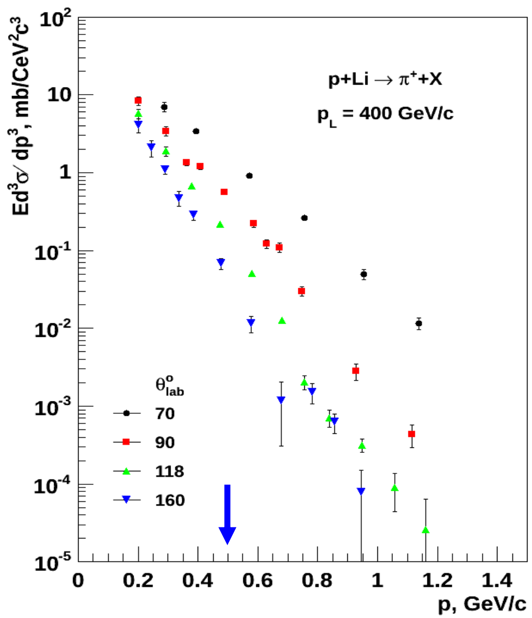
- 4-momentum
- electric charge
- baryon number
- flavor

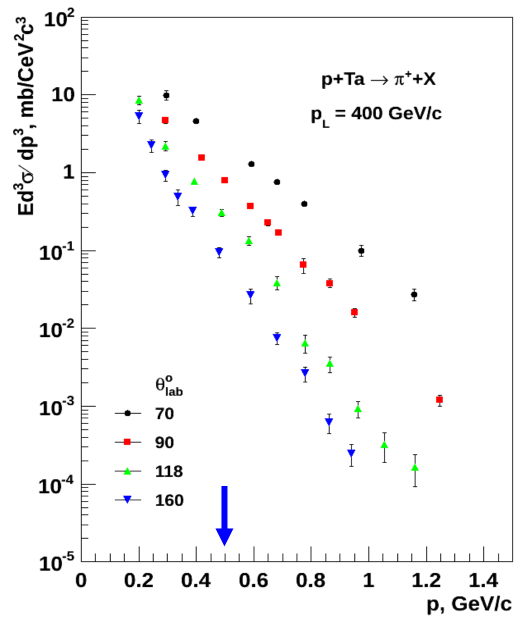
Cumulative pion spectra in p+A at FNAL



G. Leksin

p_L = 400 GeV/c, A=Li,Be,C,Al,Cu,Ta θ_{lab}=70,90,118,160 deg.

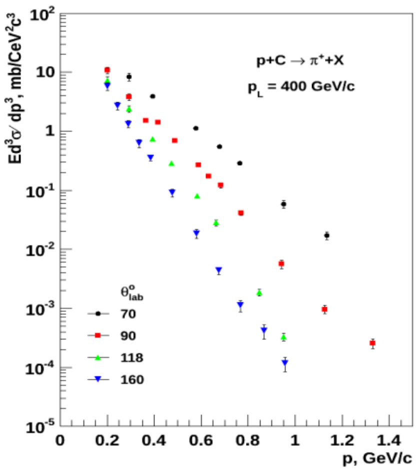


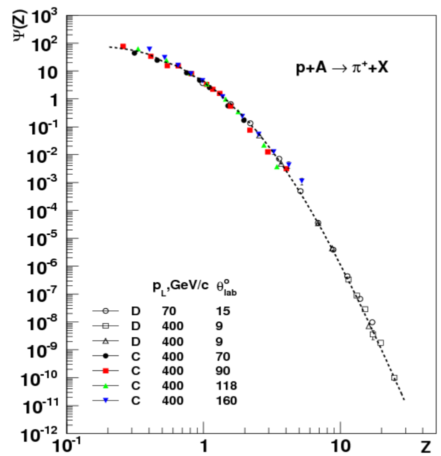


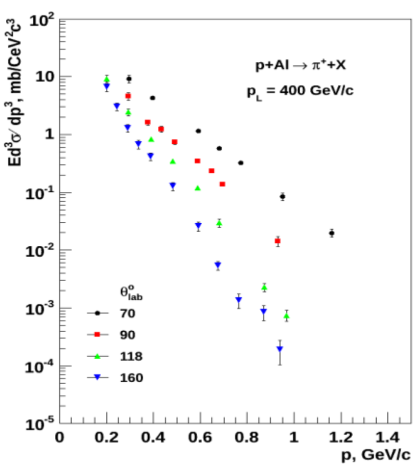
- Spectra in cumulative region: p > 0.5 GeV/c.
- Smooth behavior of spectra vs. p.
- Strong angular dependence with p.
- A-dependence of spectra (A=7-181).

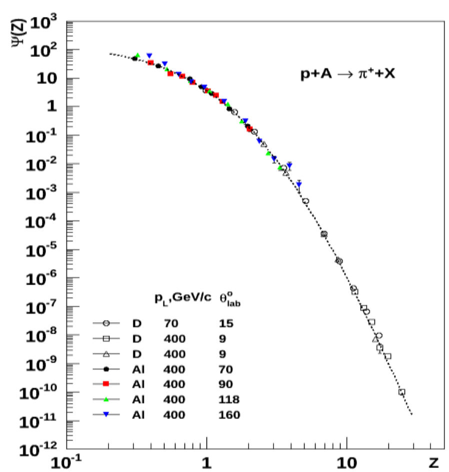
N.A. Nikiforov et al., Phys. Rev. C22 (1980)700.

High-p_T and low-p_T pion production in p+A









C, Al & D

- Collapse of data points
- Universal shape of Ψ(z)
- Self-similarity over a wide kinematic range

$\theta^{\pi}_{lab} = 180^0$

p^{π}_{max} (GeV/c)	p	D	C	Al	p_L (GeV/c)
	0.447	0.905	5.13	10.6	70
	0.456	0.928	5.53	12.2	400

Cumulation
under nucleus compression

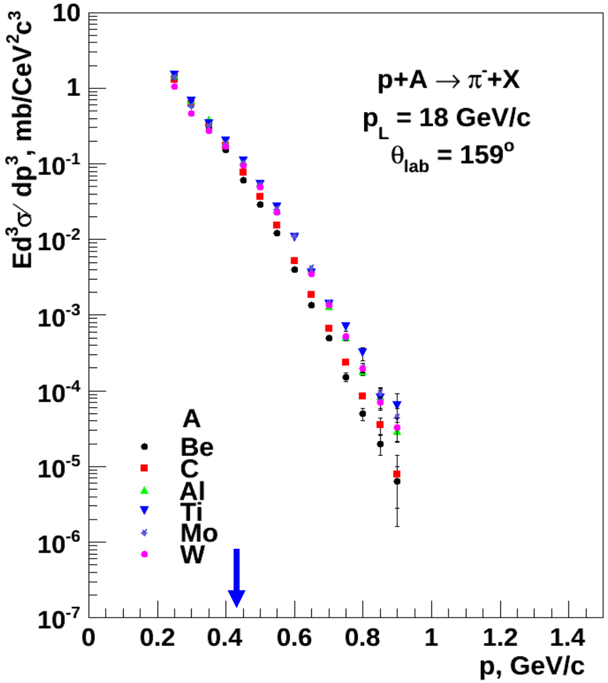
N.A. Nikiforov et al., Phys. Rev. C22 (1980) 700.
&
J.W. Cronin et.al., Phys. Rev. D11 (1975) 3105.
D. Antreasyan et al., Phys. Rev. D19 (1979) 764.
V.V. Abramov et al., Sov. J. Nucl. Phys. 41 (1985) 357.
D.E. Jaffe et al., Phys. Rev. D40 (1989) 2777.

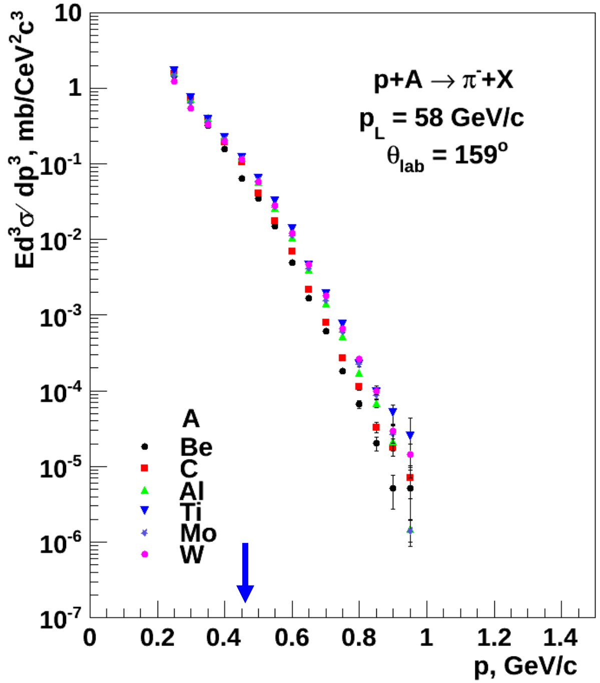
D.Toivonen, M.T. (2003)
A. Aparin, M.T. (2014)

Low-p_T cumulative pion spectra in p+A at U70

p_L =17, 58 GeV/c, A= Be,C,Al,Ti,Mo,W, θ_{lab}=159 deg.

L. Zolin





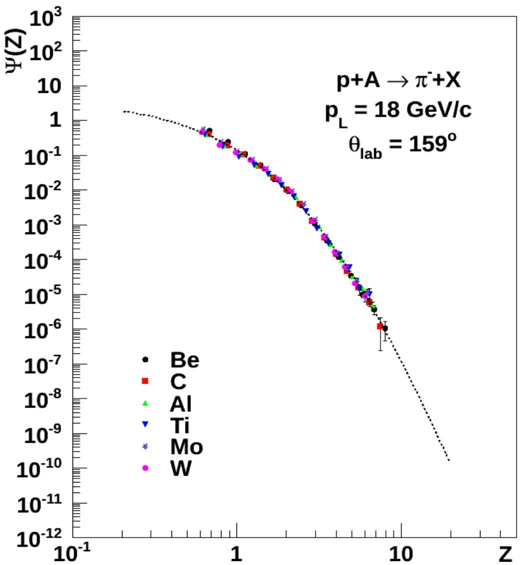
- Spectra in cumulative region: p > 0.5 GeV/c.
- Smooth behavior of spectra vs. p .
- A-dependence of spectra (A= 9-184).

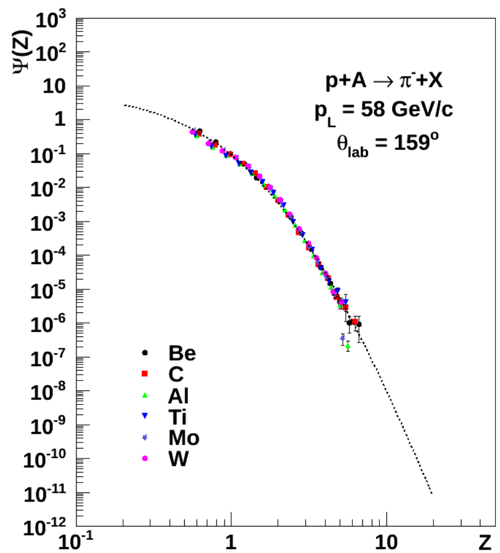
O.P. Gavrishchuk et al., Nucl. Phys. A523 (1991) 589.

Low-p_T cumulative pion spectra in p+A at U70

z-presentation of spectra

U70 (L. Zolin)





Be,C, Al,Ti,Mo,W & D

- Collapse of data points
- Universal shape of $\Psi(z)$
- Self-similarity over a wide kinematic range

$\theta_{lab} = 159^0$

	p	C	Ti	W	p_L (GeV/c)
p^{π}_{max} (GeV/c)	0.43	4.1	9.56	14.1	18
	0.46	5.2	16.3	34.5	58

Cumulation
under nucleus compression

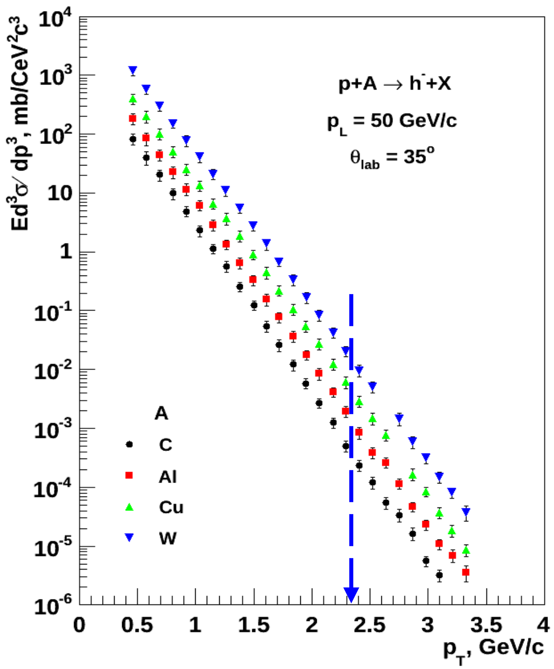
- "Universal shape " of $\Psi(z)$
- Power law for $z > 4$
- No discontinuity of δ_A

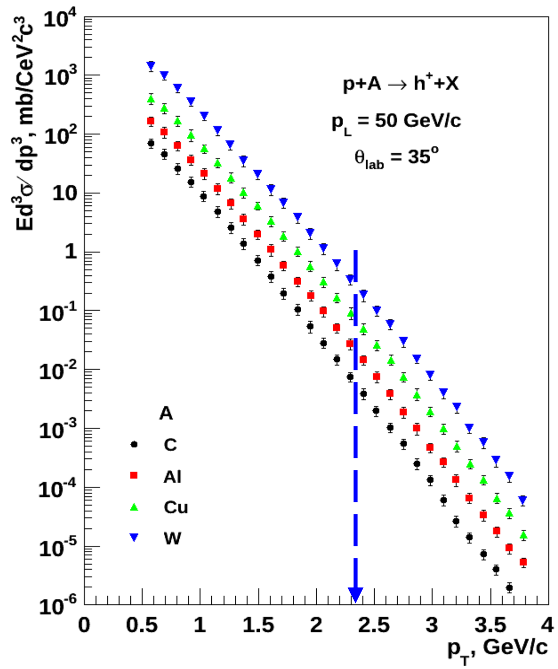
Self-similar properties
of nuclear matter
A=9-184

High-p_T cumulative hadron spectra in p+A at U70

V. Gapienko

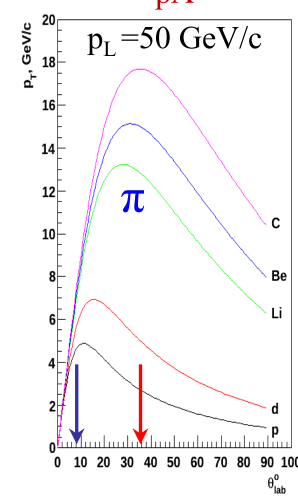
p_L =50 GeV/c, A=C,Al,Cu,W, θ_{lab}=35 deg.





V. Ammosov

pA



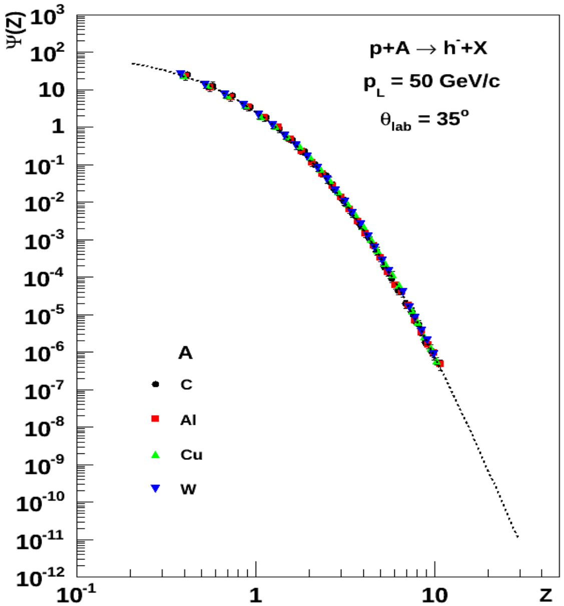
- Spectra in cumulative region: $p_T > 2.5$ GeV/c.
- Smooth behavior of spectra vs. p_T.
- A-dependence of spectra (A=12-184).

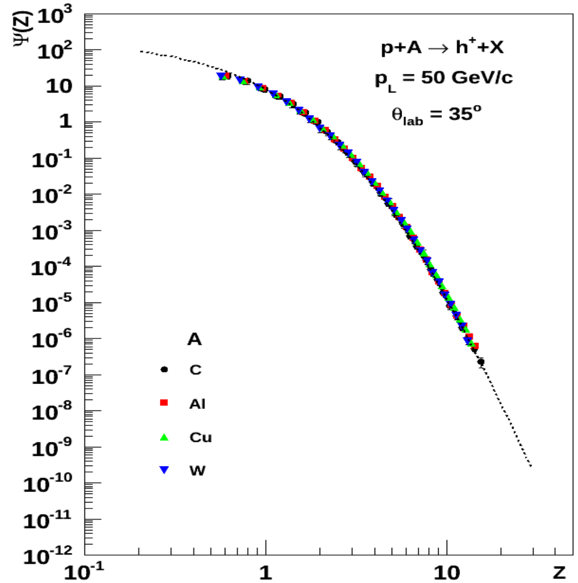
N.N.Antonov et al. (IHEP, Protvino) "Physics of Fundamental Interactions", Russian Academy of Science, ITEP,Moscow, Russia, 21-25 November, 2011.
V.V.Ammosov et al., Yad.Fiz. 76 (2013) 1276.

High-p_T cumulative hadron spectra in p+A at U70

z-presentation of spectra

FNAL (J.Cronin, D.Jaffe) & U70 (R.Sulyaev, V.Gapienko)





C, Al, Cu, W & D

- Collapse of data points
- Universal shape of $\Psi(z)$
- Self-similarity over a wide kinematic range

$p_L = 50$ (GeV/c) $\theta_{lab} = 35^0$

$p^h_{T\,max}$ (GeV/c)	p	C	Al	Cu	W
	2.62	15.6	20.7	24.4	26.7

- Universal shape of $\Psi(z)$
- Power law for $z > 4$
- No discontinuity of δ_A

Cumulation under nucleus compression

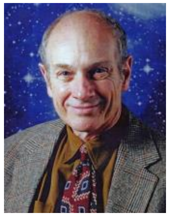
Self-similar properties of nuclear matter A=12-184

A.Aparin, MT
Phys. Part. Nucl. Lett., 11 (2014) 381.

Low-p_T cumulative pion production in p+A at FNAL

G. Leksin

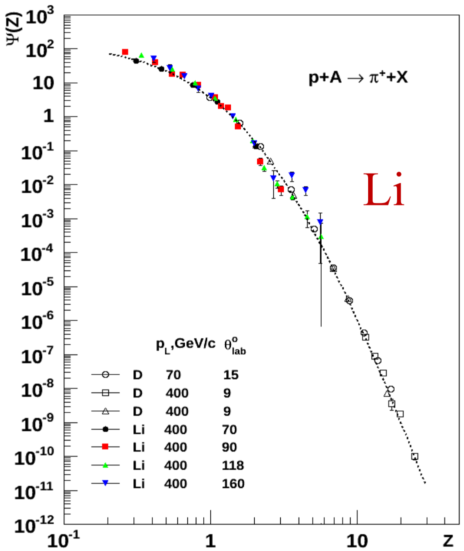
FNAL (J.Cronin, G.Leksin, D.Jaffe) & U70 (R.Sulyaev)

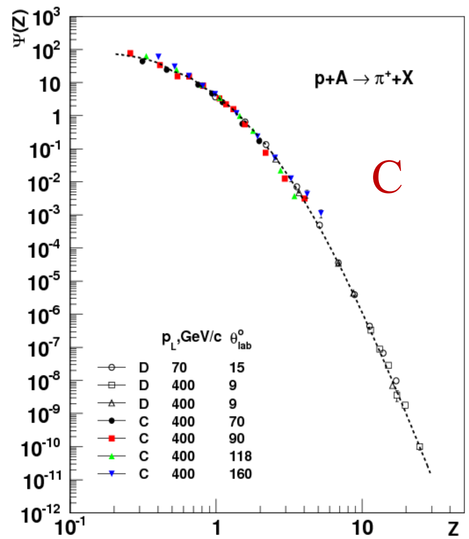


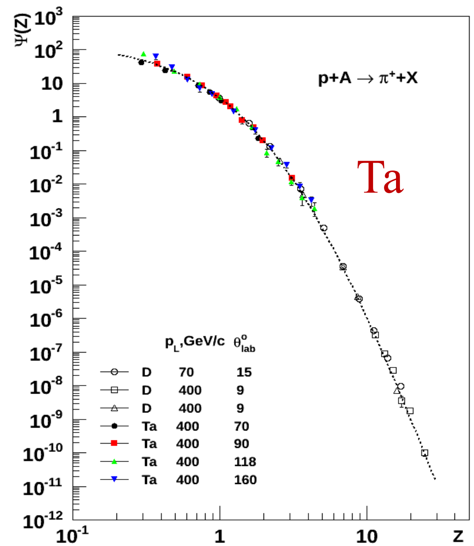
J. Cronin

R. Sulyaev

D. Jaffe







- Universal shape of $\Psi(z)$
- Power law for $z > 4$
- No discontinuity of $\delta_A = A_2\delta$

Scale invariance

Independence of the shape of the curve on $\{z,\Psi\}$ plane on scale quantities $\sqrt{s}$, p_T, θ

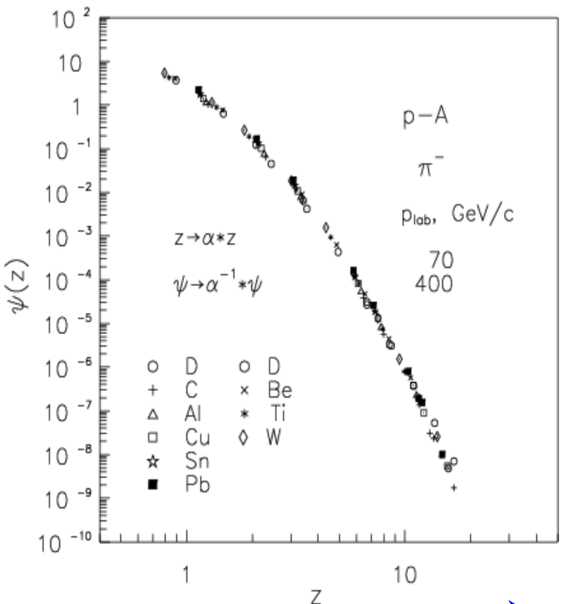
N.A. Nikiforov et al., Phys. Rev. C22 (1980) 700.
A.Aparin, MT, Phys. Part. Nucl., Lett. 11 (2014) 91

$z \to \alpha(A)z$ $\quad$ $\Psi \to \alpha^{-1}(A)\Psi$

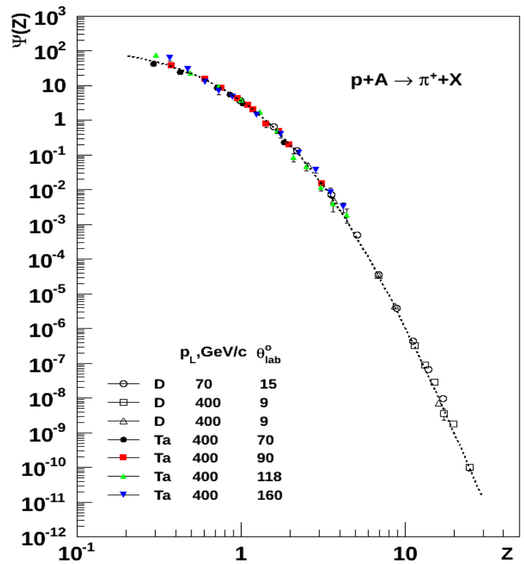
Self-similarity of hadron production in p+A

FNAL (J.Cronin, G.Leksin, D.Jaffe) & U70 (R.Sulyaev, V.Gapienko)

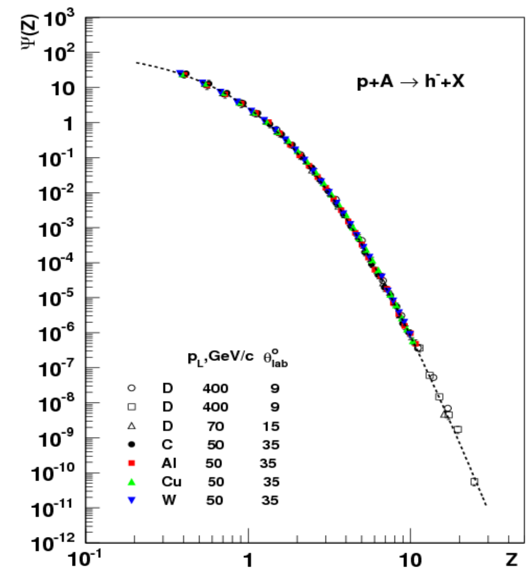
high-p_T & non-cumulative



low-p_T & cumulative



high-p_T & cumulative



- Beam Energy Scan in p+A
- Spectra of cumulative identified particles
- Multiplicity density $dN_{ch}/d\eta$ vs. $\sqrt{s}$ and η
- Centrality dependence of spectra

- Verification of z-scaling in cumulative range
- Smooth transition from high-p_T to cumulative range

Conclusions II

- Self-similarity of particle structure, constituent interactions and fragmentation process of hadron production in non-cumulative and cumulative regions in p+A collisions were verified.
- Smooth behavior of $\Psi(z)$ in the overlapping range was found.

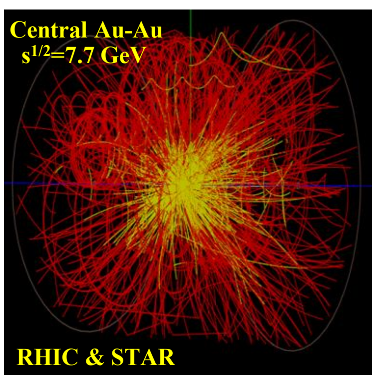
Scaling features of hadron production in Au+Au at RHIC

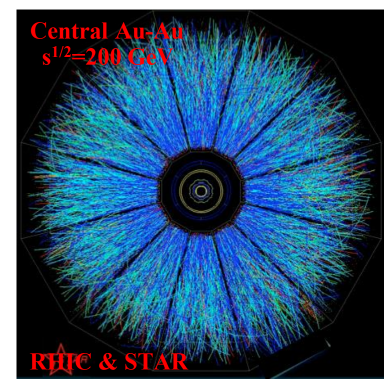
Probing microscopic structure of the hot- and high-density nuclear matter at multiple length scales

Self-similarity of hadron production

RHIC beam energy scan with Au+Au:

$\sqrt{s_{NN}}$ = 7.7, 11.5, 14.5, 19.6, 27, 39, 62, 130, 200 GeV





Int. J. Mod. Phys. (2015) 1560103
Nucl. Phys. A993 (2020) 121646
Nucl. Phys. A1025 (2022) 122492

Self-similarity in Au+Au collisions

Self-similarity parameter

$$z = z_0 \Omega^{-1}$$

$$z_0 = \frac{s_{\perp}^{1/2}}{(dN_{ch}/d\eta|_0)^{c_{AA}} m_N}$$

$$\Omega = (1-x_1)^{\delta_{A_1}}(1-x_2)^{\delta_{A_2}}(1-y_a)^{\varepsilon_{AA}}(1-y_b)^{\varepsilon_{AA}}$$

- $dN_{ch}/d\eta|_0$ - multiplicity density
- c_{AA} - "specific heat" of bulk matter
- δ_A - nucleus fractal dimension
- ε_{AA} - fragmentation dimension

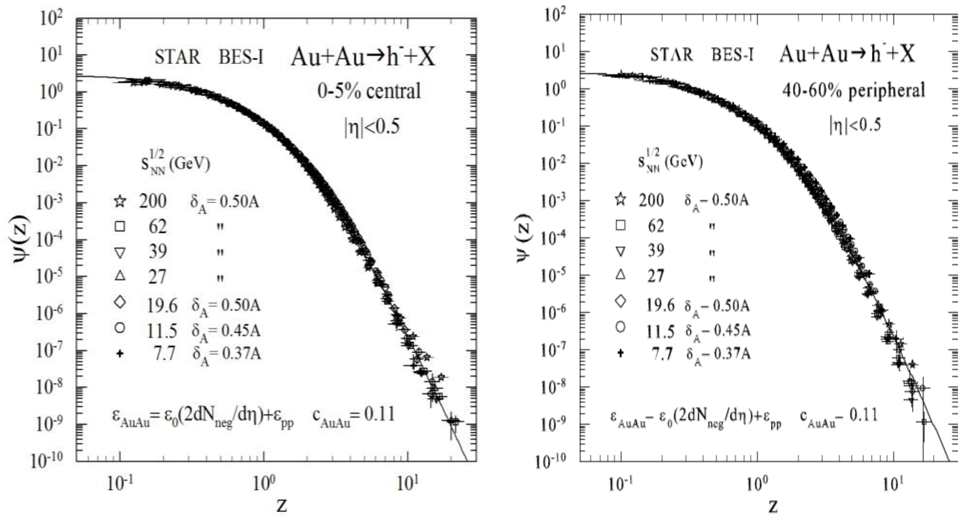
A+A collisions:

$$\delta_A = A\delta$$

$$\varepsilon_{AA} = \varepsilon_0 (dN_{AA}/d\eta) + \varepsilon_{pp}$$

$$\Psi(z) = \frac{\pi}{(dN/d\eta)\,\sigma_{inel}} J^{-1} E \frac{d^3\sigma}{dp^3}$$

"Collapse" of data points onto a single curve



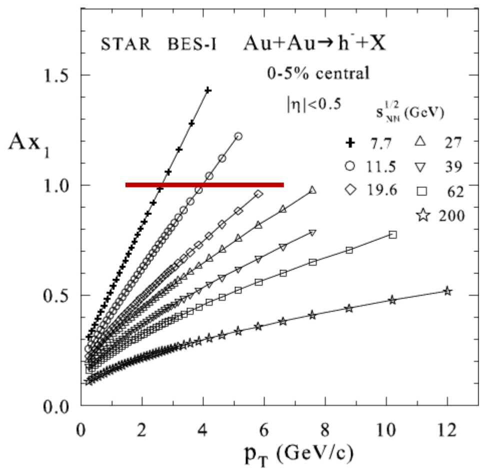
- Energy independence of $\Psi(z)$
- Centrality independence of $\Psi(z)$
- Dependence of ε_{AA} on multiplicity
- Power law at low- and high-z regions

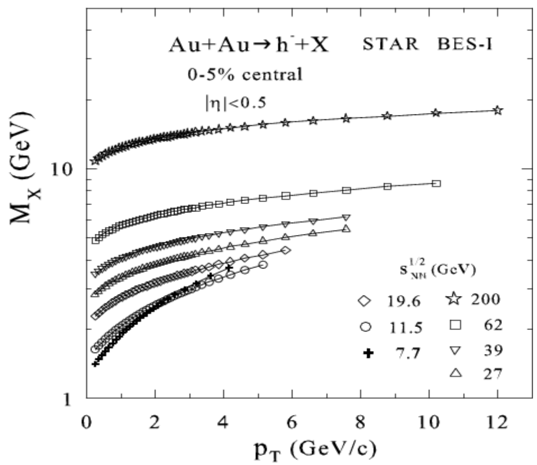
Indication of a decrease of δ for $\sqrt{s_{NN}} < 19.6$ GeV

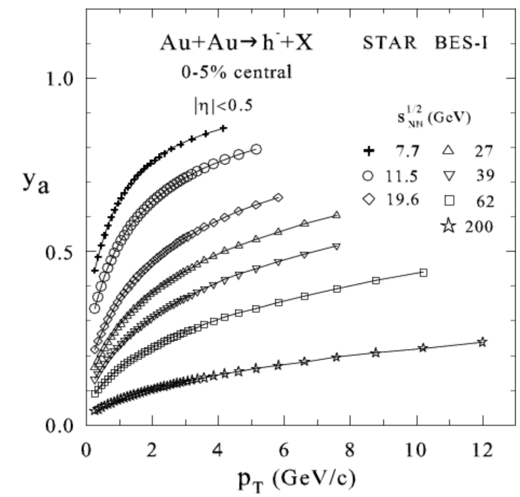
Self-similarity of h^- production in central Au+Au

Constituent sub-process in terms of

Momentum fraction Ax_1 — Recoil mass M_X — Energy loss $\Delta E/E=(1-y_a)$







Ax_1
- increases with p_T
- decreases with $\sqrt{s_{NN}}$

M_X
- increases with p_T
- increases with $\sqrt{s_{NN}}$

$\Delta E/E$
- decreases with p_T
- increases with $\sqrt{s_{NN}}$

Smooth behavior of x_1, y_a, M_X vs. p_T, centrality, collision energy

- High x_1 and p_T → compressed matter
- Large M_X → high density recoil system
- High y_a → small energy loss

Energy loss in Au+Au

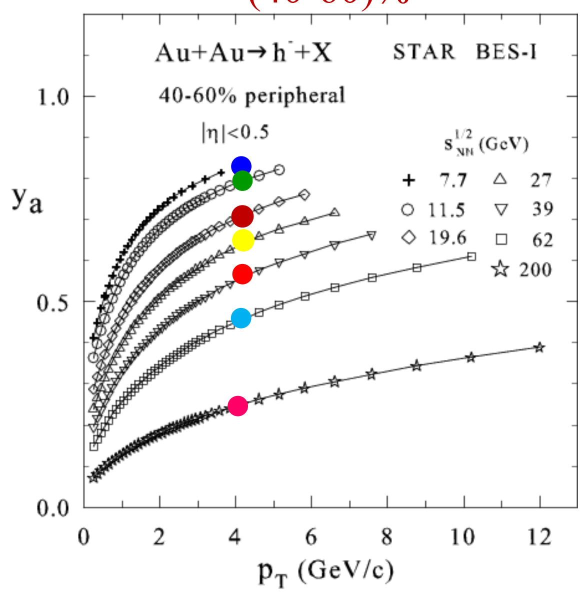
Energy loss $\Delta E/E = (1-y_a)$

p_T – transverse momentum of produced hadron
q – momentum of scattered constituent

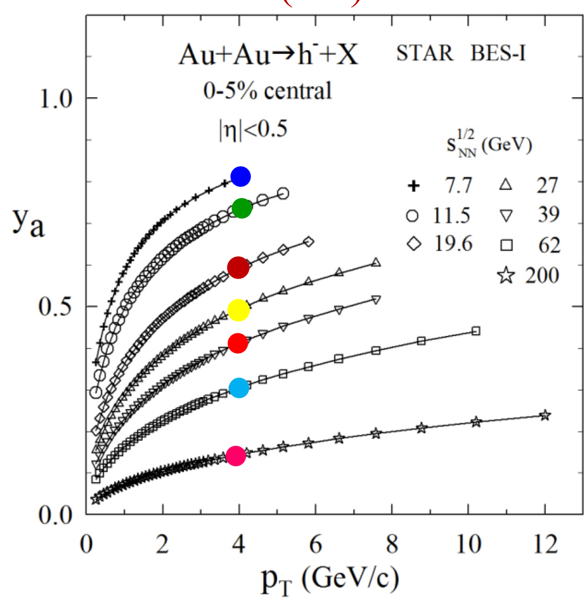
$p_T = 4$ GeV/c , $q = p_T / y_a$

(40-60)%		(0-5)%	
20 % **energy loss** $q \approx 5$ GeV/c			**20 %** **energy loss** $q \approx 5$ GeV/c
22 % **energy loss** $q \approx 5.1$ GeV/c			**25 %** **energy loss** $q \approx 5.3$ GeV/c
30 % **energy loss** $q \approx 5.7$ GeV/c			**40 %** **energy loss** $q \approx 6.7$ GeV/c
35 % **energy loss** $q \approx 6.2$ GeV/c			**50 %** **energy loss** $q \approx 8$ GeV/c
45 % **energy loss** $q \approx 7.3$ GeV/c			**60 %** **energy loss** $q \approx 10$ GeV/c
55 % **energy loss** $q \approx 8.9$ GeV/c			**70 %** **energy loss** $q \approx 13.3$ GeV/c
75 % **energy loss** $q \approx 16$ GeV/c			**85 %** **energy loss** $q \approx 26.6$ GeV/c

(40-60)%



(0-5)%



$\Delta E/E$

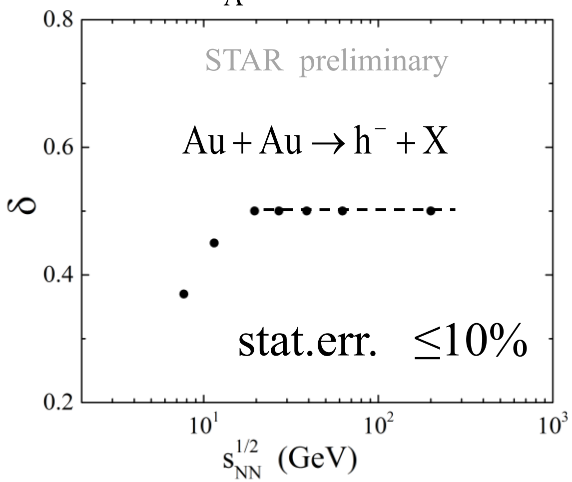
- decreases with p_T
- increases with $\sqrt{s_{NN}}$
- increases with centrality

Model parameters: δ_A, ε_{AA}, c_{AA}

Parameters δ_A, ε_{AA}, c_{AA} are determined from the requirement of scaling behavior of Ψ as a function of self-similarity parameter z

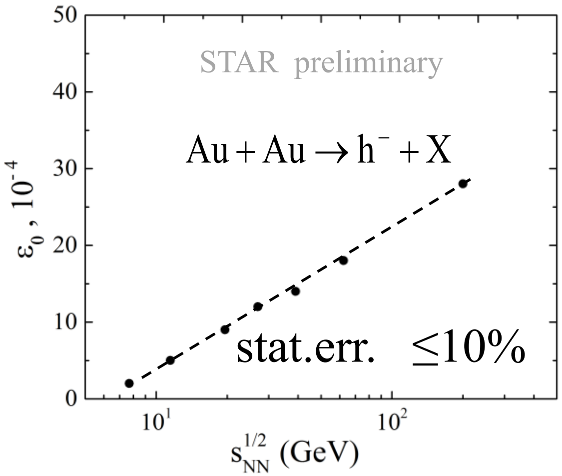
Nucleus fractal dimension

$\delta_A = A \cdot \delta$



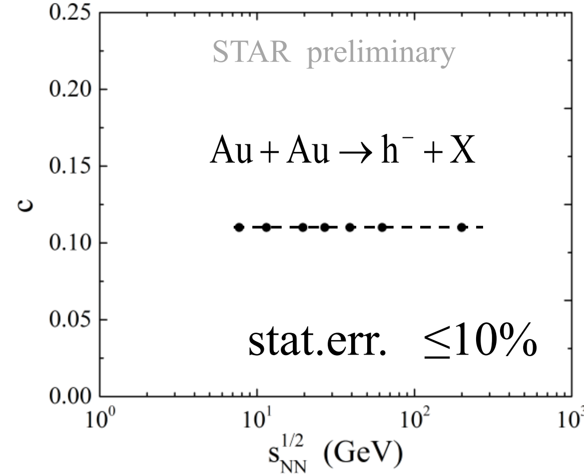
Fragmentation dimension

$\varepsilon_{AA} = \varepsilon_0 (dN_{AA}/d\eta) + \varepsilon_{pp}$



"Specific heat"

c_{AA}

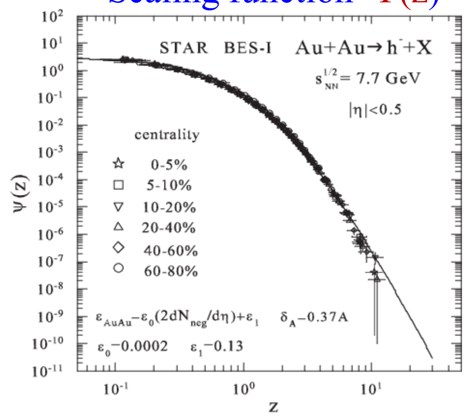


- δ_A decreases with energy for $\sqrt{s_{NN}} \le 20$ GeV
- δ_A is independent of energy for $\sqrt{s_{NN}} \ge 20$ GeV
- ε_{AA} increases with energy
- c_{AA} is independent of energy

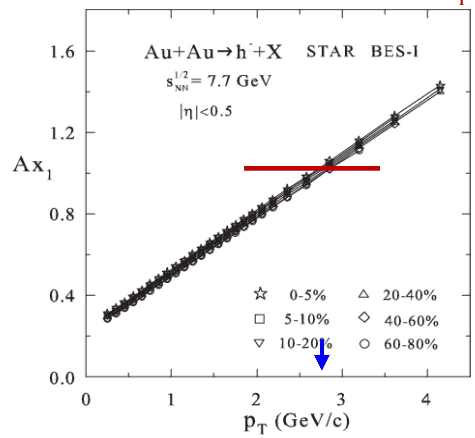
Search for discontinuity and correlations of the model parameters as signatures of Phase Transition and Critical Point.

Cumulative hadron production at RHIC in Au+Au at STAR & $\sqrt{s_{NN}}$ =7.7 GeV

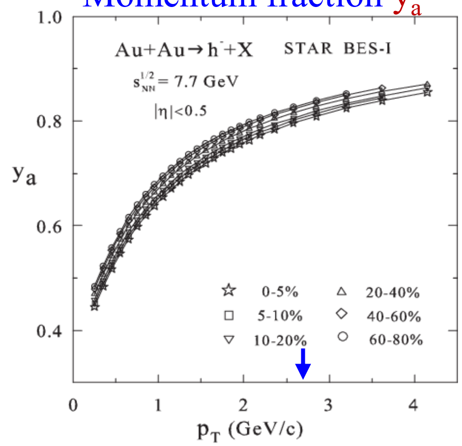
Scaling function $\Psi(z)$



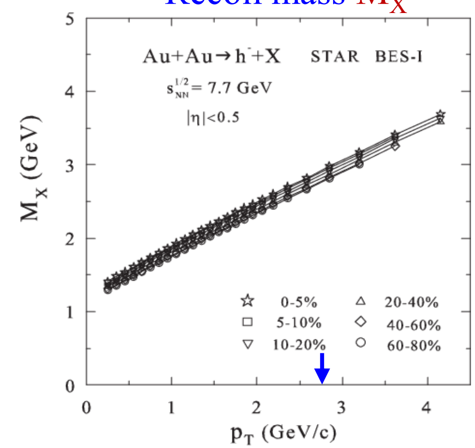
Momentum fraction Ax_1



Momentum fraction y_a



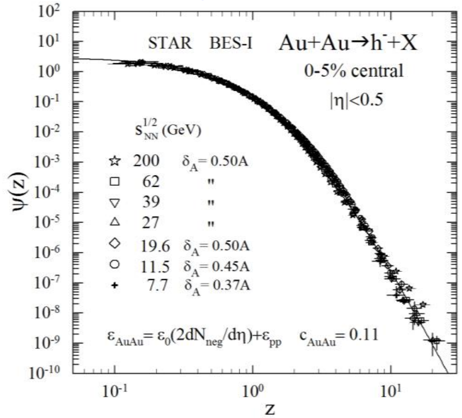
Recoil mass M_X



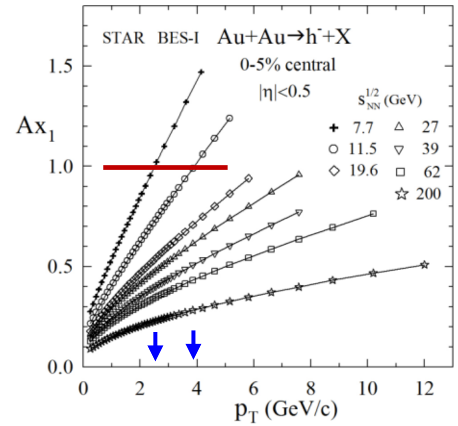
Smooth behavior of $\Psi(z)$, Ax_1, y_a, M_x vs. p_T

Central Au+Au at STAR & $\sqrt{s_{NN}}$ = 7.7 – 200 GeV

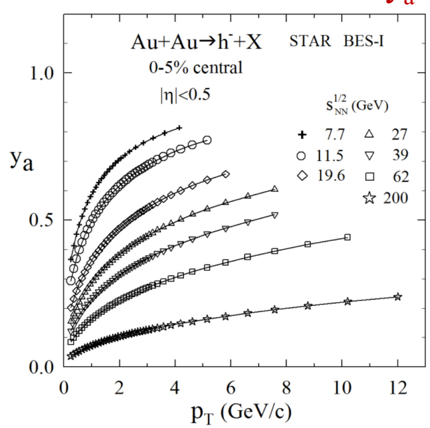
Scaling function $\Psi(z)$



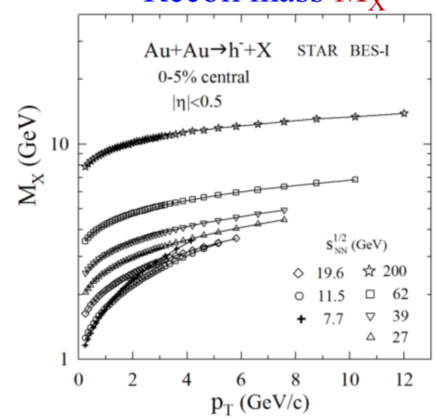
Momentum fraction Ax_1



Momentum fraction y_a



Recoil mass M_X



Cumulative region was only reached at $\sqrt{s_{NN}}$ < 11.5 GeV

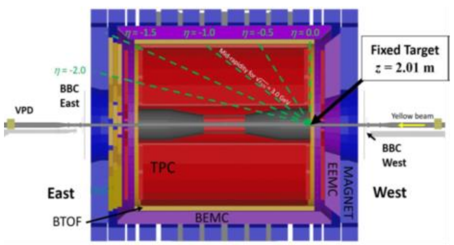
Conclusions III

- The STAR BES-I data on negative hadrons produced in Au+Au collisions cover cumulative region at $\sqrt{s_{NN}}$=7.7, 11.5 GeV.
- Results of analysis demonstrate smooth behavior in z-presentation vs. collision energy, centrality over a wide range of p_T .
- z-Scaling of particle production manifests self-similarity, locality and fractality of hadron interactions at a constituent level.

Self-similarity of cumulative production in collider and fixed target mode at RHIC & NICA ?

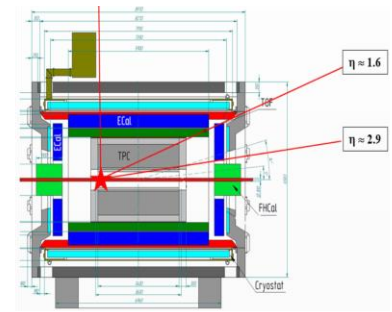
RHIC & STAR

CM: two beams, $\sqrt{s_{NN}}$ = 7.7-200 GeV
FXT: one beam, $\sqrt{s_{NN}}$ = 3.0-7.7 GeV



NICA & MPD

CM: two beams, $\sqrt{s_{NN}}$ = 4-11 GeV
FXT: one beam, $\sqrt{s_{NN}}$ = 2.4-3.5 GeV



We consider that:

- Smaller energy losses is better for localization of a Critical Point.
- High-p_T region is most preferable region to search for a Critical Point.
- Colliding ions should be not very heavy.
- Collision energy $\sqrt{s_{NN}}$ should be not very high.

G.A.Leksin
Phys. At. Nucl. 65 (2002) 1985.

I.Zborovský, MT
Phys. Part. Nucl., Lett. 7 (2010) 271.

I.Zborovský, MT, A.Aparin
Phys. Part. Nucl., Lett. 12 (2015) 221.

Self-similarity of π^- production in Au+Au

Self-similarity parameter

$$z = z_0 \Omega^{-1}$$

$$z_0 = \frac{s_\perp^{1/2}}{(dN_{ch}/d\eta|_0)^{c_{AA}} m_N}$$

$$\Omega = (1-x_1)^{\delta_{A1}}(1-x_2)^{\delta_{A2}}(1-y_a)^{\varepsilon_{AA}}(1-y_b)^{\varepsilon_{AA}}$$

- $dN_{ch}/d\eta|_0$ - multiplicity density
- c_{AA} - "specific heat" of bulk matter
- δ_A - nucleus fractal dimension
- ε_{AA} - fragmentation dimension

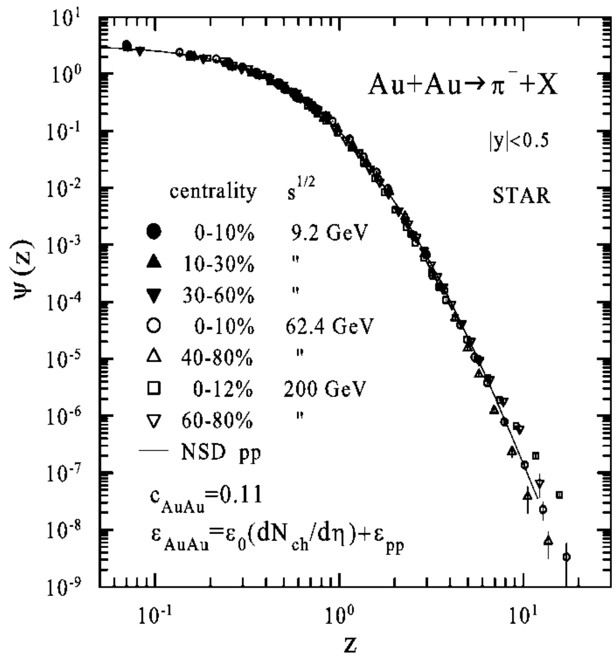
A+A collisions:

$$\delta_A = A\delta$$

$$\varepsilon_{AA} = \varepsilon_0 (dN_{AA}/d\eta) + \varepsilon_{pp}$$

$$\Psi(z) = \frac{\pi}{(dN/d\eta)\,\sigma_{inel}} J^{-1} E \frac{d^3\sigma}{dp^3}$$

"Collapse" of data points onto a single curve



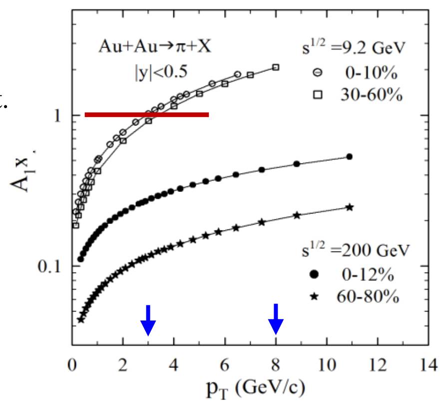
- Energy independence of $\Psi(z)$
- Centrality independence of $\Psi(z)$
- Power law at high z
- Saturation at low z

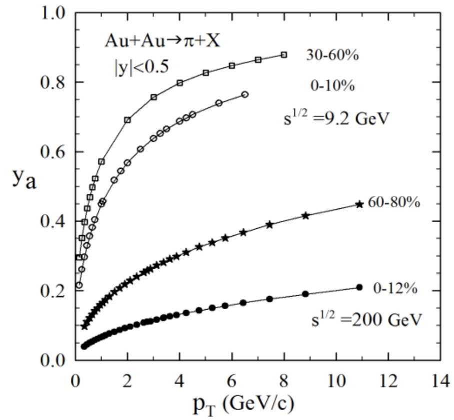
I.Zborovský, MT
Phys. Part. Nucl., Lett. 7 (2010) 271

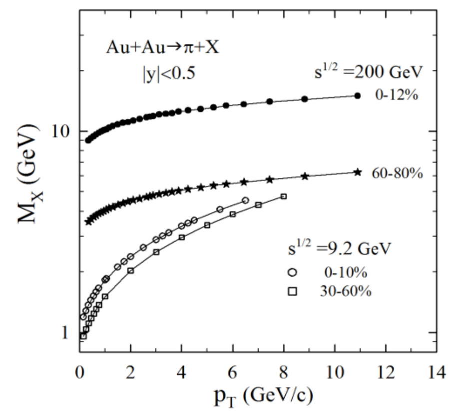
Collider mode of π^- meson production in Au+Au

Momentum fractions x_1, x_2, y_a & recoil mass M_X

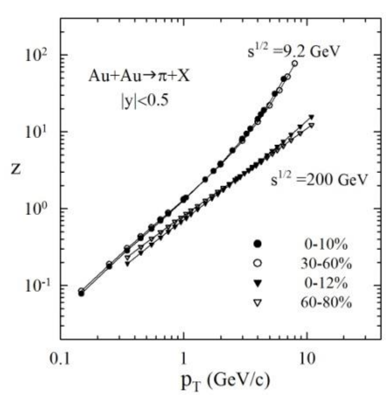
I.Zborovský & MT
Phys.Part.Nucl.,Lett.
7 (2010) 271



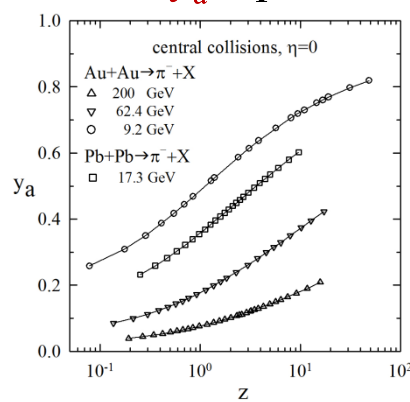




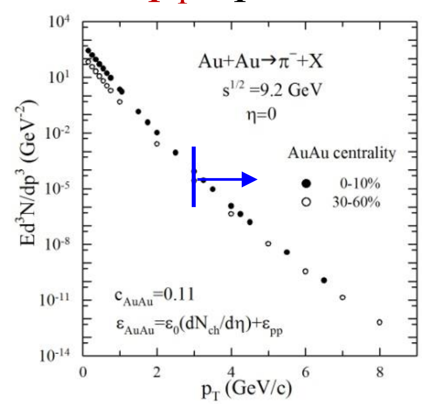
z-p_T plot



y_a-z plot



p_T -spectra



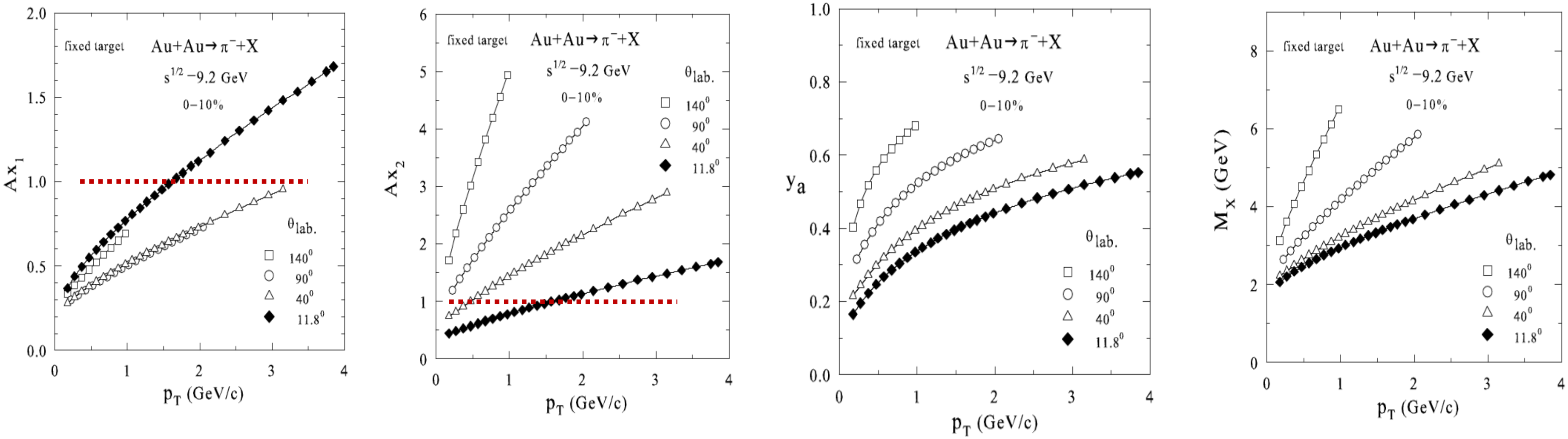
Cumulative region: $A_1x_1 > 1$, $p_T > 3$ GeV/c $\rightarrow A_1x_1 = 2$, $p_T = 8$ GeV/c

Energy loss: $\Delta E/E < 40\% \rightarrow 20\%$

Recoil mass: $M_X > 2$ GeV $\rightarrow$ 4 GeV

Microscopic scenario of pion production in Au+Au

Momentum fractions x_1, x_2, y_a & recoil mass M_X



Kinematics and event selection

- Cumulative range: $Ax_2 > 1$
- Small energy loss : $\Delta E/E = (1-y_a)$
- Probe with high p_T
- Events with high multiplicity

In cumulative region

- Nuclear matter compressed
- Phase transition not smeared

⬇

Clear signatures
of phase transition and critical point

Conclusions IV

- Pion production in collider and fixed target modes in Au+Au collisions in the framework of z-scaling approach were analyzed.
- Dependence of momentum fractions x_1, x_2, y_a and recoil mass M_X on transverse momentum and angle of inclusive particle was studied.
- Verification of self-similarity of cumulative high-p_T pion production in Au+Au collisions was suggested.
- Discontinuity of fractal dimensions of nuclei and fragmentation process and "heat capacity " is proposed to be considered as a signature of a phase transition .

Conclusions

- Data on cumulative hadron spectra obtained by G.Leksin, L.Zolin and V.Gapienko groups in p+A collisions at $\sqrt{s_{NN}} = 11.5 - 27.4$ GeV were reviewed in the framework of z-scaling approach.
- Results of this analysis were compared with previous data obtained by J.Cronin, R.Sulyaev and D.Jaffe groups.
- Indication on self-similarity of hadron production in p+A collisions at high energies in the cumulative region were found.
- Universality of the shape of $\Psi(z)$ was used to predict pion spectra in Au+Au collisions at $\sqrt{s_{NN}} = 9.2$ GeV in cumulative range.
- Cumulative production in collider and fixed target mode is "window" to search for signatures of phase transitions of nuclear matter.

The results can be used to develop programs
to search for new physics phenomena in p+A and A+A collisions
at U70, RHIC, LHC & NICA, FAIR.

Thank You for Your Attention !

Back-up slides

Self-similarity & z-scaling

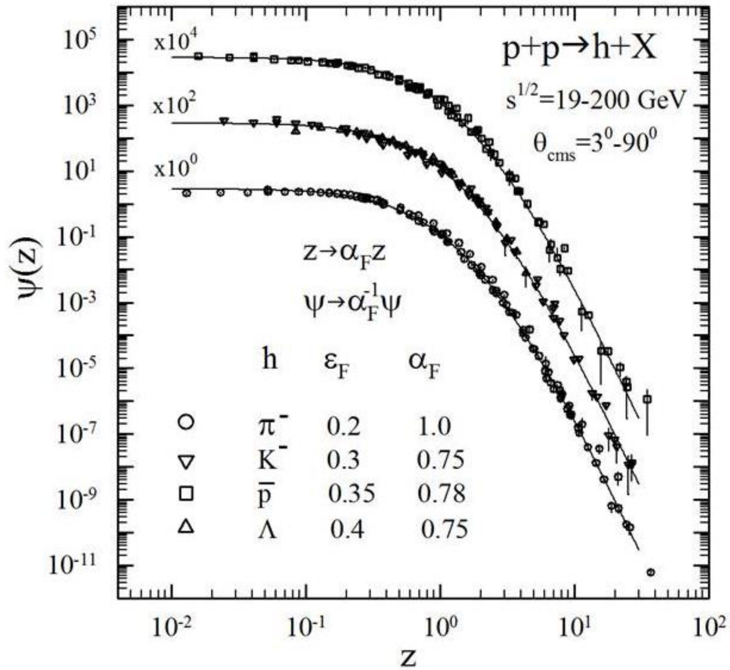
Inclusive cross sections of π^-, K^-, $\bar{p}$, Λ in pp collisions

FNAL:
PRD 75 (1979) 764

ISR:
NPB 100 (1975) 237
PLB 64 (1976) 111
NPB 116 (1976) 77
(low p_T)
NPB 56 (1973) 333
(small angles)

STAR:
PLB 616 (2005) 8
PLB 637 (2006) 161
PRC 75 (2007) 064901

Saturation at low z



Power law at high z

Energy scan of spectra at U70, ISR, S$\bar{p}$pS, SPS, HERA, FNAL(fixed target), Tevatron, RHIC, LHC

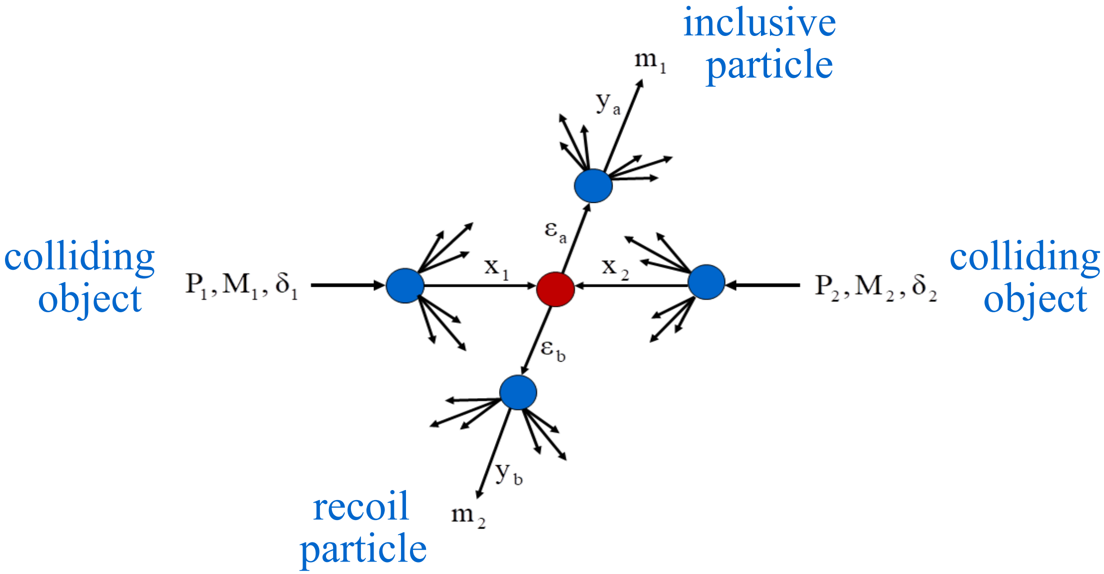
MT & I.Zborovsky
T.Dedovich
Phys.Rev.D75,094008(2007)
Int.J.Mod.Phys.A24,1417(2009)
J. Phys.G: Nucl.Part.Phys. 37,085008(2010)
Int.J.Mod.Phys.A27,1250115(2012)
J.Mod.Phys.3,815(2012)
Int.J.Mod.Phys. A32,1750029(2017)
Nucl. Phys. A993 (2020) 121646

- Energy & angular independence
- Flavor independence (π, K, $\bar{p}$, Λ)
- Saturation for $z < 0.1$
- Power law $\Psi(z) \sim z^{-\beta}$ for high $z > 4$

Scaling – "collapse" of data points onto a single curve.
Universality classes – hadron species (ε_F, α_F).

Locality

Collisions of colliding objects are expressed via interactions of their constituents



P_1, P_2, p – momenta of colliding and produced particles

M_1, M_2, m_1 – masses of colliding and produced particles

x_1, x_2 – momentum fractions of colliding particles carried by constituents

y_a, y_b – momentum fractions of scattered constituents carried by inclusive particle and its recoil

δ_1, δ_2 – fractal dimensions of colliding particles

$\varepsilon_a, \varepsilon_b$ – fractal dimensions of scattered constituents (fragmentation dimensions)

m_2 – mass of recoil particle

Elementary sub-process:

$$(x_1M_1) + (x_2M_2) \rightarrow (m_1/y_a) + (x_1M_1+x_2M_2+m_2/y_b)$$

Momentum conservation law for sub-process

$$(x_1P_1+x_2P_2-p/y_a)^2 = M_X^2$$

Mass of recoil system

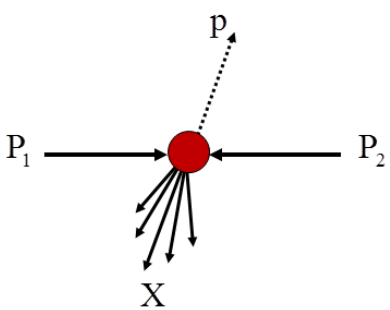
$$M_X = x_1M_1+x_2M_2+m_2/y_b$$

M.T., I.Zborovský
Yu.Panebratsev, G.Skoro
Phys.Rev.D54 5548 (1996)
Int.J.Mod.Phys.A16 1281 (2001)

Self-similarity

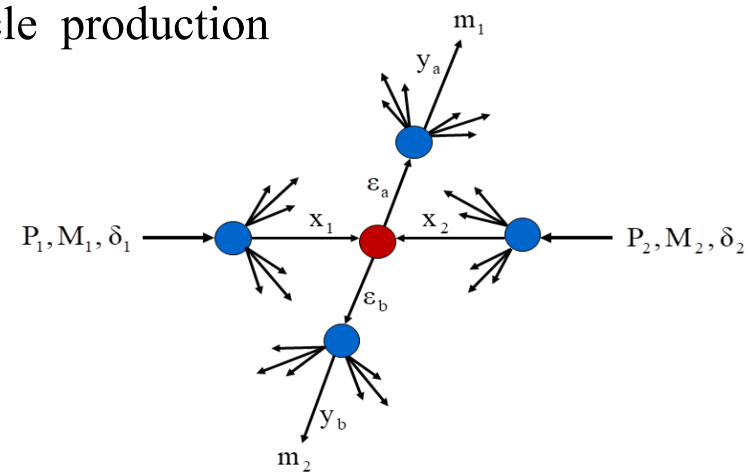
Interactions of constituents are mutually similar

The self-similarity parameter z is a dimensionless variable, expressed through the dimensional quantities P_1, P_2, p, M_1, M_2, m_1, m_2, characterizing the process of inclusive particle production



$$z = z_0 \cdot \Omega^{-1}$$

$$z_0 = \frac{s_\perp^{1/2}}{(dN_{ch}/d\eta|_0)^c m_N}$$



- Ω^{-1} is the minimal resolution at which a constituent sub-process can be singled out of the inclusive reaction
- $s_\perp^{1/2}$ is the transverse kinetic energy of the sub-process consumed on production of m_1 & m_2
- $dN_{ch}/d\eta|_0$ is the multiplicity density of charged particles at $\eta = 0$
- c is a parameter interpreted as a “specific heat” of created medium
- m_N is an arbitrary constant (fixed at the value of nucleon mass)

Fractality

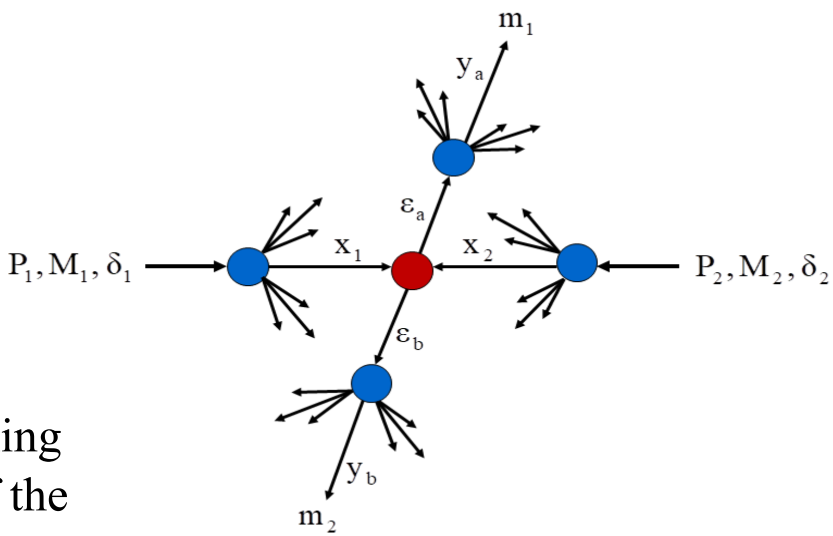
Self-similarity over a wide scale range

Fractal measure

$$z = z_0 \cdot \Omega^{-1}$$

$$\Omega = (1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon_a}(1-y_b)^{\varepsilon_b}$$

$$0 < x_1, x_2 < 1 \qquad 0 < y_a, y_b < 1$$



Ω is relative number of configurations containing a sub-process with fractions x_1, x_2, y_a, y_b of the corresponding 4-momenta

δ_1, δ_2, ε_a, ε_b are parameters characterizing structure of the colliding objects and fragmentation process, respectively

$\Omega^{-1}(x_1, x_2, y_a, y_b)$ characterizes resolution at which a constituent sub-process can be singled out of the inclusive reaction

The fractal measure z diverges as the resolution Ω^{-1} increases.

$$z(\Omega)|_{\Omega^{-1}\to\infty} \to \infty$$

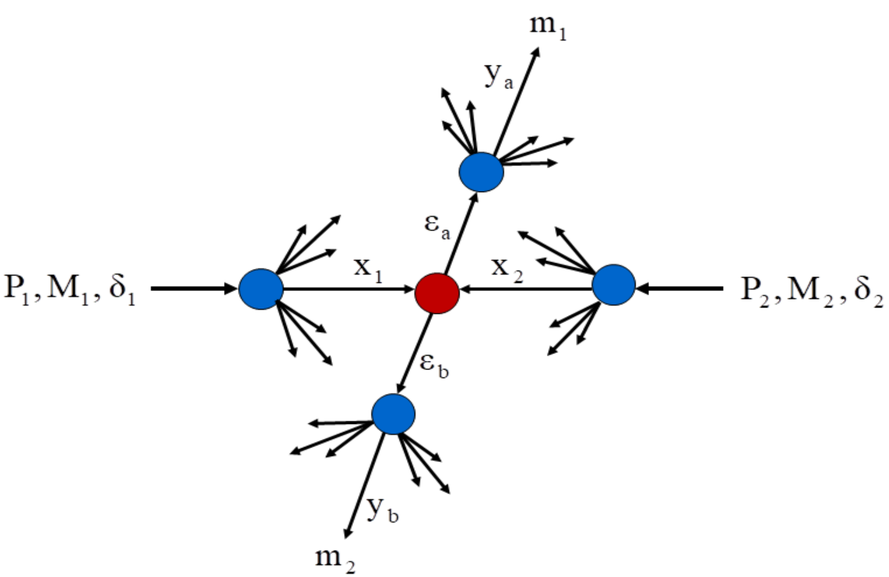
Momentum fractions: x_1, x_2, y_a, y_b

Principle of minimal resolution: The momentum fractions x_1, x_2 and y_a, y_b are determined in a way to minimize the resolution Ω^{-1} of the fractal measure z with respect to all constituent sub-processes taking into account 4-momentum conservation law:

Momentum conservation law

$$(x_1P_1+x_2P_2-p/y_a)^2 = M_X^2$$

$$\begin{cases} \partial\Omega/\partial x_1 \,|_{y_a=y_a(x_1,x_2,y_b)} = 0 \\ \partial\Omega/\partial x_2 \,|_{y_a=y_a(x_1,x_2,y_b)} = 0 \\ \partial\Omega/\partial y_b \,|_{y_a=y_a(x_1,x_2,y_b)} = 0 \end{cases}$$



Resolution of sub-process

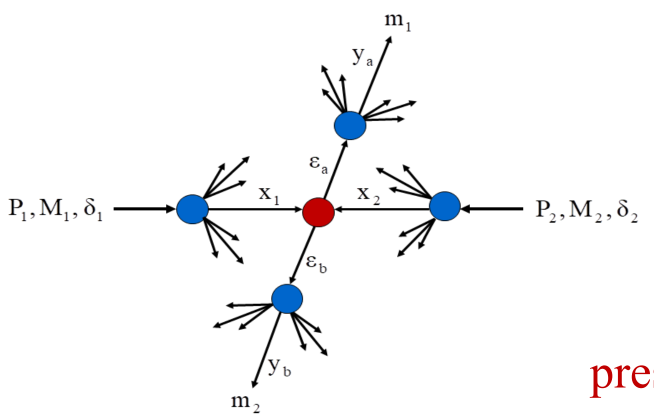
$$\Omega^{-1} = (1-x_1)^{-\delta_1}(1-x_2)^{-\delta_2}(1-y_a)^{-\varepsilon_a}(1-y_b)^{-\varepsilon_b}$$

Fractions x_1, x_2, y_a, y_b are expressed via Lorentz invariants – scalar products of 4-D momenta and particle masses.

Mass of recoil system

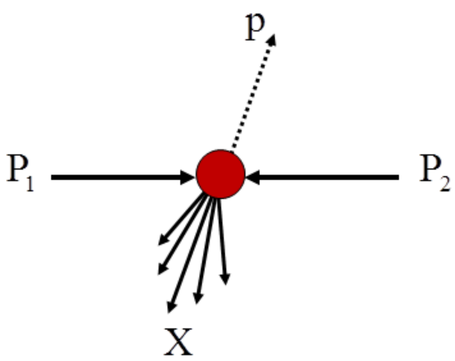
$$M_X = x_1M_1+x_2M_2+m_2/y_b$$

Scaling function $\Psi(z)$



Normalization condition

$$\int_0^\infty \Psi(z)dz = 1$$



Scale transformation

$$z \to \alpha_F \cdot z \qquad \Psi \to \alpha_F^{-1} \cdot \Psi$$

preserves the normalization condition

$$\Psi(z) = \frac{\pi}{(dN/d\eta)\cdot \sigma_{inel}} \cdot J^{-1} \cdot E\frac{d^3\sigma}{dp^3} \Longleftrightarrow \int E\frac{d^3\sigma}{dp^3} dy d^2p_\perp = \sigma_{inel} \cdot \langle N \rangle$$

- σ_{in} - the inelastic cross section
- $\langle N \rangle$ - the average multiplicity
- $dN/d\eta$ - the multiplicity density
- $J(z,\eta;p_T^2,y)$ - the Jacobian
- $Ed^3\sigma/dp^3$ - the inclusive cross section

The scaling function $\Psi(z)$ is a probability density to produce the inclusive particle with the corresponding value of self-similarity variable z.