Neutrino geometrical phase in non-dipolar magnetic fields

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Outline

1. Electromagnetic property of neutrinos
2. Neutrino's geometrical phase in twisted magnetic fields
3. Numerical results

Electromagnetic property of neutrino

In general, the Hamiltonian of neutrino electromagnetic interaction has the following form[1,3]

$$H_{em}^{(\nu)}(x) = j_{\mu}^{(\nu)}(x)A^{\mu}(x) = \sum_{f,i}^{N} \overline{\nu_f}(x)\Lambda_{\nu}^{fi}\nu_i(x)A^{\mu}(x)$$

For the neutrino interaction with magnetic field through the neutrino magnetic moment the Hamiltonian reduces to[2]

$$H_B = -\mu_{\alpha\alpha'}\overline{\nu}_{\alpha'}\vec{\Sigma}\vec{B}\nu_{\alpha} + h.c.$$

The matrix element $\Delta_{\alpha\alpha'}^{ss'}$ in spherical coordinates for a neutrino moving along an arbitrary direction is

$$\langle \nu_{\alpha,s}| H_B |\nu_{\alpha',s'}\rangle = -\frac{1}{2}\mu_{\alpha\alpha'} \int d^3x \nu_{\alpha,s}^{\dagger} \mathbf{B}\gamma_0 \begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & \boldsymbol{\sigma} \end{pmatrix} \nu_{\alpha',s'}.$$

1. C. Giunti and A. Studenikin, Rev. Mod. Phys. 87, 531 (2015).
2. A. Popov and A. Studenikin, Eur. Phys. J. C 79, 144 (2019).
3. C.Giunt, K. Kouzakov, Y.F Li, A. Studenikin, Neutrino Electromagnetic Properties, Ann.Rev. 75(2025)

Neutrino's geometrical phase in twisted magnetic fields

In this study, we are interested in oscillation between electron left-handed and right-handed states

$$\nu_e^L \to \nu_e^R.$$

The evolution equation can be written as[4]

$$i\frac{\partial}{\partial t}\begin{pmatrix}\nu_e^L\\ \nu_e^R\end{pmatrix} = \begin{pmatrix}\left(\frac{\mu}{\gamma}\right)_{ee} B_r + \frac{G_F}{\sqrt{2}}(2n_e - n_n) & \mu_{ee}(B_\theta - iB_\phi)\\ \mu_{ee}(B_\theta + iB_\phi) & -\left(\frac{\mu}{\gamma}\right)_{ee} B_r)\end{pmatrix}\begin{pmatrix}\nu_e^L\\ \nu_e^R\end{pmatrix}$$

With $B_\theta - iB_\phi = B_\perp e^{-i\Phi}$ where $B_\perp = \sqrt{B_\theta^2 + B_\phi^2}$ and $\Phi = \arctan\frac{B_\phi}{B_\theta}$ we can rewrite the Hamiltonian as

$$i\frac{\partial}{\partial t}\begin{pmatrix}\nu_e^L\\ \nu_e^R\end{pmatrix} = \begin{pmatrix}\frac{G_F}{\sqrt{2}}(2n_e - n_n) - \frac{\dot{\Phi}}{2} & \mu_{ee}B_\perp\\ \mu_{ee}B_\perp & \frac{\dot{\Phi}}{2}\end{pmatrix}\begin{pmatrix}\nu_e^L\\ \nu_e^R\end{pmatrix}$$

$$\left(\frac{\mu}{\gamma}\right)_{ee} = \frac{\mu_{11}}{\gamma_{11}}\cos^2\theta + \frac{\mu_{22}}{\gamma_{22}}\sin^2\theta + \frac{\mu_{12}}{\gamma_{12}}\sin 2\theta,$$

$$\mu_{ee} = \mu_{11}\cos^2\theta + \mu_{22}\sin^2\theta + \mu_{12}\sin 2\theta,$$

$$\gamma_{\alpha\beta}^{-1} = \frac{1}{2}(\gamma_\alpha^{-1} + \gamma_\beta^{-1}),$$

$$\gamma_\alpha^{-1} = \frac{m_\alpha}{E_\alpha}.$$

4. Mukhamedshina, A., Stankevich, K., Studenikin, A, Wang, D, *Phys. Atom. Nuclei* (2025).

Neutrino's geometrical phase in twisted magnetic fields

- Resonance condition[5]:

$$\frac{G_F}{2\sqrt{2}}(2n_e - n_n) = \dot{\Phi}$$

- Adiabatic parameter[5]:

$$\frac{8(\mu_{ee}B_\perp)^2}{\frac{G_F}{\sqrt{2}}(2\dot{n}_e - \dot{n}_n) + \ddot{\Phi}}$$

5. Smirnov, A. Y. *Physics Letters B* (1991).

Numerical results in Supernovae environment

$$A_\phi^l(r,\theta) = \frac{B_0^\theta}{(2l+1)} r \frac{r_0^{l+2}}{r^{l+2}+r_0^{l+2}} \frac{P_{l-1}(\cos\theta) - P_{l+1}(\cos\theta)}{\sin\theta},$$

$$B_\phi^l(r,\theta) = \frac{B_0^\phi}{(2l+1)} \frac{r_0^{l+2}}{r^{l+2}+r_0^{l+2}} \frac{P_{l-1}(\cos\theta) - P_{l+1}(\cos\theta)}{\sin\theta}$$

$$B_r^l = (\boldsymbol{\nabla} \times \boldsymbol{A})_r = \frac{1}{r\sin\theta} \frac{\partial}{\partial\theta} \left(\sin\theta A_\phi^l\right),$$

$$B_\theta^l = (\boldsymbol{\nabla} \times \boldsymbol{A})_\theta = -\frac{1}{r} \frac{\partial}{\partial r} \left(r A_\phi^l\right).$$ [6]

$$B_0 = 10^{12}\ \mathrm{G}$$

$$R_0 = 10\ \mathrm{km}$$

$$\rho = 10^9\ \mathrm{g/cm^3}$$

$$n_B(r) = \frac{\rho}{m_B} \frac{R_0^3}{r^3 + R_0^3}$$

$$Y_e = \frac{1}{6}$$

6. M. Bugli, J. Guilet, M. Obergaulinger, P. Cerd´a-Dur´an, and M. A. Aloy, Mon. Not. Roy. Astron. Soc. 492, 58 (2020).

Numerical results in Supernovae environment

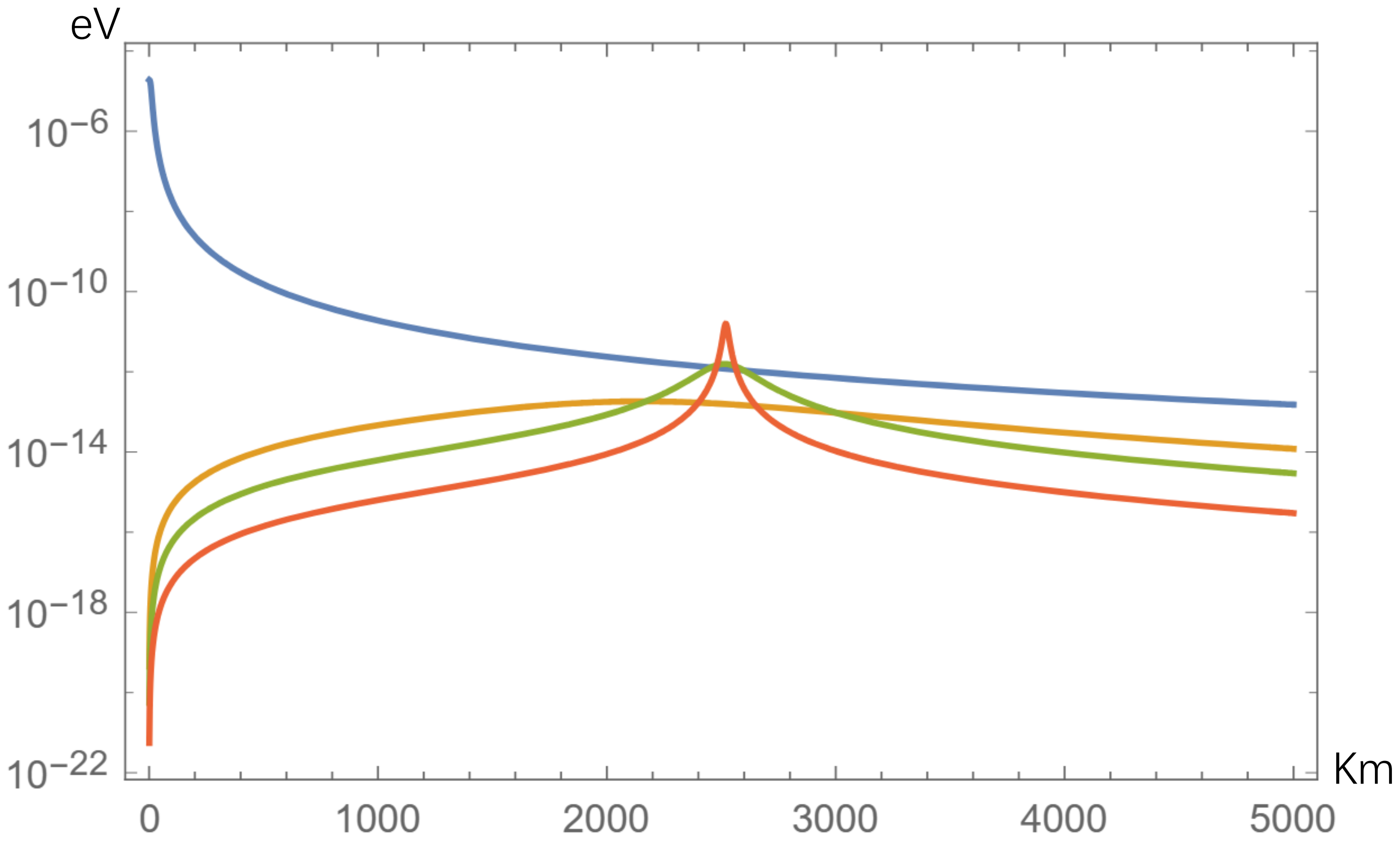


Figure 1: The matter potential V(r) (blue line), the contribution of the geometrical phase ϕ'(r) for η = 1 (orange line), η = 0.2 (green line) and η = 0.01 (red line).

Numerical results in Neutron star environment

$$\mathbf{B} = B_0 \left[\eta_p \nabla\alpha\left(r,\theta\right) \times \nabla\phi + \eta_t \beta\left(\alpha\right) \nabla\phi\right]$$

$$\alpha\left(r,\theta\right) = f(r)\sin^2\theta$$

$$f(r) = \frac{35}{8}\left(r^2 - \frac{6r^4}{5} + \frac{3r^6}{7}\right)$$

$$\beta(\alpha) = \begin{cases} (\alpha-1)^2, & \alpha \geq 1, \\ 0, & \alpha \leq 1, \end{cases}$$ [7]

$$\rho(r) = \rho_c\left(1 - r^2\right)$$

$$\rho_c = 1.8 * 10^{15} g/cm^3$$

$$Y_e(r) = 0.11556 + 0.00931\frac{\rho(r)}{\rho_0} - 0.00352\left(\frac{\rho(r)}{\rho_0}\right)^2$$ [8]

7. Mastrano, A., Melatos, A., Reisenegger, A., & Akgün, T. Monthly Notices of the Royal Astronomical Society, (2011).
8. Gao, Z. F., Shan, H., Wang, W., & Wang, N. Astronomische Nachrichten, (2017).

Numerical results in Neutron star environment

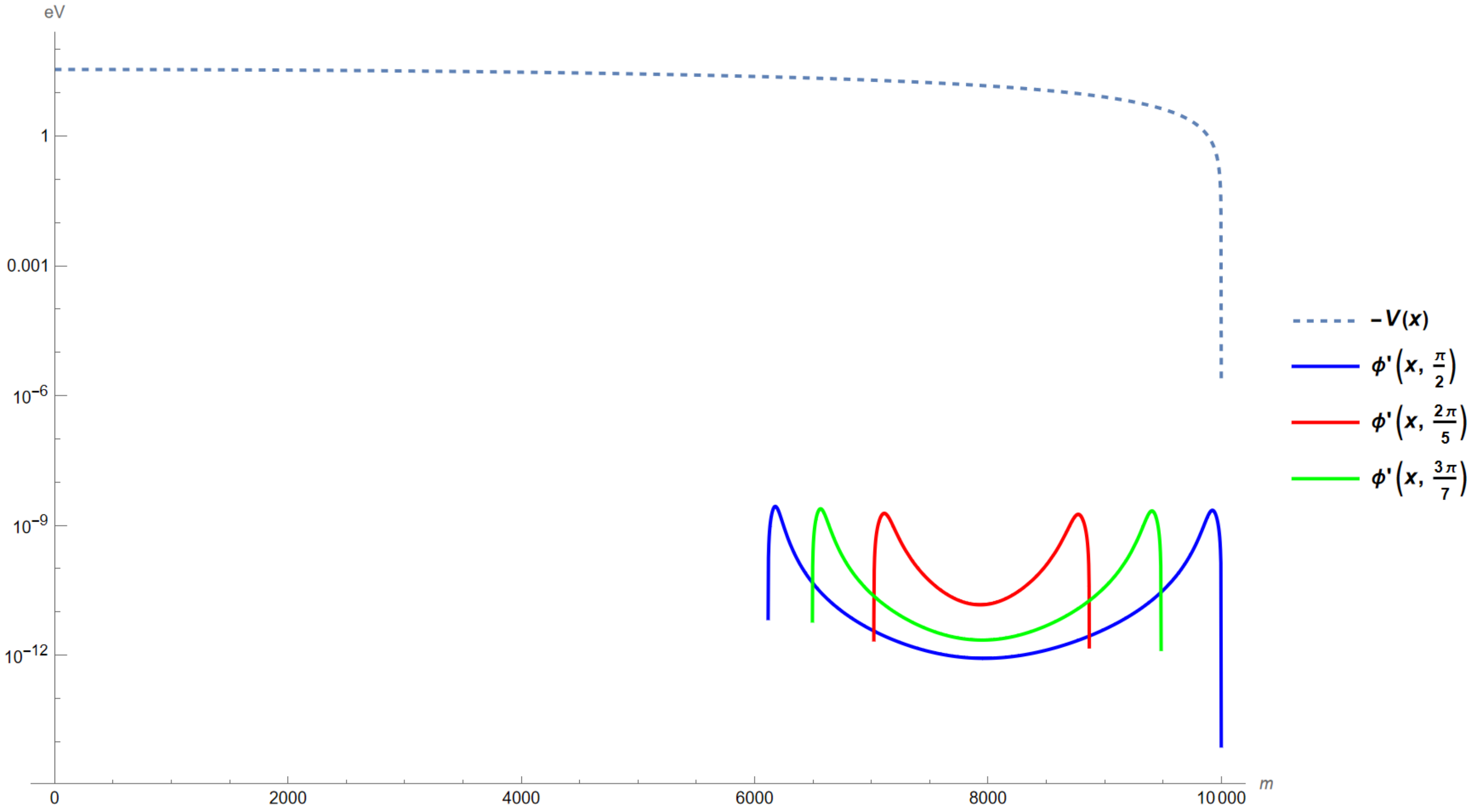


Figure 2: The matter potential V(r) (blue dashed line) and the contribution of the geometrical phase $\phi'(r)$ for
$\theta = \pi/2$(blue line);
$\theta = 2\pi/5$(red line);
$\theta = 3\pi/7$(green line).

Numerical results in Neutron star environment

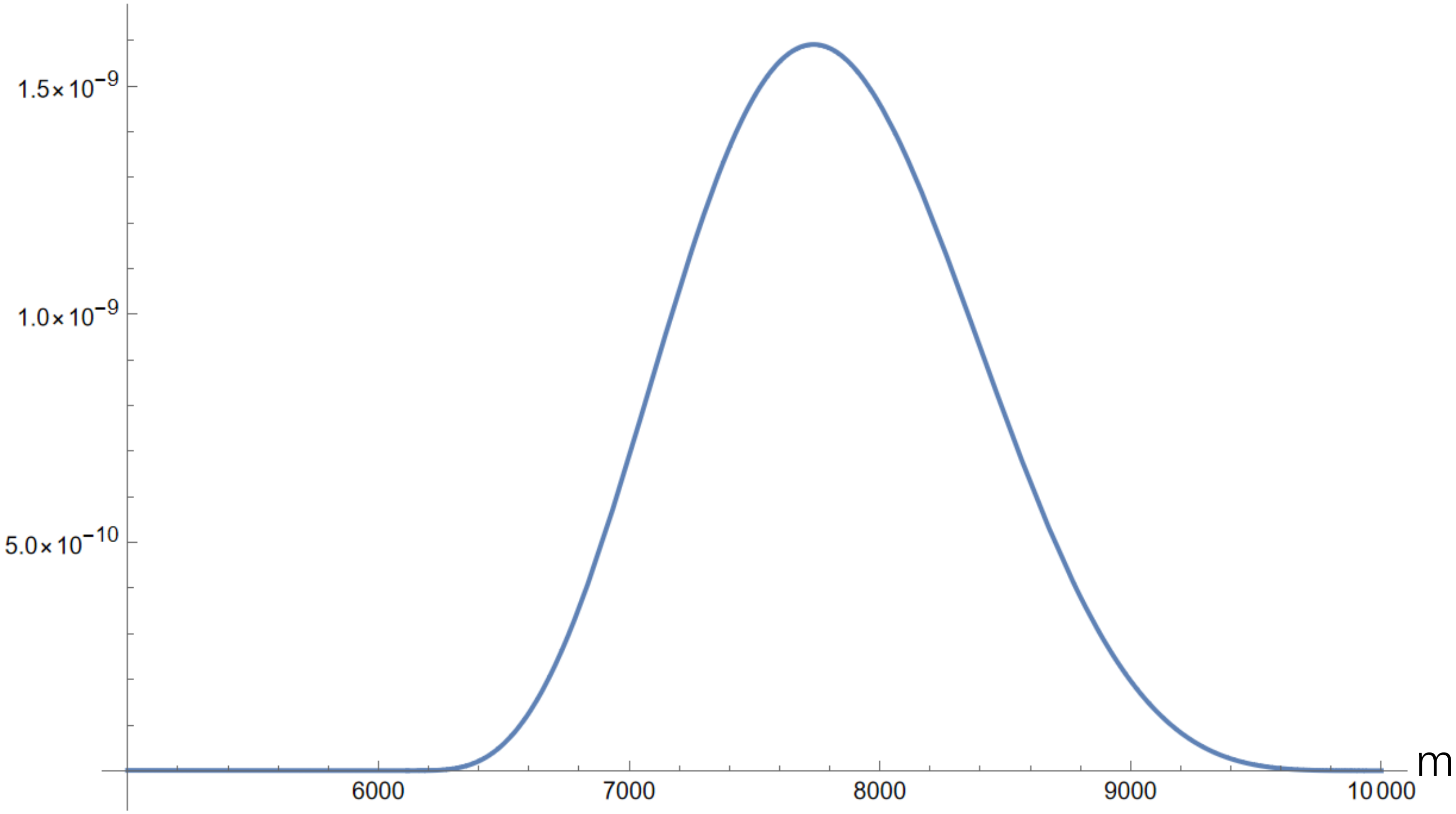


Figure 3: Adiabatic coefficient

Numerical results in Solar analog environment

For magnetic field, we use the equilibrium model constructed by Kamchatnov which has the following form[9]:

$$B_x = \frac{2\left(xz - y\right)}{\left(1 + r^2\right)^3}$$

$$B_y = \frac{2\left(yz - x\right)}{\left(1 + r^2\right)^3}$$

$$B_z = \frac{1 + 2z^2 - r^2}{\left(1 + r^2\right)^3}$$

where x, y, z are unitless Cartesian coordinate and $r^2 = x^2 + y^2 + z^2$

9. Kamchatnov, A. M. Zh. Eksp. Teor. Fiz, 82, 117-124 (1982).

Numerical results in Solar analog environment

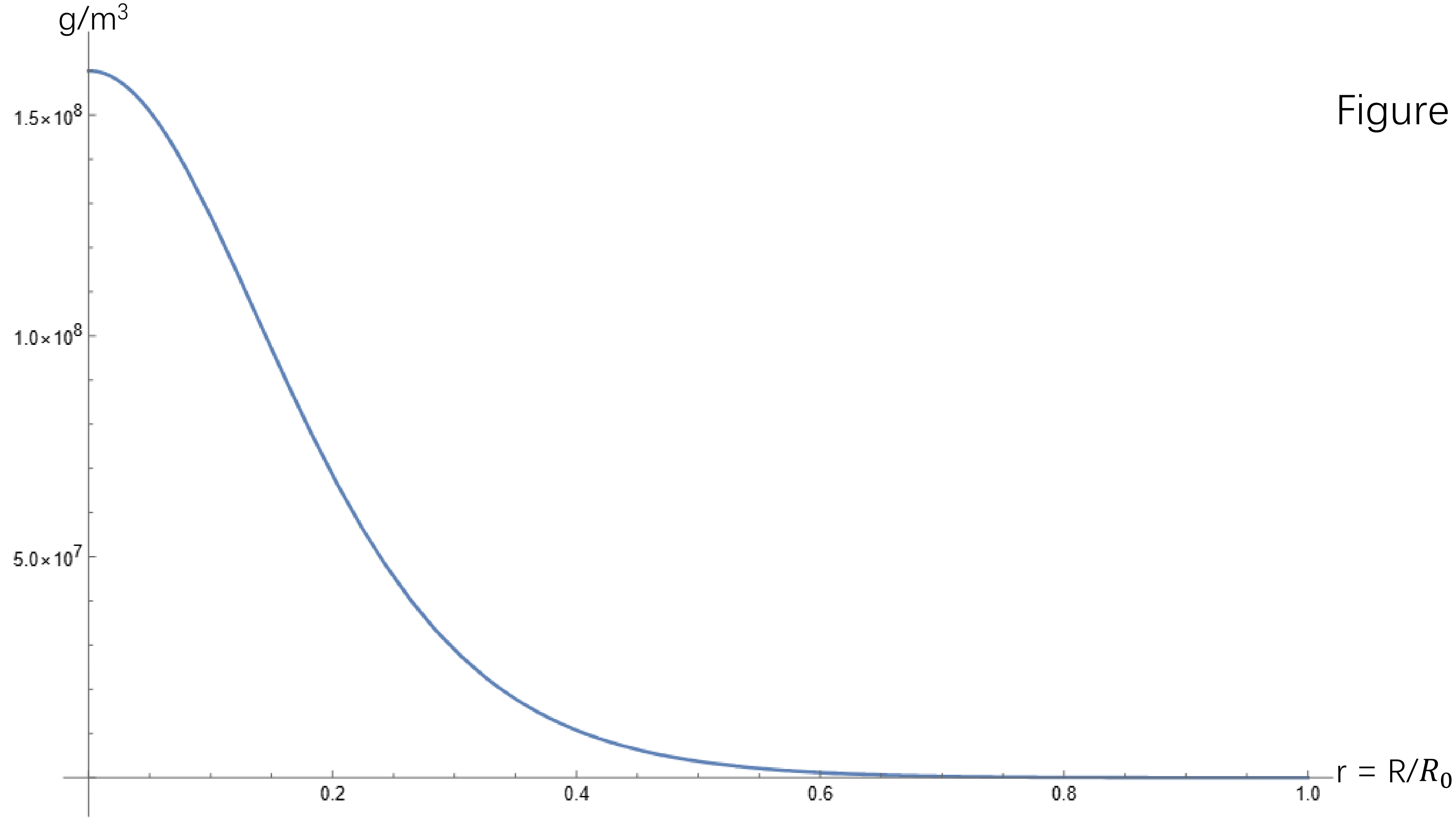


Figure 4: Density profile

Numerical results in Solar analog environment

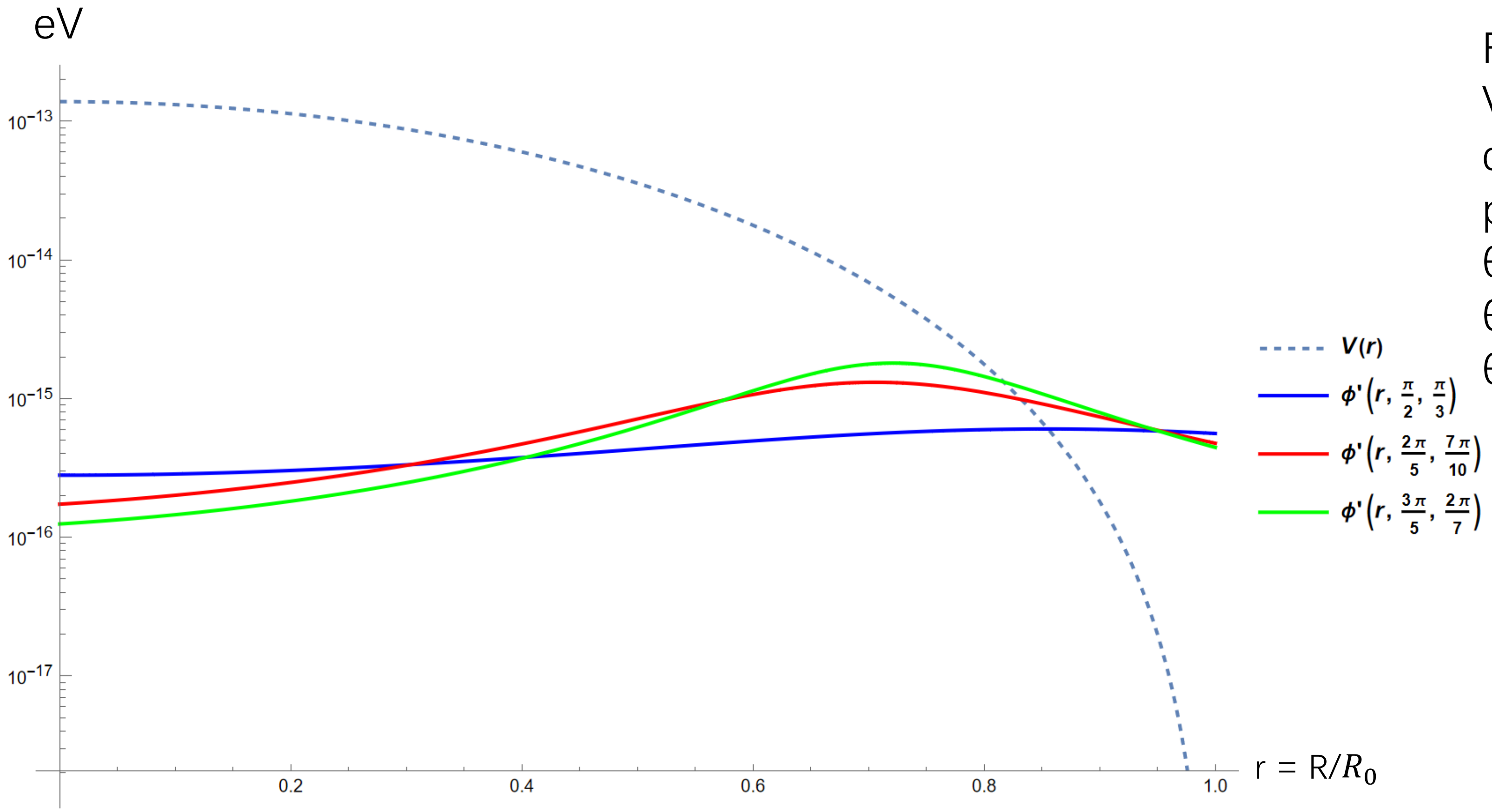
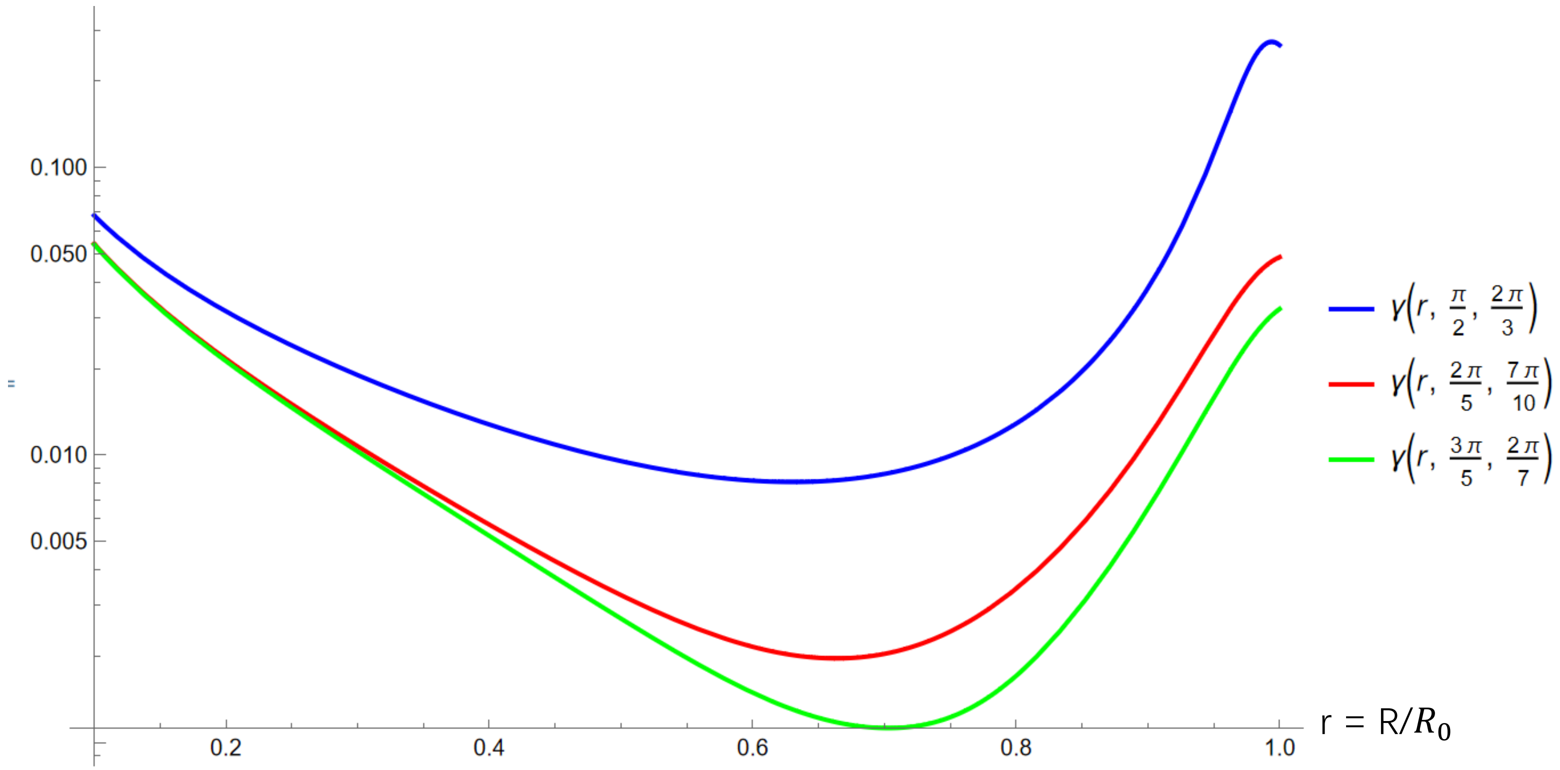


Figure 5: The matter potential V(r) (blue dashed line) and the contribution of the geometrical phase $\phi'(r)$ for
$\theta = \pi/2$, $\varphi = \pi/3$(blue line);
$\theta = 2\pi/5$, $\varphi = 7\pi/10$(red line),
$\theta = 3\pi/5$, $\varphi = 2\pi/7$(green line)

Numerical results in Solar analog environment

Figure 6: Adiabatic coefficient



Numerical results in Ap-star environment

$$\mathbf{B} = B_0\left[\eta_p \nabla\alpha\left(r,\theta\right) \times \delta\phi + \eta_t \beta\left(\alpha\right) \nabla\phi\right]$$

$$\rho(r) = \rho_c \left(\frac{\sin\left(\pi r\right)}{\pi r}\right)^3$$

$$\alpha\left(r,\theta\right) = f(r)\sin^2\theta$$

$$\rho_c = 2.6 * 10^6 g/m^3$$

$$f(r,\theta) = \left(\frac{43}{8}r^2 - \frac{33}{4}r^4 + \frac{39}{8}r^6 - r^8\right)$$

$$\beta(\alpha) = \begin{cases} (\alpha - 1)^2, & \alpha \geq 1, \\ 0, & \alpha \leq 1, \end{cases}$$ [10]

10. Becerra, L., Reisenegger, A., Valdivia, J. A., & Gusakov, M. Monthly Notices of the Royal Astronomical Society, 517(1), 560-568 (2022).

Numerical results in Ap-star environment

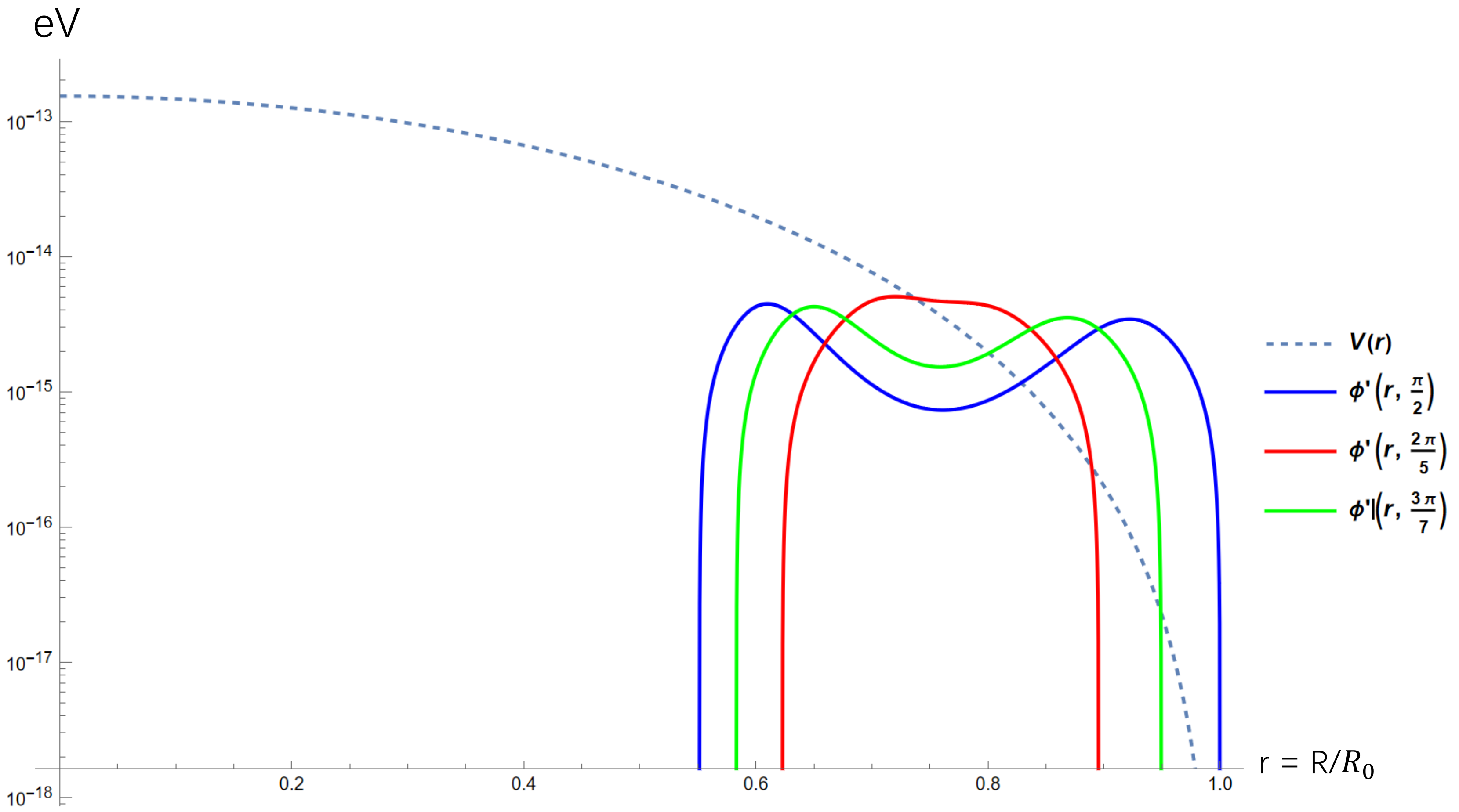


Figure 7: The matter potential V(r) (blue dashed line) and the contribution of the geometrical phase ϕ'(r) for
$\theta = \pi/2$(blue line);
$\theta = 2\pi/5$(red line);
$\theta = 3\pi/7$(green line).

Numerical results in Ap-star environment

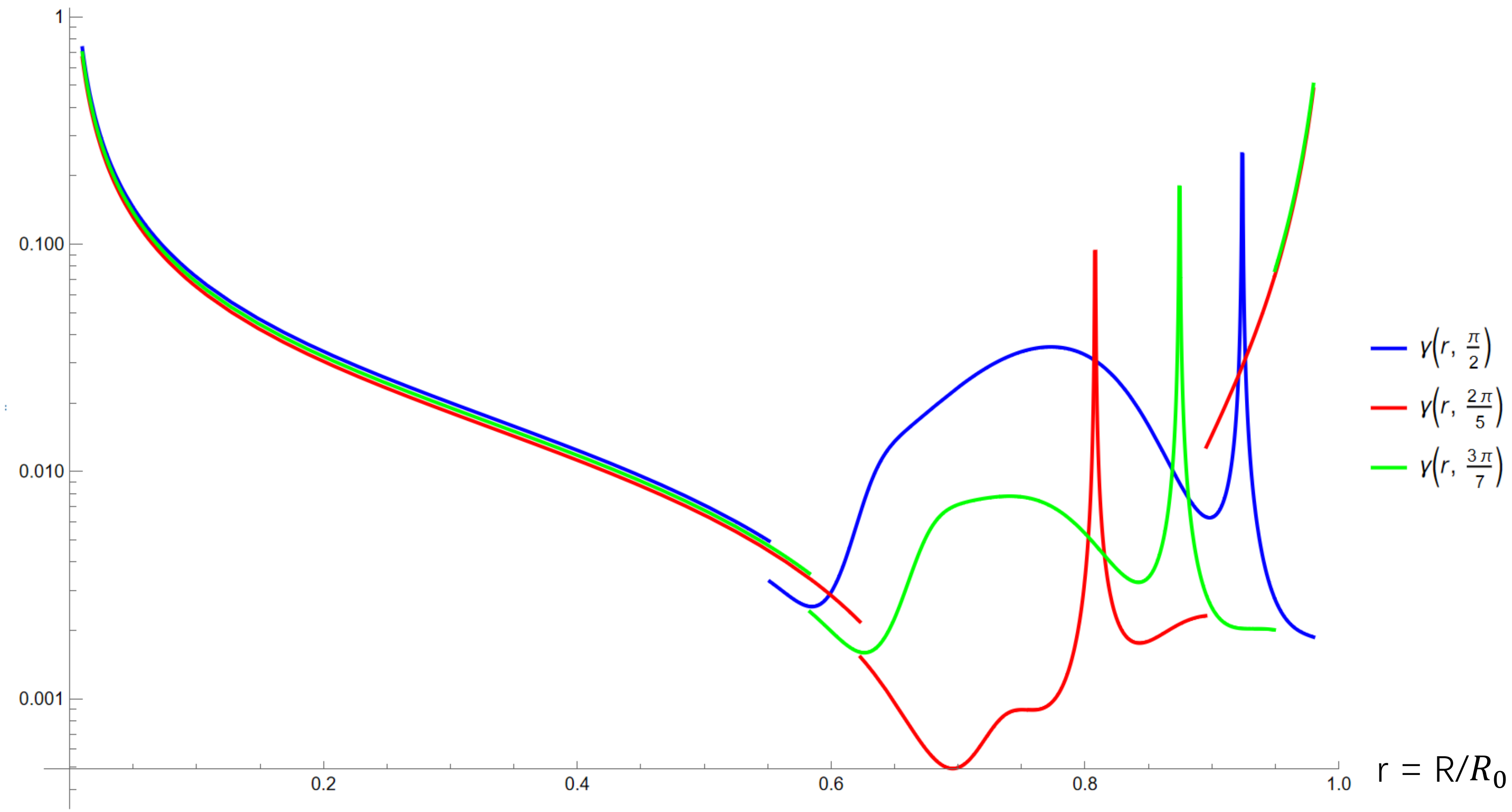


Figure 8: Adiabatic coefficient

Reference

1. Giunti, C., & Studenikin, A. (2015). Neutrino electromagnetic interactions: a window to new physics. *Reviews of Modern Physics*, *87*(2), 531-591.
2. Popov, A., & Studenikin, A. (2019). Neutrino eigenstates and flavour, spin and spin-flavour oscillations in a constant magnetic field. *The European Physical Journal C*, *79*(2), 1-7.
3. C.Giunt, K. Kouzakov, Y.F Li, A. Studenikin, Neutrino Electromagnetic Properties, Ann.Rev. 75(2025)
4. Mukhamedshina, A., Stankevich, K., Studenikin, A. *et al.* (2025). Neutrino Spin and Spin–Flavor Oscillations in Nondipolar Magnetic Fields of Astrophysical Objects. *Phys. Atom. Nuclei* **88**, 280–284.
5. Smirnov, A. Y. (1991). The geometrical phase in neutrino spin precession and the solar neutrino problem. *Physics Letters B*, *260*(1-2), 161-164.
6. Bugli, M., Guilet, J., Obergaulinger, M., Cerdá-Durán, P., & Aloy, M. A. (2020). The impact of non-dipolar magnetic fields in core-collapse supernovae. *Monthly Notices of the Royal Astronomical Society*, *492*(1), 58-71.
7. Mastrano, A., Melatos, A., Reisenegger, A., & Akgün, T. (2011). Gravitational wave emission from a magnetically deformed non-barotropic neutron star. *Monthly Notices of the Royal Astronomical Society*, *417*(3), 2288-2299.
8. Gao, Z. F., Shan, H., Wang, W., & Wang, N. (2017). Reinvestigation of the electron fraction and electron Fermi energy of neutron star. *Astronomische Nachrichten*, *338*(9-10), 1066-1072.
9. Kamchatnov, A. M. (1982). Topological solitons in magnetohydrodynamics. Zh. Eksp. Teor. Fiz, 82, 117-124.
10. Becerra, L., Reisenegger, A., Valdivia, J. A., & Gusakov, M. (2022). Stability of axially symmetric magnetic fields in stars. Monthly Notices of the Royal Astronomical Society, 517(1), 560-568.

Thank you for your attention!