Formalism of open quantum systems in neutrino scattering on Superfluid Helium

SATURNE Project

TWENTY- SECOND LOMONOSOV CONFERENCE ON ELEMENTARY PARTICLE PHYSICS

K. Kouzakov, F. Lazarev, K. Stankevich, A. Studenikin, M. Vyalkov, and A. Yukhimchuk

MSU Sarov, NCPhM

25.08.25

Presentation Outline

1. Neutrinos
   - Window to New Physics
   - Neutrino Electromagnetic Properties
   - Neutrino-Atom Scattering
2. Superfluid Helium
   - Dynamic structure factor
3. Open quantum systems
   - Formalism and principles
4. SATURNE project
   - Single quasiparticle generation
   - OQS formalism for neutrino-HeII scattering
5. Conclusion

Neutrinos - window to new physics

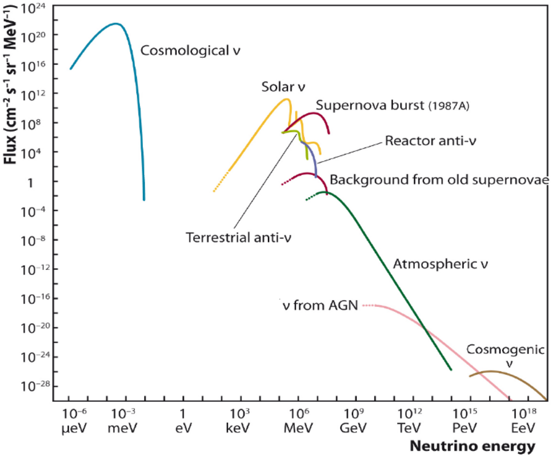
Relevance

Neutrinos are a window to new physics.

C.Guinti, A.Studenikin, "Neutrino electromagnetic interactions: a window to new physics" Rev. Mod. Phys. 87 (2015) has over 600 citations.

SATURNE project

Study of coherent elastic neutrino-atom scattering and setting record constraints on neutrino magnetic moment.



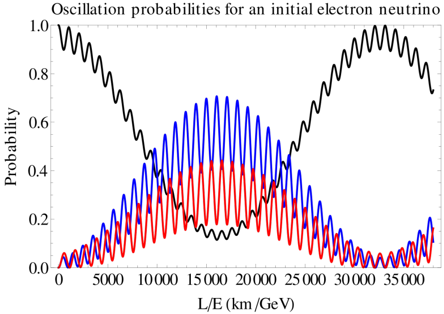
Neutrino Oscillations: Essence and Significance

Physical essence:

- Flavor transformation ($\nu_e \leftrightarrow \nu_\mu \leftrightarrow \nu_\tau$)
- Requires massive neutrinos ($m_\nu \neq 0$)

Historical milestones:

- 1957 - B. Pontecorvo predicts oscillations
- 1998 - Super-Kamiokande discovers atmospheric ν_μ oscillations
- 2001 - SNO confirms solar ν_e oscillations
- 2015 - Nobel Prize for neutrino mass discovery



Fundamental implications

- **New physics**: Standard Model requires modification

Neutrino Electromagnetic Properties

Key fact

Neutrinos have **no electric charge** in SM but massive neutrinos imply non-zero magnetic moment.

Magnetic moment

Minimally extended SM prediction

$$\mu_\nu \approx 3.2 \times 10^{-19} \mu_B$$

Experimental limits

- Borexino (solar ν): $\mu_\nu < 2.8 \times 10^{-11} \mu_B$
- GEMMA (reactor $\bar{\nu}$): $\mu_\nu < 2.9 \times 10^{-11} \mu_B$

C.Guinti, A.Studenikin, "Neutrino electromagnetic interactions: a window to new physics" Rev. Mod. Phys. 87 (2015)

Coherent Neutrino-Nucleus Scattering (CEνNS)

Physics:

- Neutrino interacts with entire nucleus
- Coherent when $qR < 1$ (q - momentum transfer)
- Cross section $\propto N^2$ (N - neutron number)
- Sensitive to:
  - Weak angle θ_W
  - Magnetic moment μ_ν
  - New interactions

Differential cross section:

$$\frac{d\sigma^{\mathrm{CE\nu NS}}}{dE_R} = \frac{G_F^2}{\pi} C_V^2 m_N \left(1 - \frac{m_N E_R}{2E_\nu^2}\right),$$

$$C_V = \frac{1}{2}\left[(1 - 4\sin^2\theta_W)ZF_Z - NF_N\right]$$

Modern experiments:

- COHERENT (USA):
  - First CEνNS observation (2017)
  - CsI[Na], Ar, Ge detectors
- CONUS (Germany):
  - Ge detectors at reactor
- MINER (USA):
  - Cryogenic detectors
- RED-100 (Russia):
  - Liquid xenon

Fundamental principles of CEνAS

Coherence condition:

$$q \cdot R_{\text{atom}} \ll 1$$

where: q – momentum transfer, R_{atom} – atomic radius

Differential cross section (nucleus):

$$\frac{d\sigma^{\text{CE}\nu\text{NS}}}{dE_R} = \frac{G_F^2}{\pi} C_V^2 m_N \left(1 - \frac{m_N E_R}{2E_\nu^2}\right),$$

$$C_V = \frac{1}{2}\left[(1 - 4\sin^2\theta_W) Z F_Z - N F_N\right]$$

Modification for atom:

$$C_V^{\text{Atom}} = C_V + \frac{1}{2}(\pm 1 + 4\sin^2\theta_W) Z F_e(q^2)$$

- $+$ for ν_e, $\bar{\nu}_e$, $-$ for other neutrinos
- $F_e(q^2)$ – electron form factor

Key effects:

- Electrons screen the weak charge of the nucleus
- Sharp dip in cross section at $T_R \sim 9$ meV
- Complete screening when $C_V^{\text{Atom}} = 0$

Why helium specifically?

- Smallest atomic radius ($R_{\text{atom}} \approx 0.5$ Å)
- Light atom $\Rightarrow$ large recoil energy
- Detection possibility via quantum evaporation

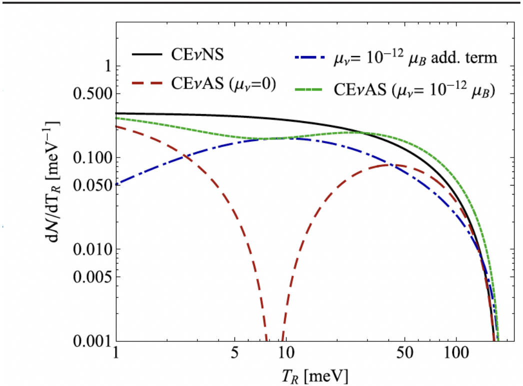
Cross section modification:

$$\left.\frac{d\sigma^{\mathrm{CE}\nu\mathrm{AS}}}{dT_R}\right|_{\mu_\nu\neq 0} \simeq \frac{d\sigma^{\mathrm{CE}\nu\mathrm{AS}}}{dT_R} + \frac{\pi\alpha^2 Z^2}{m_e^2}\left(\frac{\mu_\nu}{\mu_B}\right)^2 \cdot \left(\frac{1}{T_R}-\frac{1}{E_\nu}\right)\left(1-F_{\mathrm{e}}\left(T_R\right)\right)^2,$$

Features:

- Atomic form factor $(1-F_e(T_R))^2$
- Enhancement at low T_R
- For helium $(Z=2)$:

$$\propto \left(\frac{\mu_\nu}{\mu_B}\right)^2 \frac{1}{T_R}$$



M. Cadeddu, F. Dordei, C. Giunti, K. Kouzakov, E. Picciau and A. Studenikin, Phys. Rev. D 100 (2019)

Superfluid Helium (He-II)

Unique properties:

- Transition temperature $T_\lambda = 2.17$ K
- Density $\rho = 145$ kg/m^3
- Sound velocity $c_1 \approx 238$ m/s
- Second sound velocity $c_2 \approx 20$ m/s

Discovery history:

- 1908 - H. Kamerlingh Onnes: **Liquefaction**
- 1938 - P. Kapitsa: **Superfluidity** (Nobel Prize 1978)
- 1941 - L. Landau: **Two-fluid model**
- 1995 - E. Cornell, C. Wieman: **BEC in gases** (Nobel Prize 2001)
    - Normal component (ρ_n)
    - Superfluid component (ρ_s)

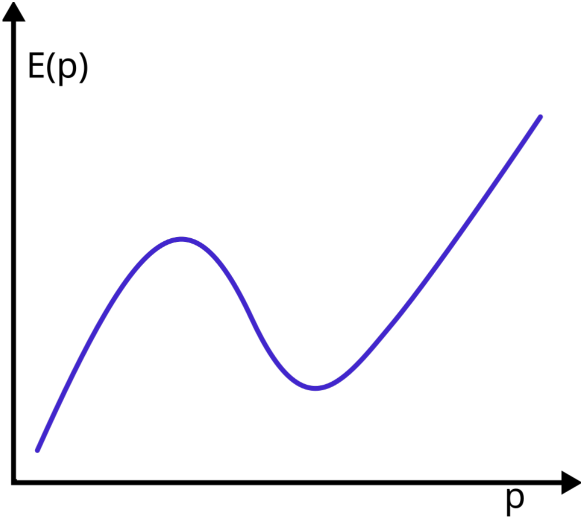
Superfluid helium: quasiparticles

Fundamental properties:

- Zero viscosity at $T < T_\lambda$
- Anomalous thermal conductivity (10^6 times > copper)
- Quantized vortices with circulation $\kappa = h/m$
- Existence of second sound

Main types of quasiparticles:

- Phonons:
  - Linear dispersion law: $\epsilon(p) = c_1 p$
  - Dominate at $T < 0.5$ K
- Rotons :
  - Minimum at $p_0 \approx 1.92$ Å^{-1}
  - Energy: $\Delta \approx 8.6$ K
  - Mass: $m^* \approx 0.16 m_{\text{He}}$



Dynamic structure factor in superfluid helium

The dynamic structure factor $S(\mathbf{q}, \omega)$ characterizes:

- System response to perturbation with wave vector $\mathbf{q}$ and frequency ω

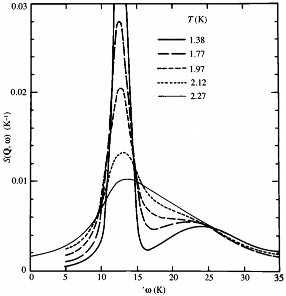
Key formulas:

- Definition:

$$S(\mathbf{q}, \omega) = \frac{1}{2\pi N} \int dt\, e^{i\omega t} \langle \rho_{\mathbf{q}}(t) \rho_{-\mathbf{q}}(0) \rangle$$

- Peaks in $S(\mathbf{q}, \omega)$ correspond to elementary excitations
- Peak width characterizes lifetime of excitations
- $$\frac{d^2\sigma}{d\Omega d\omega} = b_c^2 \frac{k'}{k} S(Q, \omega)$$

Alan Grifin, Excitations in a Bose-condensed Liquid, Cambridge University Press, 1993



$$S(q, \omega) = S_s + S_m + S_{\text{inc}}$$

Open Quantum Systems: Formalism and Applications

Definition:

- Systems interacting with environment (reservoir)
- Irreversible processes: dissipation, decoherence
- Described by reduced density matrix $\rho_s = \mathrm{Tr}_E(\rho_{\text{total}})$

Key equations:

- Master equation (Lindblad equation):

$$\dot{\rho} = -\frac{i}{\hbar}[H, \rho] + \mathcal{D}[\rho]$$

$$\mathcal{D}[\rho] = \sum_k \gamma_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\} \right)$$

Historical development:

- 1960s: Works by Haake, Zubarev
- 1976: G. Lindblad - Lindblad equation
- 1980s: Quantum trajectories (Dalibard, Harrow)
- 1990-2000: Quantum information (Kraus, Nielsen)

Application areas:

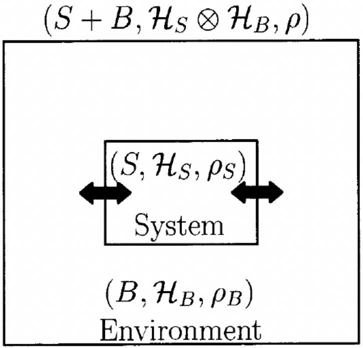
- Quantum optics: lasers, resonators
- Quantum information: qubit decoherence
- Chemical physics: quantum molecular dynamics
- Biophysics: photosynthesis
- Quantum technologies: sensors

Open quantum system scheme

Main components:

- System: Studied system
- Environment: Surrounding medium
- Interaction: H_{int}

H.-P. Breuer, F. Petruccione, "The Theory of Open Quantum Systems"



Derivation of the Lindblad equation: main steps

Step 1: Total system

- System + Environment = Isolated system
- Hamiltonian:

$$H_{\text{total}} = H_S \otimes I_E + I_S \otimes H_E + H_{\text{int}}$$

- Interaction: $H_{\text{int}} = \sum_k A_k \otimes B_k$

Step 2: von Neumann equation

$$\dot{\rho}_{\text{total}} = -\frac{i}{\hbar}[H_{\text{total}}, \rho_{\text{total}}]$$

Step 3: Approximations

- Born approximation: Weak interaction
- Markov approximation: No environment memory
- Initial state: $\rho(0) = \rho_S(0) \otimes \rho_E$

Step 4: Partial trace

$$\dot{\rho}_S = \text{Tr}_E\left(-\frac{i}{\hbar}[H_{\text{total}}, \rho_{\text{total}}]\right)$$

Step 5: Lindblad equation

$$\dot{\rho}_S = -\frac{i}{\hbar}[H_S, \rho_S] + \mathcal{D}[\rho_S]$$

where dissipator:

$$\mathcal{D}[\rho_S] = \sum_k \gamma_k \left(L_k \rho_S L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho_S\}\right)$$

Lindblad operators L_k:

- Describe dissipation channels
- γ_k - relaxation rate

SATURNE Project (NCPhM)

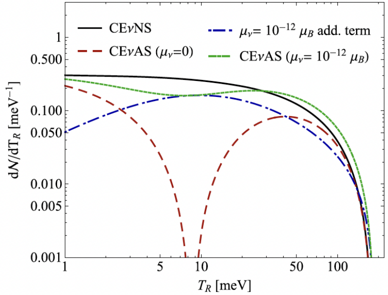
SATURNE aims to study coherent elastic neutrino-atom scattering and search for neutrino magnetic moment.

Project goals:

- First detection of neutrino-atom scattering
- Search for neutrino magnetic moment

He-II based detector:

- Sensitivity to recoil energies ∼2 meV
- Sensitivity to neutrino-atom scattering



M. Cadeddu, F. Dordei, C. Giunti, K. Kouzakov, E. Picciau and A. Studenikin, Phys. Rev. D 100 (2019)

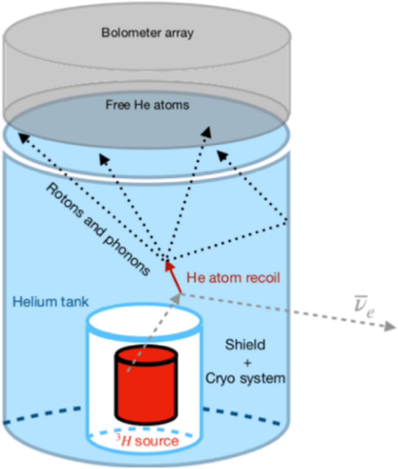
Process dynamics in SATURNE

Experimental stages:

1. Neutrino scattering
2. Recoil atom production
3. Quasiparticle generation
4. Detection

Aim:

- Obtain quasiparticle spectrum from tritium source neutrino scattering



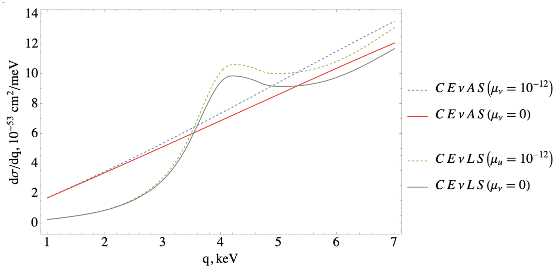
Neutrino scattering with recoil energy < 2 meV

With collective effects:

$$\frac{d\sigma}{dT} = \int \frac{d\sigma^{\text{CE}\nu\text{AS}}}{dq} S(q,T)dq$$

Dynamic structure factor:

$$S(q,\omega) = S_s$$



Key findings

- Collective effects significantly affect cross section at $T \sim 1$ meV
- Roton production shows resonant behavior at $E \approx \Delta_{\text{rot}}$

K. Kouzakov et al., Phys. Atom. Nucl (2025) (accepted)

Recoil Atom Dynamics in HeII

Before:

- Neutrino + Atom (He)

After:

- Neutrino + recoiling atom (He)
- Recoil energy: 2-100 meV

Open quantum system:

- System: Recoil atom
- Environment: Superfluid helium
- Processes:
  - Quasiparticle excitation
  - Energy dissipation

Theoretical description:

$$H = H_{\text{atom}} + H_{\text{HeII}} + H_{\text{int}}$$

Post-scattering processes

- Cascade of energetic He atoms
- Phonon/roton excitations

OQS Formalism for Neutrino Scattering on HeII

- Evolution equation:

$$\frac{d}{dt}\rho(t) = -i\left[H_I(t), \rho(t)\right]$$

- Density matrix decomposition:

$$\rho(t) \approx \rho_S(t) \otimes \rho_B$$

- Interaction:

$$H_I = \sum_\alpha A_\alpha \otimes B_\alpha \sim j_\mu J^\mu$$

K.Matchev, et al. Superfluid effective field theory for dark matter direct detection, JHEP, 2022.

- Equation for reduced density matrix:

$$\frac{d}{dt}\rho_S(t) = \int_0^\infty ds\, \mathrm{tr}_B\{H_I(t-s)\rho_S(t)\rho_B H_I(t) - H_I(t)H_I(t-s)\rho_S(t)\rho_B\} + \text{h.c.}$$

$$\frac{d}{dt}\rho_S(t) \sim \sum_{\omega,\omega'}\sum_{\alpha,\beta} e^{i(\omega'-\omega)t}\Gamma_{\alpha\beta}(\omega) \times\left[j_\beta(\omega)\rho_S(t)j_\alpha^\dagger(\omega') - j_\alpha^\dagger(\omega')j_\beta(\omega)\rho_S(t)\right] + \text{h.c.}$$

- Correlation function:

$$\Gamma_{\alpha\beta} \equiv \int_0^\infty ds\, e^{i\omega s}\langle J_\alpha^\dagger(t) J_\beta(t-s)\rangle$$

Key results

K. Stankevich, A. Studenikin, M. Vyalkov, "Generalized Lindblad master equation for neutrino evolution", Phys. Rev. D 111, **2025**

For superfluid helium

$$\Gamma_{\alpha\beta}(Q,\omega) \approx \Gamma_{00}(Q,\omega) \sim S(Q,\omega)$$

We obtain the Lindblad equation

$$\frac{d}{dt}\rho_S(t) = -i\left[H_{LS}, \rho_S(t)\right] + \mathcal{D}(\rho_S(t))$$

$$\mathcal{D}(\rho_S) = \sum_{\omega} \Gamma_{00}(\omega)\left(j_0(\omega)\rho_S j_0^{\dagger}(\omega) - \frac{1}{2}\left\{j_0^{\dagger}(\omega) j_0(\omega), \rho_S\right\}\right)$$

Conclusion

Results

- Evolution equation for the helium atom after neutrino scattering has been obtained for recoil energies above 2 meV.
- The explicit form of Lindblad operators is obtained.

Thank you for your attention