Neutrino propagation in Quantum Field Theory at short and long baselines

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Oscillation mechanism in quantum mechanics

Assumptions

- The flavor states are linear superposition of the massive states:

$$|\nu_\alpha\rangle = \sum_i V^*_{\alpha i}\,|\nu_i\rangle\,, \quad \alpha = e, \mu, \tau, \ldots$$

- All massive states have equal momenta: $\boldsymbol{p}_i = \boldsymbol{p}_\nu$.
- Neutrinos are **ultrarelativistic**, so $L \approx t$.

Schrödinger equation for the **massive** neutrino states reads:

$$i\frac{d}{dt}|\nu(t)\rangle_{\text{mass}} = \mathbf{H}_0|\nu(t)\rangle_{\text{mass}}, \quad \mathbf{H}_0 = \text{diag}(E_1, E_2, E_3, \ldots).$$

The standard quantum mechanical formula

$$\mathcal{P}\left(\nu_\alpha \to \nu_\beta\right) \equiv \mathcal{P}_{\alpha\beta} = \sum_{ij} V_{\alpha i} V_{\beta j} V^*_{\alpha j} V^*_{\beta i} \exp\left(i\,\frac{m_i^2 - m_j^2}{2E} L\right)$$

Velocity difference

- As the massive neutrino components have the same momentum $\boldsymbol{p}_\nu$, their velocities are in fact different:

$$\boldsymbol{v}_i = \boldsymbol{p}_\nu / \sqrt{\boldsymbol{p}_\nu^2 + m_i^2} \implies |\boldsymbol{v}_i - \boldsymbol{v}_j| \approx \Delta m_{ji}^2 / 2E_\nu^2.$$

- During the time T the neutrino ν_i travels the distance $L_i = |\boldsymbol{v}_i| T$.
- There must be a spread in distances of each neutrino pair

$$\delta L_{ij} = L_i - L_j \approx \Delta m_{ji}^2 \, L / 2E_\nu^2, \quad \text{where} \quad L = cT = T,$$

$$L_{ij} = 4\pi E_\nu / \Delta m_{ij}^2 \quad \text{is the oscillation length}$$

Δm_{ji}^2	E_ν	L	L_{ij}	$\lvert\delta L_{ij}\rvert$
Δm_{23}^2	1 GeV	$2R_\oplus$	$0.1R_\oplus$	$\sim 10^{-12}$ cm
Δm_{21}^2	1 MeV	1 AU	$0.25R_\oplus$	$\sim 10^{-3}$ cm

Are $|\delta L_{ij}|$ sufficiently small to preserve the coherence?

QFT approach

- Within the framework of the plane-wave formalism of Quantum Field Theory, the particles states are defined as Fock's states:

$$|\boldsymbol{k}, s\rangle = \sqrt{2E_{\boldsymbol{k}}}\, a^{\dagger}_{\boldsymbol{k}s}|0\rangle. \tag{1}$$

- The Fock states have no information about the particle coordinate.

★ To introduce dependence on the coordinate one needs to build a wave packet that is superposition of the Fock states:

$$|\boldsymbol{p}, s, x\rangle = \int \frac{d\boldsymbol{k}\, \phi(\boldsymbol{k}, \boldsymbol{p}) e^{i(k-p)x}}{(2\pi)^3 2E_{\boldsymbol{k}}} |\boldsymbol{k}, s\rangle, \quad \phi(\boldsymbol{k}, \boldsymbol{p}) \xmapsto{\text{PWL}} (2\pi)^3 2E_{\boldsymbol{p}} \delta(\boldsymbol{k} - \boldsymbol{p}).$$

- The amplitude is constructed as follows:

$$\langle \mathbf{out}|\mathbb{S}|\mathbf{in}\rangle \left(\langle \mathbf{in}|\mathbf{in}\rangle \langle \mathbf{out}|\mathbf{out}\rangle\right)^{-1/2} \stackrel{\text{def}}{=} \mathcal{A}_{\beta\alpha}.$$

CRGP wave packet model[1]

Relativistic **G**aussian **P**acket model:

$$\phi_G(\boldsymbol{k},\boldsymbol{p}) = N_\sigma \exp\left[\frac{(k_0-p_0)^2-(\boldsymbol{k}-\boldsymbol{p})^2}{4\sigma^2}\right], \quad \int \frac{d\boldsymbol{k}\,\phi_G(\boldsymbol{k},\boldsymbol{p})}{(2\pi)^3 2E_{\boldsymbol{k}}} = 1$$

Coordinate representation of wave packet

$$\psi(\boldsymbol{p},x) = \int \frac{d\boldsymbol{k}}{(2\pi)^3 2E_{\boldsymbol{k}}}\phi(\boldsymbol{k},\boldsymbol{p})e^{ikx},$$

By imposing the following conditions

$$(px)^2 \ll m^4/\sigma^4, \qquad (px)^2 - m^2x^2 \ll m^4/\sigma^4.$$

one obtain the **C**ontracted **R**elativistic **G**aussian **P**acket (CRGP) model:

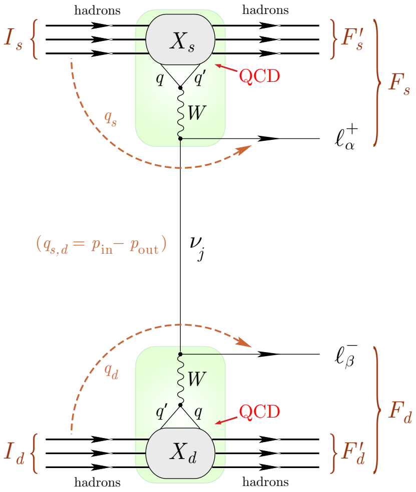
$$\psi_G(\boldsymbol{p},x) = \exp\left\{i(px) - \frac{\sigma^2}{m^2}\left[(px)^2 - m^2x^2\right]\right\}.$$

[1] D. V. Naumov and V. A. Naumov. *J. Phys.* **G 37** (2010) 105014. arXiv: 1008.0306v2 [hep-ph]

The studied process (Macroscopic Feynman Diagram)

- X_d and X_d denote the regions where the intermediate neutrino was born and detected, respectively. These regions are macroscopically separated in the space-time,
- $I_{s,d}$ denote initial particles in the source and detector vertices, respectively; $F_{s,d}$, $F'_{s,d}$ denote the sets of final particles (the notation is clear from the picture).

$$I_s \oplus I_d \to F_s \oplus F_d$$



The amplitude

$$\mathcal{A}_{\beta\alpha} \stackrel{\mathrm{def}}{=} \langle \mathbf{out}|\mathbb{S}|\mathbf{in}\rangle \left(\langle \mathbf{in}|\mathbf{in}\rangle\langle \mathbf{out}|\mathbf{out}\rangle\right)^{-1/2}.$$

$$\mathcal{L}_W(x) = -\frac{g}{2\sqrt{2}}\left[j_\ell(x)W(x) + j_q(x)W(x) + \mathrm{H.c.}\right],$$

$$j_\ell^\mu(x) = \sum_{\alpha i} V_{\alpha i}^* \, \overline{\nu}_i(x) O^\mu \ell_\alpha(x), \qquad j_q^\mu(x) = \sum_{qq'} V_{qq'}^{\prime *} \, \overline{q}(x) O^\mu q'(x).$$

First nonvanishing term:

$$\mathcal{A}_{\beta\alpha} = \frac{1}{\mathcal{N}}\left(\frac{-ig}{2\sqrt{2}}\right)^4 \langle F_s \oplus F_d | T \int dx dx' dy dy' : j_\ell(x) W(x) : : j_q(x') W^\dagger(x') : \\ \times : j_\ell^\dagger(y) W^\dagger(y) : : j_q^\dagger(y') W(y') : \mathbb{S}_h | I_s \oplus I_d \rangle, \quad \mathcal{N}^2 = \langle \mathbf{in}|\mathbf{in}\rangle\langle \mathbf{out}|\mathbf{out}\rangle$$

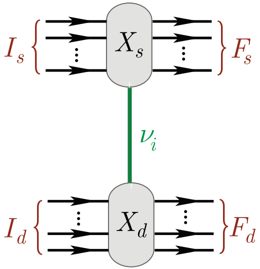
$\mathbb{S}_h = \exp\left[i\int dz \mathcal{L}_\mathrm{h}(z)\right]$, $\mathcal{L}_\mathrm{h}(z)$ is Lagrangian responsible for interaction involving hadrons

The distance dependence of the amplitude

The L dependence of the amplitude is defined by the external WP-modified ν propagator:

$$J(X) = \int \frac{d^4q}{(2\pi)^4} \frac{\widetilde{\delta}_s(q - q_s)\widetilde{\delta}_d(q + q_d)(\hat{q} + m)e^{-iqX}}{q^2 - m^2 + i\epsilon},$$

where $\widetilde{\delta}_{s,d}$ are "smeared" δ functions, responsible for approximate 4-momentum conservation in the vertices, $X = (T, \boldsymbol{L})$; $q_{s,d}$ are the 4-momentum transfers in the source/detector.



- The integral can be solved:
  1. SHORT baseline asymptotics
  2. LONG baseline asymptotics

- Short-baseline asymptotics[2]:

$$J(X) = -8i\pi^2\sqrt{\frac{\pi|\mathcal{R}|}{\mathcal{G}}}\exp(\Omega)$$

$$\times \exp\left[\sum_{a\geq 0}\sum_{b\geq 0}\sum_{c\geq 0} F_{abc}\left(i\Sigma^2\frac{\rho_{\mu\nu}q^\mu X^\nu}{\rho_{\mu\nu}q^\mu q^\nu}\right)^a\left(\frac{\Sigma^2}{E^2}\right)^b\left(\frac{\mathcal{P}q - m^2}{\rho_{\mu\nu}q^\mu q^\nu}\right)^c\right],$$

$$\frac{1}{\Sigma^2} \ll L^2 \ll \frac{|\boldsymbol{q}|^2}{\Sigma^4}$$

- Extended Grimus-Stockinger theorem[3] (long-baseline asymptotics):
  For $\Phi(\boldsymbol{q}) \in S(\mathbb{R}^3)$ and $\omega > 0$

$$J(L,\omega) \stackrel{\text{def}}{=} \int \frac{d\boldsymbol{q}}{(2\pi)^3}\,\frac{\Phi(\boldsymbol{q})e^{i\boldsymbol{qL}}}{\boldsymbol{q}^2 - \omega^2 - i\epsilon} = \frac{e^{i\omega L}}{4\pi L}\sum_{k\geq 0}\frac{(-i)^k\left[D_k\Phi(\boldsymbol{q})\right]_{\boldsymbol{q}=\omega\boldsymbol{l}}}{L^k}, \quad L \to \infty.$$

[2]Vadim A. Naumov and Dmitry S. Shkirmanov. *Eur. Phys. J. C* **82**.8 (2022) 736. arXiv: 2208.02621 [hep-ph].

[3]V. A. Naumov and D. S. Shkirmanov. *Eur. Phys. J.* **C 73** (2013) 2627. arXiv: 1309.1011 [hep-ph].

Event rate in the detector

$$\frac{N_{\beta\alpha}}{\tau_d} = \sum_{\text{spins}} \int d\boldsymbol{x} \int d\boldsymbol{y} \int d\mathfrak{P}_s \int d\mathfrak{P}_d \int d|\boldsymbol{q}| \frac{\mathcal{P}_{\alpha\beta}\left(|\boldsymbol{q}|, |\boldsymbol{y}-\boldsymbol{x}|\right)}{4(2\pi)^3 \, |\boldsymbol{y}-\boldsymbol{x}|^2}$$

$$\times \left(1 - \text{ISLV corrections}\right)$$

Differential forms are the following

$$d\mathfrak{P}_s = \prod_{a\in I_s} \frac{d\boldsymbol{p}_a f_a(\boldsymbol{p}_a, s_a, \boldsymbol{x})}{(2\pi)^3 2E_a} \prod_{b\in F_s} \frac{d\boldsymbol{p}_b}{(2\pi)^3 2E_b} (2\pi)^4 \delta_s(q - q_s) |M_s|^2,$$

$$d\mathfrak{P}_d = \prod_{a\in I_d} \frac{d\boldsymbol{p}_a f_a(\boldsymbol{p}_a, s_a, \boldsymbol{y})}{(2\pi)^3 2E_a} \prod_{b\in F_d} \frac{[d\boldsymbol{p}_b]}{(2\pi)^3 2E_b} (2\pi)^4 \delta_d(q + q_d) |M_d|^2.$$

Inverse Square Law Violation

- the neutrino event rate in the detector is proportional to the factor:

Long baseline: $$dN_\nu \propto \frac{1}{|L|^2}\left[1 - \frac{L_0^2}{|L|^2} + \dots\right], \quad L^2 \gg \frac{E_\nu^2}{\Sigma_{\text{LBL}}^4}$$

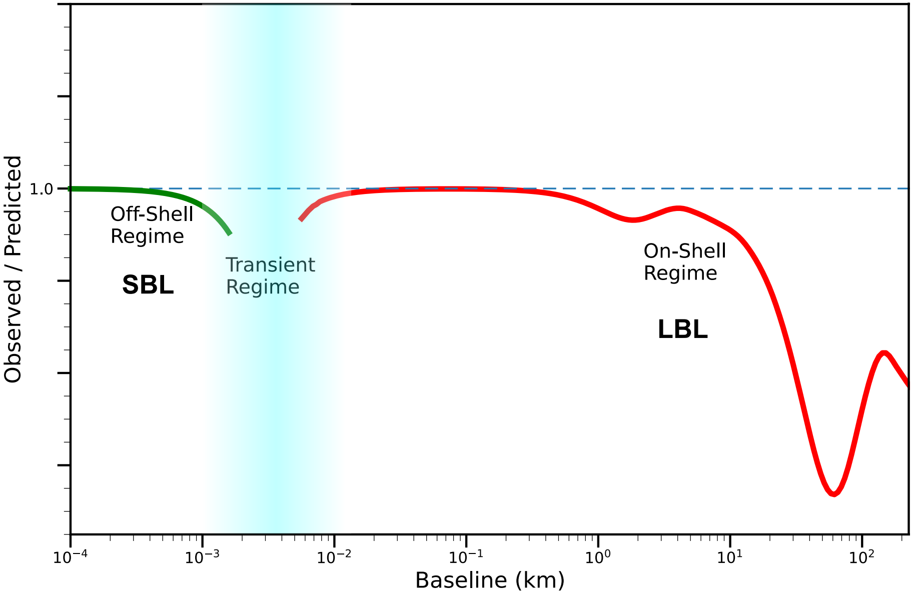
Short baseline: $$dN_\nu \propto \frac{1}{|L|^2}\left[1 - \frac{|L|^2}{L_0'^2} + \dots\right], \quad \frac{1}{\Sigma_{\text{SBL}}^2} \ll L^2 \ll \frac{|\boldsymbol{q}_\nu|^2}{\Sigma_{\text{SBL}}^4}$$

- The order of the magnitude estimation:

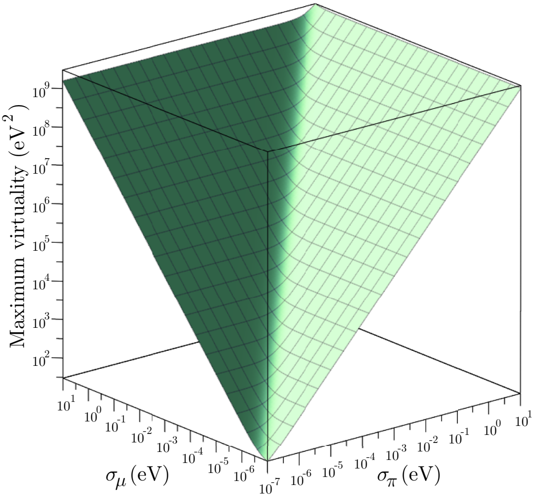
$$L_0 \sim L_0' \sim \langle E_\nu \sigma_{\text{eff}}^{-2}(E_\nu)\rangle \approx 20 \left\langle \left(\frac{E_\nu}{1\ \text{MeV}}\right)\left[\frac{\sigma_{\text{eff}}(E_\nu)}{1\ \text{eV}}\right]^{-2}\right\rangle \text{cm.} \qquad (2)$$

The ISL violation effect might be observable.

Inverse Square Law Violation: graphical representation

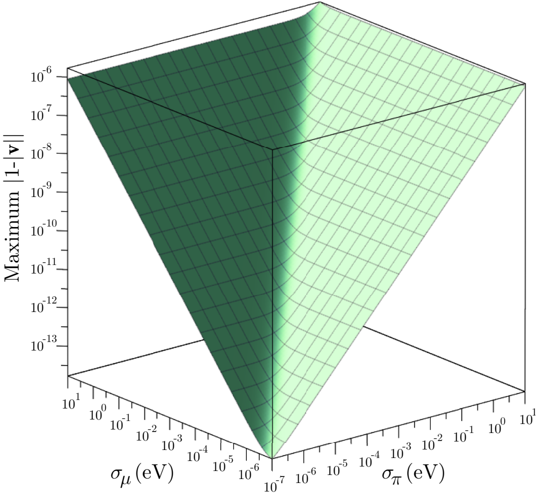


Neutrino virtuality at short baselines



Maximum neutrino virtuality $|\mathcal{P}^2 - m_j^2|$ within the pion decay model. Calculations are performed assuming the exact energy momentum conservation and zero neutrino mass.

Deviation from light speed at short baselines



Maximum deviation of neutrino speed from speed of light within the pion decay model. Calculations are performed assuming the exact energy momentum conservation and zero neutrino mass.

Compare with classical velocity of neutrino from pion decay at rest

$$v_\nu \approx 1 - 5 \times 10^{-18}$$

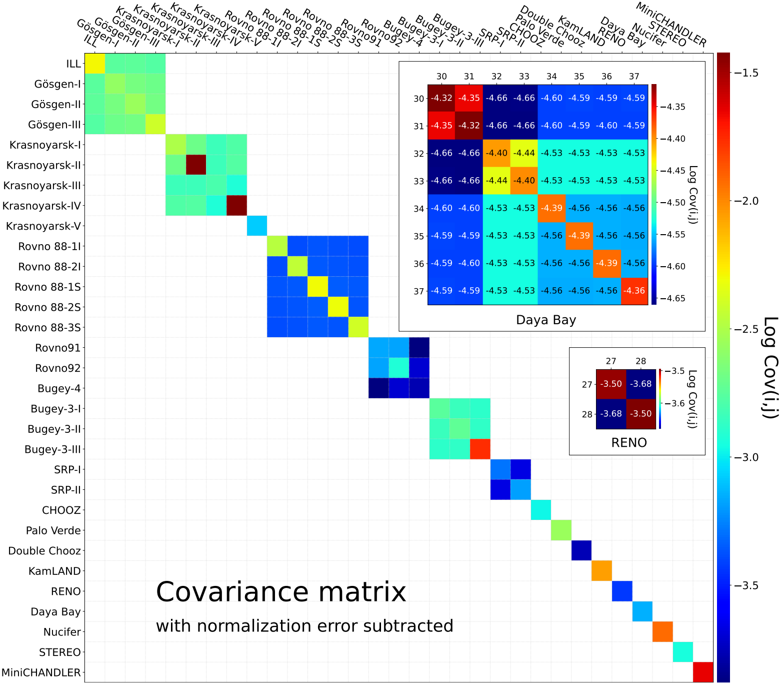
Our reanalysis of the reactor data.

- A careful analysis of the systematic error correlations in the reactor data set is performed. As a result the errors in general differ from those adopted by Mention *et al.*[4] and Zhang *et al.*[5] and are closer to those adopted by Kopp *et al.*[6].
  - correlated groups are re-ordered.
  - correlations are revisited.
- Some omitted data are added.
- New (recommended in RPP-2018) neutron lifetime $\tau_n = 880.2 \pm 1.0$ s is accounted for, instead of $\tau_n = 885.7$ s used in the mentioned analyses (this shifts down all the ratios by about 0.6%).

[4] G. Mention *et al. Phys. Rev.* **D 83** (2011) 073006. arXiv: 1101.2755 [hep-ex].

[5] C. Zhang, X. Qian, and P. Vogel. *Phys. Rev.* **D 87** (2013) 073018. arXiv: 1303.0900 [nucl-ex].

[6] J. Kopp *et al. JHEP* **05** (2013) 050. arXiv: 1303.3011v2 [hep-ph]. Note that the "Observed/Predicted" ratios listed in that paper are systematically (1 to 1.2%) smaller than ours.


ILL
Gösgen-I
Gösgen-II
Gösgen-III
Krasnoyarsk-I
Krasnoyarsk-II
Krasnoyarsk-III
Krasnoyarsk-IV
Krasnoyarsk-V
Rovno 88-1I
Rovno 88-2I
Rovno 88-1S
Rovno 88-2S
Rovno 88-3S
Rovno91
Rovno92
Bugey-4
Bugey-3-I
Bugey-3-II
Bugey-3-III
SRP-I
SRP-II
CHOOZ
Palo Verde
Double Chooz
KamLAND
RENO
Daya Bay
Nucifer
STEREO
MiniCHANDLER
Log Cov(i,j)
−1.5
−2.0
−2.5
−3.0
−3.5
Daya Bay
30 31 32 33 34 35 36 37
30 -4.32 -4.35 -4.66 -4.66 -4.60 -4.59 -4.59 -4.59
31 -4.35 -4.32 -4.66 -4.66 -4.60 -4.59 -4.60 -4.59
32 -4.66 -4.66 -4.40 -4.44 -4.53 -4.53 -4.53 -4.53
33 -4.66 -4.66 -4.44 -4.40 -4.53 -4.53 -4.53 -4.53
34 -4.60 -4.60 -4.53 -4.53 -4.39 -4.56 -4.56 -4.56
35 -4.59 -4.59 -4.53 -4.53 -4.56 -4.39 -4.56 -4.56
36 -4.59 -4.60 -4.53 -4.53 -4.56 -4.56 -4.39 -4.56
37 -4.59 -4.59 -4.53 -4.53 -4.56 -4.56 -4.56 -4.36
−4.35 −4.40 −4.45 −4.50 −4.55 −4.60 −4.65
RENO
27 28
27 -3.50 -3.68
28 -3.68 -3.50
−3.5 −3.6
Covariance matrix
with normalization error subtracted

ISL violation

Theoretical model:

$$T(L; N_0, L_0) = N_0 \cdot \langle P^{3\nu}_{\rm surv}(L)\rangle \cdot \left(1 - \frac{L_0^2}{L^2}\right). \tag{3}$$

N_0 is a free normalization parameter and

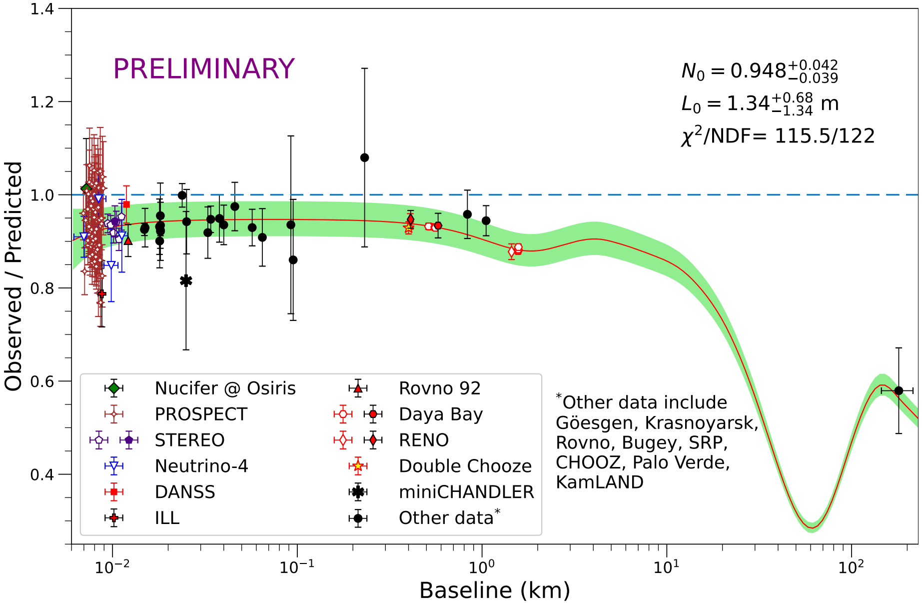
$$\langle P^{3\nu}_{\rm surv}(L)\rangle = \frac{\int\limits_0^\infty dE \sum_k f_k P^{3\nu}_{\rm surv}(L,E)\sigma(E)S_k(E)}{\int\limits_0^\infty dE \sum_k f_k \sigma(E)S_k(E)}, \tag{4}$$

f_k is the fraction of main fissile isotope contributing to the $\overline{\nu}_e$ flux with a spectrum in energy $S_k(E)$[7], $\sigma(E)$ is the IBD cross section[8]
$P^{3\nu}_{\rm surv}(L,E)$ is the $\overline{\nu}_e$ survival probability in the standard 3ν mixing scheme:

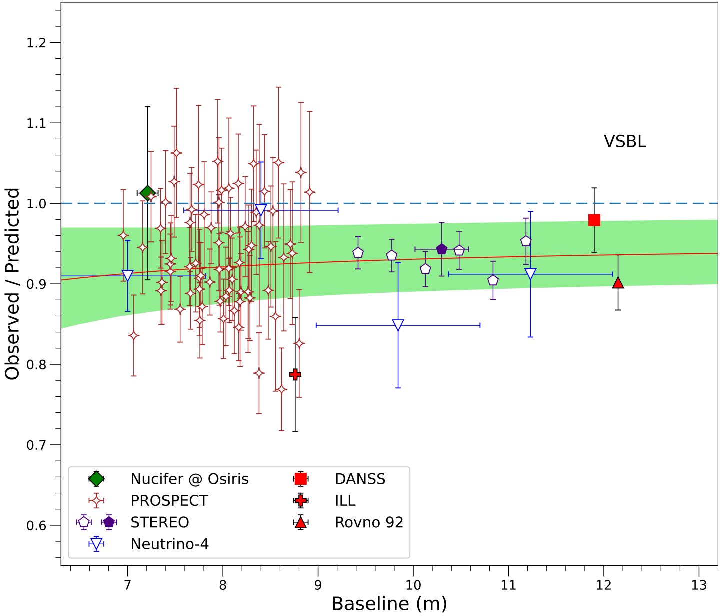
$$P^{3\nu}_{\rm surv}(L,E) = 1 - \sin^2(2\theta_{13})\left(\cos^2\theta_{12}\sin^2\Delta_{31} + \sin^2\theta_{12}\sin^2\Delta_{32}\right) - \cos^4\theta_{13}\sin^2(2\theta_{12})\sin^2\Delta_{21}, \qquad \Delta_{ij} = 1.267\Delta m^2_{ij}L/E.$$

[7] Th A. Mueller *et al. Phys. Rev.* **C 83** (2011) 054615. arXiv: 1101.2663 [hep-ex]
[8] A. N. Ivanov *et al. Phys. Rev.* **C 88** (2013) 055501. arXiv: 1306.1995v2 [hep-ph]

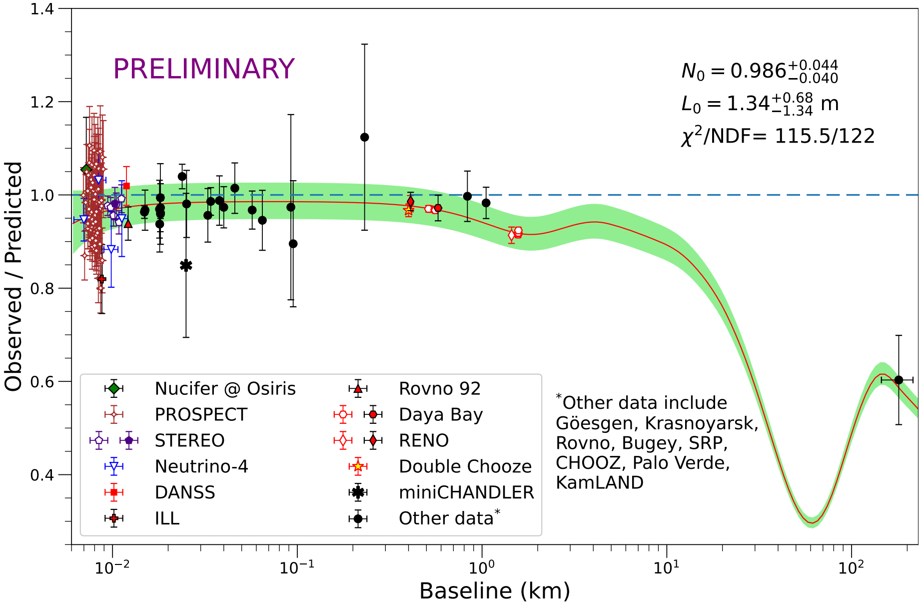
ISLV, Spectrum type = Huber-Mueller



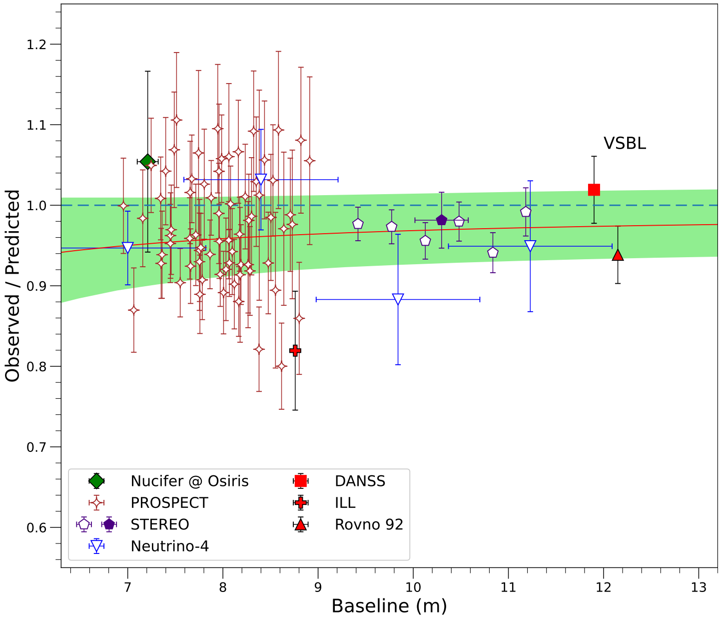
ISLV, Spectrum type = Huber-Mueller (zoom)



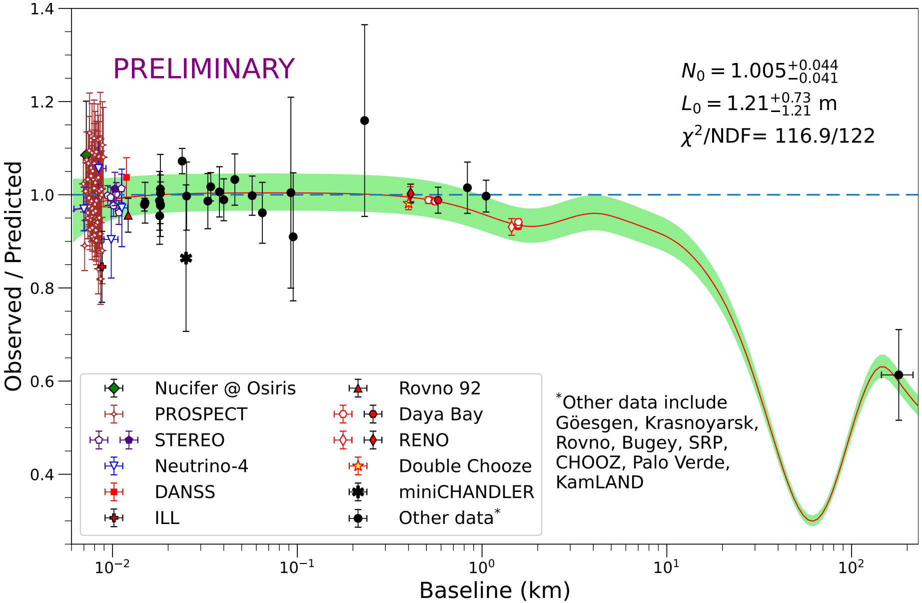
ISLV, Spectrum type = "Kurchatov Institute"



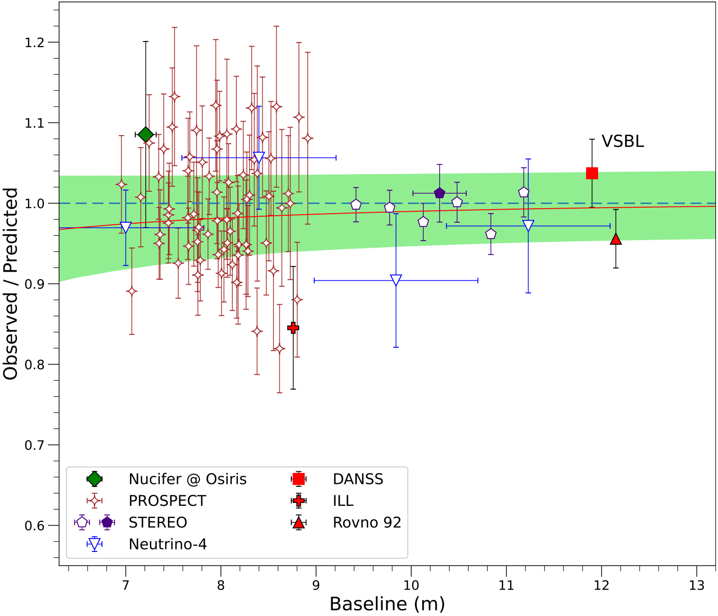
ISLV, Spectrum type = "Kurchatov Institute" (zoom)



ISLV, Spectrum type = Fallot



ISLV, Spectrum type = Fallot (zoom)



Notes and Summary

- The QFT approach predicts that the classical inverse-square law could be broken.
- Current reactor data hint that the ISLV is already seen.
- Best fit value of L_0 is formally compatible with zero (lower error is not defined). However, high-precision relative DANSS data are in preparation and have not yet been included in the analysis. Preliminary calculations show that they can define the lower error.
- Experiments with small intense $\nu/\overline{\nu}$ sources and sectioned or movable detector(s) are required (BEST-2, SOX/CeSOX, CeLAND).
- Well, what if the new data (say from DANSS) closes our explanation? It'll be a pity, but it'll not mean a confutation of the QFT approach or extended GS theorem. Rather, it'll only be an indication that σ_{eff} is above the sensitivity threshold.

Thanks for attention!

[1] D. V. Naumov and V. A. Naumov. *J. Phys.* **G 37** (2010) 105014. arXiv: 1008.0306v2 [hep-ph].

[2] Vadim A. Naumov and Dmitry S. Shkirmanov. *Eur. Phys. J. C* **82**.8 (2022) 736. arXiv: 2208.02621 [hep-ph].

[3] V. A. Naumov and D. S. Shkirmanov. *Eur. Phys. J.* **C 73** (2013) 2627. arXiv: 1309.1011 [hep-ph].

[4] G. Mention *et al. Phys. Rev.* **D 83** (2011) 073006. arXiv: 1101.2755 [hep-ex].

[5] C. Zhang, X. Qian, and P. Vogel. *Phys. Rev.* **D 87** (2013) 073018. arXiv: 1303.0900 [nucl-ex].

[6] J. Kopp *et al. JHEP* **05** (2013) 050. arXiv: 1303.3011v2 [hep-ph].

[7] Th A. Mueller *et al. Phys. Rev.* **C 83** (2011) 054615. arXiv: 1101.2663 [hep-ex].

[8] A. N. Ivanov *et al. Phys. Rev.* **C 88** (2013) 055501. arXiv: 1306.1995v2 [hep-ph].