Quantum Entanglement as a Probe of Neutrino Oscillation Parameters

(based on the paper: e-Print: 2410.05727 [hep-ph])

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Contents

Current Section

Motivation

- Are the accelerator neutrinos entangled?
- If so, what type of entanglement it is: purely tripartite or partially bipartite?
- Whether we can find best-fit value of oscillation parameters through quantum entanglement (QE) aspect!
- Will QE affect the neutrino physics sensitivity?

Current Section

Introduction

Quantum system in Hilbert space:

- In quantum mechanics (QM), every physical system is associated with a Hilbert space **H**.
- State of a system is represented by a state vector $|\psi\rangle \in \mathbf{H}$.
- The wave function $|\psi\rangle$ encodes all measurable properties of the system.
- The state of multiple quantum systems lies in a composite Hilbert space formed by the tensor product of their individual spaces: $\mathbf{H_1} \otimes \mathbf{H_2} \otimes \mathbf{H_3}....$
- The density matrix;

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|. \tag{1}$$

Separable State:

Density matrix is in the product state of its constituent system density matrices: $\rho_{123...} = \sum_j \lambda_j \rho_1^j \otimes \rho_2^j \otimes \rho_3^j \otimes ...$

Entangled State:

Can't write in product state of its constituent system density matrix: $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B - |1\rangle_A \otimes |0\rangle_B)$

Neutrino qubits[2]

3-qubit neutrino

$$|\nu_e\rangle \equiv |1\rangle_e \otimes |0\rangle_\mu \otimes |0\rangle_\tau$$
$$|\nu_\mu\rangle \equiv |0\rangle_e \otimes |1\rangle_\mu \otimes |0\rangle_\tau$$
$$|\nu_\tau\rangle \equiv |0\rangle_e \otimes |0\rangle_\mu \otimes |1\rangle_\tau$$

where $|0\rangle_{\nu_\alpha}$ and $|1\rangle_{\nu_\alpha}$ $\rightarrow$ absence and presence of ν_α respectively.

- The density matrix for the state $|\nu_\mu(t)\rangle$ [1],

$$\rho_\mu(t) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & |\tilde{U}_{\mu\tau}|^2 & \tilde{U}_{\mu\tau}\tilde{U}^*_{\mu\mu} & 0 & \tilde{U}_{\mu\tau}\tilde{U}^*_{\mu e} & 0 & 0 & 0 \\ 0 & \tilde{U}_{\mu\mu}\tilde{U}^*_{\mu\tau} & |\tilde{U}_{\mu\mu}|^2 & 0 & \tilde{U}_{\mu\mu}\tilde{U}^*_{\mu e} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{U}_{\mu e}\tilde{U}^*_{\mu\tau} & \tilde{U}_{\mu e}\tilde{U}^*_{\mu\mu} & 0 & |\tilde{U}_{\mu e}|^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

with $|\tilde{U}_{\mu\mu}|^2 = P_{\mu\mu}$, $|\tilde{U}_{\mu e}|^2 = P_{\mu e}$ and $|\tilde{U}_{\mu\tau}|^2 = P_{\mu\tau}$

[1] Nucl. Phys. B 1002 (2024) 116544

[2] Mod. Phys. Lett. A 36 (2021), no. 09 2150056.

Current Section

Entanglement Measures[3]

Entanglement of formation (EOF):

- The EOF is the most basic measurement of entanglement.
- For each pure state, the entanglement of formation is defined as the entropy of either of the two systems A and B,

$$\mathrm{EOF}(\psi) = -\mathrm{Tr}(\rho_A \log_2 \rho_A) = -\mathrm{Tr}(\rho_B \log_2 \rho_B). \tag{2}$$

Here, ρ_A is the partial trace of $|\psi\rangle\langle\psi|$ over subsystem B.

Concurrence:

The Concurrence is a quantitative measure of entanglement for a bipartite system, defined as

$$C^{\alpha} = \left[\max(\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0)\right], \tag{3}$$

where λ_i are the eigenvalues of the reduced density matrix of the bipartite system.

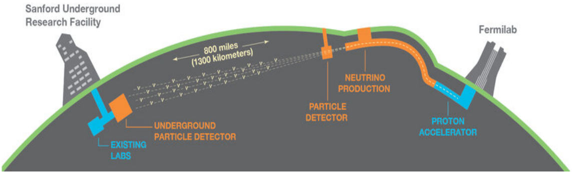
[3] Phys. Rev. Lett. 70 (Mar, 1993) 1895–1899, Phys. Rev. Lett. 80 (Mar, 1998) 2245–2248

Current Section

Experimental details: DUNE

Experiment	DUNE
Current Status	Future
Runtime	$5\nu : 5\bar{\nu}$
Baseline	1300 km
First Oscillation Peak	2.5 GeV
Detector Material	LArTPC
Fiducial Volume of FD	40 kt

Table 1: Experimental specifications for **DUNE** future experiment



Current Section

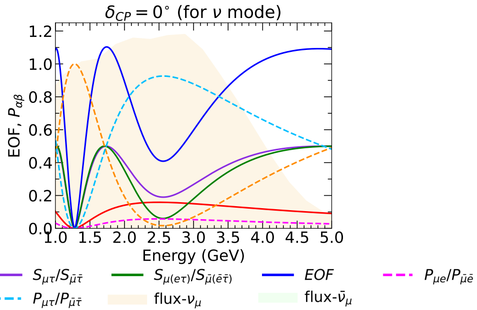
Expression of EOF

EOF in 3-qubit neutrino system,

$$\mathrm{EOF} = S_{\mu e} + S_{\mu\tau} + S_{\mu(e\tau)}. \quad (4)$$

$$\begin{aligned} S_{\mu e} &= -\frac{1}{2}\Big[P_{\mu e}\log_2 P_{\mu e} + (P_{\mu\mu}+P_{\mu\tau})\log_2(P_{\mu\mu}+P_{\mu\tau})\Big], \\ S_{\mu\tau} &= -\frac{1}{2}\Big[P_{\mu\tau}\log_2 P_{\mu\tau} + (P_{\mu\mu}+P_{\mu e})\log_2(P_{\mu\mu}+P_{\mu e})\Big], \\ S_{\mu(e\tau)} &= -\frac{1}{2}\Big[P_{\mu\mu}\log_2 P_{\mu\mu} + (P_{\mu e}+P_{\mu\tau})\log_2(P_{\mu e}+P_{\mu\tau})\Big]. \end{aligned} \quad (5)$$

Here, $S_{\mu e}$, represents the entanglement between ν_μ and ν_e and so on.



Conclusion of EOF plot

- Energy window 1 (EW1) (1.0-1.5 GeV) : EOF has zero value $\rightarrow$ global minimum $\rightarrow$ all the flavor states are pure (no entanglement)
- EW2 (1.5-2.0 GeV) : EOF has maximum value $\rightarrow$ maximum point at 1.74 GeV $\rightarrow$ maximal entanglement
- EW3 (2.0-3.0 GeV) : EOF has non-zero but small value $\rightarrow$ local minimum $\rightarrow$ partially entanglement
- As EOF is a logarithmic function, it is very difficult to analyze the nature of the entanglement within the system

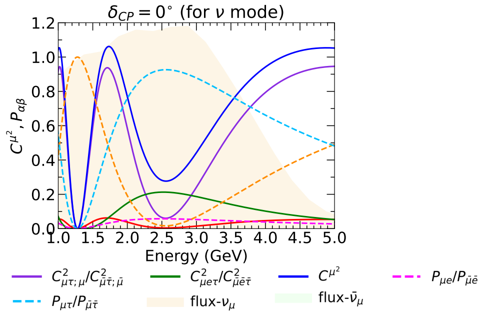
Expression of Concurrence

Concurrence in 3-qubit neutrino system,

$$C^{\mu^2} = C^2_{\mu e;\mu} + C^2_{\mu\tau;\mu} + 4\mathrm{P}_{\mu e}\mathrm{P}_{\mu\tau}, \tag{6}$$

$$\begin{aligned} C^2_{\mu e;\mu} &= 4P_{\mu e}P_{\mu\mu}, \\ C^2_{\mu\tau;\mu} &= 4P_{\mu e}P_{\mu\mu} \end{aligned} \tag{7}$$

third term $4\mathrm{P}_{\mu e}\mathrm{P}_{\mu\tau}$ describes the product of two oscillation probabilities, which we redefine as $C^2_{\mu e\tau}$.



Conclusion of Concurrence plot

- EW1 (1.0-1.5 GeV) : EOF has zero value $\rightarrow$ global minimum $\rightarrow$ all the flavor states are pure (no entanglement)
- EW2 (1.5-2.0 GeV) : EOF has maximum value $\rightarrow$ achieves maxima when $P_{\mu\tau}$ and $P_{\mu\mu}$ are equal to 0.5
- EW3 (2.0-3.0 GeV) : EOF has non-zero but small value $\rightarrow$ local minimum $\rightarrow$ Both $P_{\mu e}$ and $P_{\mu\tau}$ attains a maxima while $P_{\mu\mu}$ attains a minima

Overall conclusion (From EOF and Concurrence)

- The maximally entangled state is occurring between ν_μ and ν_τ.
- Electron flavor neutrino (ν_e) behaves as separable flavor.
- There are two minima in the entanglement measures; one is the global minimum (at 1.3 GeV) and other is the local minimum (at 2.5 GeV).
- EW1 provides a pure state for ν_μ, while EW3 offers a nearly separable state for ν_e.

Entanglement monogamy

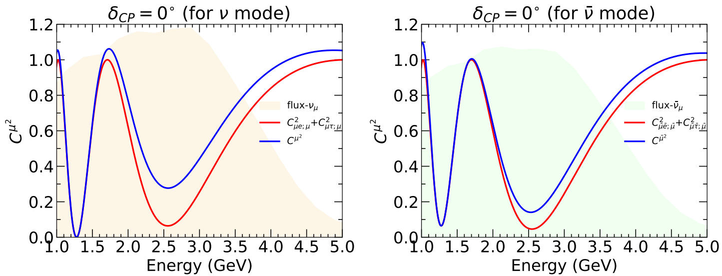
- 3-particle system ABC. Assume BC: a single composite system, then $C_{A|BC} \rightarrow$ concurrence between qubit A and the pair BC. Thus, A and BC; a pair of qubits in a pure state. The CKW inequality is then expressed as (Physical Review A 61 (Apr., 2000).):

$$C^2_{AB} + C^2_{AC} \leq C^2_{A|(BC)} \,. \tag{8}$$

C^2_{AB} and $C^2_{AC} \rightarrow$ squared entanglement between $(A; B)$ and $(A; C)$ respectively.

- For neutrino system,

$$C^2_{\mu e;\mu} + C^2_{\mu\tau;\mu} \leq C^{\mu^2} \tag{9}$$



Conclusion from monogamy plot

- Three flavor oscillation framework of neutrino clearly satisfies the CKW inequality across all the energy bins.
- Within EW1 this inequality approaches equality due to the negligible contribution from the appearance probability of electron neutrino.
- The inequality satisfies clearly in the energy bins EW2 and EW3 where $4P_{\mu e}P_{\mu\tau}$ is non zero.
- Analysis is the clear cut experimental evidence of the possibility of forming W state in the three flavor neutrino oscillation.
- **This experimental observation highlights the dominance of bipartite entanglement in neutrino flavor systems.**

Oscillation parameters at entanglement minima

Precise measurements of neutrino oscillation parameters are most effectively conducted in energy windows where one of the three flavor states is either in a pure state or nearly disentangled.

Condition	$\theta_{12}(^\circ)$	$\theta_{13}(^\circ)$	$\theta_{23}(^\circ)$	$\Delta m^2_{21}(\mathrm{eV}^2)$	$\Delta m^2_{31}(\mathrm{eV}^2)$	$\delta_{CP}(^\circ)$
Nufit v5.2	33.41	8.58	42.2	7.41×10^{-5}	2.507×10^{-3}	232
@ 2.50 GeV (0.03225)	31.31°	8.43°	45.2°	7.01×10^{-5}	2.487×10^{-3}	270
@ 1.27 GeV (0.00314)	35.71°	8.23°	45.2°	7.61×10^{-5}	2.587×10^{-3}	270

Table 2: Table presents the best fit values of the oscillation parameters from NuFit v5.2, local and global minima of concurrence.

Results

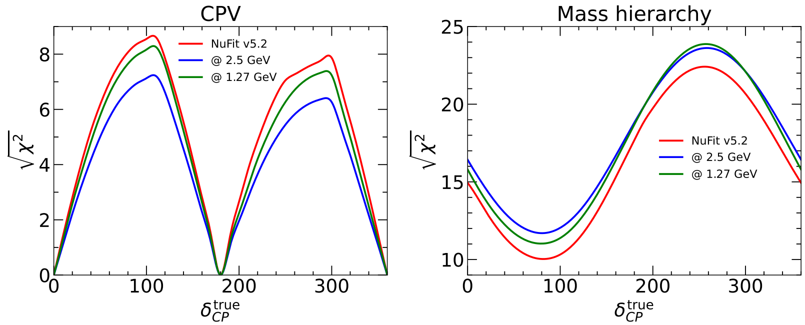


Figure 1: Physics sensitivities in the presence and absence of quantum entanglement concept. Left (right) panel is the CPV (mass hierarchy) sensitivity as a function of $\delta_{CP}^{\text{true}}$.

Main findings

- Value of the neutrino oscillation parameter at the global and local oscillation minima differs from the Nufit v5.2.
- CP sensitivity decreases at the local and global minima of concurrence than Nufit value.
- Sensitivity in the mass hierarchy increases at the minima of the concurrence.
- The value of θ_{23} angle depicts the maximal mixing in the neutrino sector.

Overall Conclusion

- Our findings show the development of maximal entanglement between the muon neutrino (ν_μ) and tau neutrino (ν_τ), while the electron neutrino (ν_e) remains nearly separable as a flavor state.
- Monogamy inequality held true in the DUNE setup, showing that the system exhibits bipartite entanglement and suggesting the potential presence of W-state type entanglement
- In the presence of quantum entanglement, CPV sensitivity was reduced from 8.5 σ to 8.2 σ and 7.0 σ for the "global" and "local" minima, respectively.
- Mass hierarchy increase when considering the concept of entanglement.
- For octant sensitivity, due to the atmospheric mixing angle approaching maximal mixing at the points of minimum entanglement, there is no significant improvement in sensitivity in the presence of entanglement.


NEUTRINO
EERIT

Backup Slides

$$|\nu_\alpha(t)\rangle = \sum_{\beta=e,\mu,\tau} \left(U_{\beta 1} U^*_{\alpha 1} e^{-iE_1 t} + U_{\beta 2} U^*_{\alpha 2} e^{-iE_2 t} + U_{\beta 3} U^*_{\alpha 3} e^{-iE_3 t} \right) |\nu_\beta\rangle, \tag{10}$$

$$|\nu_\mu(t)\rangle = \tilde{U}_{\mu e}|\nu_e\rangle + \tilde{U}_{\mu\mu}|\nu_\mu\rangle + \tilde{U}_{\mu\tau}|\nu_\tau\rangle, \tag{11}$$

where, $\tilde{U}_{\alpha\beta} = U_{\beta i} U^*_{\alpha i} e^{-iE_i t}$.
$|\tilde{U}_{\mu\mu}|^2 + |\tilde{U}_{\mu e}|^2 + |\tilde{U}_{\mu\tau}|^2 = 1$

$$\mathrm{EOF} = (S_A + S_B + S_C), \tag{12}$$

where the individual component S_A, S_B and S_C gives the entropy associated with the different flavor states in three flavor neutrino oscillations with the expression,

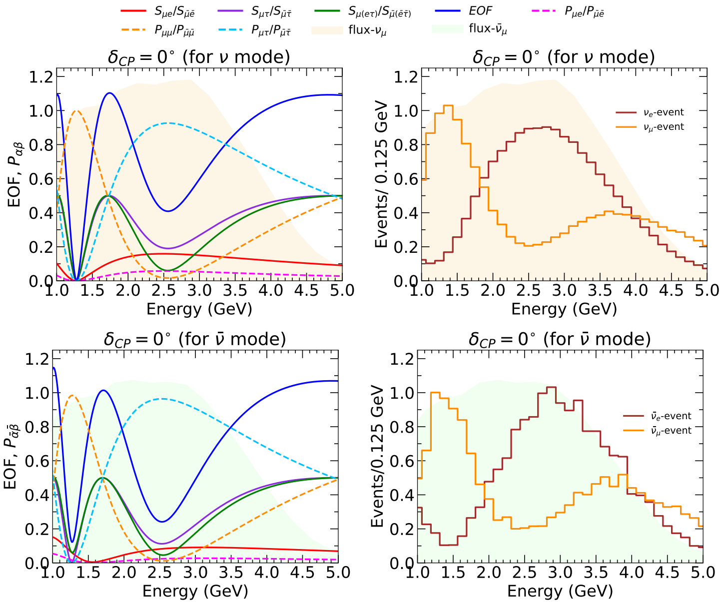
$S_A = -\frac{1}{2}\left[P_{\alpha\beta}\log_2 P_{\alpha\beta} + (P_{\alpha\alpha} + P_{\alpha\gamma})\log_2(P_{\alpha\alpha} + P_{\alpha\gamma})\right]$,
$S_B = -\frac{1}{2}\left[P_{\alpha\alpha}\log_2 P_{\alpha\alpha} + (P_{\alpha\beta} + P_{\alpha\gamma})\log_2(P_{\alpha\beta} + P_{\alpha\gamma})\right]$ and
$S_C = -\frac{1}{2}\left[P_{\alpha\gamma}\log_2 P_{\alpha\gamma} + (P_{\alpha\alpha} + P_{\alpha\beta})\log_2(P_{\alpha\alpha} + P_{\alpha\beta})\right]$

$$C^{\alpha} = \left[3 - \mathrm{Tr}(\rho_{\alpha})^2 + \mathrm{Tr}(\rho_{\beta})^2 + \mathrm{Tr}(\rho_{\gamma})^2\right]^{1/2}. \quad (13)$$

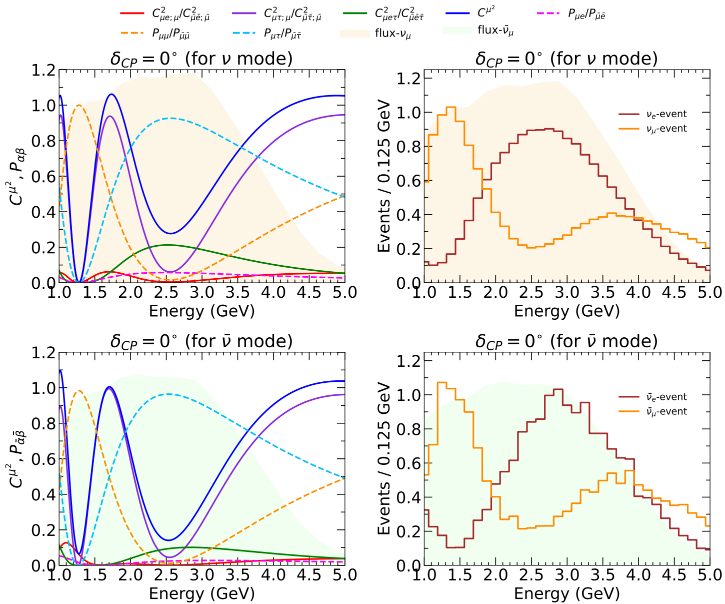
For the three-flavor neutrino system Eq. 13 modifies to the expression,

$$C^{\alpha^2} = C^2_{\beta\alpha;\alpha} + C^2_{\gamma\alpha;\alpha} + 4\mathrm{P}_{\alpha\beta}\mathrm{P}_{\alpha\gamma}\,. \quad (14)$$

Here, α denotes the initial flavor of the neutrino, while β and γ represent the oscillated flavor states.



$S_{\mu e}/S_{\bar\mu\bar e}$
$S_{\mu\tau}/S_{\bar\mu\bar\tau}$
$S_{\mu(e\tau)}/S_{\bar\mu(\bar e\bar\tau)}$
EOF
$P_{\mu e}/P_{\bar\mu\bar e}$
$P_{\mu\mu}/P_{\bar\mu\bar\mu}$
$P_{\mu\tau}/P_{\bar\mu\bar\tau}$
flux-ν_μ
flux-$\bar\nu_\mu$
$\delta_{CP} = 0^\circ$ (for ν mode)
EOF, $P_{\alpha\beta}$
Energy (GeV)
$\delta_{CP} = 0^\circ$ (for ν mode)
Events/ 0.125 GeV
ν_e-event
ν_μ-event
Energy (GeV)
$\delta_{CP} = 0^\circ$ (for $\bar\nu$ mode)
EOF, $P_{\bar\alpha\bar\beta}$
Energy (GeV)
$\delta_{CP} = 0^\circ$ (for $\bar\nu$ mode)
Events/0.125 GeV
$\bar\nu_e$-event
$\bar\nu_\mu$-event
Energy (GeV)



$C^2_{\mu e;\mu}/C^2_{\bar{\mu}\bar{e};\bar{\mu}}$
$C^2_{\mu\tau;\mu}/C^2_{\bar{\mu}\bar{\tau};\bar{\mu}}$
$C^2_{\mu e\tau}/C^2_{\bar{\mu}\bar{e}\bar{\tau}}$
C^{μ^2}
$P_{\mu e}/P_{\bar{\mu}\bar{e}}$
$P_{\mu\mu}/P_{\bar{\mu}\bar{\mu}}$
$P_{\mu\tau}/P_{\bar{\mu}\bar{\tau}}$
flux-ν_μ
flux-$\bar{\nu}_\mu$
$\delta_{CP} = 0^\circ$ (for ν mode)
$C^{\mu^2}, P_{\alpha\beta}$
Events / 0.125 GeV
ν_e-event
ν_μ-event
$\delta_{CP} = 0^\circ$ (for $\bar{\nu}$ mode)
$C^{\mu^2}, P_{\bar{\alpha}\bar{\beta}}$
$\bar{\nu}_e$-event
$\bar{\nu}_\mu$-event
Energy (GeV)

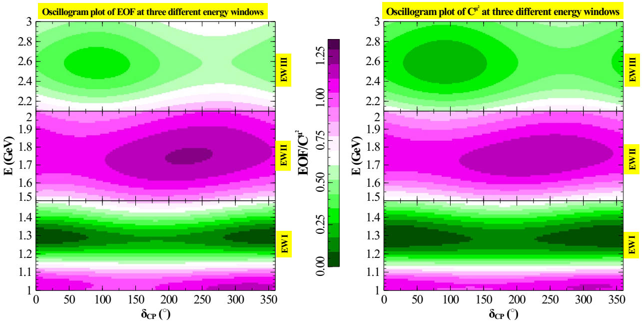
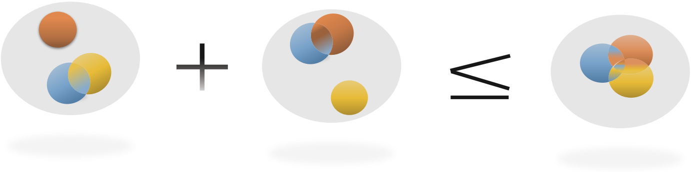
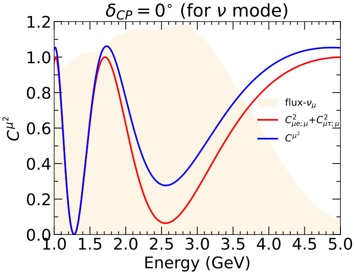


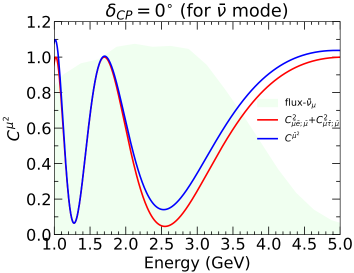
Figure 2: Left (right) panel shows the oscillogram plot of EOF (concurrence) at three different energy windows for whole δ_{CP} range.



Colour Scheme :

Muon neutrino   Tau neutrino   Electron neutrino





- Our findings show the development of maximal entanglement between the muon neutrino (ν_μ) and tau neutrino (ν_τ), while the electron neutrino (ν_e) remains nearly separable as a flavor state.
- o further investigate whether this entanglement is bipartite or genuine tripartite, we apply the MOE inequality, specifically utilizing the CKW inequality. This inequality held true in the DUNE setup, showing that the system exhibits bipartite entanglement and suggesting the potential presence of W-state type entanglement
- In the presence of quantum entanglement, CPV sensitivity was reduced from 8.5 σ to 8.2 σ and 7.0 σ for the "global" and "local" minima, respectively
- In contrast to CPV, the sensitivity to the mass hierarchy increase when considering the concept of entanglement. However, for octant sensitivity, due to the atmospheric mixing angle approaching maximal mixing at the points of minimum entanglement, there is no significant improvement in sensitivity in the presence of entanglement.