Neutrino quantum decoherence in a fluctuating ALPs field

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Outline

- Classical fluctuating ALPs field
- Equation for neutrino density matrix in a presence of ALPs fluctuations
- Dissipation matrix
- Estimations on correlation time of the ALPs fluctuations
- Estimations on neutrino-ALPs interaction
- Conclusion

Classical fluctuating ALPs field

Axion-like particles (ALPs) are pseudo-Goldstone bosons arising from spontaneous U(1) symmetry breaking, resembling axions but with independent coupling constants and masses, offering potential solutions to issues like the muon magnetic moment discrepancy and neutrino mass generation.

In case when ALPs form Bose-Einstein condensate they can be described as a real classical field. But in the literature there is a description of any ALPs as a classical field, since it is assumed that thermalization has occurred in the Universe and any ALP, acquiring mass, is already close to the vacuum of potential energy.

Classical fluctuating ALPs field

Fluctuating classical ALPs field has the following form

$$a(x) = \alpha(t)\frac{\sqrt{2\rho}}{m_a}\cos\left(m_a t - \mathbf{p}\cdot\mathbf{x}\right)$$

where ρ is an energy density of ALPs, m_a is a mass of ALPs, $\alpha(t)$ – positive random value, which obeys Rayleigh distribution

$$f(\alpha) = \alpha \exp\left(-\frac{\alpha^2}{2}\right).$$

Providing that ALPs velocity in Milky Way is order 10^{-3} c, dependence on momentum and coordinate is omitted

$$a(x) \approx \alpha(t)\frac{\sqrt{2\rho}}{m_a}\cos\left(m_a t\right).$$

Equation for neutrino density matrix in a presence of ALPs fluctuations

General equation

$$i\frac{d\rho_\nu(t)}{dt} = [H(t), \rho_\nu(t)]$$

Neutrino-ALPs interaction

$$H(t) = \int d^3\mathbf{x} j(x) a(x),$$

where

$$j(x) = i\bar{\nu}(x)\left(g_V + g_A\gamma_5\right)\nu(x)$$

while g_V and g_A are vector and axial coupling constant respectively (m_i is a neutrino mass)

$$g_V^{ij} = C_V^{ij}\frac{m_i - m_j}{F},$$
$$g_A^{ij} = C_A^{ij}\frac{m_i + m_j}{F},$$

Equation for neutrino density matrix in a presence of ALPs fluctuations

Classical ALPs field can be separated on two terms: mean field and fluctuations

$$a(t) = a_{mean}(t) + a_{fl}(t)$$

where

$$a_{mean}(t) = \frac{\sqrt{\pi\rho}}{m_a}\cos(m_a t),$$

and

$$a_{fl}(t) = \left(\alpha(t) - \sqrt{\frac{\pi}{2}}\right)\frac{\sqrt{2\rho}}{m_a}\cos(m_a t).$$

Equation for neutrino density matrix in a presence of ALPs fluctuations

To average fluctuating part, we use path integral with Rayleigh measure (similar to [1], where Gaussian perturbations are considered)

$$\langle g(a_{fl})\rangle = \int_0^{+\infty} \prod_t \left[\alpha(t) d\alpha(t)\right] g(a_{fl}) e^{-\int_0^t \frac{\alpha^2(t')}{4\tau} dt'}$$

Therefore, mean value and correlation of $a_{fl}(t)$ is equal to

$$\langle a_{fl}(t)\rangle = 0,$$
$$\langle a_{fl}(t_1) a_{fl}(t_2)\rangle = \frac{(4-\pi)\rho}{m_a^2} \cos^2(m_a t_1) \tau \delta(t_1 - t_2),$$

where τ is a correlation time.

[1] F. N. Loreti and A. B. Balantekin. Neutrino oscillations in noisy media. Phys. Rev. D, 50:4762–4770, Oct 1994

Equation for neutrino density matrix in a presence of ALPs fluctuations

The presence of mean and fluctuating terms of ALPs field allows to separate Hamiltonian

$$H(t) = a_{mean}(t)\mathcal{H}(t) + a_{fl}(t)\mathcal{H}(t),$$

where

$$\mathcal{H}(t) = \int d^3\mathbf{x} j(x).$$

It is possible to transform general equation to the following form

$$i\frac{d\rho'_\nu(t)}{dt} = a_{fl}(t)[\mathcal{H}'(t), \rho'_\nu(t)].$$

Formal solution

$$\rho'_\nu(t) = \rho'_\nu(t_0) - i\int_{t_0}^{t} a_{fl}(t_1)[\mathcal{H}'(t_1), \rho'_\nu(t_0)] - \int_{t_0}^{t} dt_1 \int_{t_0}^{t} dt_2 a_{fl}(t_1) a_{fl}(t_2)[\mathcal{H}'(t_1), [\mathcal{H}'(t_2), \rho_\nu(t_0)]] + \ldots$$

Equation for neutrino density matrix in a presence of ALPs fluctuations

After averaging it becomes

$$\rho'_\nu(t) = \rho'_\nu(t_0) - \int_{t_0}^{t} dt_1 \frac{(4-\pi)\rho}{m_a^2} \cos^2(m_a t_1)\tau[\mathcal{H}'(t_1), [\mathcal{H}'(t_1), \rho_\nu(t_0)]] + \ldots$$

Since the period of coherent ALPs oscillations $\frac{2\pi}{m_a} \approx 1.3$ year for $m_a \sim 10^{-22}$ eV and the more mass, the less period, it is possible to average out coherent oscillations. After averaging, formal solution obeys

$$\frac{d\rho'_\nu(t)}{dt} = -\frac{(4-\pi)\rho}{2m_a^2}\tau[\mathcal{H}'(t), [\mathcal{H}'(t), \rho'_\nu(t)]].$$

Finally, in Schrodinger representation

$$\frac{d\rho_\nu(t)}{dt} = -i[H_0 + a_{mean}(t)\mathcal{H}, \rho_\nu(t)] - \frac{(4-\pi)\rho}{2m_a^2}\tau[\mathcal{H}, [\mathcal{H}, \rho_\nu(t)]],$$

where H_0 is a free Hamiltonian.

Dissipation matrix

Dissipation term $D[\rho_\nu(t)] = -\frac{(4-\pi)\rho}{2m_a^2}\tau[\mathcal{H},[\mathcal{H},\rho_\nu(t)]]$ causes effect of quantum decoherence of neutrino oscillations. It is possible to decompose it into SU(3) group generators λ expressed in terms of Gell-Mann matrices (in a basis of neutrino mass states)

$$D[\rho_\nu(t)] = \sum_{a,b=0}^{8} D_{ab}\rho_a\lambda_b,$$

where $\rho_a = 2Tr(\rho_\nu(t)\lambda_a)$. The matrix element D_{ab} has the following view

$$D_{ab} = -\frac{(4-\pi)\rho}{2m_a^2}\tau \sum_{c,d,e=1}^{8} h_c h_d f_{ceb} f_{ade},$$

where $h_a = 2\,\mathrm{Tr}(\mathcal{H}\lambda_a)$ and f_{abc} is a structure constant of the SU(3) group.

Dissipation matrix

The full dissipation matrix has the following form

$$
D = -\frac{(4-\pi)\rho}{8m_a^2}\tau
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 4h_2^2+h_5^2+h_7^2 & 0 & 0 & 3h_2h_5 & 0 & -3h_2h_7 & 0 & 0 \\
0 & 0 & h_5^2+h_7^2 & 0 & 0 & -h_2h_5 & 0 & -h_2h_7 & 0 \\
0 & 0 & 0 & 4h_2^2+h_5^2+h_7^2 & -3h_2h_7 & 0 & -3h_2h_5 & 0 & 0 \\
0 & 3h_2h_5 & 0 & -3h_2h_7 & h_2^2+4h_5^2+h_7^2 & 0 & 3h_5h_7 & 0 & 0 \\
0 & 0 & -h_2h_5 & 0 & 0 & h_2^2+h_7^2 & 0 & -h_5h_7 & 0 \\
0 & -3h_2h_7 & 0 & -3h_2h_5 & 3h_5h_7 & 0 & h_2^2+h_5^2+4h_7^2 & 0 & 0 \\
0 & 0 & -h_2h_7 & 0 & 0 & -h_5h_7 & 0 & h_2^2+h_5^2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

$$
h_2 = \frac{\Delta m_{21}^2 \left(C_V^{21} - C_A^{21}\right)}{E_\nu F},
$$

$$
h_5 = \frac{\Delta m_{31}^2 \left(C_V^{31} - C_A^{31}\right)}{E_\nu F},
$$

$$
h_7 = \frac{\Delta m_{32}^2 \left(C_V^{32} - C_A^{32}\right)}{E_\nu F}.
$$

As it is seen, matrix contains non-diagonal elements!

Dissipation matrix

In the literature, dependence of the decoherence matrix on the neutrino energy has been reported

$$D_{ij}(E_\nu) = D_{ij}(E_0)\left(\frac{E_\nu}{E_0}\right)^n,$$

where E_0 is a pivot energy scale equals to 1 GeV and n is a power-law index.

- n = -1: neutrino decay [2]
- n = -2: gravitational waves [3]

As can be seen from the dependence of the dissipative matrix on the neutrino energy, the value of the power index is also −2

[2] K. Stankevich, A. Studenikin, and M. Vyalkov. Generalized Lindblad master equation for neutrino evolution. Phys. Rev. D, 111:036014, Feb 2025.
[3] A. Domi, T. Eberl, M. J. Fahn, K. Giesel, L. Hennig, U. Katz, R. Kemper, and M. Kobler, ” JCAP 11 (2024) 006.

Estimations on correlation time of the ALPs fluctuations

The limits of applicability of the method of averaging ALPs field fluctuations

$$\tau E_{max} \ll 1,$$

where E_{max} corresponds to the maximum element of matrix $[H_0, \mathcal{H}]$. Condition for τ becomes

$$\tau \frac{|\Delta m_{13}^2|^2}{8E_\nu^2 F} \ll 1.$$

Therefore, the critical correlation time τ_{cr} can be chosen

$$\tau_{cr} = \frac{0.8 E_\nu^2 F}{|\Delta m_{13}^2|^2}$$

For the case of supernova neutrino it can be estimated by 10^7 s.

Estimations on correlation time of the ALPs fluctuations

Supposed parameters

- ALPs mass: 10^{-22}-10^{-13} eV (for ultra-light Dark Matter)
- Neutrino oscillations parameters:

Parameter	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	$\sin^2\theta_{13}$	$\Delta m^2_{12}/\text{eV}^2$	$\lvert\Delta m^2_{13}\rvert/\text{eV}^2$
Value	0.307	0.561	0.022	7.49×10^{-5}	2.534×10^{-3}

- ALPs-neutrino interaction

1) V: only g_V is non-zero
2) A: only g_A is non-zero
3) V-A: $g_V = -g_A$ (left neutrinos interaction)
4) V+A: $g_V = g_A$ (right neutrinos interaction; excluded, because D = 0)

Estimations on correlation time of the ALPs fluctuations

- Best bounds on ALPs-lepton interaction from [4] ($g_{\tau\tau}$ is set to be zero)

Constant	V (GeV)	A (GeV)	V-A (GeV)
F_{ee}	-	4.6×10^9	-
$F_{\mu e}$	4.8×10^9	4.8×10^9	1.0×10^9
$F_{\mu\mu}$	-	1.3×10^8	-
$F_{\tau e}$	4.3×10^6	4.3×10^6	4.3×10^6
$F_{\tau\mu}$	3.3×10^6	3.3×10^6	3.3×10^6

where

$$F_{\alpha\beta} = \frac{F}{\sqrt{|C_V^{\alpha\beta}|^2 + |C_A^{\alpha\beta}|^2}}.$$

[4] Lorenzo Calibbi, Diego Redigolo, Robert Ziegler, and Jure Zupan. Looking forward to lepton-flavor-violating ALPs. J. High Energ. Phys., 2021, 2021.

Estimations on correlation time of the ALPs fluctuations

- Constraints on decoherence parameter (we set $D_{ij}(E_0) = \gamma_0$) from SN1987A data from [5]
  1) Time-integrated analysis (TI): $\gamma_0 = 3.1\times10^{-41}$ GeV
  2) Time-dependent analysis (TD): $\gamma_0 = 6.4\times10^{-41}$ GeV
- Substitution of the bounds on ALPs-lepton interaction in dissipation matrix gives

$$D_{max} \sim \frac{\tau}{m_a^2} \times 10^{-74} \text{ eV}$$

Therefore, for the ALPs mass range 10^{-22}-10^{-13} eV correlation time should be between 10^{-18} and 1 s

[5] Christoph A. Ternes, Giulia Pagliaroli, and Francesco L. Villante. SN1987A bounds on neutrino quantum decoherence. arXiv:2503.04573 [hep-ph], 2025

Estimations on neutrino-ALPs interaction

To obtain the most conservative estimates of the coupling constants between ALPs and neutrinos, we focused our analysis on the diagonal elements of the decoherence matrix. In literature three cases of diagonal matrices are considered

$$D_{\text{phase-pert.}} = \text{diag}(0, \Gamma, \Gamma, 0, \Gamma, \Gamma, \Gamma, \Gamma, 0),$$

$$D_{\text{state-select}} = \text{diag}(0, \Gamma, \Gamma, \Gamma, \Gamma, \Gamma, \Gamma, \Gamma, \Gamma),$$

$$D_{\nu\text{-loss}} = \text{diag}(\Gamma, \Gamma, \Gamma, \Gamma, \Gamma, \Gamma, \Gamma, \Gamma, \Gamma),$$

where $\Gamma = \gamma_0 \left(\frac{E_\nu}{E_0}\right)^n$.

Estimations on neutrino-ALPs interaction

Since the disspative matrix does not correspond to any of these cases, we have to at least constrain the diagonal elements by Γ. The conservative estimation gives the following limits on h_j^2 (if the elements with three terms are constrained by Γ then it will be the same for the elements with two ones)

$$4h_2^2 + h_5^2 + h_7^2 \leq \frac{8m_a^2\gamma_0}{(4-\pi)\rho\tau}\left(\frac{E_0}{E_\nu}\right)^2,$$
$$h_2^2 + 4h_5^2 + h_7^2 \leq \frac{8m_a^2\gamma_0}{(4-\pi)\rho\tau}\left(\frac{E_0}{E_\nu}\right)^2,$$
$$h_2^2 + h_5^2 + 4h_7^2 \leq \frac{8m_a^2\gamma_0}{(4-\pi)\rho\tau}\left(\frac{E_0}{E_\nu}\right)^2.$$

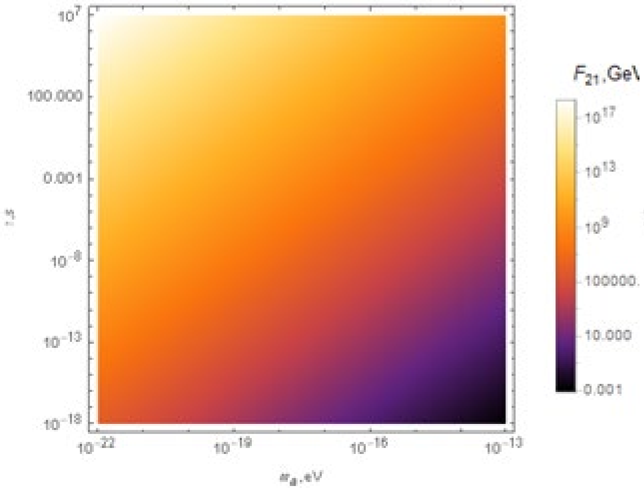
$$\Rightarrow$$

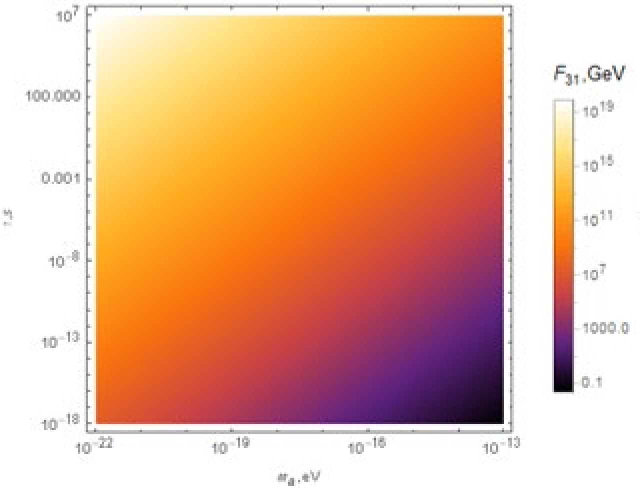
$$h_2^2 \leq \frac{4m_a^2\gamma_0}{3(4-\pi)\rho\tau}\left(\frac{E_0}{E_\nu}\right)^2,$$
$$h_5^2 \leq \frac{4m_a^2\gamma_0}{3(4-\pi)\rho\tau}\left(\frac{E_0}{E_\nu}\right)^2,$$
$$h_7^2 \leq \frac{4m_a^2\gamma_0}{3(4-\pi)\rho\tau}\left(\frac{E_0}{E_\nu}\right)^2.$$

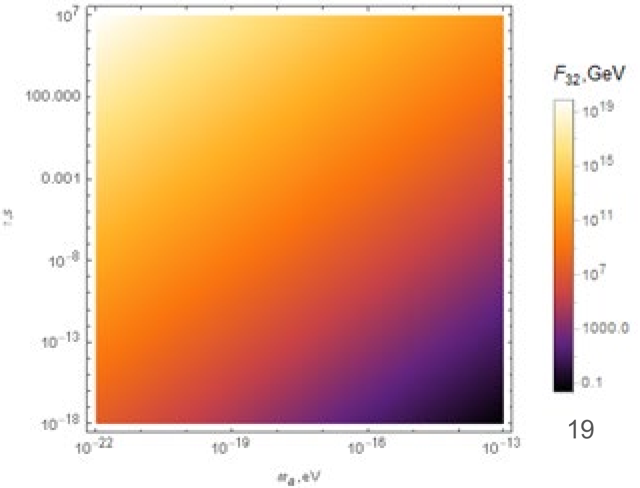
Estimations on neutrino-ALPs interaction

Therefore, we arrive at the following relation, which relates the coupling constant of neutrinos and axion-like particles to other parameters of the system

$$\frac{F}{C_V^{ij} - C_A^{ij}} \geq \frac{\Delta m_{ij}^2 \sqrt{3(4-\pi)\rho\tau}}{2 m_a E_0 \sqrt{\gamma_0}}.$$







Conclusion

The present study is devoted to the research of the quantum decoherence of neutrino oscillations under the influence of stochastic fluctuations in the classical ALPs field. It was shown that the neutrino density matrix in the case of interaction with such fluctuations is described by the Redfield equation, which made it possible to analyze the decoherent effects resulting from the interaction of neutrinos with the fluctuating ALPs field.

Using the limits on decoherence parameter from the SN1987A data, we provide the constraints on the correlation time between ALPs fluctuation and also the limits on ALPs-neutrino interaction were obtained.