Probe of transverse neutrino spin polarization with the electromagnetic interactions in neutrino scattering on electrons, nucleons and nuclei.

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TWENTY-SECOND LOMONOSOV
CONFERENCE August, 21-27, 2025
ON ELEMENTARY PARTICLE PHYSICS
MOSCOW STATE UNIVERSITY

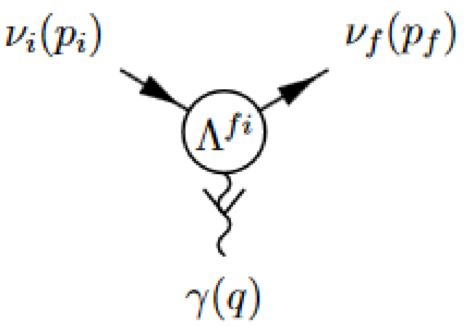
Search for electromagnetic neutrino properties

C. Giunti, A. Studenikin, Neutrino electromagnetic interactions: a window to new physics Rev. Mod. Phys. **87**, 531 (2015)
A. Studenikin, Electromagnetic neutrinos: The basic interaction processes and constraints from laboratory experiments and astrophysics. Int.J.Mod.Phys.E **33**, 2441033 (2024)

Neutrino electromagnetic properties open a window to the beyond-Standard-Model physics

- Already in SM, neutrinos have a ***charge radius***
  [Bernabeu et al, PRD (2000), PRL (2002), arXiv (2003)]

$$< r_\nu^2 > \sim 10^{-32}\mathrm{cm}^2$$



- Minimally extended SM predicts the neutrino's ***magnetic momer***
  [Fujikawa, Shrock, PRL (1980); Shrock, NPB (1982)]

$$\mu_\nu = 3{,}2 \times 10^{-19}\left(\frac{m_\nu}{1\ \mathrm{eV}}\right)\mu_B$$

- Neutrinos may also have other electromagnetic properties:
  millicharge, electric and anapole moments

$$(\Lambda_\mu(q))_{jk} = (\gamma_\mu - \frac{q_\mu \not{q}}{q^2})[(f_Q(q^2))_{jk} + \gamma_5 (f_A(q^2))_{jk} q^2] - i\sigma_{\mu\nu} q^\nu (f_M(q^2))_{jk} + \sigma_{\mu\nu} q^\nu \gamma_5 (f_E(q^2))_{jk}$$

$$f_Q^{jk}(0) = e_{jk}, \quad f_M^{jk}(0) = \mu_{jk}, \quad f_E^{jk}(0) = \epsilon_{jk}, \quad f_A^{jk}(0) = a_{jk} \quad \langle r_\nu^2 \rangle = 6\frac{d f_Q(q^2)}{dq^2}\bigg|_{q^2=0}$$

millicharge

magnetic dipole moment

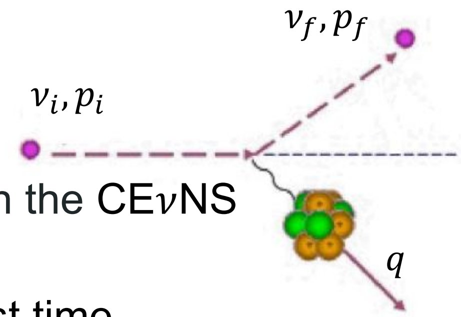
electric dipole moment

anapole moment

charge radius

Coherent elastic neutrino-nucleus scattering

CEνNS



Electromagnetic properties of neutrinos can be probed experimentally with the CEνNS

- In 2017 COHERENT collaboration observed CEνNS process for the first time
- Data of COHERENT and Dresden-II experiments have been already used to obtain limits for neutrino's millicharge, charge radius and magnetic moment

Charge radii ($10^{-32} cm^2$)

	Fixed R_n			Free R_n		
	1σ	2σ	3σ	1σ	2σ	3σ
			CsI + Ar			
$\langle r^2_{\nu_{ee}} \rangle$	$-56 \div -2$	$-68 \div 11$	$-78 \div 22$	$-55 \div -4$	$-67 \div 14$	$-77 \div 25$
$\langle r^2_{\nu_{\mu\mu}} \rangle$	$-64 \div 6$	$-68 \div 12$	$-71 \div 17$	$-64 \div 9$	$-67 \div 15$	$-71 \div 19$
$\langle r^2_{\nu_{e\mu}} \rangle$	<27	<33	<36	<25	<32	<36
$\langle r^2_{\nu_{e\tau}} \rangle$	<27	<40	<50	<26	<40	<50
$\langle r^2_{\nu_{\mu\tau}} \rangle$	<36	<40	<44	<36	<40	<44

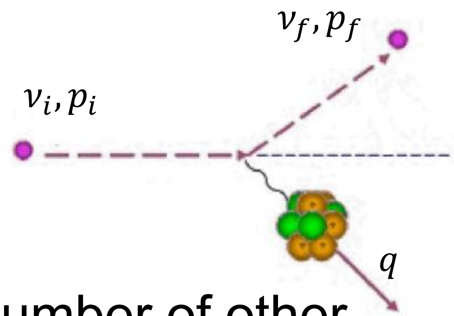
Magnetic moments ($10^{-10} \mu_B$)

	Fixed R_n			Free R_n		
	1σ	2σ	3σ	1σ	2σ	3σ
			CsI + Ar			
$\lvert\mu_{\nu_e}\rvert$	<27	<44	<56	<33	<48	<60
$\lvert\mu_{\nu_\mu}\rvert$	$5 \div 27$	<34	<41	$12 \div 31$	<37	<43

Cadeddu, M., Dordei, F., Giunti, C., Li, Y. F., Picciau, E., & Zhang, Y. Y. (2020). *PRD*, *102*(1), 015030.
Cadeddu, M., Giunti, C., Kouzakov, K. A., Li, Y. F., Studenikin, A. I., & Zhang, Y. Y. (2018). *PRD*, 98(11), 113010.

Coherent elastic neutrino-nucleus scattering

CEνNS

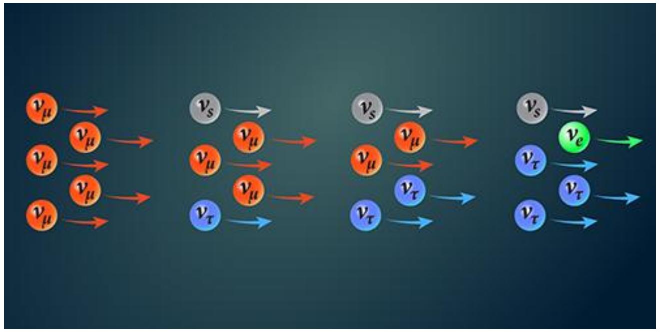


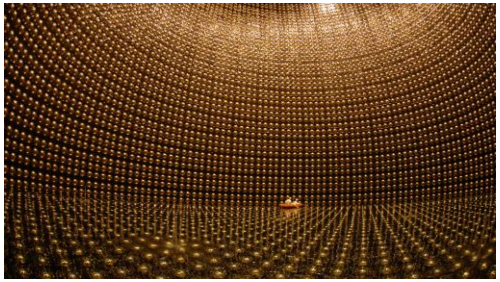
- In addition to COHERENT and Dresden-II there is a number of other CEvNS experiments:
  vGEN, CONUS, CONNIE, NU-CLEUS, MINER, RED-100, CEVENS, Ricochet, TEXONO, ,…

- In order to investigate neutrino electromagnetic properties in CEνNS experiments we need a theoretical apparatus, which takes into account ALL form factors of the neutrino and nucleus

- A proton is the simplest nuclear target. Moreover, elastic neutrino-proton scattering is a promising tool for detecting supernova neutrinos (JUNO yellow book arXiv:1507.05613)

Astrophysical neutrino's state in the detector on Earth





- source
- oscillation
- detector

Due to interaction of the neutrino magnetic moment with a magnetic field in the astrophysical source and/or with interstellar/intergalactic one the spin-flavor neutrino oscillations arise

Therefore, in the most general case the neutrino state in the detector before scattering on a nucleon is described by the spin-flavor density matrix (written in the mass basis)

$$\rho_{ij} = \frac{1}{2}\not{k}\left(\tilde{\rho}_{ij} - \zeta^{\parallel}_{ij}\gamma_5 + (\boldsymbol{\zeta}^{\perp}_{ij}\cdot\boldsymbol{\gamma}_{\perp})\gamma_5\right)$$

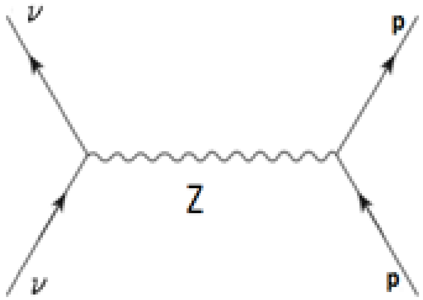
$\tilde{\rho}_{ij}$ is a reduced density matrix in the neutrino mass space

$\boldsymbol{\zeta}_{ij}$ form the matrix of spin polarizations of the neutrino in its rest frame

K.A. Kouzakov, F.M. Lazarev, A.I. Studenikin, Phys. Rev. D, 111 (2025) 3, 035025

Neutrino-nucleon scattering

The matrix element of the process with account for all the neutrino and nucleon form factors:



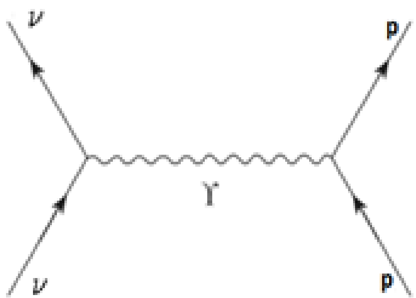
$$= -\frac{G_F}{\sqrt{2}} \bar{u}^{(\nu_n)}_{k',r'} \gamma^\mu (1-\gamma^5)\delta^{ni} u^{(\nu_i)}_{k,r}\, \bar{u}^{(N)}_{p',s'} \Lambda_\mu^{(\mathrm{NC};N)}(-q)\, u^{(N)}_{p,s}$$

Nucleon neutral weak vertex

+

Neutrino electromagnetic vertex

Nucleon electromagnetic vertex



$$= \frac{4\pi\alpha}{q^2} \bar{u}^{(\nu_n)}_{k',r'} \Lambda_\mu^{(\mathrm{EM};\nu)ni}(q)\, u^{(\nu_i)}_{k,r}\, \bar{u}^{(N)}_{p',s'} \Lambda_\mu^{(\mathrm{NC};N)}(-q)\, u^{(N)}_{p,s}$$

Neutrino vertex and form factors

Neutrino electromagnetic vertex:

$$\Lambda_\mu^{(\mathrm{EM};\nu)fi}(q) = (\gamma_\mu - q_\mu \not{q}/q^2)[f_Q^{fi}(q^2) + f_A^{fi}(q^2)q^2\gamma_5] - i\sigma_{\mu\nu}q^\nu[f_M^{fi}(q^2) + if_E^{fi}(q^2)\gamma_5]$$

$$f_Q^{jk}(0) = e_{jk}, \quad f_M^{jk}(0) = \mu_{jk}, \quad f_E^{jk}(0) = \epsilon_{jk}, \quad f_A^{jk}(0) = a_{jk} \quad \langle r_\nu^2 \rangle = 6\frac{df_Q(q^2)}{dq^2}\Big|_{q^2=0}$$

- e_{jk}: millicharge
- μ_{jk}: magnetic dipole moment
- ϵ_{jk}: electric dipole moment
- a_{jk}: anapole moment
- $\langle r_\nu^2 \rangle$: charge radius

Nucleon vertexes and form factors

Nucleon electromagnetic vertex:

$$\Lambda_\mu^{(\mathrm{EM};N)}(q) = \gamma_\mu F_Q(q^2) - \frac{i}{2m_N}\sigma_{\mu\nu}q^\nu F_M(q^2) + \frac{1}{2m_N}\sigma_{\mu\nu}q^\nu\gamma_5 F_E(q^2) - \left(q^2\gamma_\mu - q_\mu \not{q}\right)\gamma_5\frac{F_A(q^2)}{(2m_N)^2}$$

Charge — Magnetic — Electric — Anapole

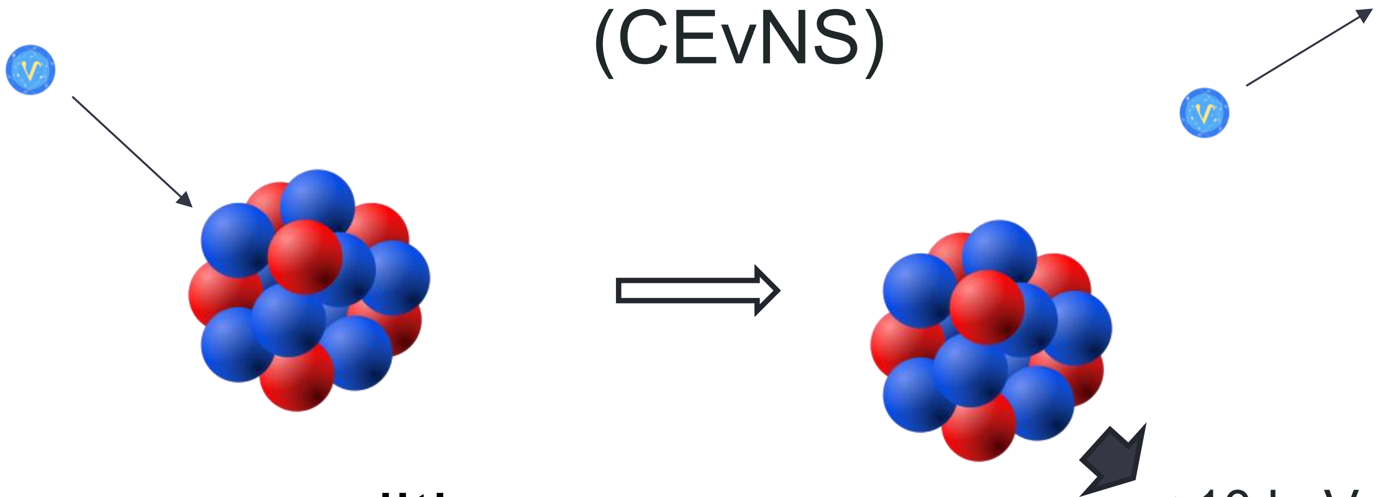
Nucleon neutral weak vertex:

$$\Lambda_\mu^{(\mathrm{NC};N)}(q) = \gamma_\mu F_1(q^2) - \frac{i}{2m_N}\sigma_{\mu\nu}q^\nu F_2(q^2) - \gamma_\mu\gamma_5 G_A(q^2) + \frac{1}{2m_N}G_P(q^2)q^\mu\gamma_5$$

Dirac — Pauli — Axial — Pseudoscalar

We omit terms containing the pseudoscalar form factor in the cross section due to a small neutrino mass

Coherent elastic neutrino nucleus scattering (CEvNS)



$\lesssim$ 10 keV recoil energy

Coherence condition

$$|\,\vec{\mathbf{q}}\,| \cdot R_{\mathrm{Nucl}} \ll 1$$

momentum transfer

nuclear radius

$$\frac{d\sigma_{CE\nu NS}}{dT} \propto N^2$$

T – is the recoil energy

number of neutrons

$$\mathcal{J}^{(EM),\mu}(q) = 2M\,\{\mathcal{F}_Q(q)\,,\,\mathbf{0}\}\,,$$

$$\mathcal{J}^{(NC),\mu}(q) = 2M\,\{\mathcal{F}_1(q)\,,\,-\mathcal{G}_A(q)\}\,,$$

$$\mathcal{F}_Q(\boldsymbol{q}) = \int d^3r e^{i(\boldsymbol{q}\cdot\boldsymbol{r})}\langle 00|\sum_{a=1}^{A} Q_a \delta(\boldsymbol{r}-\boldsymbol{r}_a)|00\rangle,$$

$$\mathcal{F}_1(\boldsymbol{q}) = \int d^3r e^{i(\boldsymbol{q}\cdot\boldsymbol{r})}\langle 00|\sum_{a=1}^{A} g_V^a \delta(\boldsymbol{r}-\boldsymbol{r}_a)|00\rangle.$$

The neutrino-nucleon cross section

$$\frac{d^2\sigma}{dT d\varphi} = A(T) - B_s(T)\sin\varphi + B_c(T)\cos\varphi$$

$$A(T) = \frac{G_F^2 m_N}{2\pi^2}\left[C_V^L + C_A^L + (C_A^L - C_V^L)\frac{m_N T}{2E_\nu}\right] + \frac{\alpha^2|\mu_\nu|^2}{2m_e^2 E_\nu}\left[\frac{E_\nu}{T}F_Q^2\right]$$

$$B_s(T) = -\frac{G_F\alpha}{m_e\pi}\sqrt{\frac{m_N}{T}}\,\mathrm{Im}K^{\perp}_{\mu_\nu,Z^0}F_1F_Q - \frac{4\sqrt{2}\alpha^2}{m_e}\sqrt{\frac{m_N}{T}}\,\mathrm{Im}K^{-}F_Q^2,$$

$$B_c(T) = -\frac{G_F\alpha}{m_e\pi}\sqrt{\frac{m_N}{T}}\,\mathrm{Re}K^{\perp}_{\mu_\nu,Z^0}F_1F_Q - \frac{4\sqrt{2}\alpha^2}{m_e}\sqrt{\frac{m_N}{T}}\,\mathrm{Re}K^{-}F_Q^2,$$

Numerical results based on general expressions, which can be found in K.A. Kouzakov, F.M. Lazarev, A.I. Studenikin, Phys. Rev. D, 111 (2025) 3, 035025

$$C_V^L = \frac{1}{2}Tr\left[\left(F_1^N - F_Q^N Q^L\right)^2\tilde{\rho}\right], \quad C_A^L = \frac{1}{2}\left(G_A^N\right)^2$$

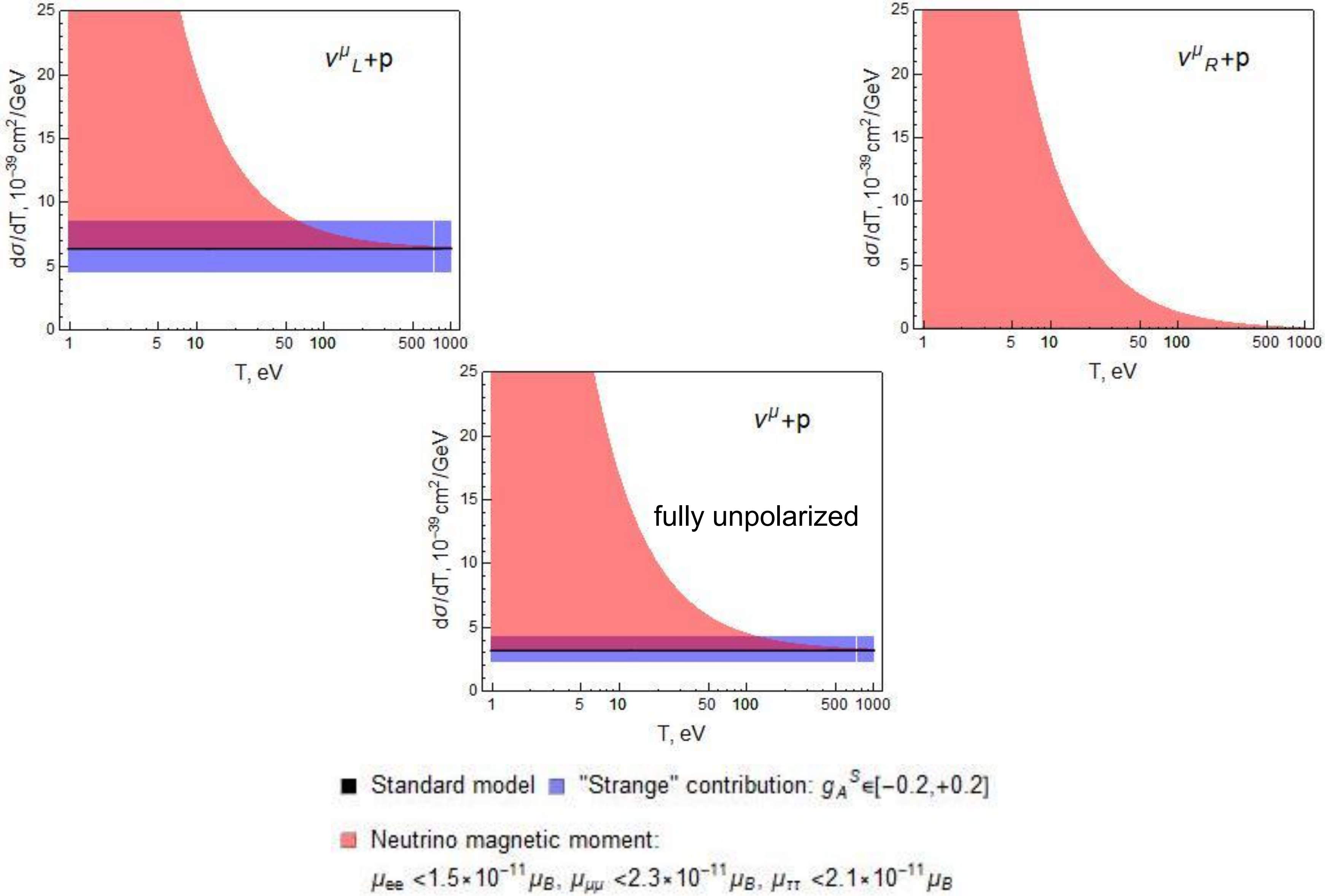
$$Q^L = \frac{2\sqrt{2}\pi\alpha}{G_F}\left(\frac{1}{6}\langle r_\nu^2\rangle - a_\nu\right), \quad |\mu_\nu|^2 = Tr\left[\left(f^M\right)^2\tilde{\rho}\right]$$

$$K^{\perp}_{\mu_\nu,Z^0} = Tr\left[\left(f^M\right)\kappa^\dagger\right], \quad K^{-} = Tr\left[a_\nu f^M \kappa^\dagger\right]$$

$$\kappa_{ij} = \frac{1}{2}\left(\zeta_{ij}^x + i\zeta_{ij}^y\right)$$

transverse neutrino polarization components

Numerical results for the ν_μ scattering on a nucleon: The effect of the **diagonal magnetic moment**



The neutrino-nucleus cross section

$$\frac{d^2\sigma}{dT d\varphi} = A(T) - B_s(T)\sin\varphi + B_c(T)\cos\varphi$$

$$A(T) = \frac{G_F^2 M}{2\pi^2}\left(1 - \frac{MT}{2E_\nu}\right)\mathcal{C}_V^L + \frac{\alpha^2|\mu_\nu|^2}{2m_e^2 E_\nu}\left[\frac{E_\nu}{T}\mathcal{F}_Q^2\right],$$

$$B_s(T) = -\frac{G_F\alpha}{m_e\pi}\sqrt{\frac{M}{T}}\mathrm{Im}K^{\perp}_{\mu_\nu,Z^0}\mathcal{F}_1\mathcal{F}_Q - \frac{4\sqrt{2}\alpha^2}{m_e}\sqrt{\frac{m_N}{T}}\mathrm{Im}K^{-}\mathcal{F}_Q^2,$$

$$B_c(T) = -\frac{G_F\alpha}{m_e\pi}\sqrt{\frac{M}{T}}\mathrm{Re}K^{\perp}_{\mu_\nu,Z^0}\mathcal{F}_1\mathcal{F}_Q - \frac{4\sqrt{2}\alpha^2}{m_e}\sqrt{\frac{m_N}{T}}\mathrm{Re}K^{-}\mathcal{F}_Q^2,$$

$$\mathcal{C}_V^L = \frac{1}{2}Tr\left[\left(-\mathcal{F}_1 + \mathcal{F}_Q Q^L\right)^2\tilde{\rho}\right]$$

$$Q^L = \frac{2\sqrt{2}\pi\alpha}{G_F}\left(\frac{1}{6}\langle r_\nu^2\rangle - a_\nu\right), \quad |\mu_\nu|^2 = Tr\left[\left(f^M\right)^2\tilde{\rho}\right]$$

$$K^{\perp}_{\mu_\nu,Z^0} = Tr\left[\left(f^M\right)\kappa^\dagger\right], \quad K^{-} = Tr\left[a_\nu f^M \kappa^\dagger\right]$$

$$\kappa_{ij} = \frac{1}{2}\left(\zeta^x_{ij} + i\zeta^y_{ij}\right)$$

The neutrino-electron cross section

$$\frac{d^2\sigma}{dT d\varphi} = A(T) - B_s(T)\sin\varphi + B_c(T)\cos\varphi$$

$$A(T) = \frac{G_F^2 m_e}{2\pi^2}\left[C_V^L(e^-) + C_A^L(e^-) + \left(C_A^L(e^-) - C_V^L(e^-)\right)\frac{m_e T}{2E_\nu}\right] + \frac{\alpha^2|\mu_\nu|^2}{2m_e^2 E_\nu}\left[\frac{E_\nu}{T}\right],$$

$$B_s(T) = \frac{G_F\alpha}{\pi\sqrt{m_e T}}\mathrm{Im}K^{\perp}_{\mu_\nu,Z^0} g_V^e + \frac{G_F\alpha}{\pi\sqrt{m_e T}}\mathrm{Im}K^{\perp}_{\mu_\nu,W} - \frac{4\sqrt{2}\alpha^2}{\sqrt{m_e T}}\mathrm{Im}K^{-},$$

$$B_c(T) = \frac{G_F\alpha}{\pi\sqrt{m_e T}}\mathrm{Re}K^{\perp}_{\mu_\nu,Z^0} g_V^e + \frac{G_F\alpha}{\pi\sqrt{m_e T}}\mathrm{Re}K^{\perp}_{\mu_\nu,W} - \frac{4\sqrt{2}\alpha^2}{\sqrt{m_e T}}\mathrm{Re}K^{-},$$

$$C_V^L(e^-) = \frac{1}{2}Tr\left[\left(-\hat{g}_V^e - Q^L\right)^2\tilde{\rho}\right], \quad C_A^L(e^-) = \frac{1}{2}Tr\left[\left(\hat{g}_A^e\right)^2\tilde{\rho}\right], \quad (\hat{g}_{V,A}^e)_{ij} = g_{V,A}^e\delta_{ij} + U_{ei}^* U_{ej}$$

$$Q^L = \frac{2\sqrt{2}\pi\alpha}{G_F}\left(\frac{1}{6}\langle r_\nu^2\rangle - a_\nu\right), \quad |\mu_\nu|^2 = Tr\left[\left(f^M\right)^2\tilde{\rho}\right]$$

$$K^{\perp}_{\mu_\nu,Z^0} = Tr\left[\left(f^M\right)\kappa^\dagger\right], \quad K^- = Tr\left[a_\nu f^M\kappa^\dagger\right] \quad K^{\perp}_{\mu_\nu,W} = Tr\left[\left(\hat{g}_V^e - g_V^e\right)f^M\kappa^\dagger\right]$$

$$\kappa_{ij} = \frac{1}{2}\left(\zeta_{ij}^x + i\zeta_{ij}^y\right)$$

transverse neutrino polarization components

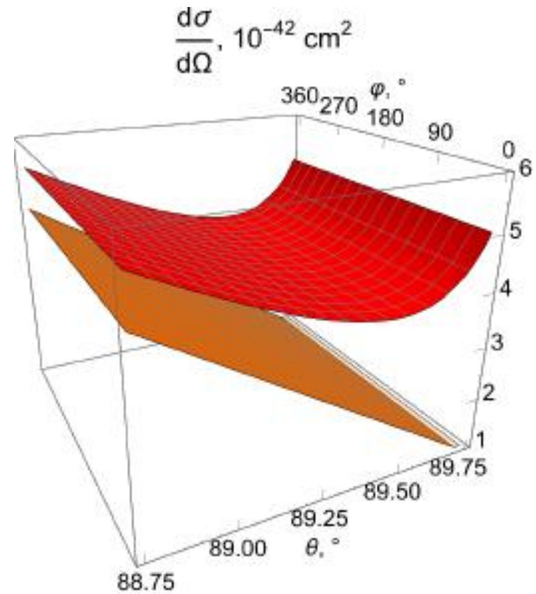
Scale of different contributions

Particle	$\frac{G_F^2 m}{2\pi^2} C_w$	$\frac{\alpha^2 \lvert\mu_\nu\rvert^2}{2m_e^2 E_\nu} Q^2$	$\frac{G_F \alpha}{m_e \pi} \mu_\nu Q_w Q$	$\frac{4\sqrt{2}\alpha^2}{m_e} a_\nu \mu_\nu Q^2$
e^-	4.5×10^{-25} (2.0×10^{-24})	2.3×10^{-27}	3.0×10^{-26} (7.7×10^{-25})	4.7×10^{-26}
p	8.2×10^{-22}	2.3×10^{-27}	3.0×10^{-26}	4.7×10^{-26}
n	1.6×10^{-21}	0	0	0
^{40}Ar	1.4×10^{-17}	7.4×10^{-25}	1.5×10^{-22}	1.5×10^{-23}
^{132}Xe	5.8×10^{-16}	6.7×10^{-24}	1.6×10^{-21}	1.4×10^{-22}

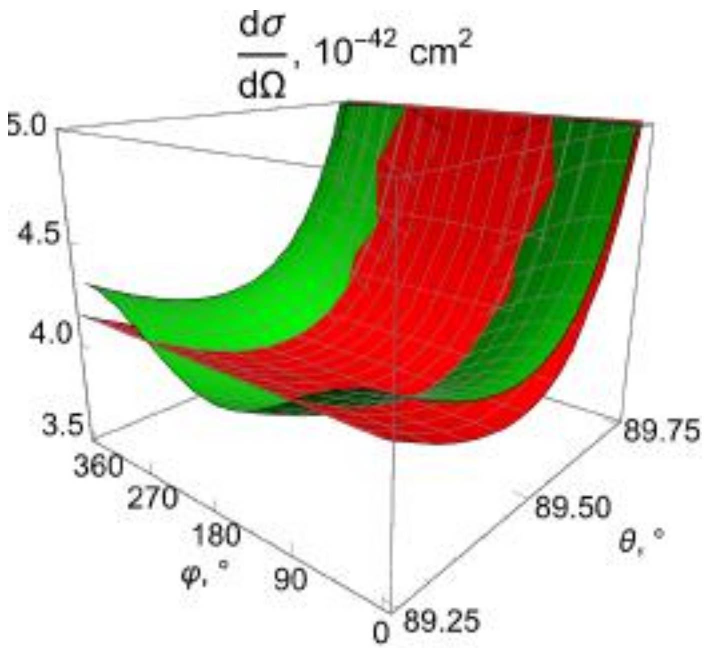
Particle	m	Q	Q_w	C_w
e^-	511 кэВ	-1	$g_V^e = \mp\frac{1}{2} + 2s_W^2 = -0.038(0.962)$	0.13(0.57)
p	938 МэВ	1	$g_V^p = \frac{1}{2} - 2s_W^2 = 0.038$	0.13
n	940 МэВ	0	$g_V^n = -0.5$	0.25
^{40}Ar	37.2 ГэВ	18	$\mathcal{F}_1 = 18g_V^p + 22g_V^n = -10.324$	53.29
^{132}Xe	122.9 ГэВ	54	$\mathcal{F}_1 = 54g_V^p + 78g_V^n = -36.971$	683.43

Numerical results for the ν scattering on a nucleon:
The effect of the **diagonal magnetic moment** and **transverse neutrino polarization**

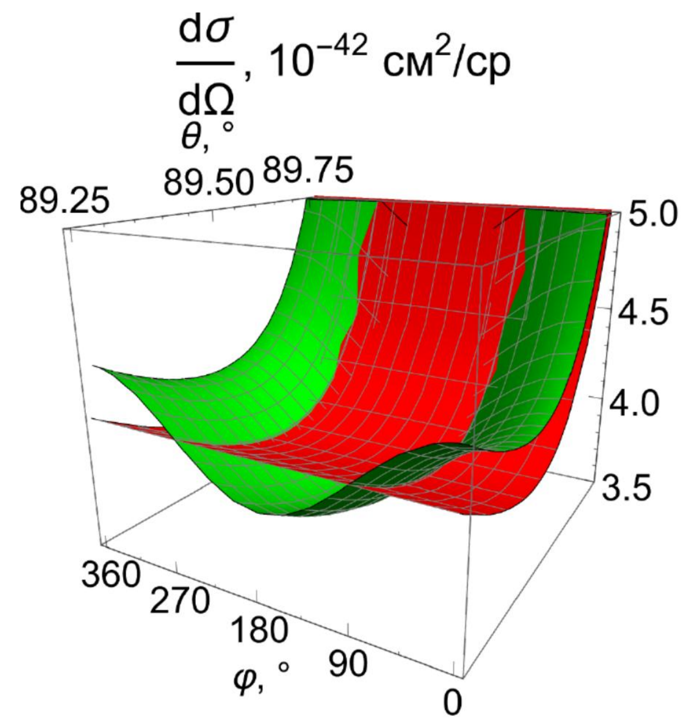
Orange surface is fully unpolarized neutrino in the SM



Red surfaces are fully unpolarized neutrino with magnetic moment

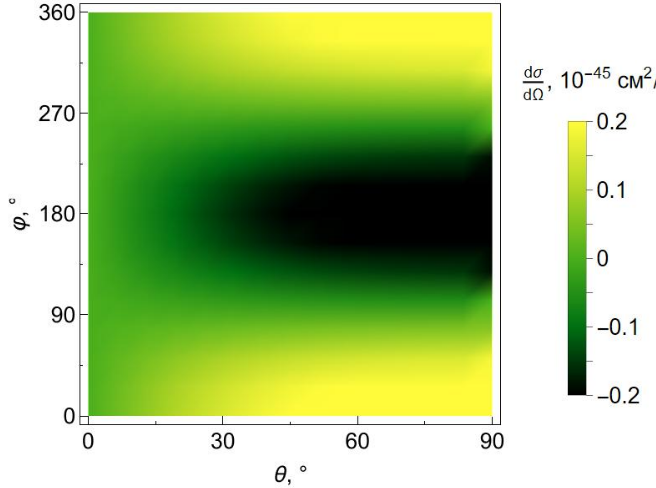


Green surfaces are transversely polarized neutrino with magnetic moment

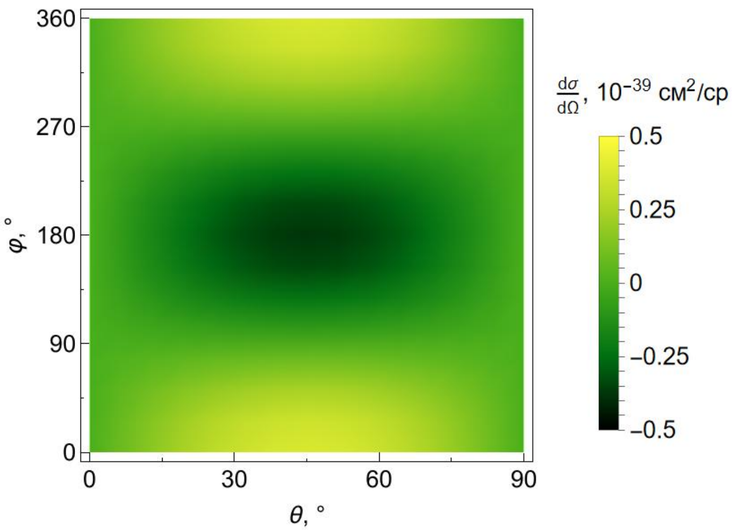


Numerical results for the ν scattering on a nucleon and nucleus:

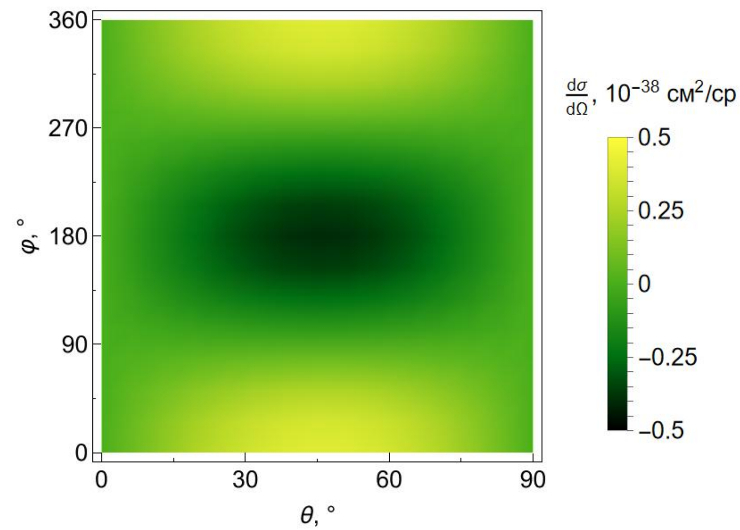
The effect of the **diagonal magnetic moment** and **transverse neutrino polarization**



ν+p



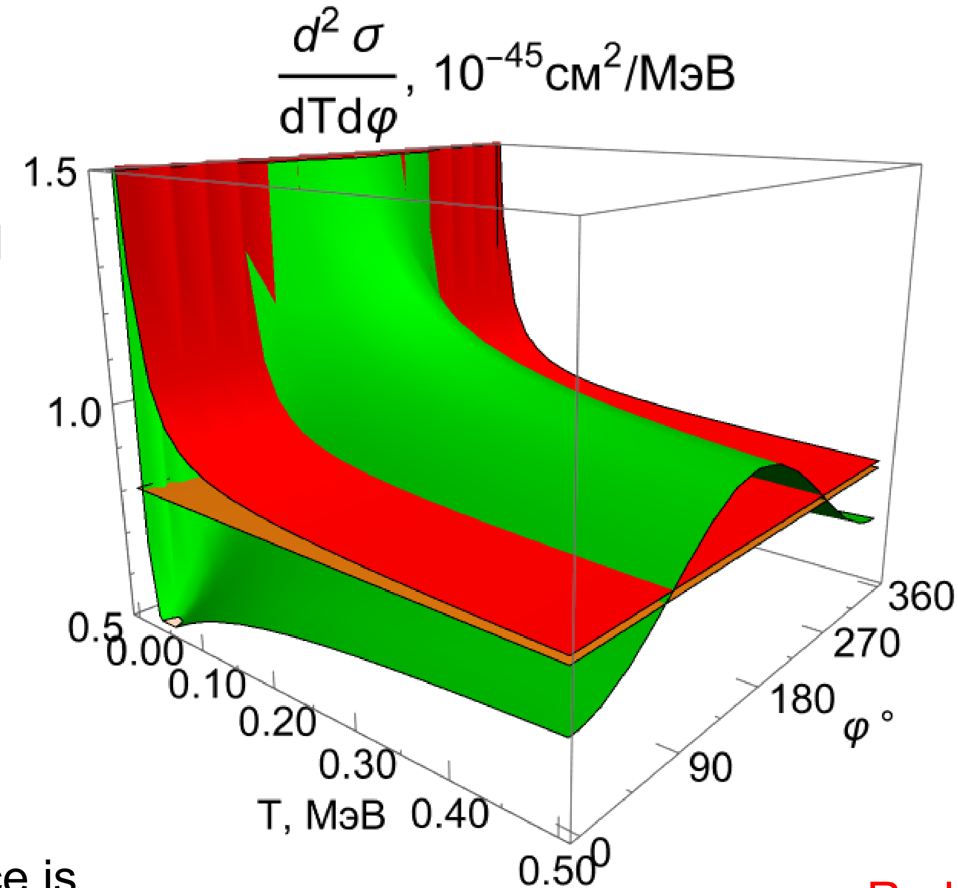
ν+ ^{40}Ar



ν+ ^{132}Xe

Numerical results for the electron ν scattering on an electron:
The effect of the diagonal magnetic moment and transverse neutrino polarization

Orange surface is fully unpolarized neutrino in the SM



Green surface is transversely polarized neutrino with magnetic moment

Red surface is fully unpolarized neutrino with magnetic moment

Summary

- Theoretical study of the processes of elastic neutrino-electron, nucleon and nucleus scattering has been carried out taking into account the electromagnetic form factors of neutrinos and the form factors of target particles, as well as the effects of neutrino spin polarizations.

- Our numerical results demonstrate the effect of azimuthal asymmetry of cross sections for elastic neutrino scattering on different targets, which can be seen in the case of measurement of recoil angle of target particles. The observation of this effect in an experiment would indicate on nonzero neutrino magnetic moment as well as on neutrino spin oscillations.

Thank you for your attention!

Parametrization of nucleon form factors

For this purpose we use the Sachs form factors →

$$F_Q^N(q^2) = \frac{G_E^N(q^2) - \frac{q^2}{4m_N^2} G_M^N(q^2)}{1 - \frac{q^2}{4m_N^2}},$$

$$F_M^N(q^2) = \frac{G_M^N(q^2) - G_E^N(q^2)}{1 - \frac{q^2}{4m_N^2}},$$

Parametrization of nucleon form factors (see Papoulias D. K., Kosmas T. S. Advances in High Energy Physics 2016 (2016) and references therein)

$$\frac{G_M^N}{\mu_N} = \frac{1 - \frac{q^2}{4m_N^2} a_M^N}{1 - \frac{q^2}{4m_N^2} b_{M1}^N + (\frac{q^2}{4m_N^2})^2 b_{M2}^N - (\frac{q^2}{4m_N^2})^3 b_{M3}^N},$$

$$G_E^p = \frac{1 - \frac{q^2}{4m_N^2} a_E^p}{1 - \frac{q^2}{4m_N^2} b_{E1}^p + (\frac{q^2}{4m_N^2})^2 b_{E2}^p - (\frac{q^2}{4m_N^2})^3 b_{E3}^p},$$

$$G_E^n = \frac{-\frac{q^2}{4m_N^2}\lambda_1}{1 - \frac{q^2}{4m_N^2}\lambda_2}(1 - \frac{q^2}{M_V^2})^{-2},$$

$$G_A^a = g_A(1 - \frac{q^2}{M_A^2})^{-2}$$

$$m_N = 938MeV, \quad \mu_p = 2.793, \quad \mu_n = -1.913,$$
$$M_V = 843MeV, \quad g_A = 1.267, \quad M_A = 1049MeV,$$
$$a_E^p = -0.19, \quad b_{E1}^p = 11.12, \quad b_{E2}^p = 15.16, \quad b_{E3}^p = 21.25,$$
$$a_M^p = 1.09, \quad b_{M1}^p = 12.31, \quad b_{M2}^p = 25.57, \quad b_{M3}^p = 30.61,$$
$$\lambda_1 = 1.68, \quad \lambda_2 = 3.63,$$
$$a_M^n = 8.28, \quad b_{M1}^n = 21.3, \quad b_{M2}^n = 77, \quad b_{M3}^n = 238.$$

Parametrization of the strange form factor (see Butkevich A. V. Phys. Rev. D. 2023. 107, N 7. 073001)

$$F_1^S(q^2) = \frac{\frac{q^2}{6}\langle r_S^2\rangle}{(1 - \frac{q^2}{4m_N^2})}(1 - \frac{q^2}{M_V^2})^{-2}$$

$$F_2^S(q^2) = \frac{\mu_S}{(1 - \frac{q^2}{4m_N^2})}(1 - \frac{q^2}{M_V^2})^{-2}$$

$$F_A^S(q^2) = g_A^S(1 - \frac{q^2}{M_A^2})^{-2}$$

$g_A^S = \in [-0{,}2, 0{,}2]$, other «strange» parameters equal zero

Connection between neutral weak and electromagnetic form factors of nucleon

According to the hypothesis of vector current conservation, vector neutral weak form factors can be expressed via electromagnetic ones

$$F^p_{1,2}(q^2) = (\frac{1}{2} - 2\sin^2\theta_W)F^p_{Q,M} - 1\frac{1}{2}F^n_{Q,M} - \frac{1}{2}F^S_{1,2}$$
$$F^n_{1,2}(q^2) = (\frac{1}{2} - 2\sin^2\theta_W)F^n_{Q,M} - 1\frac{1}{2}F^p_{Q,M} - \frac{1}{2}F^S_{1,2}$$

$F^S_{1,2}$ – strange form factors of nucleon

Here we restrict ourselves only with charge and magnetic form factors in the electromagnetic vertex

In the axial case, one can also factorize an axial strange form factor

$$G^N_{A,P}(q^2) = \frac{\tau_3}{2}G^a_{A,P}(q^2) - \frac{1}{2}G^S_{A,P}(q^2)$$

The cross section

The full cross section:

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma^L}{d\Omega} + \frac{d\sigma^R}{d\Omega} + \frac{d\sigma^\perp}{d\Omega} \qquad \frac{d\sigma^K}{d\Omega} = \frac{d\sigma^K_{\rm hp}}{d\Omega} + \frac{d\sigma^K_{\rm hf}}{d\Omega}, \quad K = \{L, R\}$$

$$\frac{d\sigma^K_{\rm hp}}{d\Omega} = \frac{G_F^2 t^2 (s - m_N^2)}{16\pi^2 m_N^2 (s + m_N^2)} \left(1 - \frac{4m_N^2}{t}\right)^{3/2} \left[2\left(1 + \frac{st}{(s - m_N^2)^2}\right)\left(C_V^K + C_A^K - \frac{t}{4m_N^2}(C_M^K + C_E^K)\right) - \right.$$
$$- \frac{4m_N^2 t}{(s - m_N^2)^2}\left(C_A^K - \frac{t}{4m_N^2} C_M^K\right) + \frac{t^2}{(s - m_N^2)^2}\left(C_V^K + C_A^K - 2{\rm Re}\, C^K_{V\&M}\right) \pm$$
$$\left. \pm \frac{2t}{s - m_N^2}\left(2 + \frac{t}{s - m_N^2}\right) {\rm Re}\left(C^K_{V\&A} - C^K_{A\&M}\right)\right],$$

The effects of form factors and oscillations are contained in the following coefficients:

$$C_V^K = Tr\left[\left(-F_1^N \delta_L^K + F_Q^N Q^K\right)^2 \rho^K\right], \quad C_A^K = Tr\left[\left(\delta_L^K G_A^N - \frac{t F_A^N Q^K}{m_N^2}\right)^2 \rho^K\right],$$

$$C^K_{V\&A} = Tr\left[\left(\delta_L^K G_A^N - \frac{t F_A^N Q^K}{m_N^2}\right)\left(-F_1^N \delta_L^K + F_Q^N Q^K\right) \rho^K\right], \quad C_M^K = Tr\left[\left(\delta_L^K F_2^N - F_M^N Q^K\right)^2 \rho^K\right],$$

$$C^K_{V\&M} = Tr\left[\left(\delta_L^K F_2^N - F_M^N Q^K\right)\left(-F_1^N \delta_L^K + F_Q^N Q^K\right) \rho^K\right], \qquad C_E^K = Tr\left[\left(F_E Q^K\right)^2 \rho^K\right],$$

$$C^K_{A\&M} = Tr\left[\left(\delta_L^K F_2^N - F_M^N Q^K\right)\left(\delta_L^K G_A^N - \frac{t F_A^N Q^K}{m_N^2}\right) \rho^K\right],$$

$$\rho^{L,R} = \frac{1}{2}(\tilde{\rho} \mp \zeta^{\parallel}), \quad Q^{L,R} = \frac{2\sqrt{2}\pi\alpha}{G_F t}\left(f^Q \mp t f^A\right)$$

Neutrino-helicity-flipping cross section

$$\frac{d\sigma_{\rm hf}^K}{d\Omega} = \frac{\alpha^2 t^2 (s-m_N^2)}{8m_e^2 m_N^2 (s+m_N^2)} \left(1 - \frac{4m_N^2}{t}\right)^{3/2} |\mu_\nu^K|^2 \Bigg[-\frac{2m_N}{t}\left(1+\frac{t}{s-m_N^2}\right)(F_Q^N)^2 - \\ -\frac{2t}{m_N^3}\left(1+\frac{st}{(s-m_N^2)^2}\right)(F_A^N)^2 + \frac{m_N t}{(s-m_N^2)^2} F_Q^N F_M^N + \frac{1}{8m_N}\left(4+\frac{4st+t^2}{(s-m_N^2)^2}\right)(F_M^N)^2 + \\ + \frac{1}{8m_N}\left(2+\frac{t}{s-m_N^2}\right)^2 (F_E^N)^2 \Bigg],$$

$$|\mu_\nu^{L,R}|^2 = Tr\left[\left(f^M \pm i f^E\right)\left(f^M \mp i f^E\right)\rho^{L,R}\right]$$

are effective left- and right-handed neutrino magnetic moments

The transverse neutrino polarization part of the cross section

$$\frac{d\sigma^{\perp}}{d\Omega} = \frac{\sqrt{2}G_F\alpha(s-m_N^2)\,(4m_N^2-t)^{3/2}}{8\pi m_e m_N^2(s+m_N^2)}\sqrt{1+\frac{st}{(s-m_N^2)^2}}\left\{\frac{2t}{s-m_N^2}\frac{tF_A^N}{m_N^2}(F_Q^N+F_M^N)C^{+}_{\mu_\nu,Q}+\right.$$

$$+C^{\perp}_{\mu_\nu,Z^0}\left[\left(2+\frac{t}{s-m_N^2}\right)\left(F_1^N F_Q^N+\frac{tF_A^N}{m_N^2}G_A^N-t\frac{F_2^N F_M^N}{4m_N^2}\right)-\frac{t}{s-m_N^2}\left(\frac{tF_A^N}{m_N^2}(F_1^N+F_2^N)+G_A^N(F_Q^N+F_M^N)\right)\right]-$$

$$\left.-\left(2+\frac{t}{s-m_N^2}\right)\left((F_Q^N)^2+\left(\frac{tF_A^N}{m_N^2}\right)^2-t\left(\frac{F_M^N}{2m_N}\right)^2-t\left(\frac{F_E^N}{2m_N}\right)^2\right)C^{-}_{\mu_\nu,Q}\right\},$$

$$Q^{L,R}=\frac{2\sqrt{2}\pi\alpha}{G_F t}\left(f^Q\mp t f^A\right)$$

$$C^{\perp}_{\mu_\nu,Z^0}=\mathrm{Re}\,Tr\left[\left(f^M+if^E\right)\kappa' e^{i\varphi}\right] \qquad C^{\pm}_{\mu_\nu,Q}=\mathrm{Re}\,Tr\left[\left[Q^L\left(f^M+if^E\right)\pm\left(f^M+if^E\right)Q^R\right]\kappa' e^{i\varphi}\right]$$

dependence on azimuthal angle of transverse recoil momentum

$$\kappa_{ij}=\frac{1}{2}\left(\zeta^x_{ij}+i\zeta^y_{ij}\right)$$

transverse neutrino polarization components