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# Spin light of neutrino in astrophysical media

# *Neutrino mass and magnetic moment*

Neutrino mass bound: $m < 0{,}45\ \mathrm{eV}$ [KATRIN Collab., *Nature Physics* **18** (2022) 160]

$m \neq 0 \Rightarrow$ **neutrino magnetic moment**

In Minimally extended Standard Model [*Phys. Rev. Lett.* **45** (1980) 963] :

$$\mu = \frac{3eG_F m}{8\sqrt{2}\pi^2} \simeq 3.2 \times 10^{-19} \mu_B \left(\frac{m}{1\ \mathrm{eV}}\right),$$

$$\mu_B = e/2m_e$$

Dirac neutrinos in the flavor basis can have both diagonal and transition $\mu_\nu$; Majorana neutrinos can only have transition $\mu_\nu$.

Current experimental limit (BOREXINO and GEMMA collab.):

$$\mu \leq (2.8\text{–}2.9) \times 10^{-11} \mu_B$$

Astrophysical data limit [*Phys. Rev. Lett.* **111** (2013) 231301]:

$$\mu \leq (1.1\text{–}2.6) \times 10^{-12} \mu_B$$

# *Matter impact on neutrino motion*

Wolfenstein showed that active neutrino $\nu_L$ wave function in matter acquires additional phase that can be treated as appearance of an energy shift [L.Wolfenstein'78]:

$$E = \sqrt{p^2 + m^2} + A$$

$A$ – effective matter potential

For instance, for $\nu_e$ and matter of electrons: $A = \frac{1}{\sqrt{2}} G_F (1 + 4 \sin^2 \theta_W)$.

$\nu_R$ dispersion is not changed.

QFT description of neutrino in matter: **the modified Dirac equation**
- accounts for the neutrino net interaction with matter particles, provided that their macroscopic amount occurs on the scale of the neutrino de Broglie wave length.

L.Chang, R.Zia,'88; J.Panteleone,'91; K.Kiers, N.Weiss, M.Tytgat,'97-'98; P.Manheim,'88; D.Nötzold, G.Raffelt,'88; J.Nieves,'89; V.Oraevsky, V.Semikoz, Ya.Smorodinsky,89; W.Naxton, W-M.Zhang'91; M.Kachelriess,'98; A.Kusenko, M.Postma,'02.

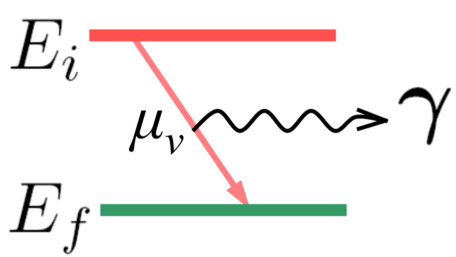
Correspondence between helicity and chiral states: $\nu_{s=-1} \simeq \nu_L$ and $\nu_{s=+1} \simeq \nu_R$

# *The spin light of neutrino (SLν) mechanism*

[A.Studenikin, A.Ternov,PLB 608 (2005) 107]

From the point of view of quantum theory, the SL$\nu$ in matter is due to two different phenomena:

- the shift of neutrino energy levels in matter corresponding to different helicity states;
- emission of a photon during the transition between helicity states due to neutrino magnetic moment (the dipole electromagnetic transition).




Quasiclassical treatment: rotation of the neutrino spin-vector in matter and radiation according to classical formula $I = \frac{2}{3}|\ddot{\boldsymbol{\mu}}_\nu|^2$ [A.Lobanov,A.Studenikin, PLB 564 (2003) 27]

# *The calculation tool: exact solutions method*

This is an effective method for studying and calculating quantum processes in various external conditions (matter and fields).

**The method:** representation of external conditions (fields, material environment) by classical non-operator fields included in the effective interaction Lagrangian of quantized fields for particles under study.

**At the Feynman diagrams level** – replacing free wave functions of particles in the initial and final states with wave functions of particles in external conditions. For fermions, the latter are the solutions of modified Dirac equation.

**It was first used** within the framework of QED (Furry's representation), where it served as the basic tool for constructing the theory of synchrotron radiation.

**The advantage of the method** is that the effects of the interaction of a particle with its environment are taken into account exactly in each order of perturbation theory.

## *The exact solution method in QED*

Dirac equation in external e/m field: $\{i\gamma^\mu \partial_\mu - e\gamma^\mu A_\mu - m\}\,\psi = 0$

Solutions of the equation, $\psi_i$

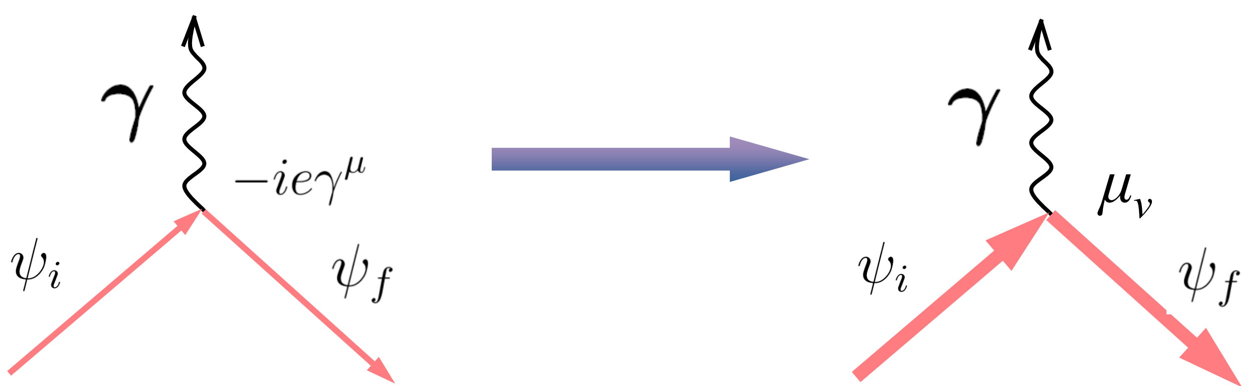
( $\psi_{n,p_y,p_z,\sigma}$ for synchrotron radiation )

Interaction Lagrangian:

$\mathcal{L}_{int} = -e j_\mu A^\mu, \quad j^\mu = \bar{\psi}\gamma_\mu\psi$

Амплитуда процесса

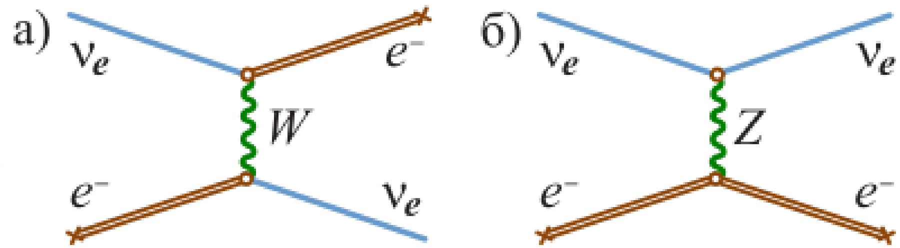
$$S_{fi} = -ie\sqrt{4\pi}\int d^4x \bar{\psi}_f(x)\gamma^\mu e^*_\mu \frac{e^{ikx}}{\sqrt{2\omega L^3}}\psi_i(x)$$



# The modified Dirac equation for neutrino in matter

# *The modified Dirac equation for neutrino in matter 1*

Interaction of $\nu_e$ with electron matter:




Moderate matter temperatures, **elastic scattering, |p$^{/}$-p|<<p**

Fermi 4-fermion interaction. The amplitude (the charged current):

$$\mathcal{A} = \frac{4G_F}{\sqrt{2}} \left\{ \bar{e}_L(p_e)\gamma^\lambda e_L(p_e) \right\} \left\{ \bar{\nu}_L(p)\gamma_\lambda \nu_L(p) \right\}$$

Averaging over the matter particles :

$$\mathcal{L}_{eff} = \frac{4G_F}{\sqrt{2}} \left\langle \bar{e}\gamma^\lambda \left( \frac{1+\gamma^5}{2} \right) e \right\rangle \left\{ \bar{\nu}_L(p)\gamma_\lambda \nu_L(p) \right\}$$

<u>The applicability</u>:

- elastic scattering
- neutrino coherent interaction with matter $\lambdabar n_0^{1/3} \gg 1$
- W-boson production $E_\nu \lesssim m_W^2/2m_e \simeq 6,3 \cdot 10^{15}$эВ

In the reference frame of matter:

$\langle e^+\gamma^0\gamma^\mu e \rangle \sim \{n_0, 0\}$,

$\langle e^+\gamma^5\gamma^0\gamma^\mu e \rangle \sim \{0, n_0\boldsymbol{\zeta}_e\}$

$\boldsymbol{\zeta}_e$- polarization vector

# The modified Dirac equation for neutrino in matter 2

In arbitrary reference frame ($\mathbf{v_e}$ is the matter velocity):

$$\lambda_e^\mu = \left\{ n_e\,(\boldsymbol{\zeta}_e \mathbf{v}_e)\,,\ n_e \boldsymbol{\zeta}_e \sqrt{1-v_e^2} + \frac{n_e \mathbf{v}_e\,(\boldsymbol{\zeta}_e \mathbf{v}_e)}{1+\sqrt{1-v_e^2}} \right\}$$

$$j_e^\mu = \{n_e,\ n_e \mathbf{v}_e\}\,, \qquad n_e = n_0(1-v_e^2)^{-1/2}$$

$$\mathcal{L}_{\text{eff}} = -f^\mu \left( \bar{\nu}(x)\,\gamma^\mu \frac{1}{2}\,(1+\gamma^5)\,\nu(x) \right)$$

$$f^\mu = \frac{G_F}{\sqrt{2}} \left[ \left(1+4\sin^2\theta_W\right) j_e^\mu - \lambda_e^\mu \right]$$

**The equation for Dirac neutrino:**

$$\left\{ i\gamma_\mu \partial^\mu - \frac{1}{2}\gamma_\mu (1+\gamma_5) f^\mu - m \right\} \Psi(x) = 0.$$

In the general case of several matter components, $f = e, p, n$:

$$f^\mu = G_F \sqrt{2} \sum_{f=e,p,n} \left( j_f^\mu q_f^{(1)} + \lambda_f^\mu q_f^{(2)} \right)$$

$$q_f^{(1)} = (I_{3L}^{(f)} - 2Q^{(f)} \sin^2\theta_W + \delta_{ef}\delta_{\nu\nu_e}), \qquad q_f^{(2)} = -(I_{3L}^{(f)} + \delta_{ef}\delta_{\nu\nu_e}),$$

$$\delta_{ef} = \begin{cases} 1, & \text{for } f = e, \\ 0, & \text{for } f = n, p, \end{cases} \qquad \delta_{\nu\nu_e} = \begin{cases} 1, & \text{for } \nu = \nu_e, \\ 0, & \text{for } \nu = \nu_\mu, \nu_\tau. \end{cases}$$

$I_{3L}^{(f)}$ - the third isospin component of fermion $f$,
$Q^{(f)}$ - its charge.

# *Solutions of the modified Dirac equation*

- a Dirac-type electron neutrino $\nu_e$
- uniform non-moving and unpolarized electron matter

The Hamiltonian form of the Dirac equation: $i\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \widehat{\mathrm{H}}_{\mathrm{med}}\Psi(\mathbf{r},t),$

$$\widehat{\mathrm{H}}_{\mathrm{med}} = (\boldsymbol{\alpha}, \hat{\mathbf{p}}) + \gamma_0 m + \frac{\tilde{G}_F n}{2\sqrt{2}}(1+\gamma_5), \quad \tilde{G}_F = G_F(1+4\sin^2\theta_W)$$

Neutrino helicity: $\frac{(\boldsymbol{\Sigma}\mathbf{p})}{|\mathbf{p}|}\Psi(\mathbf{r},t) = s\Psi(\mathbf{r},t), \quad s = \pm 1$

**Neutrino dispersion:**

$$E_\varepsilon = \varepsilon\sqrt{(p-s\tilde{n})^2+m^2}+\tilde{n} \qquad \varepsilon = \pm 1 \qquad \tilde{n} = \frac{G_F(1+4\sin^2\theta_W)}{2\sqrt{2}}n$$

The solution:

$$\Psi_{\varepsilon,\mathbf{p},s}(\mathbf{r},t) = \frac{e^{-i(E_\varepsilon t - \mathbf{p}\mathbf{r})}}{2L^{\frac{3}{2}}}\begin{pmatrix} \sqrt{1+\frac{m}{E_\varepsilon-\tilde{n}}}\sqrt{1+s\frac{p_3}{p}} \\ s\sqrt{1+\frac{m}{E_\varepsilon-\tilde{n}}}\sqrt{1-s\frac{p_3}{p}}e^{i\delta} \\ s\varepsilon\sqrt{1-\frac{m}{E_\varepsilon-\tilde{n}}}\sqrt{1+s\frac{p_3}{p}} \\ \varepsilon\sqrt{1-\frac{m}{E_\varepsilon-\tilde{n}}}\sqrt{1-s\frac{p_3}{p}}e^{i\delta} \end{pmatrix},$$

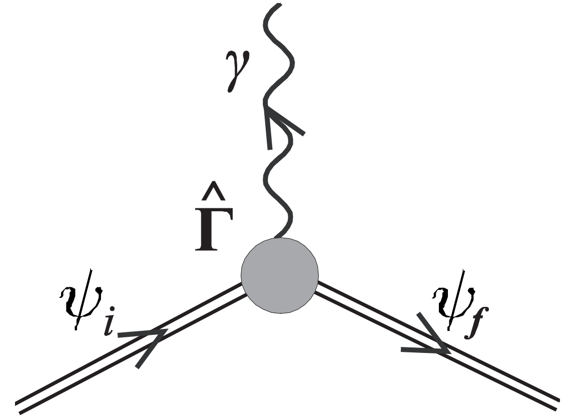
In the general case
(the matter consisting of e, p and n):

$$\tilde{n}_{\nu_e} = \frac{G_F}{2\sqrt{2}}\Big(n_e(1+4\sin^2\theta_W) + n_p(1-4\sin^2\theta_W) - n_n\Big)$$

$$\tilde{n}_{\nu_\mu,\nu_\tau} = \frac{G_F}{2\sqrt{2}}\Big(n_e(4\sin^2\theta_W - 1) + n_p(1-4\sin^2\theta_W) - n_n\Big)$$

# Quantum theory of neutrino spin light in matter

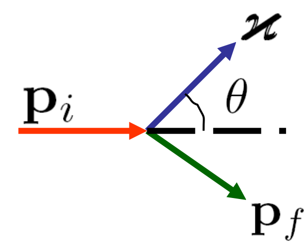
# *Quantum theory of neutrino spin light in matter*




**The process amplitude:**

$$
\begin{cases}
S_{fi} = -\mu\sqrt{4\pi}\int d^4x\bar{\psi}_f(x)(\hat{\mathbf{\Gamma}}\mathbf{e}^*)\dfrac{e^{ikx}}{\sqrt{2\omega L^3}}\psi_i(x), \\
\hat{\mathbf{\Gamma}} = i\omega\big\{\big[\mathbf{\Sigma}\times\boldsymbol{\varkappa}\big] + i\gamma^5\mathbf{\Sigma}\big\}, \qquad k^\mu = (\omega, \mathbf{k}), \quad \boldsymbol{\varkappa} = \mathbf{k}/\omega,
\end{cases}
$$

where $\mathbf{k}$ is the momentum, $\mathbf{e}$ is the polarization vector of the photon.




**The photon energy:**

Conservation laws:

$$E_i = E_f + \omega, \quad \mathbf{p}_i = \mathbf{p}_f + \boldsymbol{\varkappa}$$

$$E_\nu = \sqrt{(p - s\tilde{n})^2 + m_\nu^2} + \tilde{n}$$

$$s = \pm 1$$

$$\omega = \frac{2\tilde{n}p\left[(E_\nu - \tilde{n}) - (p + \tilde{n})\cos\theta\right]}{(E_\nu - \tilde{n} - p\cos\theta)^2 - \tilde{n}^2}$$

The spin is reversed: $s_i = -1 \longrightarrow s_f = 1.$

# *The total rate and power of the radiation*

The angular distribution of the transition rate

$$\frac{d\Gamma}{d\Omega} = \frac{4}{\pi}\mu^2\,\alpha^3 m^3\,p^3\,\widetilde{E}^3 \frac{(1-\tilde{\beta}\,\cos\theta)^2\,\left((1+\tilde{\beta}^2)\,(1-\cos\theta)^2 + 2\,\cos\theta\,(1-\tilde{\beta})^2\right)}{\left((\widetilde{E}-p\,\cos\theta)^2-\alpha^2 m^2\right)^3},$$

The angular distribution of the radiation power

$$\frac{dI}{d\Omega} = \frac{8}{\pi}\mu^2\,\alpha^4 m^4\,p^4\,\widetilde{E}^4 \frac{(1-\tilde{\beta}\,\cos\theta)^3\,\left((1+\tilde{\beta}^2)\,(1-\cos\theta)^2 + 2\,\cos\theta\,(1-\tilde{\beta})^2\right)}{\left((\widetilde{E}-p\,\cos\theta)^2-\alpha^2 m^2\right)^4}.$$

Here $\alpha = \frac{1}{2\sqrt{2}}\tilde{G}_F\frac{n}{m}$, $\tilde{\beta} = \frac{p+\alpha m}{E-\alpha m}$ and $\widetilde{E} = E - \alpha m.$

# *Complete expressions for the probability and power*

$$\begin{aligned}
\Gamma \; = \; & \frac{1}{2\,(E-p)^2\,(E+p-2\alpha m)^2\,(E-\alpha m)\,p^2} \\
& \times \Big\{ \left(E^2-p^2\right)^2 \left(p^2-6\alpha^2 m^2+6E\alpha m-3E^2\right) \left((E-2\alpha m)^2-p^2\right)^2 \\
& \times \ln\left[\frac{(E+p)\,(E-p-2\alpha m)}{(E-p)\,(E+p-2\alpha m)}\right] + 4\alpha m p\left[16\alpha^5 m^5 E\left(3E^2-5p^2\right)\right. \\
& -8\alpha^4 m^4\left(15E^4-24E^2p^2+p^4\right)+4\alpha^3 m^3 E\left(33E^4-58E^2p^2+17p^4\right) \\
& -2\alpha^2 m^2\left(39E^2-p^2\right)\left(E^2-p^2\right)^2+12\alpha m E\left(2E^2-p^2\right)\left(E^2-p^2\right)^2 \\
& \left.-\;\left(3E^2-p^2\right)\left(E^2-p^2\right)^3\right]\Big\},
\end{aligned}$$

$$\begin{aligned}
I = & \frac{5}{2\,(E-p)^3\,(E+p-2\alpha m)^3\,p^2} \times \Big\{(E+p)^2(E-m)^3(E+p-2\alpha m)^3 \\
& \times (E-p-2\alpha m)^2\Big(2\alpha^2m^2-2\alpha m(E+\frac{1}{5}p)+E^2-\frac{3}{5}p^2\Big) \\
& \times \ln\left(\frac{(2\alpha m-p-E)(E-p)}{(2\alpha m+p-E)(E+p)}\right)-4\alpha m p\Big(32\alpha^6m^6\Big(E^4-pE^3-\frac{5}{3}p^2E^2+\frac{5}{3}p^3E+\frac{8}{15}p^4\Big) \\
& -96\alpha^5m^5\Big(E^5-\frac{23}{30}pE^4-\frac{83}{45}p^2E^3+\frac{11}{9}p^3E^2+\frac{38}{45}p^4E-\frac{1}{10}p^5\Big) \\
& +128\alpha^4m^4\Big(E^6-\frac{47}{80}pE^5-\frac{511}{240}p^2E^4+\frac{127}{120}p^3E^3+\frac{157}{120}p^4E^2-\frac{89}{240}p^5E-\frac{7}{48}p^6\Big) \\
& -96(E^2-p^2)\alpha^3m^3\Big(E^5-\frac{53}{120}pE^4-\frac{3}{2}p^2E^3+\frac{89}{180}p^3E^2+\frac{47}{90}p^4E-\frac{19}{360}p^5\Big) \\
& +42(E^2-p^2)^2\alpha^2m^2\Big(E^4-\frac{32}{105}pE^3-\frac{314}{315}p^2E^2+\frac{4}{21}p^3E+\frac{17}{105}p^3\Big) \\
& -10\alpha m(E^2-p^2)^3\Big(E^3-\frac{4}{25}pE^2-\frac{17}{25}p^2E+\frac{2}{25}p^3\Big)+(E^2-p^2)^4\Big(E^2-\frac{3}{5}p^2\Big)\Big)\Big\}
\end{aligned}$$

## *Limiting cases*

**1. The case** $p \gg m$ (relativistic neutrino).

$$\Gamma = \begin{cases} \frac{64}{3}\mu^2\alpha^3 p^2 m, \\ 4\mu^2\alpha^2 m^2 p, \\ 4\mu^2\alpha^3 m^3, \end{cases} \quad I = \begin{cases} \frac{128}{3}\mu^2\alpha^4 p^4, & \text{for } \alpha \ll \frac{m}{p}, \\ \frac{4}{3}\mu^2\alpha^2 m^2 p^2, & \text{for } \frac{m}{p} \ll \alpha \ll \frac{p}{m}, \\ 4\mu^2\alpha^4 m^4, & \text{for } \alpha \gg \frac{p}{m}, \end{cases}$$

Matter density parameter:

$$\alpha = \frac{1}{2\sqrt{2}}\tilde{G}_F \frac{n}{m}$$

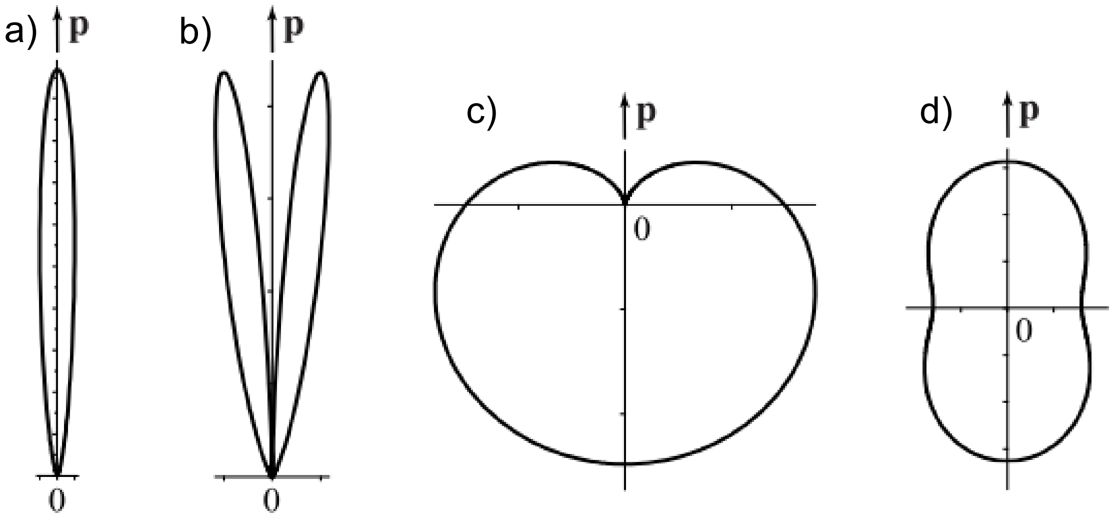
For $m \ll p$ the radiation is confined within narrow cone with the angle:

$$\theta \lesssim \sqrt{\alpha \frac{m}{p}}.$$

**2. The case** $p \ll m$.

$$\Gamma = \begin{cases} \frac{64}{3}\mu^2\alpha^3 p^3, \\ \frac{512}{5}\mu^2\alpha^6 p^3, \\ 4\mu^2\alpha^3 m^3, \end{cases} \quad I = \begin{cases} \frac{128}{3}\mu^2\alpha^4 p^4, & \text{for } \alpha \ll 1, \\ \frac{1024}{3}\mu^2\alpha^8 p^4, & \text{for } 1 \ll \alpha \ll \frac{m}{p}, \\ 4\mu^2\alpha^4 m^4, & \text{for } \alpha \gg \frac{m}{p}. \end{cases}$$

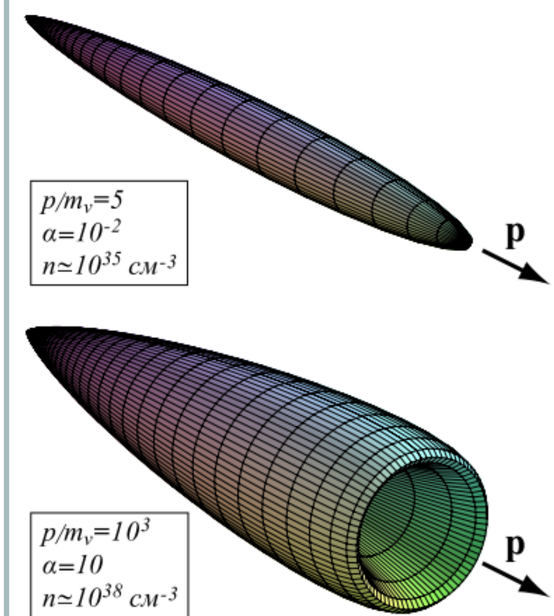
# *Angular distribution of radiation power*




Two-dimensional distributions of radiation power:

(a): $\mathrm{p}/m_\nu = 5,\ \ \alpha = 0.01$; (b): $\mathrm{p}/m_\nu = 10^3,\ \ \alpha = 10$

(c): $\mathrm{p}/m_\nu = 5,\ \ \alpha = 50$; (d): $\mathrm{p}/m_\nu = 0.01,\ \ \alpha = 0.01$




**The average energy of emitted photons:** $\langle\omega\rangle = \dfrac{I}{\Gamma}$

$$p/m_\nu \gg 1: \quad \langle\omega\rangle \simeq \begin{cases} 2\alpha\frac{p^2}{m_\nu}, & \text{for} \quad \alpha \ll m_\nu/p, \\ \frac{1}{3}p, & \text{for} \quad m_\nu/p \ll \alpha \ll p/m_\nu, \\ \alpha m_\nu, & \text{for} \quad \alpha \gg p/m_\nu, \end{cases}$$

$$p/m_\nu \ll 1: \quad \langle\omega\rangle \simeq \begin{cases} 2\alpha p, & \text{for} \quad \alpha \ll 1, \\ \frac{10}{3}\alpha^2 p, & \text{for} \quad 1 \ll \alpha \ll m_\nu/p, \\ \alpha m_\nu, & \text{for} \quad \alpha \gg m_\nu/p. \end{cases}$$

*SL* $\nu$ can be an effective source of neutrino energy losses in dense matter.

# *SLν polarization properties*

## Linear polarization

$$\mathbf{e}_1 = \frac{[\boldsymbol{\varkappa} \times \mathbf{j}]}{\sqrt{1-(\boldsymbol{\varkappa}\mathbf{j})^2}}, \quad \mathbf{j} = \frac{\boldsymbol{\beta}}{\beta}$$

$$\mathbf{e}_2 = \frac{\boldsymbol{\varkappa}(\boldsymbol{\varkappa}\mathbf{j}) - \mathbf{j}}{\sqrt{1-(\boldsymbol{\varkappa}\mathbf{j})^2}}, \quad \mathbf{j} \perp \mathbf{e_1}$$

$$\mathbf{e}_2 = \{\cos\phi\cos\theta, \sin\phi\cos\theta, -\sin\theta\}$$

$$\mathbf{e}_1 = \{\sin\phi, -\cos\phi\}$$

$$I^{(1),(2)} = \mu^2 \int_0^{\pi} \frac{\omega^4}{1+\beta' y} \left(\frac{1}{2} S \mp \Delta S\right) \sin\theta d\theta$$

where

$$S = \frac{(\beta\beta'+1)(1-y\cos\theta) - (\beta+\beta')(\cos\theta - y)\sin\theta}{1+\beta' y} \quad \text{and} \quad \Delta S = \frac{m^2 p \sin^2\theta}{2\,(E'-\alpha m)\,(E-\alpha m)\,p'} \;;$$

$$\tilde{\beta} = \frac{p+\alpha m}{E-\alpha m}, \quad \tilde{\beta}' = \frac{p'-\alpha m}{E'-\alpha m}, \quad y = \frac{\omega - p\cos\theta}{p'}, \quad K = \frac{E-\alpha m - p\cos\theta}{\alpha m},$$

$$E' = E - \omega, \quad p' = K\omega - p$$

- In the limit of **low** matter density:

$$\begin{pmatrix} I^{(1)} \\ I^{(2)} \end{pmatrix} \simeq 32 \begin{pmatrix} 1/3 \\ 1 \end{pmatrix} \mu^2 \alpha^4 p^4$$

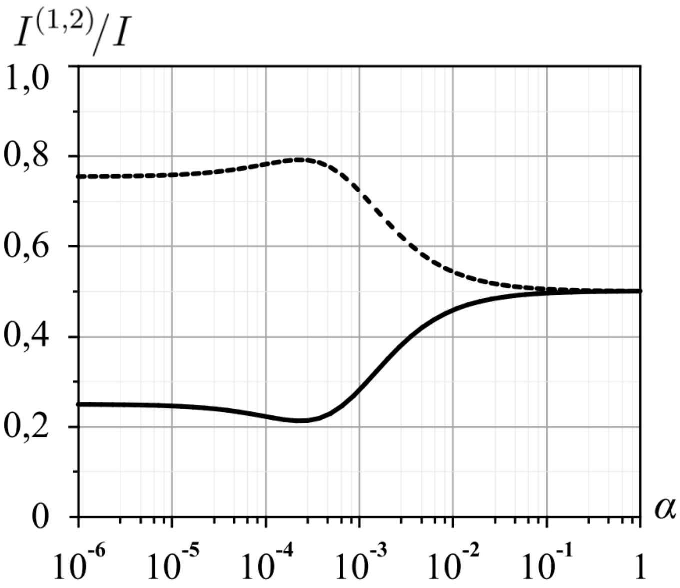
→ $SL\boldsymbol{\nu}$ is linearly polarized

- For **high** matter density (rest of the cases):

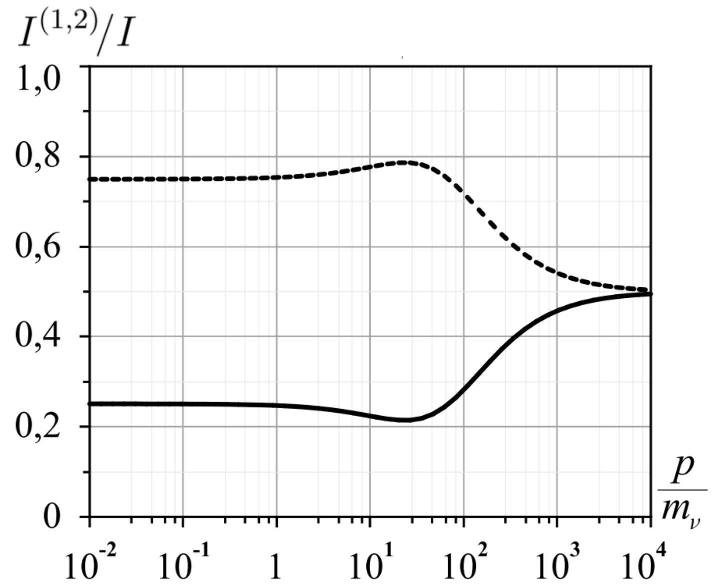
$$I^{(1)} \simeq I^{(2)} \simeq \frac{1}{2}\left(I^{(1)} + I^{(2)}\right)$$

→ $SL\boldsymbol{\nu}$ is non-polarized

# *SLν linear polarization behavior*




Linear polarization components of the radiation power depending on matter density parameter $\alpha$: $I^{(1)}/I$ (solid line), $I^{(2)}/I$ (dashed line); $p$=1 keV, $m$=1 eV




Linear polarization components of the radiation power depending on matter density parameter $\alpha$: $I^{(1)}/I$ (solid line), $I^{(2)}/I$ (dashed line); $\alpha=10^{-2}$, $m$=1 eV

# $SL\nu$ polarization properties

## Circular polarization

$$\mathbf{e}_l = \frac{1}{\sqrt{2}}(\mathbf{e}_1 + il\mathbf{e}_2).$$

$l = \pm 1$ defines right and left circular polarizations, respectively

$$I^{(l)} = \mu^2 \int_0^{\pi} \frac{\omega^4}{1+\beta' y} S_l \sin\theta d\theta$$

where:

$$S_l = \frac{1}{2}\left(1 + l\beta'\right)\left(1 + l\beta\right)\left(1 - l\cos\theta\right)\left(1 + ly\right)$$

- In the limit of **low** matter density and relativistic ν :

$$I^{(l)} \simeq \frac{64}{3}\mu^2\alpha^4 p^4 \left(1 - l\frac{p}{2E_0}\right)$$

$$E_0 = \sqrt{p^2 + m^2}$$

→ $SL\boldsymbol{\nu}$ is circular polarized:

$$\frac{I^{(-1)} - I^{(+1)}}{I} \simeq \frac{1}{2} \to 50\%$$

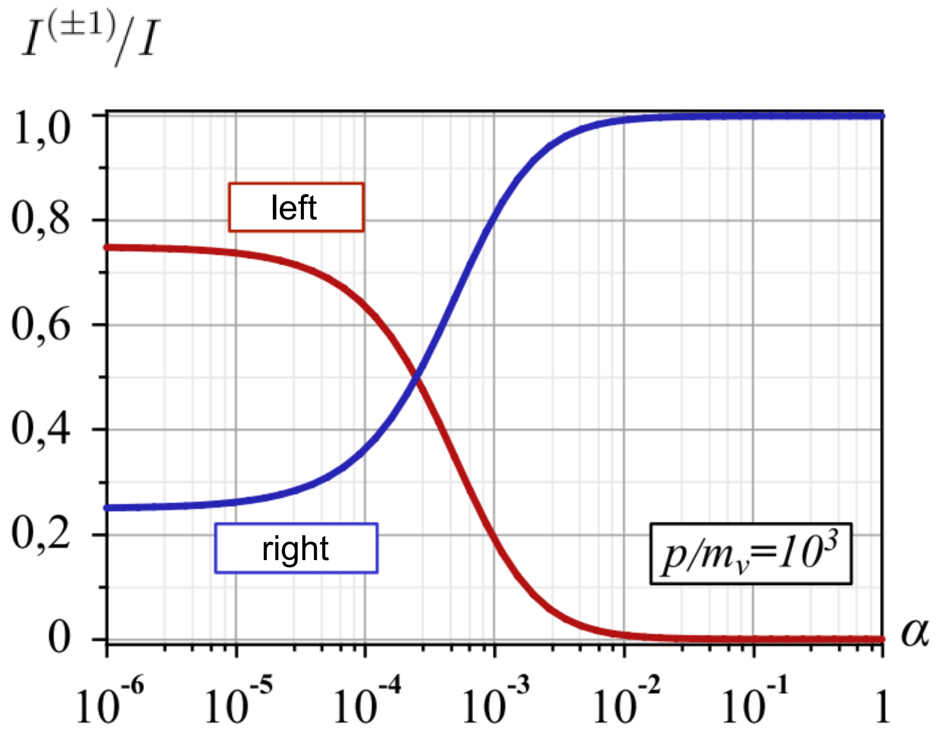
- For **high and extremely high** matter density:

($m/p \ll \alpha$ for $m \ll p$ and $1 \ll \alpha$ for $p \ll m$)

$$\begin{aligned} I^{(+1)} &\simeq I \\ I^{(-1)} &\simeq 0 \end{aligned}$$

→ $SL\boldsymbol{\nu}$ is completely right-hand polarized

# SLν circular polarization behavior




In a dense (in the case of a relativistic neutrino) and in an extremely dense matter, the *SLν* has almost complete right-handed circular polarization.

This property may be important from the point of view of the possibility to identify the radiation experimentally from dense astrophysical objects.

# *Accounting for photon dispersion*

$$\omega = \sqrt{\mathbf{k}^2 + m_\gamma^2},$$ where $m_\gamma$ is the plasmon mass

Conservation laws:

$$E_i = E_f + \omega, \quad \mathbf{p}_i = \mathbf{p}_f + \boldsymbol{\varkappa}$$

$$E_\nu = \sqrt{(p - s\tilde{n})^2 + m_\nu^2} + \tilde{n}$$

The threshold condition:

$$p > \frac{m_\gamma^2 + 2m_\gamma m_\nu}{4\tilde{n}}$$

In the limit of vanishing $m_\nu$:

$$k_{1,2} = \frac{\left[4\,(p+\tilde{n})\,\tilde{n} + m_\gamma^2\right] p\cos\theta \pm (p+2\tilde{n})\sqrt{\left[4\,(p+\tilde{n})\,\tilde{n} - m_\gamma^2\right]^2 - 4p^2 m_\gamma^2 \sin^2\theta}}{2[(p+2\tilde{n})^2 - p^2\cos^2\theta]}$$

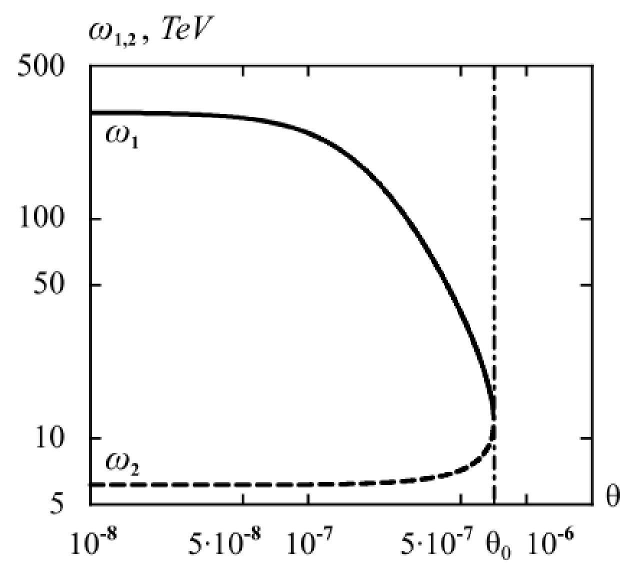
The "threshold parameter":

$$a = \frac{m_\gamma^2}{4\tilde{n}p} < 1$$

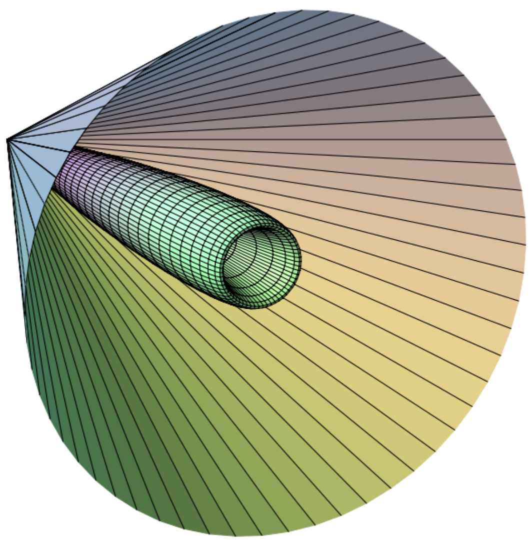
$$p = 3\times 10^2 \ \ TeV$$

$$m_\gamma = 8.87 \ MeV$$

$$\tilde{n} = 3.2 \ eV$$



# *Accounting for photon dispersion*



1. The angular distribution of the radiation is modified – the surrounding "funnel" appeared. The radiation is confined within the angle $\theta_0$:

$$\theta \leqslant \theta_0 = \arcsin \left[ \frac{4(p+\widetilde{n})\widetilde{n} - m_\gamma^2}{2pm_\gamma} \right] \simeq \frac{2\widetilde{n}}{m_\gamma}.$$

2. For a relativistic neutrino with momentum $p_{th} \ll p$ the expressions for the total rate and power reduce to the ones obtained without $m_\gamma$.

## The total rate and power

$$a = m_\gamma^2/4\widetilde{n}p,$$

Far from the threshold ($0 < a < 1$):

$$\Gamma = 4\mu^2 \widetilde{n}^2 p \left[(1-a)(1+7a) + 4a(1+a)\ln a\right],$$

$$I = \frac{4}{3}\mu^2 \widetilde{n}^2 p^2 \left[(1-a)(1-5a-8a^2) - 12a^2 \ln a\right]$$

Close to the threshold ($a \to 0$):

$$\Gamma = 4\mu^2 \widetilde{n}^2 (1-a)\left[(1-a)p + 2\widetilde{n}\right],$$

$$I = 4\mu^2 \widetilde{n}^2 p\,(1-a)\left[(1-a)p + 2\widetilde{n}\right].$$

$$\langle\omega\rangle = \frac{I}{\Gamma} \simeq p \simeq E_\nu$$

# Neutrino spin light in neutron star matter

- The medium consists mainly of neutrons with $n_n \sim 10^{38}$–$10^{39}$ cm$^{-3}$.
- The density of the electron fraction is significantly lower, $Y_e = n_e / n_b \sim (0{,}05 - 0{,}1) n_n$.
- The electron fraction is a degenerate Fermi gas.

The effective potential of a neutrino in such a medium is negative => we consider the $SL\nu$ emitted by electron antineutrinos $\bar{\nu}_e$.

$$\tilde{n} = \frac{1}{2\sqrt{2}} G_F n_n \simeq 3.2 \times \left( \frac{n_n}{10^{38}\,\mathrm{cm}^{-3}} \right) \mathrm{eV}$$

$$\mu_e = (3\pi^2 n_e)^{1/3} \simeq 130 \times \left( \frac{n_e}{10^{37}\,\mathrm{cm}^{-3}} \right)^{1/3} \mathrm{MeV} \gg m_e \simeq 0.51\,\mathrm{MeV}.$$

$$m_\gamma = \left( \frac{2\alpha}{\pi} \right)^{1/2} \mu_e \simeq 8.87 \times \left( \frac{n_e}{10^{37}\,\mathrm{cm}^{-3}} \right)^{1/3} \mathrm{MeV}$$

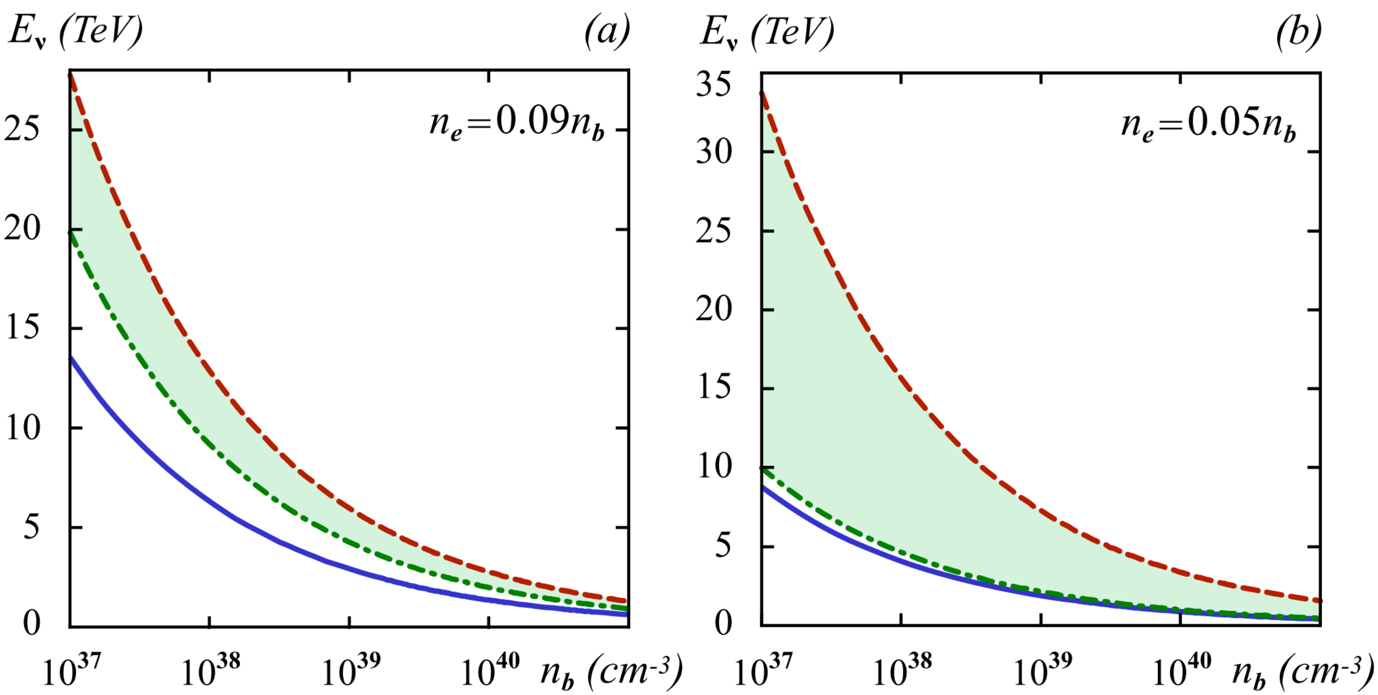
The threshold energy: $E_\nu > p_{\mathrm{th}} \simeq E_{\mathrm{th}} \simeq 28.5 \times \frac{Y_e^{2/3}}{1 - Y_e} \left( \frac{10^{38}\,\mathrm{cm}^{-3}}{n_n} \right)^{1/3} \mathrm{TeV}$

For $Y_e = 0{,}1 n_n$: $E > 6,82 \left( \frac{n_n}{10^{38}\mathrm{cm}^{-3}} \right)^{-1/3} \mathrm{TeV}$

# Neutrino spin light in neutron star matter

High neutrino momentum => propagator effects in interaction of neutrinos with particles of the medium (contact approximation violation): scattering of electron antineutrinos on electrons of the medium occurs in the s-channel and therefore the cross section is characterized by the Glashow resonance. The presence of resonance is associated with the possibility of the birth of a W boson, the threshold of this process is

$$\varepsilon_W = \frac{m_W^2}{4\mu_e} \simeq 5.77 \times \left(\frac{10^{38}\,\mathrm{cm}^{-3}}{Y_e n_n}\right)^{1/3} \mathrm{TeV}$$




The allowed range of electron antineutrino energies for the $SL\nu$ in the matter of a neutron star depending on the neutron density. Solid line: the $SL\nu$ process threshold without account for the $\bar{\nu}_e e$-scattering; dash-dotted line: the $SL\nu$ process threshold with account for the $\bar{\nu}_e e$-scattering; dashed line: the threshold for the $W$ boson production. (a) $Y_e = 0.09$; (b) $Y_e = 0.05$. The allowed regions are marked in green.

[Grigoriev et al.'2017]

# *Neutrino spin light in neutron star matter*

**For $\overline{\nu}_{\mu}$ and $\overline{\nu}_{\mu}$**: interaction with matter particles through the Z-boson in t-channel, no propagator effects =>

no competing processes, $SL_{\nu}$ is possible if the neutrino energy is above the threshold

The radiation time for $\bar{\nu}_{\mu}$ and $\bar{\nu}_{\mu}$:

$$\tau_{\mathrm{SL}\nu} \simeq 2.17 \times 10^{6} \left(\frac{10^{-11}\mu_B}{\mu}\right)^{2} \left(\frac{10^{38}\,\mathrm{cm}^{-3}}{n_n}\right)^{2} \left(\frac{10\ \mathrm{TeV}}{E_{\nu}}\right) \mathrm{s}$$

For the parameters $\mu \simeq 2.9 \times 10^{-11}\mu_B,\ Y_e = 0.1,\ n_n = 10^{38}\,\mathrm{cm}^{-3},\ E_{\nu} \simeq 10\,\mathrm{PeV}$:

$$\tau_{\mathrm{SL}\nu} \simeq 320\ \mathrm{s}\,, \quad \ell = c\tau_{\mathrm{SL}\nu} \simeq 9.6 \times 10^{12}\ \mathrm{cm}$$

Neutron star radius:

$R \simeq (1.0\text{--}1.4)\times 10^{6}\,\mathrm{cm}$

$$n_n = 10^{39}\,\mathrm{cm}^{-3},\ E_{\nu} \simeq 10^{20}\,\mathrm{eV} \implies \tau_{SL_{\nu}} \sim 1\ \mathrm{s}\,, \quad \ell \simeq R$$

Hypothetical objects – quark and hybrid stars: $n_b = 10^{41}\,\mathrm{cm}^{-3}$

$$\tau_{\mathrm{SL}\nu} \simeq 2.6 \times 10^{-4}\,\mathrm{s}\,, \quad \ell \simeq R$$

Potential source for observation: galactic clusters

Characteristic feature: 100% circular polarization

# *Neutrino spin light in supernova matter*

**The "hot bubble model" within the "delayed burst mechanism".**

[*Nature* **341** (1989) 489, *Phys. Rept.* **442** (2007) 38, *Phys. Rev.* **D 74** (2006) 105014]

Effective neutrino density at distance $r$ from the center of the star

$$n^{\text{eff}}_{\nu_e}(r) = n_{\nu_e}(r) - n_{\bar\nu_e}(r) \simeq 7.8\times 10^{32}\left[1-\sqrt{1-\left(\frac{11\,\text{km}}{r}\right)^2}\,\right]\text{cm}^{-3}, \qquad n^{\text{eff}}_{\nu_\mu} = n^{\text{eff}}_{\nu_\tau} = 0$$

Electron density: $n_e(r) \simeq 2.4\times 10^{29}\left(\frac{10\,\text{km}}{r}\right)^3 \text{cm}^{-3}$

Temperature profile: $T \simeq 1.96\left(\frac{100}{S}\right)\left(\frac{10\,\text{km}}{r}\right)\text{MeV}$

**1. Electron neutrino near the neutrinosphere :** $r \simeq 13$ km

$$n_e \simeq 10^{29}\text{cm}^{-3},\quad p_F = 0,28\,\text{MeV},\quad T \simeq 0,6\,\text{MeV},\quad n^{\text{eff}}_{\nu_e} \simeq 3,9\times 10^{32}\text{cm}^{-3}$$

The electron gas is relativistic and hot : $m_\gamma = \sqrt{\frac{2\pi\alpha}{3}}\,T \simeq 1.24\times 10^{-1}\left(\frac{T}{1\,\text{MeV}}\right)\text{MeV} \simeq 74\,\text{keV} \;\Longrightarrow\; E_{\text{th}} \simeq 46\,\text{TeV}$

The competitive process : $\nu_e\bar\nu_e \to Z$ . $\Gamma_Z = \Gamma_{\text{SL}\nu}$ for $E$ =7,8 TeV => SLν is suppressed

**2. Electron neutrino motion far from the neutrinosphere,** $r \simeq 100$ km:

$$n_e \simeq 2,4\times 10^{26}\text{cm}^{-3},\quad p_F = 0,037\,\text{MeV},\quad T \simeq 0,07\,\text{MeV},\quad n^{\text{eff}}_{\nu_e} \simeq 4,8\times 10^{30}\text{cm}^{-3}$$

The electron gas is non-relativistic: $m_\gamma = \sqrt{\frac{4\pi\alpha n_e}{m_e}} \simeq 3.69\times 10^2\left(\frac{n_e}{10^{26}\,\text{cm}^{-3}}\right)^{1/2}\text{eV} \simeq 570\,\text{eV} \;\Longrightarrow\; p_{th} \simeq 270\ \text{ГэВ}$

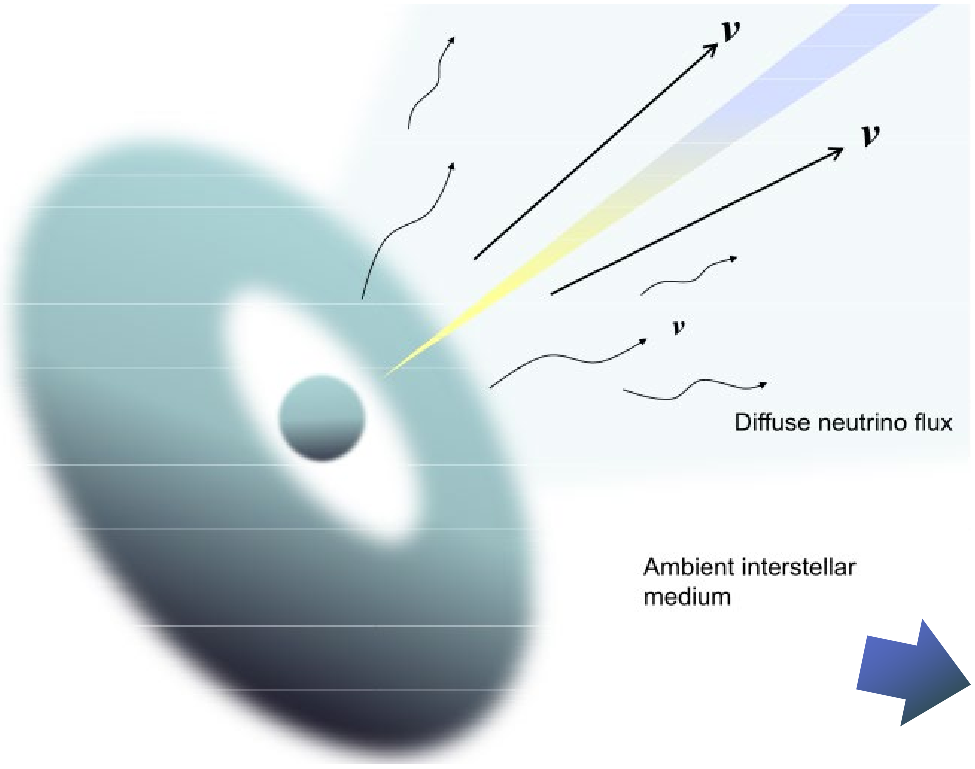
$\Gamma_Z \simeq \Gamma_{\text{SL}\nu}$ for $E$ =6,9 TeV $\Longrightarrow$ a "window" in values of $E$ for which SLν is allowed

$$E_\nu \simeq 1\,\text{TeV} \;\Longrightarrow\; \tau_{\text{SL}\nu} \simeq 2.9\times 10^{20}\ \text{s} = 9.1\times 10^{12}\ \text{years}$$

# *Neutrino spin light and gamma-ray bursts*

GRBs are considered the most likely sources of ultra-high-energy neutrinos $E \gtrsim 1$ PeV [*Ann. Rev. Nucl. Part. Sci.* **67** (2017) 45]

<u>The model of short GRB</u>




Factors for the most efficient generation of $\mathrm{SL}\nu$:

- Neutrino high energy and intensity of the flux
- High background density of neutral matter in the medium
- Low density of the charged component
- Low temperature of the charged component
- Significant extent of the medium

$\mathrm{SL}\nu$ of ultra-high energy neutrinos in the diffuse neutrino wind emanating from a neutron star merger

# *Neutrino spin light and gamma-ray bursts*

Characteristics of the medium [*Mon. Not. Roy. Astron. Soc.* **443** (2014) 3134]

- neutrino density:

$$n_\nu \sim 10^{32}\ \mathrm{cm}^{-3}$$

- Electron density:

$$T = 0.1\ \mathrm{MeV},\ Y_e = 0.01,\ n_e \simeq 3 \times 10^{25}\ \mathrm{cm}^{-3},\ p_F \simeq 0.01\ \mathrm{MeV}$$

⇩

$$m_\gamma \simeq 10^{-3}\ \mathrm{MeV} \qquad E_{\mathrm{th}} \simeq 1\ \mathrm{GeV}$$

SL$\nu$ radiation time:

$$\tau_{\mathrm{SL}\nu} \simeq 5.4 \times 10^{15} \left(\frac{10^{-11}\mu_B}{\mu}\right)^2 \left(\frac{10^{32}\,\mathrm{cm}^{-3}}{n_{\nu_e}}\right)^2 \left(\frac{1\ \mathrm{PeV}}{E_\nu}\right)\ \mathrm{s}.$$

$$E_\nu \sim 10^{12}\text{–}10^{18}\ \mathrm{eV} \quad \Longrightarrow \quad \tau_{\mathrm{SL}\nu} \simeq 6.4 \times (10^{11}\text{–}10^{17})\ \mathrm{s} = 2 \times (10^{4}\text{–}10^{10})\ \mathrm{years}$$

The problem of GRB radiation polarization [*Astron. Astrophys. Trans.* **29** (2016) 205]

~100% circular polarization of SLv in dense matter

⇨

Contribution of SLv to the circularly polarized component of GRB emission?

# *Neutrino spin light in moving matter*

A.Studenikin, Phys.Atom.Nucl. 67 (2004) 993:

generation of neutrino spin oscillations by transversal matter current.

V. Cirigliano, G. Fuller, and A. Vlasenko, Phys. Lett. B 747, 27 (2015);
C. Volpe, Int.J.Mod.Phys. E 24, 1541009 (2015);
A. Kartavtsev, G. Raffelt, and H. Vogel, Phys.Rev. D 91, 125020 (2015)

Account for transversal matter motion in SL$\nu$:

$$\left\{ i\gamma_\mu \partial^\mu - \frac{1}{2}\gamma_\mu (1+\gamma_5) f^\mu - m \right\} \Psi(x) = 0.$$

$$f^\mu = \frac{G_F}{\sqrt{2}} \left[ \left(1 + 4\sin^2\theta_W\right) j_e^\mu \right.$$

$$j_e^\mu = (n, n\mathbf{v})$$

$$\Gamma = 4\mu^2 \tilde{n}^2 p \left( \frac{p^2 + \tilde{n} p \mathrm{v}^2 + \tilde{n}^2 \mathrm{v}^2}{p^2 + \tilde{n}^2 \mathrm{v}^2} \right), \quad \tilde{n} = \tilde{n}_0 / \sqrt{1 - \mathrm{v}^2}$$

The SL$\nu$ is amplified: $\Gamma' = \Gamma / \gamma^2$

[A.Grigoriev,A.Studenikin,A.Ternov, Russ Phys J 67, 1864–1877 (2024)]

# Thank you for your attention!