Dualities of QCD phase diagram and inhomogeneous phases

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Group and Acknowlegements

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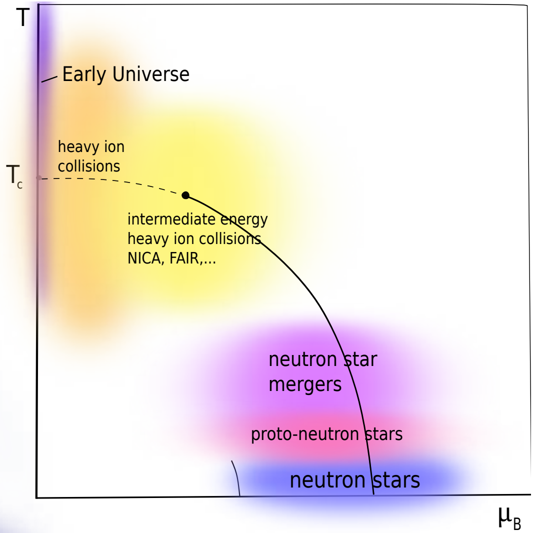
QCD Dhase Diagram

QCD at T and μ

(QCD at extreme conditions)

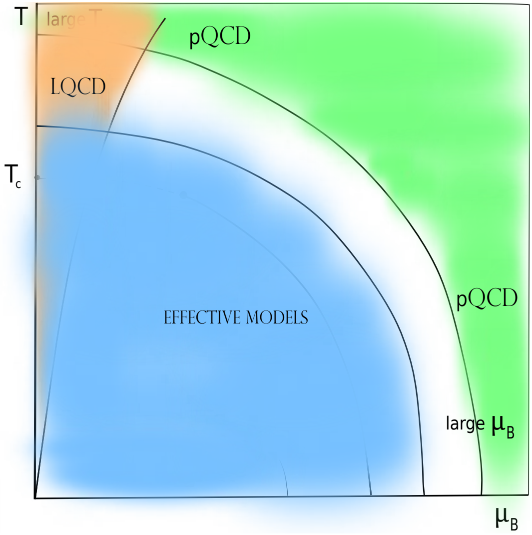
- Early Universe
- heavy ion collisions
- neutron stars
- proto- neutron stars
- neutron star mergers



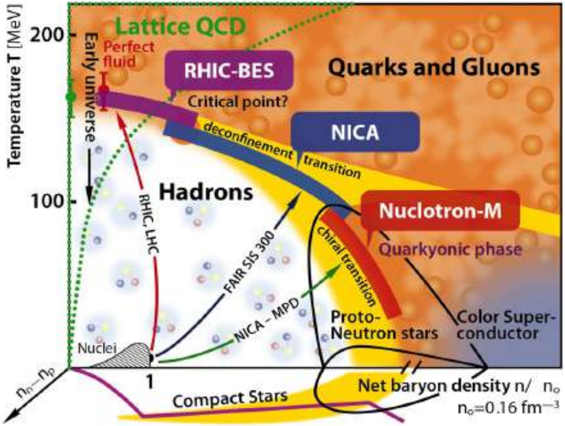
QCD Dhase Diagram and Methods

Methods of dealing with QCD

- Perturbative QCD
- First principle calculation – lattice QCD
- Effective models
- DSE, FRG
- Gauge/Gravity duality
- .....

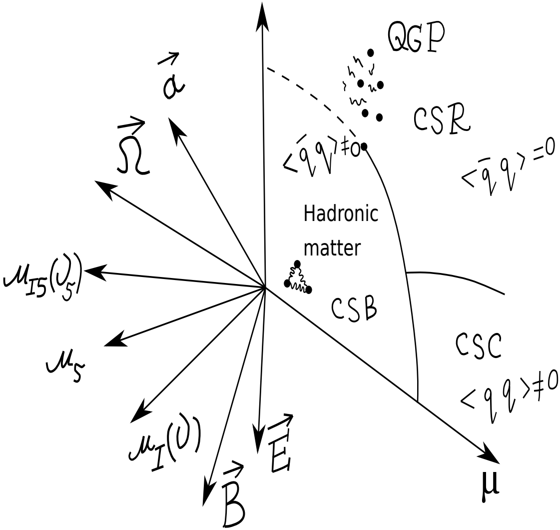


QCD Phase Diagram

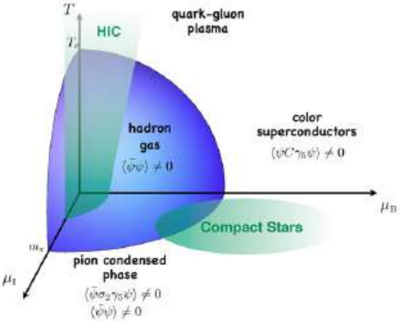


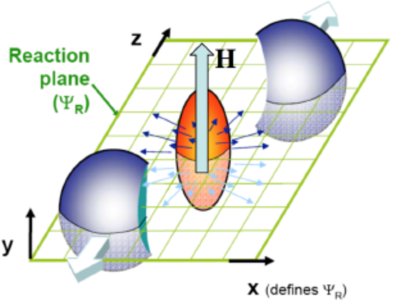
More than just QCD at (μ, T)

- more chemical potentials μ_i
- magnetic fields *(see talk by K. Rannu)*
- rotation of the system $\vec{\Omega}$
- acceleration $\vec{a}$
- finite size effects (finite volume and boundary conditions)



Chemical potentials





$$\mu = \frac{\mu_B}{3}, \qquad \nu = \frac{\mu_I}{2}, \qquad \mu_5, \qquad \nu_5 = \frac{\mu_{I5}}{2}$$

Chemical potentials

- **Isotopic chemical potential** μ_I

$$\frac{\mu_I}{2}\bar{q}\gamma^0\tau_3 q = \nu\,(\bar{q}\gamma^0\tau_3 q), \qquad n_I = n_u - n_d$$

Neutron stars, intermediate energy heavy-ion collisions, neutron star mergers

- **Chiral (axial) chemical potential** μ_5

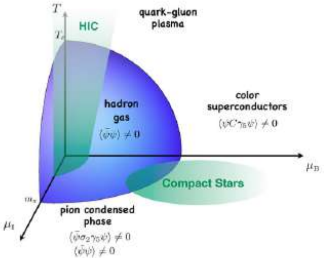
$$\mu_5\,\bar{q}\gamma^0\gamma^5 q, \qquad n_5 = n_R - n_L, \;\; \mu_5 = \mu_R - \mu_L$$

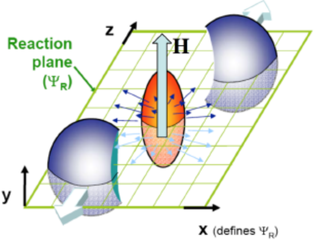
- **Chiral isospin chemical potential** μ_{I5}

$$\mu_5^u \neq \mu_5^d \qquad\qquad \mu_{I5} = \mu_5^u - \mu_5^d$$

$$\frac{\mu_{I5}}{2}\bar{q}\tau_3\gamma^0\gamma^5 q = \nu_5(\bar{q}\tau_3\gamma^0\gamma^5 q)$$

$$n_{I5} = n_{u5} - n_{d5}, \qquad\qquad n_{I5} \quad \longleftrightarrow \quad \nu_5$$





- **Recalling the dualities of phase diagram**
- **Dualities in QCD and QC_2D from first principles**
- **Wide swathes of application of dual**
  - **Speed of sound in quark matter with different properties**
  - **Inhomogeneous phases**

Recall that in NJL model **in** $1/N_c$ **approximation or in the mean field** there have been found **dualities**

(*It is not related to holography or gauge/gravity duality*)

Chiral symmetry breaking $\Longleftrightarrow$ pion condensation

Isospin imbalance $\Longleftrightarrow$ Chiral imbalance

The TDP

$$\Omega(T, \mu, \mu_i, ..., \langle \bar{q}q \rangle, ...) \qquad \Omega(T, \mu, \nu, \nu_5, ..., M, \pi, ...)$$

$$\mathcal{D}: \; M \longleftrightarrow \pi, \quad \nu \longleftrightarrow \nu_5$$

- A lot of densities and imbalances
  baryon, isospin, chiral, chiral isospin imbalances
- Finite temperature $T \neq 0$
- Physical pion mass $m_\pi \approx 140$ MeV
- Inhomogeneous phases (case)

$$\langle \sigma(x) \rangle = M(x), \quad \langle \pi_\pm(x) \rangle = \pi(x), \quad \langle \pi_3(x) \rangle = 0.$$

- Inclusion of color superconductivity phenomenon

Dualities in QC_2D

Similarity of SU(2) and SU(3)

- similar phase transitions:
  confinement/deconfinement, chiral symmetry breaking/restoration
- A lot of physical quantities coincide with some accuracy
  Critical temperature, shear viscosity etc.
- There is **no sign problem** in SU(2) case and lattice simulations at non-zero baryon density are possible — $(Det(D(\mu)))^\dagger = Det(D(\mu))$

It is a great playground for studying dense matter

$$\sigma(x) = -2H(\bar{q}q), \qquad \Delta(x) = -2H\left[\overline{q^c} i\gamma^5 \sigma_2 \tau_2 q\right]$$

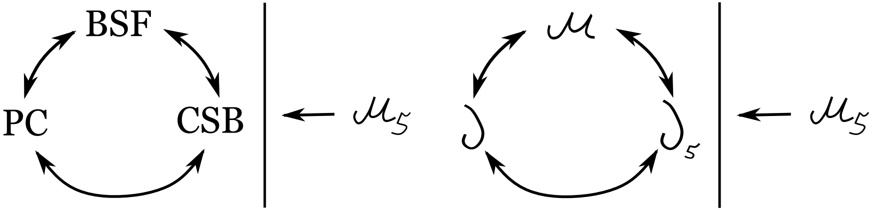
$$\vec{\pi}(x) = -2H(\bar{q}i\gamma^5\vec{\tau}q), \qquad \Delta^*(x) = -2H\left[\bar{q} i\gamma^5 \sigma_2 \tau_2 q^c\right]$$

Condensates and phases

$M = \langle\sigma(x)\rangle \sim \langle\bar{q}q\rangle,$ CSB phase: $M \neq 0,$

$\pi_1 = \langle\pi_1(x)\rangle = \langle\bar{q}\gamma^5\tau_1 q\rangle,$ PC phase: $\pi_1 \neq 0,$

$\Delta = \langle\Delta(x)\rangle = \langle qq\rangle = \langle q^T C\gamma^5\sigma_2\tau_2 q\rangle,$ BSF phase: $\Delta \neq 0.$



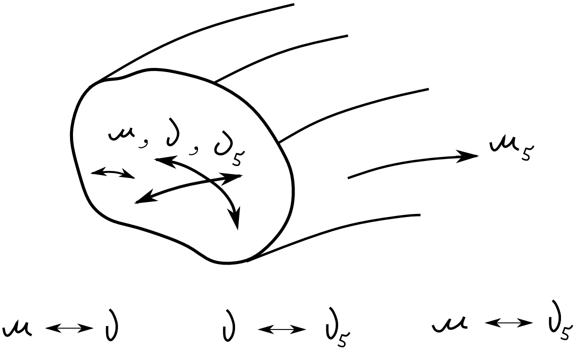
$$\text{(I)} \qquad \mathcal{D}_1 : \quad \mu \longleftrightarrow \nu, \quad \pi_1 \longleftrightarrow \Delta, \qquad \text{PC} \longleftrightarrow \text{BSF}$$

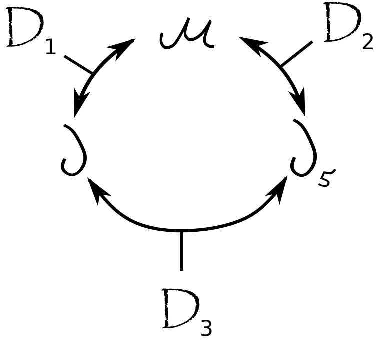
$$\text{(II)} \qquad \mathcal{D}_3 : \quad \nu \longleftrightarrow \nu_5, \quad M \longleftrightarrow \pi_1, \qquad \text{PC} \longleftrightarrow \text{CSB}$$

$$\text{(III)} \qquad \mathcal{D}_2 : \quad \mu \longleftrightarrow \nu_5, \quad M \longleftrightarrow \Delta, \qquad \text{CSB} \longleftrightarrow \text{BSF}$$

The phase diagram of (μ, ν, μ_5, ν_5)

The phase diagram is foliation of dually connected cross-section of (μ, ν, ν_5) along the μ_5 direction





$D_1 : \ \mu \longleftrightarrow \nu$

$D_2 : \ \mu \longleftrightarrow \nu_5$

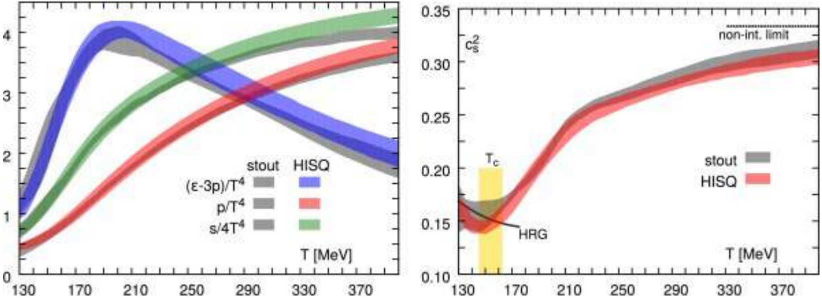
$D_3 : \ \nu \longleftrightarrow \nu_5$

$$\mathcal{D}_{\rm II}: \qquad \langle \bar{\psi}\psi \rangle \longleftrightarrow \langle i\bar{\psi}\gamma^5\tau_1\psi \rangle, \quad M \longleftrightarrow \pi, \quad \nu \leftrightarrow \nu_5$$

From first principles

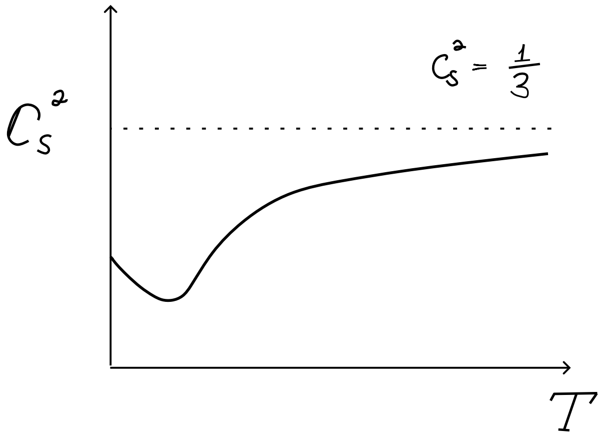
Speed of sound c_s^2

Thermodynamic properties could be calculated in lattice QCD



A. Bazavov et al. [HotQCD], Phys. Rev. D ***90*** *(2014), 094503*

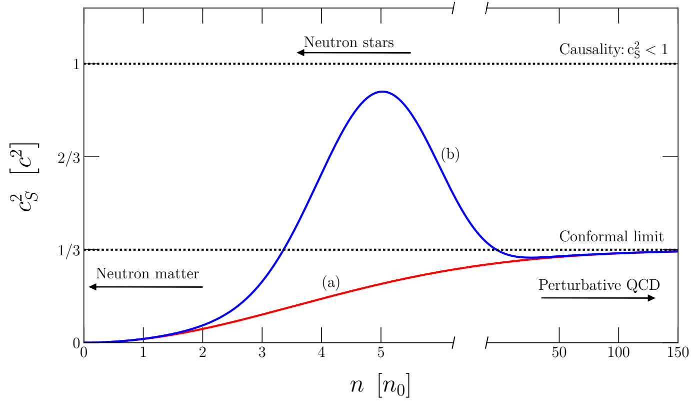
Sound speed in QCD at finite T: skematic



There was discussed bound from holography

A. Cherman, T. D. Cohen and A. Nellore, Phys. Rev. D 80 (2009), 066003

Two possible scenario of **speed of sound** at **non-zero baryon density**



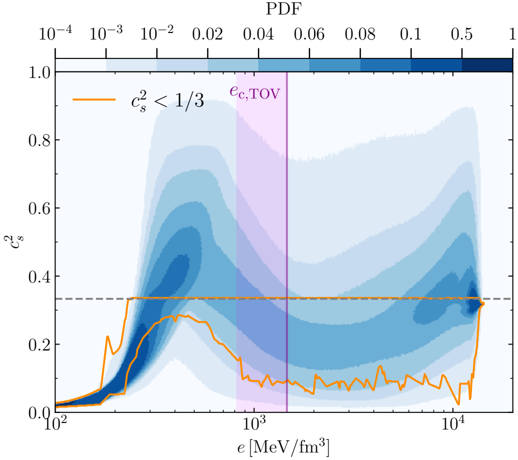
taken from S. Reddy et al, Astrophys. J. ***860*** *(2018) no.2, 149*

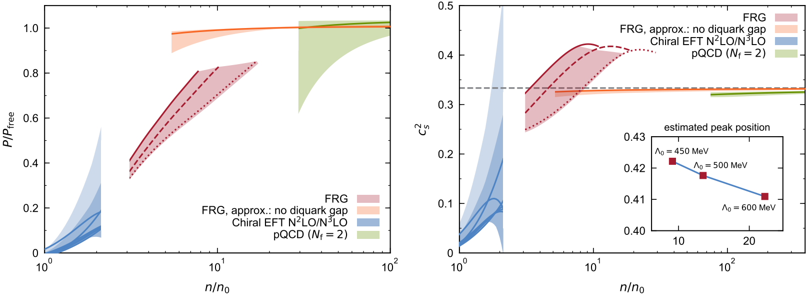
Sound speed in QCD with non-zero baryon density

EOS with continuous c_s^2 consistent not only with nuclear theory and perturbative QCD, but also with astrophysical observations.

EOS with sub-conformal sound speeds, i.e.,$c_s^2 < 1/3$ are **possible in principle but very unlikely in practice**

L. Rezzolla et al, Astrophys.J.Lett. 939 (2022) 2, L34





- **Sound speed** squared has been obtained from **FRG approach**

Phys.Rev.Lett. 125 (2020) 14, 142502

$$Z = \int D[\text{gluons}]\, D[\text{quarks}]\, e^{-S^E_{QCD}}$$

$$Z = \int D[\text{gluons}]\, Det\, D(\mu)\, e^{-S^E_{gluons}}$$

It is well known that **at non-zero baryon chemical potential μ_B lattice simulation** is quite challenging due to the **sign problem**
complex determinant

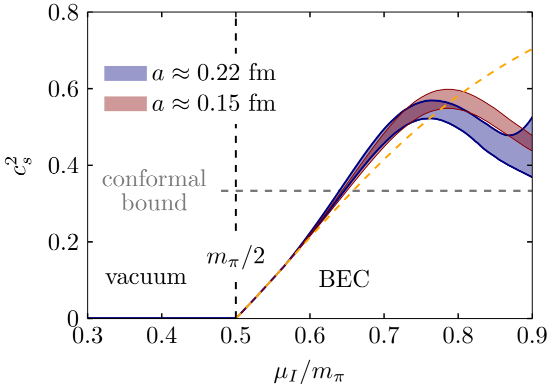
$$Det(D(\mu))^\dagger = Det(D(-\mu))$$

For isospin chemical potential μ_I

$$Det(D(\mu_I))^\dagger = Det(D(\mu_I))$$

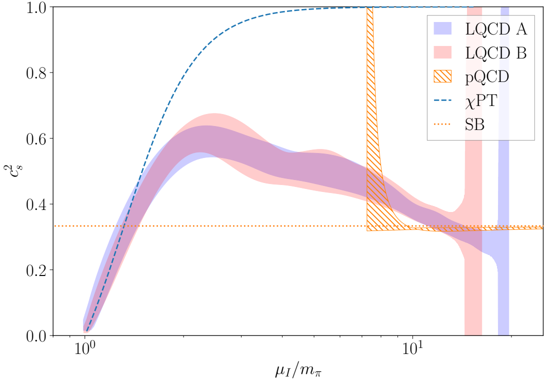
- **Sound speed** squared has been obtained from **lattice QCD simulations** for **QCD with non-zero isospin** μ_I

B. B. Brandt, F. Cuteri and G. Endrodi, JHEP 07, 055 (2023)



- **Sound speed** squared has been obtained from **lattice QCD simulations** for **QCD with non-zero isospin** μ_I for values of μ_I up to $10 m_\pi$

R. Abbott et al. [NPLQCD], Phys. Rev. D 108, no.11, 114506 (2023)



Duality between chiral symmetry breaking and pion condensation

$$\mathcal{D}: \; M \longleftrightarrow \pi, \quad \nu \longleftrightarrow \nu_5$$

The TDP of the quark matter

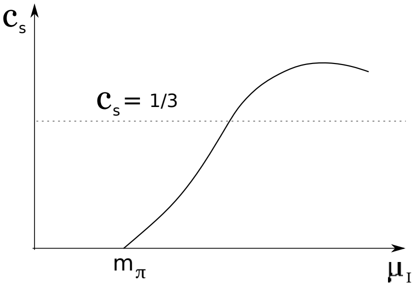
$$\Omega(T,\, \mu,\, \nu,\, \nu_5,\, \mu_5,\, |\, M,\, \pi) = \mathrm{inv}$$

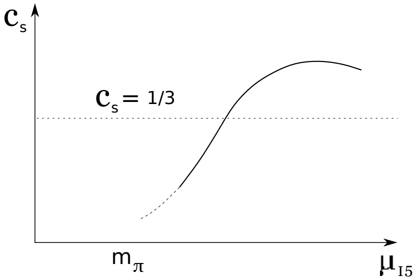
The speed of sound $$c_s^2 = \frac{dp}{d\epsilon}$$

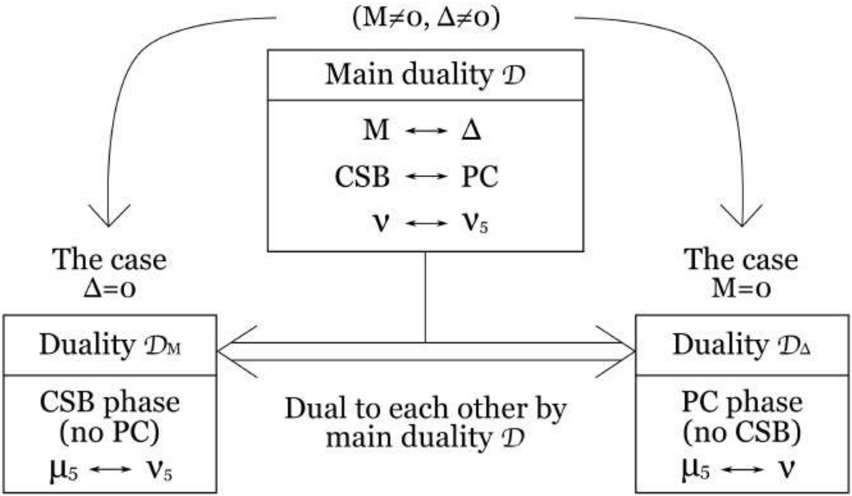
$$\Omega(T, ...) \Longrightarrow c_s^2(T, ...)$$

The speed of sound $c_s^2 = \dfrac{dp}{d\epsilon}$, $\Omega(T,...) \Longrightarrow c_s^2(T,...)$

$$\Omega(T,...,\nu) = \Omega(T,...,\nu_5) \Longrightarrow c_s^2(T,...,\nu) = c_s^2(T,...,\nu_5)$$



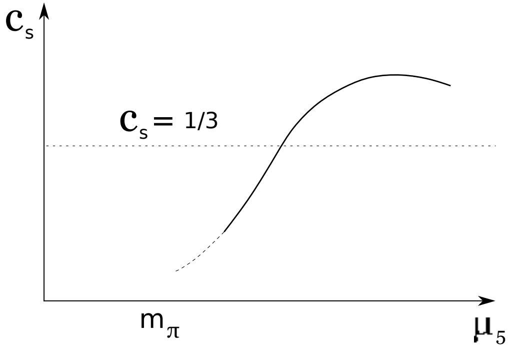



(M≠0, Δ≠0)
Main duality $\mathcal{D}$
M ⟷ Δ
CSB ⟷ PC
ν ⟷ ν_5
The case
Δ=0
The case
M=0
Duality $\mathcal{D}_M$
CSB phase
(no PC)
μ_5 ⟷ ν_5
Dual to each other by
main duality $\mathcal{D}$
Duality $\mathcal{D}_\Delta$
PC phase
(no CSB)
μ_5 ⟷ ν

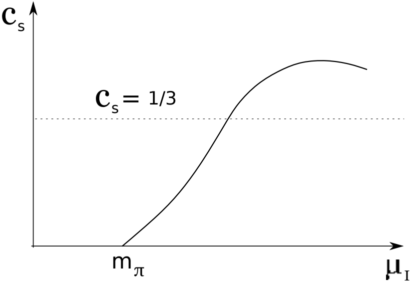
Duality

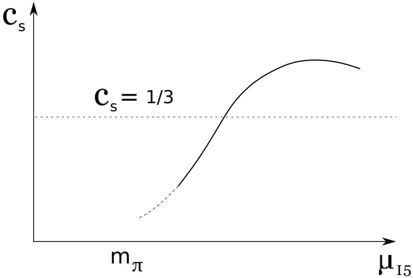
$$\nu_5 \longleftrightarrow \mu_5, \qquad M \neq 0, \quad \langle \pi \rangle = \langle \Delta \rangle = 0$$

- **Sound speed** squared for **QCD with non-zero chiral imbalance** μ_5 **only in the framewwork of effective model**



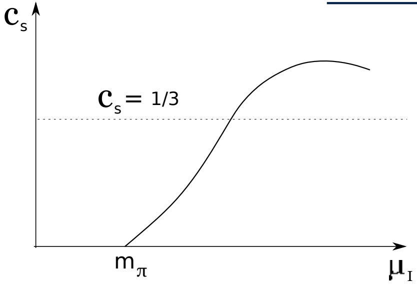

c_s^2
$c_s = \frac{1}{3}$
μ

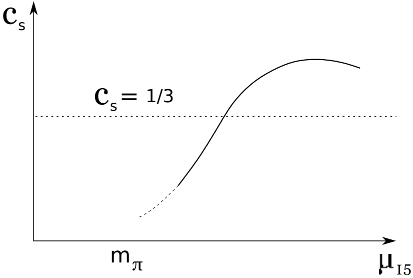

c_s
$c_s = 1/3$
m_π
μ_I

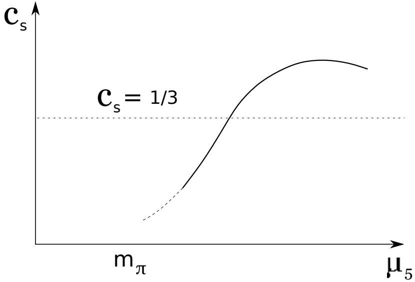

c_s
$c_s = 1/3$
m_π
μ_{I5}


c_s^2
$c_s = \frac{1}{3}$
μ_5


c_s^2
μ

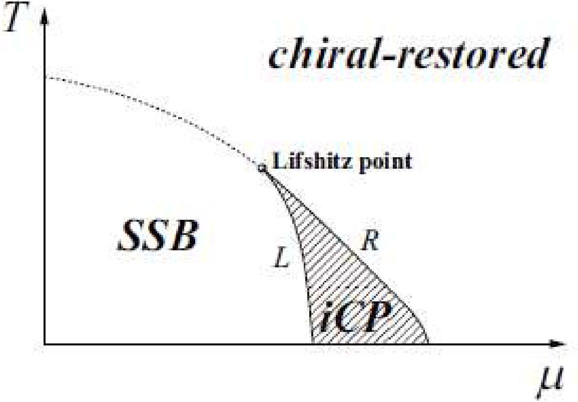

c_s
$c_s = 1/3$
m_π
μ_I


c_s
$c_s = 1/3$
m_π
μ_{I5}


c_s
$c_s = 1/3$
m_π
μ_5

Inhomogeneous phases in QCD and QC_2D

It is open question if there is inhomogeneous chiral symmetry breaking phase at $\mu_B \neq 0$



$$\langle \bar{q}q \rangle \sim M(x)$$

- **Inhomogeneous phase was predicted in: (1+1)-dimensional Gross-Neveu (GN) model**

  M. Thies, ...

  A. Wipf, M. Wagner, M. Winstel, L. Pannullo etc.

- **Inhomogeneous phase in (3+1)-dimensional effective models**

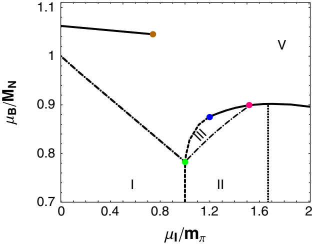
- **Inhomogeneous phase in effective models: dependence on the chosen regularization scheme**

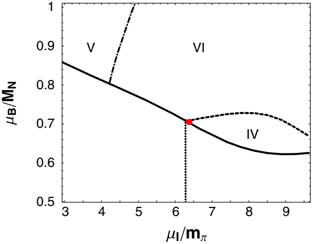
  M. Wagner et al, Phys. Rev. D 110 (2024) 7, 076006

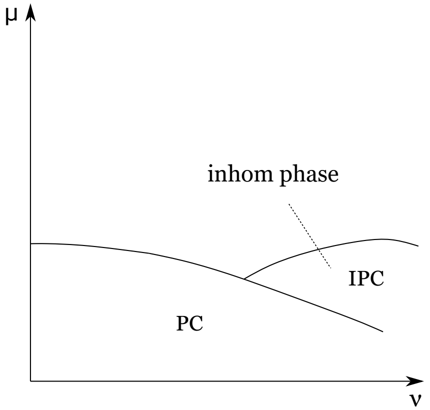
- **Inhomogeneous phase shown in functional approach**

  C. Fischer et al, Phys. Rev. D 108 (2023) 11, 114019, Phys.Rev.D 110 (2024) 7, 074014

Inhomogeneous PC at μ_I: skematic





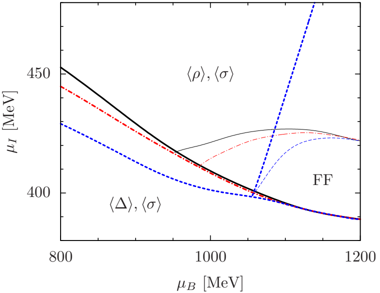


Lianyi He et al, Phys.Rev.D 82 (2010) 056006

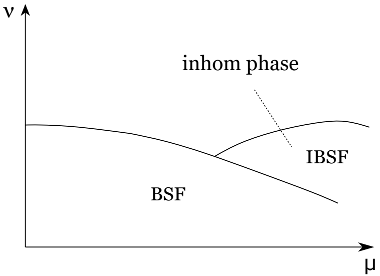
Inhomogeneous phase at μ_I: skematic

Inhomogeneous diquark condensation found in two color case

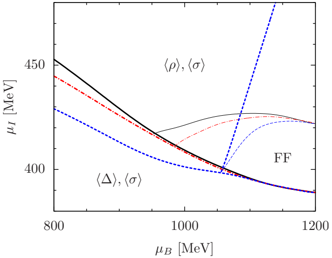
in the framework of effective models

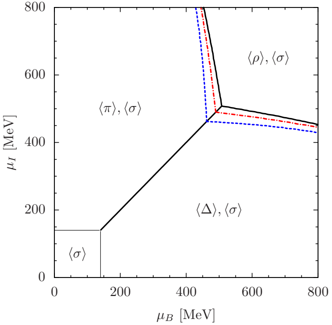


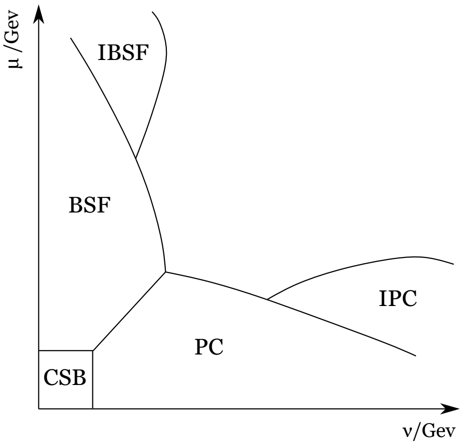
J. Andersen et al Phys.Rev.D 81 (2010) 096004



Inhomogeneous phase at μ_I: skematic





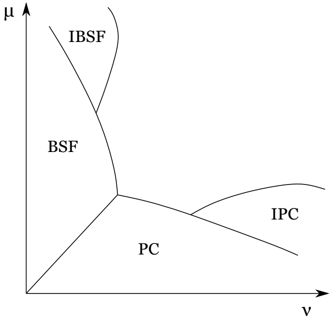


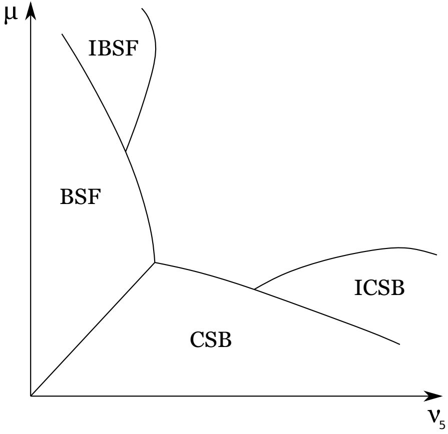
$$\mu \longleftrightarrow \nu, \qquad \pi_1 \longleftrightarrow \Delta$$

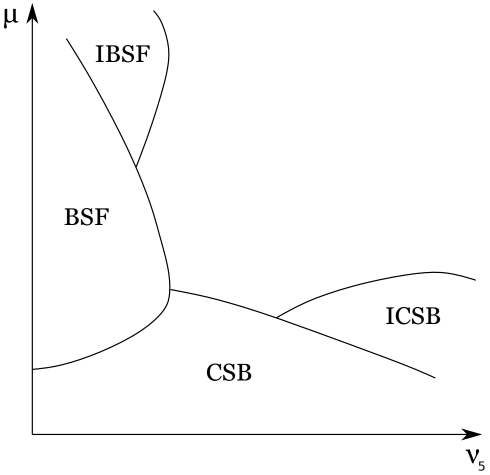
$$\text{PC} \longleftrightarrow \text{BSF}, \qquad \text{IPC} \longleftrightarrow \text{IBSF}$$

Inhomogeneous phase: skematic

$$\nu \longleftrightarrow \nu_5, \qquad M \longleftrightarrow \pi_1, \qquad \text{CSB} \longleftrightarrow \text{PC}, \qquad \text{ICSB} \longleftrightarrow \text{IPC}$$







$$\mu \longleftrightarrow \nu_5, \quad M \longleftrightarrow \Delta, \qquad \text{CSB} \longleftrightarrow \text{BSF}, \qquad \text{ICSB} \longleftrightarrow \text{IBSF}$$

Inhomogeneous phases

Homogeneous case

$$\langle \sigma(x) \rangle = M, \quad \langle \pi_{\pm}(x) \rangle = \pi, \quad \langle \pi_3(x) \rangle = 0.$$

Inhomogeneous phases (three color case)

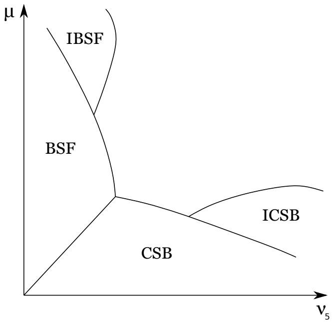
$$\langle \sigma(x) \rangle = M(x), \quad \langle \pi_{\pm}(x) \rangle = \pi(x), \quad \langle \pi_3(x) \rangle = 0.$$

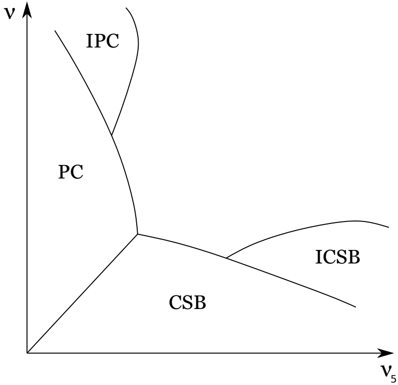
$$\mathcal{D}: \; M(x) \longleftrightarrow \pi(x), \quad \nu \longleftrightarrow \nu_5$$

$$\text{ICSB} \longleftrightarrow \text{IPC} \quad \nu \longleftrightarrow \nu_5$$

Inhomogeneous phase: skematic

$$\mu \longleftrightarrow \nu, \quad \pi_1 \longleftrightarrow \Delta, \quad \text{PC} \longleftrightarrow \text{BSF}, \quad \text{IPC} \longleftrightarrow \text{IBSF}$$

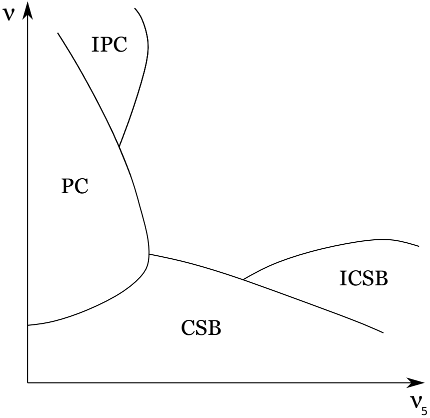




Inhomogeneous phase at zero μ_B: skematic

Inhomogeneous phases exist usually at $\mu_B \neq 0$

Inhomogeneous phase in two color case exist at $\mu_B = 0$



$\nu \longleftrightarrow \nu_5, \quad M \longleftrightarrow \pi_1, \quad \text{CSB} \longleftrightarrow \text{PC}, \quad \text{ICSB} \longleftrightarrow \text{IPC}$

Dualities has been proven from first principles

Speed of sound exceeding the conformal limit is rather **natural** and taking place in a lot of systems, **with various chemical potentials**

And it is natural if it has similar structure in QCD at non-zero baryon density, the most interesting case

Inhomogeneous phases in two and three color case have been studied, in two color case exist at $\mu_B = 0$