# Effects of QED higher-order radiative corrections in electron-positron annihilation process at future colliders

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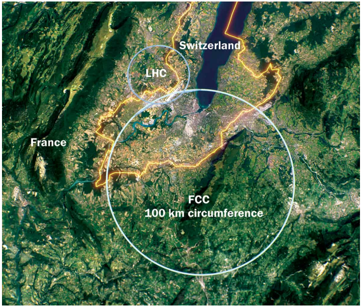
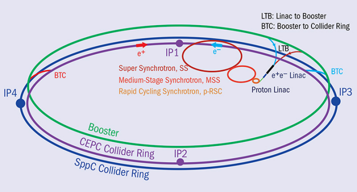
# Outline

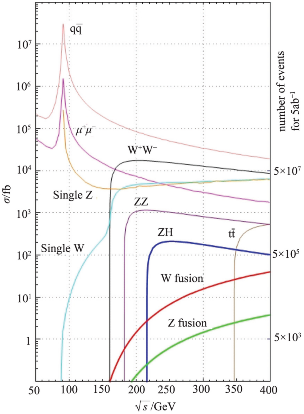
# Introduction

- Standard Model is the most successful theory of particles and their interactions, but it is not fundamental theory
- We need new experiments with high precision and sensitivity to test the SM, its boundaries, and possibly find new physics beyond the SM $\rightarrow$ new colliders
- Lepton colliders ($e^+e^-$, $\mu^+\mu^-$) allow experiments with higher precision than hadron ones
- $e^+e^-$-annihilation is one of the main processes at $e^+e^-$-colliders
- We need precise theoretical calculations to predict the results of these experiments (calculation of highter order corrections)
- Calculation of higher order radiative corrections is a complicated problem
- We need methods to simplify this calculation (e.g. PDF approach)
- Resummation of calculated corrections to improve convergence of perturbation theory series $\rightarrow$ choose a factorization scale

# Future $e^+e^-$-colliders




- $\sqrt{s} = M_z = 91.1876$ GeV - Z-peak
- $\sqrt{s} = 160$ GeV - $W\bar{W}$
- $\sqrt{s} = 240$ GeV - Higgs vev, ZH
- $\sqrt{s} = 350, 365$ GeV - $t\bar{t}$

**See the talk of A. Drutskoy**

From Mo et al., CPC 03 (2016) 033001

# Parton distribution functions (PDFs) approach

**See the talk of A. B. Arbuzov today**

- Parton distribution functions approach allow to calculate only corrections enhanced by the large logarithm:

$$L = \ln \frac{\mu_F^2}{\mu_R^2},$$

$\mu_F$ - factorization scale, $\mu_R$ - renormalization scale

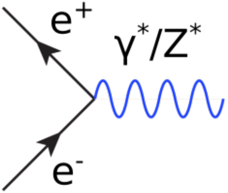
- Evolution equation of PDFs in QED is analogous to DGLAP equation in QCD

$$D_{ba}(x, \mu^2, \mu_0^2) = \delta(1-x)\delta_{ba} + \sum_{i=e,\bar{e},\gamma} \int\limits_{\mu_0^2}^{\mu^2} \frac{dt\alpha(t)}{2\pi t} \int\limits_x^1 \frac{dy}{y} D_{ia}(y, t, \mu_0) P_{bi}\left(\frac{x}{y}\right)$$

- Equations are solved by iterations (U.V., A.Arbuzov, JPG 2023) with initial conditions defined by subtraction scheme (here $\overline{\mathrm{MS}}$)
- Process independent PDFs are convoluted with functions containing information about the process

# $e^+e^-$-annihilation

$e^+e^- \to \gamma^*/Z^*$




LO and NLO ISR corrections to the order $\alpha^5 L^6$ (Ablinger at al., NPB 2020)
We recalculated and corrected some mistakes (A.B. Arbuzov, U.V., PRD 2024, arXiv: 2405.03443 [hep-ph])

$$\sigma^{\rm NLO}_{\bar{e}e}(s') = \sum_{i,j=e,\bar{e},\gamma} \int\limits_{\bar{z_1}}^{1} \int\limits_{\bar{z_2}}^{1} dz_1 dz_2 D^{\rm str}_{ie}\left(z_1, \frac{\mu_R^2}{\mu_F^2}\right) D^{\rm str}_{j\bar{e}}\left(z_2, \frac{\mu_R^2}{\mu_F^2}\right) \times$$
$$\times \left(\sigma^{(0)}_{ij}(sz_1z_2) + \bar{\sigma}^{(1)}_{ij}(sz_1z_2) + \mathcal{O}(\alpha^2 L^0)\right) \delta(s'-sz) + \mathcal{O}\left(\frac{\mu_R^2}{\mu_F^2}\right)$$

$$d\sigma^{\rm NLO}_{ab\to cd} \quad = \quad d\sigma^{(0)}_{ab\to cd}\left\{1 + \sum_{i=1}^{\infty}\left(\frac{\alpha}{2\pi}\right)^i \sum_{j=i-1}^{i} c_{ij} L^j + \mathcal{O}(\alpha^i L^{i-2})\right\}$$

## $e^+e^-$-annihilation

$$d\sigma^{\mathrm{NLO}}_{ab\to cd} = d\sigma^{(0)}_{ab\to cd}\left\{1+\sum_{i=1}^{\infty}\left(\frac{\alpha}{2\pi}\right)^i\sum_{j=i-1}^{i}c_{ij}L^j+\mathcal{O}(\alpha^i L^{i-2})\right\}$$

$$h_{ij}=\left(\frac{\alpha}{2\pi}\right)^i L^j c_{ij}$$

$$\sigma_{e^+e^-}=\sum_{i,j}\left(h^{\Delta}_{ij}\sigma^{(0)}(1)+\int\limits_{z_{min}}^{1-\Delta}dz\sigma^{(0)}(z)h^{\theta}_{ij}(z)\right)$$

$$h^{num}_{ij}=h^{\Delta}_{ij}\sigma^{(0)}(1)+\int\limits_{z_{min}}^{1-\Delta}dz\sigma^{(0)}(z)h^{\theta}_{ij}(z)$$

$\Delta=10^{-7},\ 10^{-8}$

# Numerical estimations, %, for $\mu_F^2 = s$

$z_{min}$ is defined by experimental conditions

$\sqrt{s} = 160$ GeV, $z_{min} = 0.5$

| $h_{11}$ | $h_{10}$ | $h_{22}$ | $h_{21}$ | $h_{33}$ | $h_{32}$ | $h_{44}$ | $h_{43}$ | $h_{55}$ |
|---|---|---|---|---|---|---|---|---|
| $\gamma$ | | | | | | | | |
| 9.81624 | 0.26017 | -1.28618 | 0.20722 | -0.00845 | -0.00714 | 0.00530 | -0.00153 | -0.00020 |
| Pairs | | | | | | | | |
| 0 | 0 | 0.13182 | -0.05573 | -0.03105 | 0.01278 | -0.00171 | 0.00046 | -0.00100 |
| Full | | | | | | | | |
| 9.81624 | 0.26017 | -1.15435 | 0.15149 | -0.03949 | 0.00563 | 0.00359 | -0.00107 | -0.00120 |

$\sqrt{s} = 240$ GeV, $z_{min} = 0.5$

| $h_{11}$ | $h_{10}$ | $h_{22}$ | $h_{21}$ | $h_{33}$ | $h_{32}$ | $h_{44}$ | $h_{43}$ | $h_{55}$ |
|---|---|---|---|---|---|---|---|---|
| $\gamma$ | | | | | | | | |
| 3.45872 | 0.51562 | -1.05495 | 0.14324 | 0.02506 | -0.01176 | 0.00273 | -0.00067 | -0.00018 |
| Pairs | | | | | | | | |
| 0 | 0 | 0.05956 | -0.02900 | -0.02628 | 0.01132 | -0.00057 | -0.00017 | -0.00080 |
| Full | | | | | | | | |
| 3.45872 | 0.51562 | -0.99539 | 0.11424 | -0.00122 | -0.00044 | 0.00216 | -0.00084 | -0.00098 |

# Numerical estimations, %, for $\mu_F^2 = s$

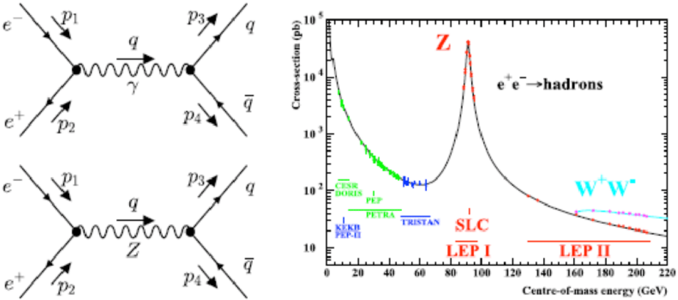
$\sqrt{s} = 160$ GeV, $z_{min} = 0.1$

| $h_{11}$ | $h_{10}$ | $h_{22}$ | $h_{21}$ | $h_{33}$ | $h_{32}$ | $h_{44}$ | $h_{43}$ | $h_{55}$ |
|---|---|---|---|---|---|---|---|---|
| $\gamma$ | | | | | | | | |
| 335.7996 | -12.6201 | 17.1479 | 0.6467 | -1.8223 | 0.5781 | -0.0348 | -0.0102 | 0.0052 |
| Pairs | | | | | | | | |
| 0 | 0 | 8.3539 | -1.8438 | 0.0043 | -0.0647 | -0.0606 | 0.0357 | -0.0922 |
| Full | | | | | | | | |
| 335.7996 | -12.6201 | 25.5019 | -1.1971 | -1.8182 | 0.5135 | -0.0953 | 0.0255 | -0.0870 |

$\sqrt{s} = 240$ GeV, $z_{min} = 0.1$

| $h_{11}$ | $h_{10}$ | $h_{22}$ | $h_{21}$ | $h_{33}$ | $h_{32}$ | $h_{44}$ | $h_{43}$ | $h_{55}$ |
|---|---|---|---|---|---|---|---|---|
| $\gamma$ | | | | | | | | |
| 324.1463 | -11.7621 | 27.1298 | -0.9909 | -0.6242 | 0.6140 | -0.0664 | 0.0149 | 0.0012 |
| Pairs | | | | | | | | |
| 0 | 0 | 25.3337 | -1.4772 | -0.1401 | -0.0207 | -0.0269 | 0.0460 | -0.1792 |
| Full | | | | | | | | |
| 324.1463 | -11.7621 | 52.4635 | -2.4680 | -0.7643 | 0.5934 | -0.0932 | 0.0609 | -0.1780 |

# Cross section

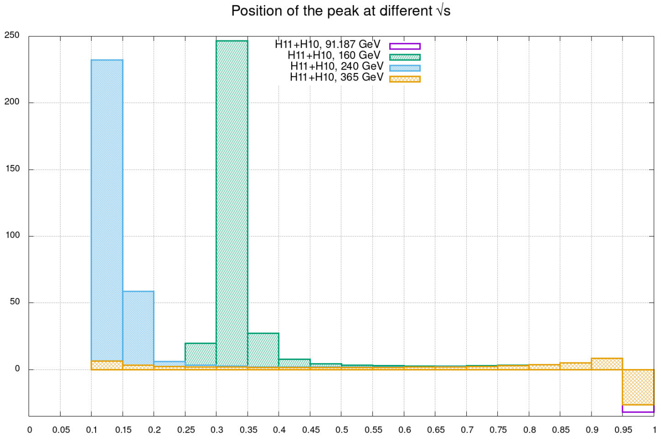



(a) (b)



*From M. Stoeltzner, 2022*

# Position of the peak at $z_{min} = 0.1$




Position of $Z$-peak: $sz = M_z^2$

| $\sqrt{s}$ | 91.1876 | 160 | 240 | 365 |
|---|---|---|---|---|
| Peak, $z$ | 1 | 0.3234 | 0.1443 | 0.0624 |
| $h_{11}$, % | $-32.7365$ | 335.7996 | 324.1463 | 23.5582 |

# Factorization scale choice, $\mathcal{O}(\alpha^1)$

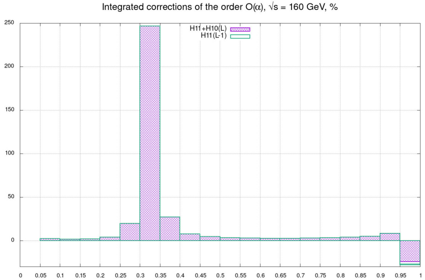
Compare 3 factorization scales: $\mu_F^2$, $\mu_F^2/e$, $\mu_F^2 z$

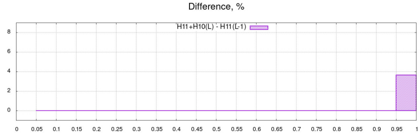
$$\mu_F^2 \leftrightarrow \mu_F^2/e, \quad e = 2.71828...$$
$$L \leftrightarrow L - 1$$

$\mathcal{O}(\alpha^1)$ :

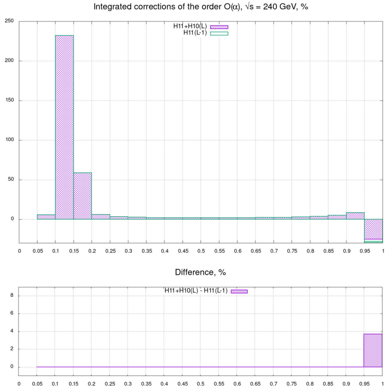
$$c_{11} = 2\frac{1+z^2}{1-z}$$
$$c_{10} = -2\frac{1+z^2}{1-z} = -c_{11}$$

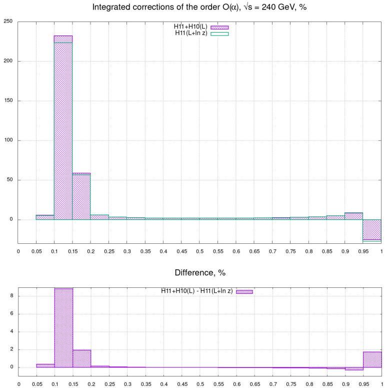
$$h_{11}(L) = \frac{\alpha}{2\pi} L c_{11}$$
$$h_{11}(L-1) = \frac{\alpha}{2\pi}(L-1)c_{11} =$$
$$= \frac{\alpha}{2\pi} L c_{11} - \frac{\alpha}{2\pi} c_{11} =$$
$$= h_{11}(L) + h_{10}(L)$$



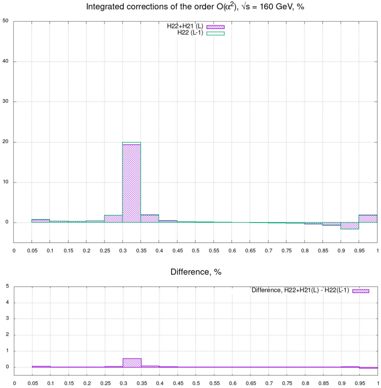



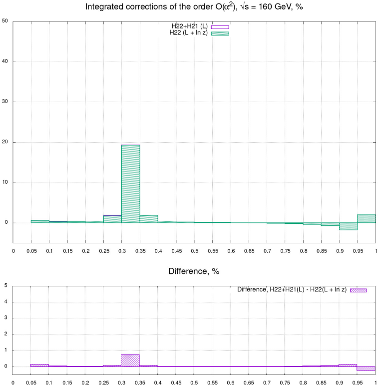
# Factorization scale choice, $\mathcal{O}(\alpha^1)$



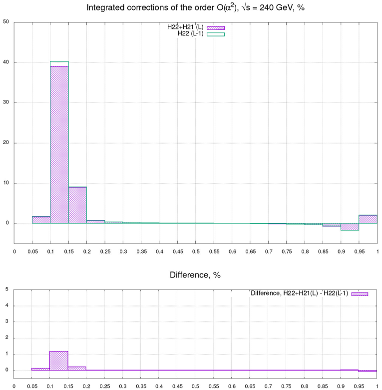



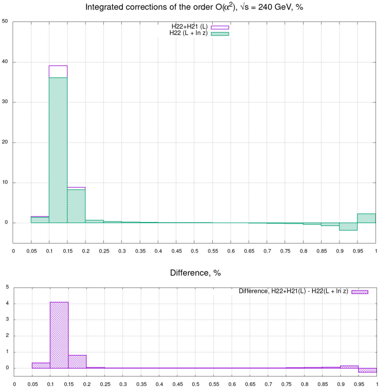
# Factorization scale choice, $\mathcal{O}(\alpha^2)$, NLO



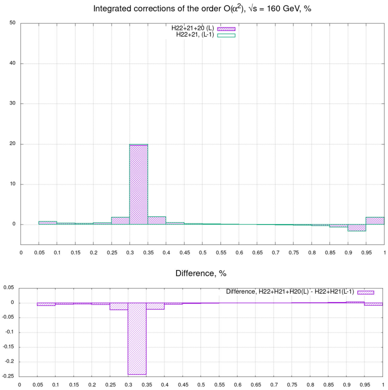



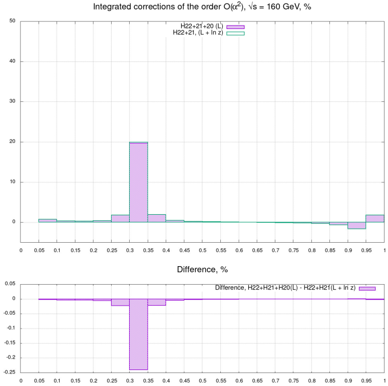
# Factorization scale choice, $\mathcal{O}(\alpha^2)$, NLO



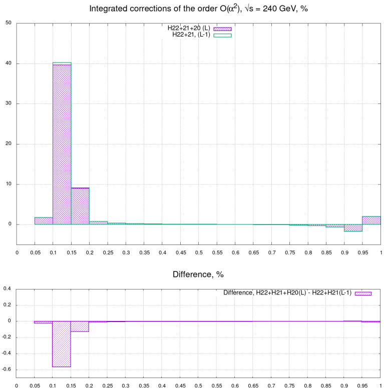



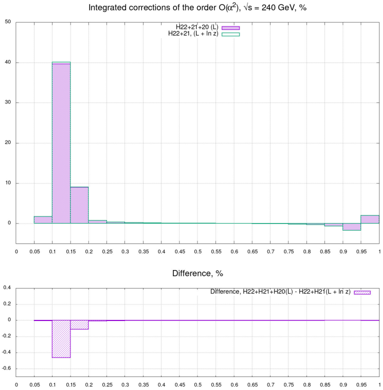
# Factorization scale choice, $\mathcal{O}(\alpha^2)$, NNLO



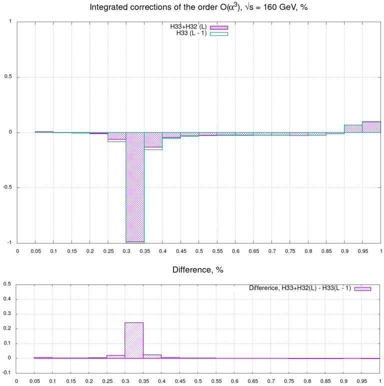



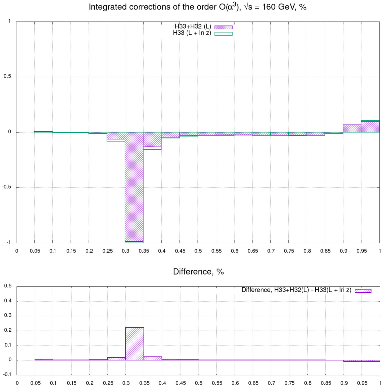
# Factorization scale choice, $\mathcal{O}(\alpha^2)$, NNLO



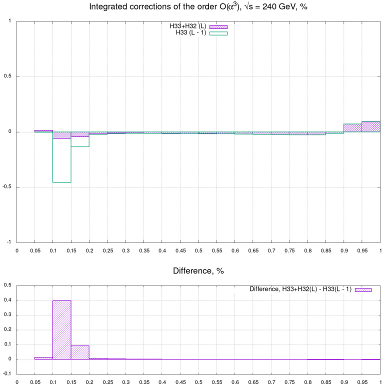



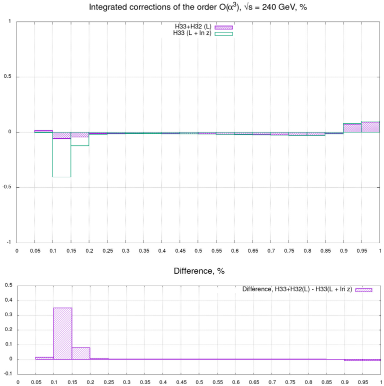
# Factorization scale choice, $\mathcal{O}(\alpha^3)$



# Factorization scale choice, $\mathcal{O}(\alpha^3)$



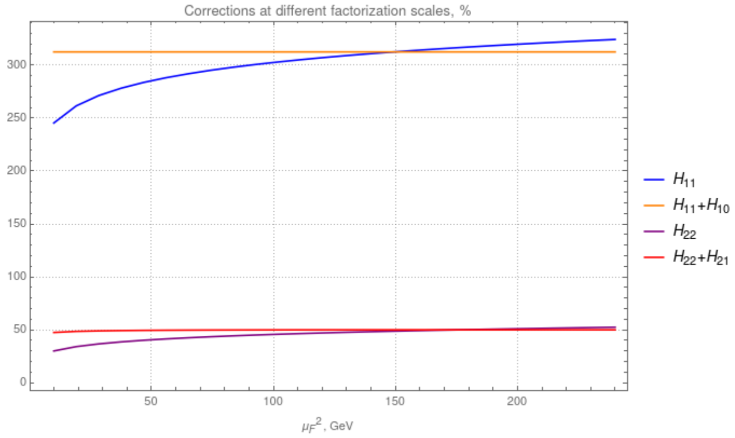



# Conclusion

- Numerical estimations of the ISR corrections to $e^+e^-$-annihilation cross section at future collider energies were obtained using PDF approach
- Factorization scale choice in QED is important
- At $\sqrt{s} = M_Z$ and the factorization scale $\mu_F^2 = s$ Z-peak can't be seen
- A factorization scale that improves convergence of perturbation theory series the order $\mathcal{O}(\alpha^2)$ was suggested
- Further investigation of different factorization schemes is needed

Thank you for your attention!

# $\mathcal{O}(\alpha^1)$ and $\mathcal{O}(\alpha^2)$ corrections, $z_{min} = 0.1$, $\sqrt{s} = 240$ GeV



# Numerical estimations, %

$$\sqrt{s} = M_z$$

| $Z_{min} = 0.1$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $h_{11}$ | $h_{10}$ | $h_{22}$ | $h_{21}$ | $h_{33}$ | $h_{32}$ | $h_{44}$ | $h_{43}$ | $h_{55}$ |
| $\gamma$ | | | | | | | | |
| -32.73654 | 2.00167 | 4.88428 | -0.59516 | -0.37760 | 0.07104 | 0.00344 | -0.00185 | 0.00315 |
| Pairs | | | | | | | | |
| | | -0.30571 | 0.15847 | 0.08753 | -0.04599 | 0.00157 | 0.00375 | -0.00009 |
| Full | | | | | | | | |
| -32.73654 | 2.00167 | 4.57858 | -0.43669 | -0.29007 | 0.02505 | 0.00501 | 0.00190 | 0.00306 |

$$\sqrt{s} = 365 \text{ GeV}$$

| $h_{11}$ | $h_{10}$ | $h_{22}$ | $h_{21}$ | $h_{33}$ | $h_{32}$ | $h_{44}$ | $h_{43}$ | $h_{55}$ |
|---|---|---|---|---|---|---|---|---|
| $Z_{min} = 0.1$ | | | | | | | | |
| $\gamma$ | | | | | | | | |
| 23.55818 | -0.22585 | 0.53154 | 0.10899 | -0.05905 | 0.02875 | -0.00094 | -0.00042 | 0.00008 |
| Pairs | | | | | | | | |
| 0 | 0 | 1.30566 | -0.11950 | -0.02707 | 0.00843 | -0.00378 | 0.00281 | -0.01137 |
| Full | | | | | | | | |
| 23.55818 | -0.22585 | 1.83721 | -0.01051 | -0.08977 | 0.03718 | -0.00472 | 0.00239 | -0.01129 |

# Contributions of different orders

$$\frac{d\sigma_{\bar{e}e}}{ds'} = \sigma^{(0)} \Big[ D_{\bar{e}\bar{e}} \otimes D_{ee} \otimes \sigma_{\bar{e}e} + D_{\gamma e} \otimes D_{ee} \otimes \sigma_{e\gamma} + D_{ee} \otimes D_{e\bar{e}} \otimes \sigma_{ee} + \\ + D_{\gamma e} \otimes D_{\bar{e}\bar{e}} \otimes \sigma_{\bar{e}\gamma} + D_{\gamma e} \otimes D_{\gamma \bar{e}} \otimes \sigma_{\gamma\gamma} + D_{\gamma e} \otimes D_{e\bar{e}} \otimes \sigma_{e\gamma} \\ + D_{\bar{e}e} \otimes D_{\bar{e}\bar{e}} \otimes \sigma_{\bar{e}\bar{e}} + D_{\bar{e}e} \otimes D_{\gamma\bar{e}} \otimes \sigma_{\bar{e}\gamma} + D_{\bar{e}e} \otimes D_{e\bar{e}} \otimes \sigma_{\bar{e}e} \Big]$$

Contributions of different orders

| i \ j | $\bar{e}$ | $\gamma$ | $e$ |
|---|---|---|---|
| $e$ | $D_{ee}D_{\bar{e}\bar{e}}\sigma_{e\bar{e}}$ LO $(1)$ | $D_{ee}D_{\gamma\bar{e}}\sigma_{e\gamma}$ NLO $(\alpha^2 L)$ | $D_{ee}D_{e\bar{e}}\sigma_{ee}$ NNLO $(\alpha^4 L^2)$ |
| $\gamma$ | $D_{\gamma e}D_{\bar{e}\bar{e}}\sigma_{\gamma\bar{e}}$ NLO $(\alpha^2 L)$ | $D_{\gamma e}D_{\gamma\bar{e}}\sigma_{\gamma\gamma}$ NNLO $(\alpha^4 L^2)$ | $D_{\gamma e}D_{e\bar{e}}\sigma_{\gamma e}$ NLO $(\alpha^4 L^3)$ |
| $\bar{e}$ | $D_{\bar{e}e}D_{\bar{e}\bar{e}}\sigma_{\bar{e}\bar{e}}$ NNLO $(\alpha^4 L^2)$ | $D_{\bar{e}e}D_{\gamma\bar{e}}\sigma_{\bar{e}\gamma}$ NLO $(\alpha^4 L^3)$ | $D_{\bar{e}e}D_{e\bar{e}}\sigma_{\bar{e}e}$ LO $(\alpha^4 L^4)$ |

# Factorization in NLO

In *Berends(1987)*, *Blumlein(2011)* factorization scale was chosen

$$\mu_F^2 = sz.$$

Then the large log

$$L = \ln(s/m_e^2) + \ln z$$

In the expression for one-loop cross section in *Blumlein(2011)*, the variable $y = z/x$ was exchanged into $x$:

$$\left[\bar{\delta}_{\bar{e}e}^{(1)}(sx)\right]^* = \frac{\alpha}{\pi}\left\{\left[\frac{1+y^2}{1-y}\right]_+ \ln y + 2(1+y^2)\left[\frac{\ln(1-y)}{1-y}\right]_+ + \right.$$
$$\left. +\delta(1-y)\left(2\zeta_2 - \frac{1}{2}\right)\right\}$$

.

In the approachof *Berends(1987)*, *Blumlein(2011)* this logarithm is integrated. In our calculations $\ln z$ is nt integrated in convolutions (the variable of integration is $y$):

$$\bar{\delta}_{\bar{e}e}^{(1)}(sx) = \frac{\alpha}{\pi}\left\{\left[\frac{1+y^2}{1-y}\right]_+ (\ln z - \ln y) + 2(1+y^2)\left[\frac{\ln(1-y)}{1-y}\right]_+ + \right.$$
$$\left. +\delta(1-y)\left(2\zeta_2 - \frac{1}{2}\right)\right\}$$