On holographic β-function for light quarks

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Based on a joint work with I.Ya. Arefeva, A. Hajilou, P. Slepov

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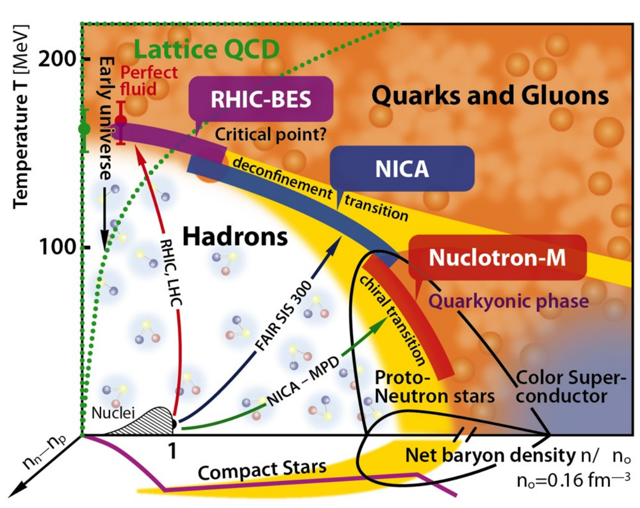

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AdS/CFT-correspondence for AdS$_5$

Gravity in 5-dimensional $AdS_5 \Leftrightarrow$ 4d conformal field theory

The 5-dimensional Einstein-Maxwell-scalar action

$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\left[R - \frac{\mathfrak{f}_0(\varphi)}{4}F^2 - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \mathcal{V}(\varphi)\right],$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Anzatz for the metric:

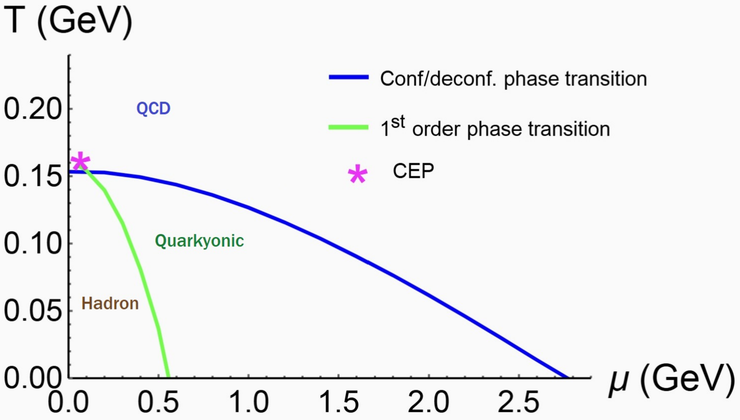
$$\begin{aligned} ds^2 &= B^2(z)\left[-g(z)dt^2 + d\vec{x}^2 + \frac{dz^2}{g(z)}\right], \\ B(z) &= \frac{e^{A(z)}}{z}, \quad \varphi = \varphi(z), \quad A_\mu = \left(A_t(z)\ , \vec{0}, 0\right) \\ A_{LQ}(z) &= -a\log\left(bz^2 + 1\right) \\ \mathfrak{f}_{0,LQ}(z) &= e^{-c\,z^2 - A(z)} \end{aligned}$$

fitting with experimental data:

$$a = 4.046, b = 0.01613 GeV^2, c = 0.227 GeV^2$$

(Li et.al.'17; Aref'eva et. al.'21, '22)

Phase structure of the model



Holographic isotropic model

EOM

$$\varphi'' + \left(\frac{g'}{g} + 3A' - \frac{3}{z}\right)\varphi' + \left(\frac{z^2 e^{-2A} A_t'^2 f_{0,\varphi}}{2g} - \frac{e^{2A} V_\varphi}{z^2 g}\right) = 0,$$

$$A_t'' + \left(\frac{f_0'}{f_0} + A' - \frac{1}{z}\right) A_t' = 0,$$

$$A'' - A'^2 + \frac{2}{z}A' + \frac{\varphi'^2}{6} = 0,$$

$$g'' + \left(3A' - \frac{3}{z}\right) g' - e^{-2A} z^2 f_0 A_t'^2 = 0,$$

$$A'' + 3A'^2 + \left(\frac{3g'}{2g} - \frac{6}{z}\right) A' - \frac{1}{z}\left(\frac{3g'}{2g} - \frac{4}{z}\right) + \frac{g''}{6g} + \frac{e^{2A} V}{3z^2 g} = 0$$

Applied boundary conditions: $A_t(0) = \mu, \; A_t(z_h) = 0, \quad g(0) = 1, \; g(z_h) = 0,$

$$\boxed{\varphi(z)\Big|_{z=z_0} = 0, \quad \text{where} \quad z_0 = \mathfrak{z}(z_h)} \Longleftarrow \mathfrak{z}_{LQ}(z_h) = 10\exp[-z_h/4] + 0.1$$

(I. Aref'eva, K.Rannu, P.Slepov'21)

General EOM solutions

$$\varphi'(z) = \sqrt{-6\left(A'' - A'^2 + \frac{2}{z}A'\right)}, \quad A_t(z) = \mu \frac{e^{cz^2} - e^{cz_h^2}}{1 - e^{cz_h^2}}$$

$$V(z) = -3z^2 g e^{-2A}\left[A'' + 3A'^2 + \left(\frac{3g'}{2g} - \frac{6}{z}\right)A' - \frac{1}{z}\left(\frac{3g'}{2g} - \frac{4}{z}\right) + \frac{g''}{6g}\right]$$

Holographic RG flows

Domain wall solution:

$$ds^2 = e^{2A(w)}\eta_{ij}dx^i dx^j + dw^2, \quad \varphi = \varphi(w)$$

E. Akhmedov'98; de Boer, Verlinde, Verlinde'98, Skenderis et. al.'98; Skenderis'99

Solution under consideration:

$$ds^2 = B(z)^2\left(\eta_{ij}dx^i dx^j + dz^2\right), \quad B(z) = \frac{e^{A(z)}}{z}, \quad \varphi = \varphi(z)$$

The holographic dictionary

- $\boxed{\alpha = e^{\varphi(z)}}$ is identified as running coupling of the field theory
- the energy scale E of the dual field theory (Galow et. al. '09)

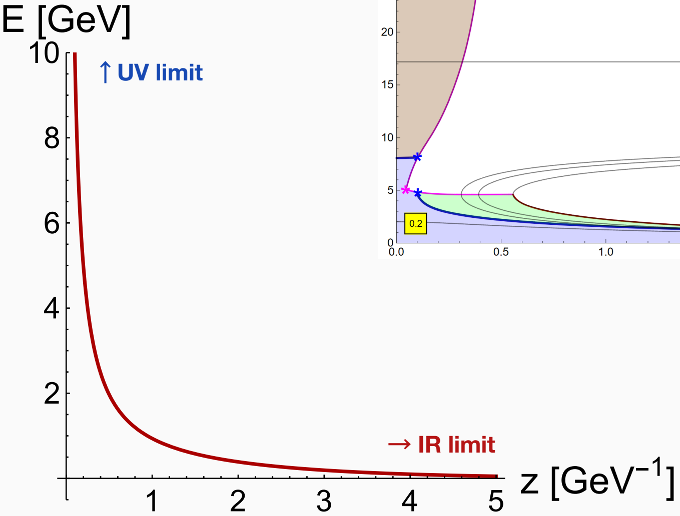
$$\boxed{\mathrm{E} = \mathrm{B(z)}} \qquad E_{LQ} = \frac{1}{z\,(1+bz^2)^a}$$

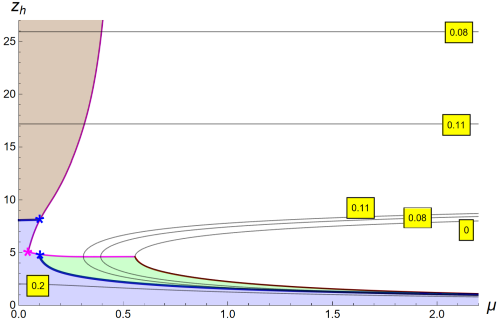
- β-function (DeWolfe et. al. '14, Kiritsis et.al.'14)

$$\beta = \frac{d\alpha}{d\log E}\bigg|_{QFT} = \alpha\frac{d\varphi}{d\log B}\bigg|_{Holo}$$

I.Ya. Aref'eva, A.Hajilou, P.S. Slepov, MU, TMF (2024), PRD (2025)

Energy scale in holographic prescription





Change of variables

- The overstretched β-function analogue: $X = \frac{\dot{\varphi}}{3}\frac{B}{\dot{B}}, \quad \beta = 3\,\alpha\,X$
- The temperature $T(z_h) = \frac{|g'|}{4\pi}|_{z=z_h}$ analogue: $Y = \frac{1}{4}\frac{\dot{g}}{g}\frac{B}{\dot{B}}$
- The chemical potential μ analogue: $H = \frac{\dot{A}_t}{B^2}$

Thus, holographic RG flows equations:

$$\begin{aligned}
\frac{d\mathcal{X}}{d\varphi} &= -\frac{4}{3}\left(1 - \frac{3}{8}\mathcal{X}^2 + \mathcal{Y}\right)\left(1 + \frac{1}{\mathcal{X}}\,\frac{2\partial_\varphi\mathcal{V}(\varphi) - \mathcal{H}^2\partial_\varphi\mathfrak{f}_0}{2\mathcal{V}(\varphi) + \mathcal{H}^2\mathfrak{f}_0}\right),\\
\frac{d\mathcal{Y}}{d\varphi} &= -\frac{4\mathcal{Y}}{3\mathcal{X}}\left(1 - \frac{3}{8}\mathcal{X}^2 + \mathcal{Y}\right)\left(1 + \frac{3}{2\mathcal{Y}}\,\frac{\mathcal{H}^2\mathfrak{f}_0}{2\mathcal{V}(\varphi) + \mathcal{H}^2\mathfrak{f}_0}\right),\\
\frac{d\mathcal{H}}{d\varphi} &= -\left(\frac{1}{\mathcal{X}} + \frac{\partial_\varphi\mathfrak{f}_0}{\mathfrak{f}_0}\right)\mathcal{H},
\end{aligned}$$

where $\quad X = \mathcal{X}(\varphi(z)), \quad Y = \mathcal{Y}(\varphi(z)), \quad H = \mathcal{H}(\varphi(z))$

Scalar potential and gauge kinetic function

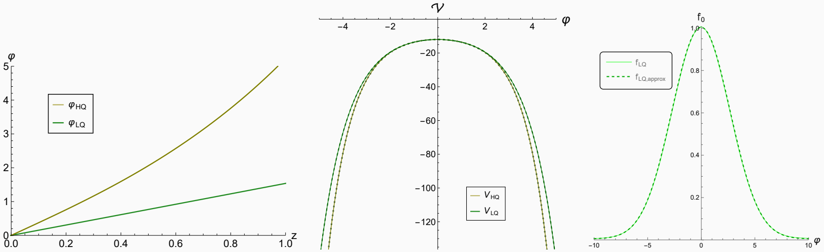
After subtraction of the scale factor into EOM:

$$\varphi_{LQ}(z,\varphi_0) = \varphi_0 + 2\sqrt{3a}\left[\sqrt{2a+1}\,\text{arcsinh}\left(\sqrt{\frac{b(2a+1)}{3}}\,z\right) - \sqrt{2(a-1)}\;\text{arctanh}\left(\sqrt{\frac{2b(a-1)}{(2a+1)bz^2+3}}\,z\right)\right]$$

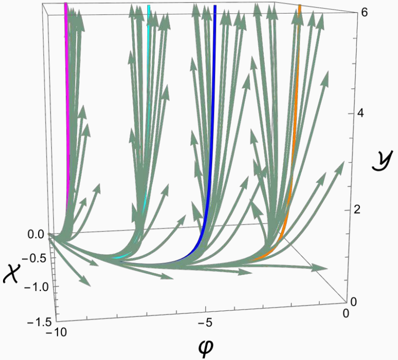
$$V_{LQ}(z) = -6\left(bz^2+1\right)^{2a-2}\left[bz^2\left((a(6a+7)+2)bz^2+5a+4\right)+2\right]$$

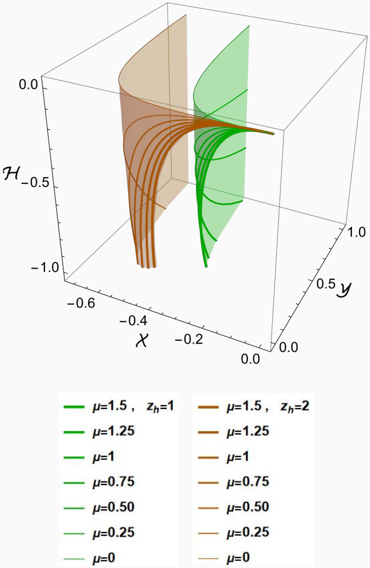
$$f_{LQ}(z) = e^{-cz^2+a\log\left(bz^2+1\right)}$$

Reconstruction: $\boxed{\mathrm{V}(\varphi) = \sum_{i=0}^{18} c_i\varphi^i}$ $\boxed{\mathrm{f}_0(\varphi) = \frac{1}{0.39931\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\varphi-\varphi_0}{2.70895}\right)^2\right]}$



hRG flows respected T and μ

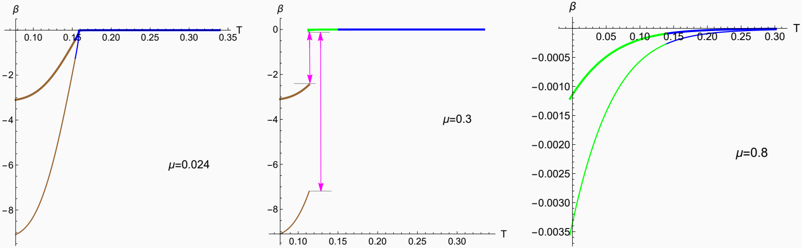


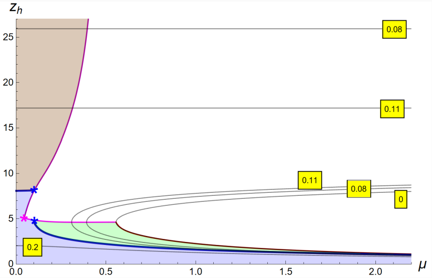


- differently colored brunches of solutions correspond to different z_h
- The scale factor A (therefore, the energy scale) decreases along the flow

β-function respecting the phase structure

$$\text{b.c.: } z_0 = \mathfrak{z}_{LQ}(z_h) = 10\, e^{(-\frac{z_h}{4})} + 0.1$$





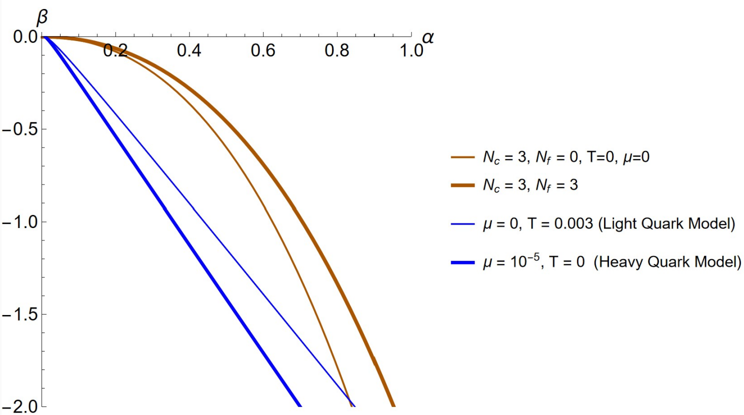
Agreement of the holographic β-function with 2-loop QCD

S. He et. al.'11; T. van Ritbergen et.al.'97

The QCD β-function at 2-loop level has the following form

$$\beta(\alpha) = -b_0\,\alpha^2 - b_1\,\alpha^3$$

where $b_0 = \frac{1}{2\pi}\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right)$ and $b_1 = \frac{1}{8\pi^2}\left(\frac{34}{3}N_c^2 - \left(\frac{13}{3}N_c - \frac{1}{N_f}\right)\right)$.



Conclusion

Results

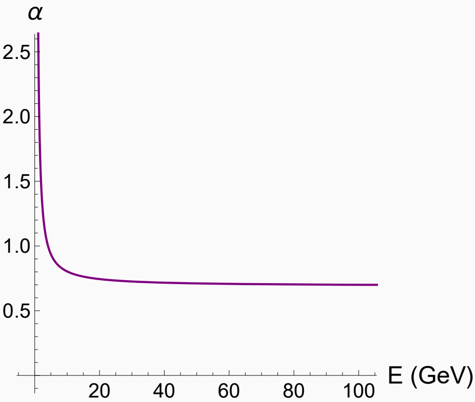
- in all regions β is negative
- on the 1-st order phase transition line, β has a jump
  - magnitude of the jump is zero at the CEP
  - this magnitude increases by decreasing the probe energy scale E
- both for hadronic and QGP regimes at fixed μ and energy scale z, β increases with increasing T
- The choice of different boundary conditions simply shifts the RG, without leading to significant changes

Prospective questions

- Exact numerical correspondence of β_{holo} with β_{QCD}

Thank you for your attention!

I.Ya. Aref'eva, A. Hajilou, P. Slepov, MU, Phys.Rev.D 110 (2024) 12, 126009



ultra-UV regime cannot be reached

BUT strongly coupled and
a small part of weakly coupled regime are covered