On cubic interaction vertices for fermionic and bosonic higher-spin fields from string field theory

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- I.L. Buchbinder, A.R, General Cubic Interacting Vertex for Massless Integer HS Fields, PLB (2021), [arXiv:2105.12030],
- I.L. Buchbinder, A.R, Covariant Cubic Interacting Vertices for Massless and Massive Integer HS Fields, Symmetry (2023), Correction (2025) [arXiv:2212.07097] ,
- A.R., BRST–BV approach for interacting HS fields, TMPh (2023) [arXiv:2303.02870],
- I.L. Buchbinder, A.R, Consistent Lagrangians for irreducible interacting HS fields with holonomic constraints [arXiv:2304.10358] PEPAN (2023)
- A.R, Covariant Cubic Interacting Vertices for fermionic and bosonic HS Fields,(2025) & in progress

"22-nd LOMONOSOV CONFERENCE ON ELEMENTARY PARTICLE PHYSICS- 2025", Moscow State University, 25.08.2025

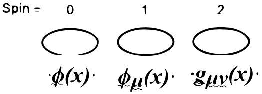
Wigner-Bargmann (1939, 1948) classification (1939, 1948, $d = 4$) of UIRs $ISO(1, d-1)$ ($d > 4$ L. Brink, A. Khan, P. Ramond 2002, A.Isaev, 2024) is characterized by $[(d+1)/2]$ Casimirs

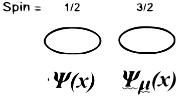
1. $P^2 = m^2, W^2 = -m^2 s(s+1)$ - massive Unitary irrep (UIR) with (half)integer spin;

2a. $P^2 = 0, W^2 = 0, W^\mu = \lambda P^\mu$ - massless with (half)integer helicity UIR;

2b. $P^2 = 0, W^2 = \mu^2$ - massless continuous (fermionic or bosonic) spin UIR;

Lower Spin refers to consistent classical field theories ($s \leqslant 2$)

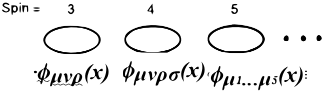




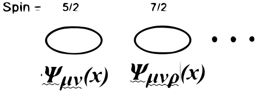
Higgs; (dark) photon, W, Z-bosons, gluons; graviton

leptons,quarks; gravitino (SYM, SUGRA)

Higher Spin (HS) stands for problematic construction ($s > 2$)



Fronsdal '78



Fang-Fronsdal '79

Cubic vertices in SM for lower spins ($m = (\neq)0$): (1,1,1), (0,0,1), $(\frac{1}{2}, \frac{1}{2}, 0)$, $(\frac{1}{2}, \frac{1}{2}, 1)$

$$S_{\mathsf{SM}} = \int d^4x\ \mathcal{L}_{\mathsf{SM}}\ , \quad \mathcal{L}_{\mathsf{SM}} = \mathcal{L}_{\text{gauge fields}} + \mathcal{L}_{\text{leptons}} + \mathcal{L}_{\text{quarks}} + \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{\text{Higgs}}\ , \quad (1)$$

$$\mathcal{L}_{\text{leptons}} = \sum_{k=1}^{3} \left[\bar{l}^k_L i\gamma^\mu \left(\partial_\mu - i\frac{g}{2} A^{\hat{a}}_\mu \tau_{\hat{a}} + i\frac{g'}{2} A_\mu \right) l^k_L + \bar{l}^k_R i\gamma^\mu \left(\partial_\mu + ig' A_\mu \right) l^k_R \right]\ ,$$

$$\mathcal{L}_{\text{quarks}} = \sum_{k=1}^{3} \left\{ \left[\begin{array}{c} \bar{u}_k \\ \bar{d}'_k \end{array} \right]_L i\gamma^\mu \left[\partial_\mu - i\frac{g_s}{2} A^{\underline{\alpha}}_\mu \lambda_{\underline{\alpha}} - i\frac{g}{2} A^{\hat{a}}_\mu \tau_{\hat{a}} - i\frac{g'}{6} A_\mu \right] \left[\begin{array}{c} u_k \\ d'_k \end{array} \right]_L \right.$$
$$\left. + \bar{u}^k_R i\gamma^\mu \left[\partial_\mu - i\frac{g_s}{2} A^{\underline{\alpha}}_\mu \lambda_{\underline{\alpha}} - i\frac{2g'}{3} A_\mu \right] u^k_R + \bar{d}'^k_R i\gamma^\mu \left[\partial_\mu - i\frac{g_s}{2} A^{\underline{\alpha}}_\mu \lambda_{\underline{\alpha}} + i\frac{g'}{3} A_\mu \right] d'^k_R \right\}\ ,$$

$$d'^k = U^{kk'}_{\mathsf{CKM}} d^{k'}\ , \quad u^k = (u, c, t)\ , \quad d^k = (d, s, b)\ ,$$

The masses of particles are generated by the Yukawa interaction term

$$\mathcal{L}_{\text{Yukawa}} = -\frac{1}{\sqrt{2}} \sum_{k=1}^{3} \left\{ f^u_k \left[\begin{array}{c} \bar{u}^k \\ \bar{d}^k \end{array} \right]_L \varphi u^k_R + f^d_k \left[\begin{array}{c} \bar{u}^k \\ \bar{d}^k \end{array} \right]_L \varphi d^k_R + f^l_k \bar{l}^k_L \varphi l^k_R + \text{h.c.} \right\}\ ,$$

$$\mathcal{L}_{\text{Higgs}} = \frac{1}{2} \left| \left(i\partial_\mu + (g/2)\, A^{\hat{a}}_\mu \tau_{\hat{a}} + (g'/2)\, A_\mu \right) \right| \varphi^2 - \frac{\mu^2}{2} |\varphi|^2 - \frac{\lambda}{4} |\varphi|^4\ ,$$

Cubic interacting vertex

Known results on cubic vertices

- metric formalism ($g^{\mu\nu}(x)$, ...) F. Berends, J. Van Reisen, NPB164 (1980), Berends, G. Burgers, H Van Dam, Nucl. Phys. B271 (1986); A. K. H. Bengtsson, I. Bengtsson, L. Brink, NPB (1983), E.S. Fradkin, M.A. Vasiliev, NPB 291 (1987), R. Manvelyan, K. Mkrtchyan, W. Ruhl, PLB 696 (2011), [arXiv:1009.1054 [hep-th]], E. Joung, M. Taronna, NPB 861 (2012) 145, arXiv:1110.5918[hep-th], I. Buchbinder, V. Krykhtin, M. Tsulaia, D. Weissman, Cubic Vertices for $\mathcal{N}=1$, NPB 967 (2021), arXiv:2103.08231; NPB 859 (2012) arXiv:0712.3526[hep-th];
- within (with algebraic constraints, cov.) BRST approach with incomplete BRST operator (or constrained BRST approach) for integer spins -R.R. Metsaev, (2013);
- in BRST approach with (in)complete BRST operator for irreps $ISO(1,d-1)$ bosonic fields by I.Buchbinder, A.R. (2021-2025) ;
- in frame-like (e^a_μ,ω^{ab}_μ,...) approach M. Vasiliev, Cubic Vertices for Symmetric higher spin Gauge Fields in (A)dS$_d$, NPB 862 (2012) 341 , arXiv:1108.5921[hep-th] arXiv:2208.02004, M. Khabarov, Yu. Zinoviev. JHEP 02 (2021);
- C.V. in $N=2$ harmonic SUSY in matter hypermultiplet interacting with $N=1$ gauge superfields (Buchbinder, E.Ivanov, N.Zaigraev 2022-24)
- ***Covariant Cubic vertex for irrep half-integer HS fields not found*** *(in BRST approach with (in)complete $Q_{(c)}$)*, e.g. to find new approach to DM problem due to SFT spectrum properties

Contents

- Interaction vertices in the gauge theories: deformation procedure
- BRST approach with incomplete BRST operator Q_c for irreducible free integer and half-integer higher spins on $R^{1,d-1}$;
- Deformation procedure with Q_c for interacting higher-spin fields;
- General solution of BRST equations for cubic vertices for constrained of helicities $(n_1 + 1/2, n_2 + 1/2, \lambda_3)$ HS fields
  1. BRST-closed linear on oscillators operator $L^{(i)}$;
  2. BRST-closed cubic on oscillators operators $Z \equiv Z_{111}$;
  3. BRST-closed zeroth and first orders operators $K^{(1,2)}, F^{(3)}$;
- Cubic vertex for massless HS fields with helicities $(n_1 + 1/2, n_2 + 1/2, s)$;
- On Cubic vertex for (ir)reducible fields within BRST with incomplete Q_c in the first order formalism

Noether's procedure (G.Barnich, M.Henneaux 1998, A.R. $L > 1$ 2021):
Gauge theory of 0th -stage reducibility in de Witt condensed notations

$$S_0[A] - \texttt{classical action of fields } A^i, i = 1, ..., n,\ \varepsilon(A^i) = \varepsilon_i = (0, 1),$$
$$\delta_0 S_0 = 0,\ \delta_0 A^i = R_0{}^i_\alpha \xi^\alpha,\ \alpha = 1, ..., m, \Longrightarrow \overleftarrow{\partial}_i S_0 R_0{}^i_\alpha = 0,\ \delta_0^{(0)} \xi^\alpha = 0$$

Deformation of k, $[k = N^2 - 1$ in $SU(N)]$ copies of LF for fields $A^{i(p)}$, $p = 1, .., k$ with quadratic $\sum_p S_0^{(p)}[A^{(p)}]$ of free fields $A^{i(p)}$ (maybe $[i(p_1)] \neq [i(p_2)]$) with rank condition

$$\boxed{N = \sum_p (n^p - m^p) \quad \texttt{where} \quad \mathrm{rank} \| \overleftarrow{\partial}_i \overrightarrow{\partial}_j S_0^{(p)} \|_{\overleftarrow{\partial}_i S_0 = 0} = n^p - m^p}$$

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$$\boxed{N=\sum_p (n^p - m^p) \quad \texttt{where} \quad \mathrm{rank}\|\overleftarrow{\partial}_i \overrightarrow{\partial}_j S_0^{(p)}\|_{\overleftarrow{\partial}_i S_0 = 0} = n^p - m^p}$$

$$S_{int} = \sum_{p=1}^k S_0^{(p)}[A^{(p)}] + g^1 S_1 + g^2 S_2 + \dots g^r S_r,\ \boxed{deg_A S_r = r+2},$$

$$\delta_{[l]} A^{i(p)} = \delta_0 A^{i(p)} + g\delta_1 A^{i(p)} + \dots + g^l \delta_1 A^{i(p)} = R_{[l]}{}^{i(p)}_{\alpha(t)} \xi^{\alpha(t)},\ \boxed{deg_A R_l{}^{i(p)}_{\alpha(t)} = l},$$

$$\text{initial condition } R_0{}^{i(p)}_{\alpha(t)} \equiv R_0{}^i_\alpha \delta^p_t.$$

Interaction vertices in the irreducible gauge theories: deformation procedure

Noether's identities as system in powers of g from $\delta_\Sigma S_{int} = 0$: $\boxed{\delta_\Sigma \equiv \sum_{l=0}^{\infty} \delta_l}$

$$g^1: \qquad \delta_0 S_1 + \delta_1 \overline{S}_0 = 0, \tag{2}$$

$$g^2: \qquad \delta_0 S_2 + \delta_1 S_1 + \delta_2 \overline{S}_0 = 0, \quad \overline{S}_0 \equiv \sum_{p=1}^{k} S_0^{(p)}$$

for the cubic vertex for irreducible GTh

$$S_{int} = \sum_{p=1}^{3} S_0^{(p)}[A^{(p)}] + g^1 S_1, \ \boxed{deg_A S_1 = 3},$$

$$\delta_{[1]} A^{i(p)} = \delta_0 A^{i(p)} + g\delta_1 A^{i(p)} = R_{[1]}{}^{i(p)}_{\alpha(t)} \xi^{\alpha(t)}, \tag{3}$$

Interaction vertices in the irreducible gauge theories: deformation procedure

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$$g^1: \qquad \delta_0 S_1 + \delta_1 \overline{S}_0 = 0, \tag{2}$$

$$g^2: \qquad \delta_0 S_2 + \delta_1 S_1 + \delta_2 \overline{S}_0 = 0, \quad \overline{S}_0 \equiv \sum_{p=1}^{k} S_0^{(p)}$$

for the cubic vertex for irreducible GTh

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$$\delta_{[1]} A^{i(p)} = \delta_0 A^{i(p)} + g\delta_1 A^{i(p)} = R_{[1]}{}^{i(p)}_{\alpha(t)} \xi^{\alpha(t)}, \tag{3}$$

The equations (2) pass to

$$g^1: \qquad \delta_0 S_1 + \delta_1 \textstyle\sum_{p=1}^{3} S_0^{(p)} = o(g), \quad \texttt{N-A Gauge algebra} \ [\delta_{[1]}, \delta_{[1]}] \sim \delta_{[1]} \tag{4}$$

In 4d for spins $(0, s_1), (0, s_2), (0, s_3)$... **in light-cone formalism**, *first classification* :
A. Bengtsson, I. Bengtsson, L. Brink, NPB (1983), A. Bengtsson, I. Bengtsson, N. Linden (1987)

BRST approach with incomplete BRST operator Q_c

due to tensionless limit ($d = 26, 10$ **SFT, M.Green, E.Witten, C. Thorn, 1989; W. Taylor, B. Zwiebach,hep-th/0311017): QG problem [**$r < 10^{-15}m$**] (G.Bonelli (2003), A. Sagnotti, M. Tsulaia, (2004)) now** ($Q_c^2 = 0\ \forall d$).

$\Longrightarrow \mathcal{Q} \stackrel{\alpha' \to \infty}{\longrightarrow} Q_c$: {∞}many HS fields $\phi_\mu(x), .., \phi_{\mu(s)}(x)$ in string spectra

$S_{str} = -(2\pi\alpha')^{-1} \int d^2\sigma \mathcal{L}(X(\sigma))$ In BRST approach with incomplete Q_c (S. Ouvry, J. Stern, A. Bengtsson, 1987, G. Barnich, M.Grigoriev, A.Semikhatov 2004, A.R. 2018) instead of direct problem for generalized canonical quantization of Constrained Dyn.S. by the aim inverse problem - is an construction of GI LF for HS fields with (m, s)

irrep conditions ISO(1,d-1), (SO(2,d-1))	$\xrightarrow{\text{SFT}}$	(super)algebra$\{o_I(x)\} = \{\partial^2 + m^2; a^+_{i\mu}\partial^\mu, a_{i\mu}\partial^\mu; \underline{o_a}, o^+_a\}$ $\{o_I(x)\} : \mathcal{H}, \quad [o_I, o_J\} = f^K_{IJ}(o)o_k$

$\xrightarrow[\text{I.B.,E.Fradkin, G.V., M.Henneaux}]{\text{BFV}}$ BRST operator $\{o_I\}$:$Q_c(x)$

$$Q_c = C^A o_A + \frac{1}{2} C^A C^B F^D_{AB} \mathcal{P}_D (-1)^{\varepsilon(o_A)+\varepsilon(o_D)}$$

$\xrightarrow{\text{LF}}$

$$Q_c^2 = 0, \quad [Q_c, \sigma_c\} = [Q_c, \mathcal{O}_a\} = 0, \quad [\mathcal{O}_a, \sigma_c\} \sim \mathcal{O}_a$$

EoM: $Q_c|\chi_c\rangle = 0, \mathrm{gh}(|\chi_c\rangle) = 0 \Rightarrow$ action: $S_c = \int d\eta_0 \langle\chi_c|Q_c|\chi_c\rangle \sim \phi_{\mu(s)}(\partial^2 + ...)\phi^{\mu(s)}$

spin: $(g_0 + \text{more})(|\chi_c\rangle, |\Lambda_c\rangle, ...) = (s - d/2 + ...)(|\chi\rangle, |\Lambda\rangle, ...)$

gauge symmetry: $\delta|\chi_c\rangle = Q_c|\Lambda_c\rangle, \delta|\Lambda_c\rangle = 0, ...,$ constr: $\mathcal{O}_a(|\chi\rangle, |\Lambda\rangle, ...) = 0$

Q_c - for 1-st class constraints without holonomic ones with auxiliary fields on 2 stage

Talk devoted to (off-shell) covariant general Lagrangian (cubic: $g\psi_1^{A\mu(s1)}\psi_2^{A\nu(s2)}\phi_3^{\rho(s3)}$) vertices for irreducible HS fields on $R^{1,d-1}$. We developed a concept of deformation Noether's procedure of free GTh on a base of BRST-BFV $\equiv$ BRST (Becchi, Rouett, Stora, Tyutin, 1975) approach with incomplete BRST operator & holonomic constraints.

$$\texttt{particle}\ (m,s):\quad \left(\partial^2+m^2,\ \partial^{\mu_1},\ \underline{\eta^{\mu_1\mu_2}}\right)\phi_{\mu(s)} = (0,0,0) \quad \Longleftrightarrow$$

$$\left(l_0,\ l_1,\ \underline{l_{11}},\ g_0 - d/2\right)|\phi\rangle = (0,0,0,s)|\phi\rangle.$$

$\operatorname{diag}\eta^{\mu\nu} = (+,-,...,-)$, String-like vector $|\phi\rangle \in \mathcal{H}$, operators $l_0,\ l_1,\ l_{11}, g_0$ are:

$$|\phi\rangle = \sum_{s\geq 0}\frac{\imath^s}{s!}\phi^{\mu(s)}\prod_{i=1}^{s}a^+_{\mu_i}|0\rangle,\quad [a_\nu, a^+_\mu] = -\eta_{\mu\nu},$$

$$\left(l_0,\ l_1,\ l_{11}, g_0\right) = \left(\partial^\nu\partial_\nu + m^2,\ -\imath a^\nu\partial_\nu,\ \tfrac{1}{2}a^\mu a_\mu, -\tfrac{1}{2}\{a^+_\mu,\ a^\mu\}\right).$$

$$\overset{complete}{Q} = \eta_0 l_0 + \eta_1^+\check{l}_1 + \check{l}_1^+\eta_1 + \imath\eta_1^+\eta_1\mathcal{P}_0 + \eta_{11}^+\widehat{L}_{11} + \widehat{L}_{11}^+\eta_{11} \equiv \overset{incomplete}{Q_c} + Tr,$$

$$\mathcal{S}_{0|s}[|\chi_c\rangle] = \int d\eta_0\, {}_s\langle\chi_c|Q_c|\chi_c\rangle_s \sim \int d^dx\phi_{\mu(s)}\Big[(\partial^2+m^2)\phi^{\mu(s)} + \phi_{1\mu(s-1)} + ...\Big],\ \ \delta|\chi_c\rangle_s = Q_c|\Lambda_c\rangle_s$$

$$\mathcal{L}_{11}\left(|\chi_c\rangle, |\Lambda^0_c\rangle\right) = \left(l_{11} - 1/2d^2 + \eta_1\mathrm{P}_1\right)\left(|\chi_c\rangle, |\Lambda^0_c\rangle\right) = (0,0),\ \text{c}\ \ l_{11} = 1/2a^\mu a_\mu$$

$$\left(|\chi_c\rangle_s,\ |\Lambda^0_c\rangle_s\right) = \left(|\Phi\rangle_s - \mathcal{P}_1^+\{\eta_0|\Phi_1\rangle_{s-1} + \eta_1^+|\Phi_2\rangle_{s-2}\},\ \ \mathcal{P}_1^+|\Xi\rangle_{s_i-1}\right);$$

$$\{\eta_0, \mathcal{P}_0\} = \imath, \quad \{\eta_1, \mathcal{P}_1^+\} = \{\eta_1^+, \mathcal{P}_1\} = 1.$$

Note, off-shell holonomic constraints in BRST approach: (Barnich, Grigoriev, Semikhatov, Tipunin 2004

Generating equations for LF with Q_c for irreducible HS field:

$$\boxed{[Q_c, \mathcal{L}_{11}] = 0, \quad [Q_c, \sigma_c] = 0, \quad [\mathcal{L}_{11}, \sigma_c] = 2\mathcal{L}_{11}}$$

with $|\Phi_{...}\rangle_{...} \equiv |\Phi(a^+, d^+)_{...}\rangle_{...}$: . $|\Phi\rangle_s|_{(d^+=0)} = |\phi\rangle_s$

Resolution of traceless constraint and algebraic EoM $\Rightarrow$ LF with single vector with $s-1$ auxiliary fields

$$\mathcal{S}^m_{C|s}(\phi, ...) = {}_s\langle\Phi|\left(l_0 - \check{l}_1^+\check{l}_1 - (\check{l}_1^+)^2\check{l}_{11} - \check{l}_{11}^+\check{l}_1^2 - \check{l}_{11}^+(l_0 + \check{l}_1\check{l}_1^+)\check{l}_{11}\right)|\Phi\rangle_s,$$

$$\delta|\Phi\rangle_s = \check{l}_1^+|\Xi\rangle_{s-1} \quad \text{and} \quad \check{l}_{11}\left(\check{l}_{11}|\Phi\rangle, |\Xi\rangle\right) = (0, 0),$$

LF has smooth massless limit for $m = d^{(+)} = 0$ resulting to Fronsdal f. (1978) $(0, s)$

$$\mathcal{L}_{C|(s)}(\phi) = (-1)^s\phi^{(\nu)_s}\Big[\Big\{\partial^2\phi_{(\nu)_s} - s\partial_{\nu_s}\partial^\mu\phi_{(\nu)_{s-1}\mu} + s(s-1)\partial_{\nu_s}\partial_{\nu_{s-1}}\phi_{(\nu)_{s-2}\mu}{}^\mu\Big\} - \tfrac{1}{2}s(s-1)\eta_{\nu_s\nu_{s-1}}\Big\{\partial^2\phi_{(\nu)_{s-2}\mu}{}^\mu + \tfrac{1}{2}(s-2)\partial_{\nu_{s-2}}\partial^\rho\phi_{(\nu)_{s-3}\rho\mu}{}^\mu\Big\}\Big], \quad (5)$$

$$\delta\phi^{(\mu)_s} = -\sum^s \partial^{\mu_i}\epsilon^{\mu_1\cdots\mu_{i-1}\mu_{i+1}\cdots\mu_s}, \quad \eta_{\mu_1\mu_2}\epsilon^{(\mu)_{s-1}} = \phi^{(\mu)_{s-4}\mu\nu}{}_{\mu\nu} = 0 \quad (6)$$

In A. R, JHEP (2018) 1803.04678 such LF was derived

$$\texttt{particle}\ (0, n+1/2): \quad \big(\imath\gamma^\mu\partial_\mu,\ \underline{\gamma^{\mu_1}}\big)\psi_{A\mu(n)} = (0,0) \quad \Longleftrightarrow$$

$$\big(t_0,\ \underline{t_1}, g_0 - d/2\big)|\psi\rangle = (0,0,n)|\psi\rangle.$$

$$\{\tilde{\gamma}^\mu, \tilde{\gamma}^\nu\} = 2\eta^{\mu\nu}, \qquad \{\tilde{\gamma}^\mu, \tilde{\gamma}\} = 0, \qquad \tilde{\gamma}^2 = -1, \ \text{so that } \gamma^\mu = \tilde{\gamma}^\mu\tilde{\gamma},$$

$$|\psi\rangle = \sum_{n\geq 0} \frac{\imath^n}{n!}\phi^{\mu(n)} \prod_{i=1}^{n} a^+_{\mu_i}|0\rangle, \quad [a_\nu, a^+_\mu] = -\eta_{\mu\nu},$$

$$\big(t_0, l_0,\ l_1,\ t_1, g_0\big) = \big(\imath\tilde{\gamma}^\nu\partial_\nu,\ \partial_\nu\partial^\nu, -\imath a^\nu\partial_\nu,\ \tilde{\gamma}^\nu a_\nu,\ \tfrac{1}{2}a^\mu a_\mu, -\tfrac{1}{2}\{a^+_\mu,\ a^\mu\}\big).$$

$$\begin{aligned} Q_{F|c} &= q_0 t_0 + \eta_0 l_0 + \eta_1^+ l_1 + l_1^+\eta_1 + \imath\big(\eta_1^+\eta_1 - q_0^2\big)\mathcal{P}_0, \quad [q_0, p_0] = \imath \\ \{\mathcal{T}_1, \mathcal{L}_{11}\} &= \{t_1 - \imath\eta_1 p_0 - 2q_0\mathcal{P}_1,\ l_{11} + \eta_1\mathcal{P}_1\}, \\ \widehat{\sigma}_c &= g_0 + \eta_1^+\mathcal{P}_1 - \eta_1\mathcal{P}_1^+ \end{aligned}$$

Generating equations for LF with $Q_{F|c}$ for irreducible HS field:

$$[Q_{F|c}, \mathcal{T}_1] = 0, \quad [Q_{F|c}, \sigma_c] = 0, \quad [\mathcal{T}_1, \sigma_c] = \mathcal{T}_1$$

The second order EoM for field vector

$$Q_{F|c}|\chi_c^0\rangle_n = 0,\ \delta|\chi_c^0\rangle_n = Q_{F|c}|\Lambda_c\rangle_n, \qquad \mathcal{T}_1\Big(|\chi_c^0\rangle_n, |\Lambda_c\rangle_n\Big) = 0,$$

BRST approach with Q_c for half-integer HS field on $R^{1,d-1}$

The corresponding BRST-like second order constrained gauge-invariant action

$$\mathcal{S}^{(2)}_{C|n} = \int d\eta_{0n} \langle \tilde{\chi}^0_c | Q_{F|c} | \chi^0_c \rangle_n, \qquad |\chi^0_c\rangle_n = |\chi^0_{0|c}\rangle_n + q_0 |\chi^1_{0|c}\rangle_n + \eta_0 \tilde{\gamma} |\chi^0_{1|c}\rangle_n,$$

$$|\chi^0_{0|c}\rangle_n = |\Psi\rangle_n + \eta_1^+ \mathcal{P}_1^+ |\chi\rangle_{n-2}, \quad |\chi^1_{0|c}\rangle_n = \mathcal{P}_1^+ \tilde{\gamma} |\chi_1\rangle_{n-1} =,$$

$$|\Lambda_{0|c}\rangle_n = \mathcal{P}_1^+ |\xi\rangle_{n-1},$$

Note, there exists triplet first order LF, which equivalent to Fang-Fronsdal LF, 1978

$$\begin{aligned}
\mathcal{L}_{C|(n)}(\Psi) \;=\;& (-1)^n \overline{\Psi}^{(\nu)_n} \Big\{ -\imath\gamma^\mu \partial_\mu \Psi_{(\nu)_n} + \tfrac{1}{4} n(n-1) \eta_{\nu_{n-1}\nu_n} (\imath\gamma^\mu \partial_\mu) \Psi_{(\nu)_{n-2}\mu}{}^{\mu} \\
-\;& n\Big(\gamma_{\nu_n} (\imath\gamma^\mu \partial_\mu) \gamma^{\mu_n} - (\imath\partial_{\nu_n}) \gamma^{\mu_n} - (\imath\partial^{\mu_n}) \gamma_{\nu_n} \Big) \Psi_{(\nu)_{n-1}\mu_n} \\
-\;& \tfrac{1}{2} n(n-1) \Big(\gamma_{\nu_{n-1}} (\imath\partial_{\nu_n}) \eta^{\mu_{n-1}\mu_n} + \eta_{\nu_{n-1}\nu_n} \gamma^{\mu_{n-1}} (\imath\partial^{\mu_n}) \Big) \Psi_{(\nu)_{n-2}\mu_{n-1}\mu_n} \Big\},
\end{aligned} \tag{7}$$

$$\delta\Psi^{(\mu)_n} \;=\; -\sum_{i=1}^{n} \partial^{\mu_i} \xi^{\mu_1 \cdots \mu_{i-1}\mu_{i+1} \cdots \mu_n}, \quad \gamma_{\mu_1} \xi^{(\mu)_{n-1}} = \gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} \Psi^{(\mu)_n} = 0 \tag{8}$$

An equivalence of the LFs with incomplete & complete BRST operators for any irrep with (half)integer spin on $\mathbb{R}^{1,d-1}$ is (cohomologically) established in A. R, JHEP (2018) 1803.04678, but for interacting theory of the same HS fields it has not yet been solved.

Deformation procedure with Q_c for interacting higher-spin fields

Aim is to find covariant form of cubic vertex for massless fermionic $(n_1+1/2), (n_2+1/2)$ and 1 bosonic s_3 fields. $\big|V^{(3)}\rangle^{(0)_3}_{(s)_3} \in \mathcal{H}^{(1)} \otimes \mathcal{H}^{(2)} \otimes \mathcal{H}^{(3)}$ found in light-cone [R. Metsaev, 2007] with preserving the irreducibility for the fields on the interacting level for each copy of interacting HS fields. ($i = 1,2,3$ enumerating the copy of fields, for massless $(m)_3 = (0,0,0)$ and spins $(s)_3 = (n_1+1/2, n_2+1/2, s_3)$)

Cubic vertex for HS fields $(n_1+1/2, n_2+1/2, s_3)$ within BRST approach with incomplete Q_c includes 2 copies of vectors for fermionic $|\chi^{(i)}\rangle_{n_i}$, $|\Lambda^{(i)}\rangle_{n_i}$, and 1 bosonic $|\chi^{(3)}_B\rangle_{s_3}$, $|\Lambda^{(3)}_B\rangle_{s_3}$ with $|0\rangle^i, |0\rangle^3$ and oscillators $a^{(i)\mu+}$..., $i = 1,2,\ldots$.

Deformed (in second order formalism) action and gauge transformations $(n_3 \equiv s_3)$

$$S_{[1]|C|(s)_3}[\chi^{(1)}, \chi^{(2)}, \chi^{(3)}_B] = \sum_{i=1}^{2} \mathcal{S}^{(2)}_{0|C|n_i} + \mathcal{S}_{0|C|s_3} + g\int \prod_{e=1}^{3} d\eta_0^{(e)} \Big({}_{n_e}\langle\chi^{(e)}\big|V^{(3)}\rangle_{(s)_3} + h.c.\Big),$$

$$\delta_{[1]}\big|\chi^{(i)}\rangle_{n_i} = Q^{(i)}_{F|c}\big|\Lambda^{(i)}\rangle_{n_i} - g\int \prod_{e=1}^{2} d\eta_0^{(i+e)} \Big({}_{n_{i+1}}\langle\Lambda^{(i+1)}\big|{}_{n_{i+2}}\langle\chi^{(i+2)}\big|$$
$$+(i+1 \leftrightarrow i+2)\Big)\big|\widetilde{V}^{(3)}\rangle_{(s)_3},$$

$$\delta_{[1]}\big|\chi^{(3)}_B\rangle_{s_3} = Q^{(3)}_c\big|\Lambda^{(3)1}\rangle_{s_3} - g\int \prod_{e=1}^{2} d\eta_0^{(e)} \Big({}_{n_1}\langle\Lambda^{(1)}\big|{}_{n_2}\langle\chi^{(2)}\big|$$
$$+(1 \leftrightarrow 2)\Big)\big|\widehat{V}^{(3)}\rangle_{(s)_3}$$

obeying x-locality

$$\big|V^{(3)}\rangle_{(s)_3} = \prod_{i=2}^{3}\delta^{(d)}\big(x_1 - x_i\big)\mathbf{V}^{(\mathbf{3})}_{(\mathbf{s})_\mathbf{3}}(\mathbf{a}^{(\mathbf{i})+}, \eta^{(\mathbf{i})+}, \mathcal{P}^{(\mathbf{i})+})\prod_{j=1}^{3}\eta_0^{(j)}|0\rangle, \quad |0\rangle \equiv \otimes_{e=1}^{3}|0\rangle^{e},$$

Proposition 1 (generating equations for cubic vertices)

The Noether identities for the cubic deformation $(\Psi^{(1)}\Psi^{(2)}\Phi^{(3)})$ of LF for the particles (m_i, s_i), $i = 1, 2, 3$

$$g^{1}: \qquad \delta_0 S_{1|(s)_3} + \delta_1 \sum_{i=1}^{3}\mathcal{S}_{0|s_i} = 0,$$

transforms to the local system of equations, which for coinciding $\widetilde{V}^{(3)} = \widehat{V}^{(3)} = V^{(3)}$ has universal form

$$\boxed{\Big(Q_c^{tot},\ \mathcal{T}_1^{(a)},\ \mathcal{L}_{11}^{(3)},\ \sigma_c^{(j)} - \tfrac{d-2}{2}\Big)\big|V^{(3)}\rangle_{(s)_3} = (\vec{0}, n_j).}, \qquad Q_c^{tot} = \sum_{k=1}^{2} Q^{(k)}_{F|c} + Q_c^{(3)}\cdot 1_{2[d/2]}$$

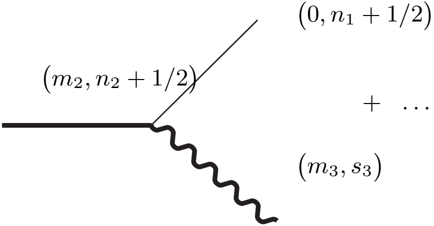
Thus, the vertex should be BRST-closed, traceless and gamma-traceless and composed from $a^{(i)+}_{\mu}, \eta^{(i)+}_1, \mathcal{P}^{(i)+}_1, \mathcal{P}^{(i)}_0$, $q^{(a)}_0, p^{(a)}_0$, $a = 1, 2$ of spin $(n_1 + 1/2, n_2 + 1/2, s_3)$

General solution of BRST equations for cubic vertices for HS fields of helicities $(n_1 + 1/2, n_2 + 1/2, s_3)$

We derive 2 types of the cubic vertices in the approach with Q_c, in following different cases:

- $(0, n_1 + 1/2)$ – $(0, n_2 + 1/2)$–$(0, \lambda_3)$ A.R, (2025) & in progress ;
- $(0, n_1 + 1/2)$ – $(0, n_2 + 1/2)$ –(m_3, s_3)
- $(0, n_1 + 1/2)$ – $(m_2, n_2 + 1/2)$ – $(0, \lambda_3)$
- $(0, n_1 + 1/2)$ – $(m_2, n_2 + 1/2)$ – (m_3, s_3)

$|V^{(3)}\rangle^{(0,m_2,m_3)}_{(n_1,n_2,s_3)} \equiv$



$+ \;\; \ldots\ldots$

;

and derive from light-cone (cubic) vertices for irreducible reps of $ISO(1, d-1)$ (Metsaev 2012) the covariant ones

General solution for cubic vertices massless HS fields of helicities $(n_1 + 1/2, n_2 + 1/2, s_3)$

seek Q_c^{tot}-BRST - closed solution,
$|V^{(3)}\rangle \sim Z^p \sum \prod_i^3 (L^{(i)})^{k_i}$ of specific homogeneous in oscillators (linear in ∂_μ) operators (1), Q^{tot}-BRST- closed monomials $\underline{L^{(i)}}$, $i = 1, 2, 3$ and 3-rd order $\underline{Z}$

$$L^{(i)} = (p_\mu^{(i+1)} - p_\mu^{(i+2)}) a^{(i)\mu +} - \imath \big(\mathcal{P}_0^{(i+1)} - \mathcal{P}_0^{(i+2)}\big) \eta_1^{(i)+}, \quad p_\mu^{(i)} = -i\partial_\mu^{(i)}$$

$deg_{(a^+,\eta^+)} L^{(i)} = 1$: $L^{(1)} \overline{\psi}^{(1)}_{\mu(n_1)} \psi^{(2)}_{\nu(n_2)} \phi^{(3)}_{\rho(s_3)} \mapsto \overline{\psi}^{(1)}_{\mu(n_1-1)\mu} \partial^\mu \psi^{(2)}_{\nu(n_2)} \phi^{(3)}_{\rho(s_3)}$,
(2)

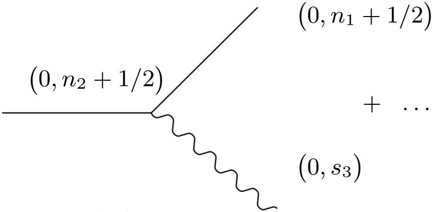
$$Z = L_{11}^{(12)+} L^{(3)} + L_{11}^{(23)+} L^{(1)} + L_{11}^{(31)+} L^{(2)}$$
$$L_{11}^{(ii+1)+} = a^{(i)\mu +} a_\mu^{(i+1)+} - \frac{1}{2} \mathcal{P}_1^{(i)+} \eta_1^{(i+1)+} - \frac{1}{2} \mathcal{P}_1^{(i+1)+} \eta_1^{(i)+}.$$

$deg_{(a^+,\eta^+)} Z = 3$: $Z \overline{\psi}^{(1)}_{\mu(n_1)} \psi^{(2)}_{\nu(n_2)} \phi^{(3)}_{\rho(s_3)} \mapsto \overline{\psi}^{(1)}_{\mu(n_1-1)\mu} \partial^\rho \psi^{(2)}_{\nu(n_2-1)}{}^{\mu} \phi^{(3)}_{\rho(s_3-1)\rho} + c.c.(1, 2, 3)$
(3)

$$\mathbf{K^{(12)}} = \tilde{\gamma}^\mu \widehat{p}^{(3)}_\mu - \imath q_0^{(1)} \big[\mathcal{P}_0^{(3)} - \mathcal{P}_0^{(2)} - 3\mathcal{P}_0^{(1)}\big] + \imath q_0^{(2)} \big[\mathcal{P}_0^{(3)} - \mathcal{P}_0^{(1)} - 3\mathcal{P}_0^{(2)}\big],$$
$$\mathcal{F} = \Big(L^{(3)} - \frac{1}{2}\big[\mathbf{K^{(12)}}, \mathcal{T}_1^{(3)}\big]_s\Big), \text{ where}$$
$$\mathcal{T}_1^{(3)} = \Big(1 - \imath\big[q_0^{(2)} p_0^{(1)} + q_0^{(2)} p_0^{(1)}\big]\Big) \tilde{\gamma}^\mu a_\mu^{+(3)} + \imath (p_0^{(1)} + p_0^{(2)}) \eta_1^{(3)+} - 2\mathcal{P}_1^{(3)+} (q_0^{(1)} + q_0^{(2)})$$

to get them we have used momenta conservation law, $\sum_{i=1}^{3} p_\mu^{(i)} = 0$

$$|V^{(3)}\rangle^{(0,0,0)}_{(n_1,n_2,s_3)} \equiv$$



$+ \;\ldots\ldots$

;

- $\mathbf{K}$-vertex in SM $(\frac{1}{2}, \frac{1}{2}, 0)$: (ψ, ψ, φ)

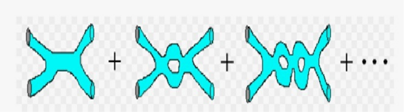
$$V^{M(3)}_{K|(s)_3} = \mathbf{K}^{(\mathbf{12})} \sum_k Z^{\frac{1}{2}(s-k+1)} \prod_{i=1}^{3} (L^{(i)})^{n_i - \frac{1}{2}(s-k+1)};$$

- $\mathbf{F}$-vertex in SM $(\frac{1}{2}, \frac{1}{2}, 1)$: $(\psi, \psi, \gamma^\mu A_\mu)$

$$V^{M(3)}_{F|(s)_3} = \mathbf{F} \sum_k Z^{\frac{1}{2}(s-k)} (L^{(3)})^{-1} \prod_{i=1}^{3} (L^{(i)})^{n_i - \frac{1}{2}(s-k)}$$

k-parametric family,e.g. for $\mathbf{K}$-vertex: $s - 2s_{\min} + 1 \leq k \leq s + 1$, $s = \sum s_i$

Conclusion



- BRST approachs with incomplete BRST operator for irreducible interacting HS fields (in second order approximation) is developed) for Lagrangian covariant cubic vertices with massless fermionic and bosonic irreducible fields in Minkowski spaces;
- What to do? - one should to reduce the interacting theory into first order (for free fermionic part) formalism, which for $d = 4$ may be effectively done in spinor formalism (I.Buchbinder, A.Isaev, V. Krykhtin, S. Fedoruk, 2019-2025);
- examples of cubic vertices for lower spins $(\frac{3}{2}, \frac{3}{2}, 1)$, ... and quantization procedure? Welcome for collaborations.

Thank you very much