String tension and phase transitions in anisotropic HQCD with magnetic field

K.A. Rannu

Peoples Friendship University of Russia (RUDN)
Steklov Mathematical Institute (MI RAS)

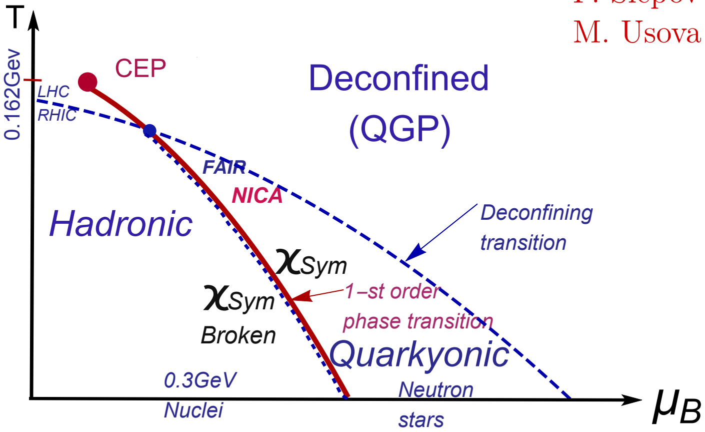
22nd Lomonosov Conference on Elementary Particle Physics

MSU
25.08.2025

with I.Ya. Aref'eva, P. Slepov
arXiv:2505.09580 [hep-th]

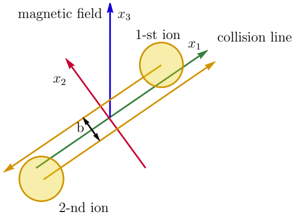
Holographic QCD phase diagram

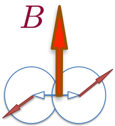
Talks by I.Ya. Aref'eva
P. Slepov
M. Usova

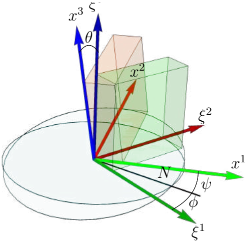


Heavy-Ion Collisions

- Origins of anisotropy
  - Primary anisotropy – longitudinal and 2 transversal directions
    - Multiplicity dependencies *ALICE* $\mathcal{M}(s) \sim s^{0.155(4)}$
    - *Aref'eva, Golubtsova, JHEP (2014)* $\mathcal{M}(s) \sim s^{1/(\nu+2)} \Rightarrow \nu \approx 4.5$
  - Secondary anisotropy (in strong magnetic field, eB $\sim 5 - 10\ m_\pi^2$
    m_π – pion mass)







Peripheral HIC

Holographic model of anisotropic plasma in magnetic field at nonzero chemical potential

I.Aref'eva, KR'18; IA, KR, P.Slepov'21

$$S = \int d^5x \sqrt{-g} \left[R - \frac{f_0(\phi)}{4} F_{(0)}^2 - \frac{f_1(\phi)}{4} F_{(1)}^2 - \frac{f_B(\phi)}{4} F_{(3)}^2 - \frac{1}{2} \partial_M \phi \partial^M \phi - V(\phi) \right]$$

$$ds^2 = \frac{L^2}{z^2} \mathfrak{b}(z) \left[- g(z)\, dt^2 + dx^2 + \left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dy_1^2 + e^{c_B z^2} \left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dy_2^2 + \frac{dz^2}{g(z)} \right]$$

$$A_{(1)\mu} = A_t(z) \delta_\mu^0 \qquad A_t(0) = \mu \qquad F_{(1)} = q_1\, dy^1 \wedge dy^2 \qquad F_{(B)} = q_3\, dx \wedge dy^1$$

$\mathfrak{b}(z) = e^{2\mathcal{A}(z)}$ $\Leftrightarrow$ quarks mass

"Bottom-up approach"

Heavy quarks (c, b)

$\mathcal{A}(z) = -\, cz^2/4$ — *Andreev, Zakharov'06*

$\mathcal{A}(z) = -\, cz^2/4 + (p - c_B\, q_3) z^4$ — *IA, Hajilou, Rannu, Slepov' 23*

Light quarks (u, d, s)

$\mathcal{A}(z) = -\, a \ln(bz^2 + 1)$ — *Li, Yang, Yuan'17*

$\mathcal{A}(z) = -\, a \ln((bz^2 + 1)(dz^4 + 1))$ — *Zhu, Chen, Zhou, Zhang, Huang'25*

Solution for "heavy" quarks for $(p - c_B q_3) z^4$

$$g(z) = e^{c_B z^2} \left[1 - \frac{I_1(z)}{I_1(z_h)} + \frac{\mu^2 \big(2R_{gg} + c_B(q_3 - 1)\big) I_2(z)}{L^2 \left(1 - e^{(R_{gg} + \frac{c_B(q_3-1)}{2}) z_h^2}\right)^2} \left(1 - \frac{I_1(z)}{I_1(z_h)} \frac{I_2(z_h)}{I_2(z)}\right) \right]$$

$$I_1(z) = \int_0^z e^{(R_{gg} - \frac{3c_B}{2})\xi^2 + 3(p - c_B q_3)\xi^4} \xi^{1+\frac{2}{\nu}} \, d\xi$$

$$I_2(z) = \int_0^z e^{(R_{gg} + \frac{c_B}{2}(\frac{q_3}{2} - 2))\xi^2 + 3(p - c_B q_3)\xi^4} \xi^{1+\frac{2}{\nu}} \, d\xi$$

$$T = \left| - \frac{e^{(R_{gg} - \frac{c_B}{2}) z_h^2 + 3(p - c_B q_3) z_h^4} z_h^{1+\frac{2}{\nu}}}{4\pi \, I_1(z_h)} \times \right.$$

$$\left. \times \left[1 - \frac{\mu^2 \big(2R_{gg} + c_B(q_3 - 1)\big) \left(e^{(R_{gg} + \frac{c_B(q_3-1)}{2}) z_h^2} I_1(z_h) - I_2(z_h) \right)}{L^2 \left(1 - e^{(R_{gg} + \frac{c_B(q_3-1)}{2}) z_h^2}\right)^2} \right] \right|$$

$$s = \frac{1}{4} \left(\frac{L}{z_h}\right)^{1+\frac{2}{\nu}} e^{-(R_{gg} - \frac{c_B}{2}) z_h^2 - 3(p - c_B q_3) z_h^4}$$

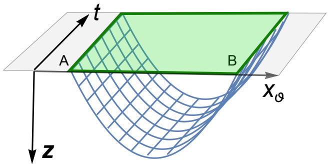
Aref'eva et al. Eur.Phys.J.C ***83*** *12 (2023) arXiv:2305.06345 [hep-th]*

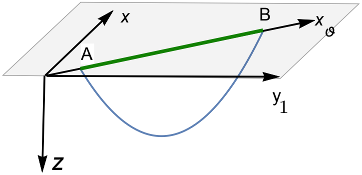
Temporal Wilson loop

$$W[C_\vartheta] = e^{-S_{\vartheta,t}} \qquad \vec{n}: \qquad n_x = \cos\vartheta, \quad n_{y_1} = \sin\vartheta, \quad n_{y_2} = 0$$

ℓ

x – collision axes





Two special cases:

- $\vartheta = 0$ WL (longitudinal)
- $\vartheta = \pi/2$ WT (transversal)

$\ell \to \infty \qquad S \sim \sigma_{DW}\ \ell$

$\swarrow$

the string tension

$$\sigma_{DW} = \frac{b(z) e^{\sqrt{\frac{2}{3}}\phi(z,z_0)}}{z^2} \sqrt{g(z)\left(z^{2-\frac{2}{\nu}}\sin^2(\vartheta) + \cos^2(\vartheta)\right)}\Bigg|_{z=z_{DW}}, \qquad \frac{\partial\sigma}{\partial z}\Bigg|_{z=z_{DW}} = 0$$

*Aref'eva, K.R., Slepov PLB **792**, 470 (2019) arXiv:1808.05596 [hep-th]*

Temporal Wilson Loops for $(p - c_B q_3) z^4$-term

$$\phi(z,z_0) = \int_{z_0}^{z} \frac{\sqrt{2}}{\nu\xi} \Bigg[2(\nu-1) + \big(6R_{gg}\nu + (2-3\nu)c_B\big)\nu\xi^2 + \left(\frac{4}{3}R_{gg}^2 - c_B^2 + 60(p - c_B\, q_3)\right)\nu^2\xi^4 + + 16R_{gg}(p - c_B\, q_3)\nu^2\xi^6 + 48(p - c_B\, q_3)^2\nu^2\xi^8 \Bigg]^{1/2} d\xi, \qquad z_0 \neq 0$$

$$\text{WL}x_1 : \quad -\frac{4}{3}R_{gg}z - 8(p - c_B q_3)z^3 + \sqrt{\frac{2}{3}}\,\phi'(z) + \frac{g'}{2g} - \frac{2}{z}\,\Big|_{z=z_{DWx_1}} = 0$$

$$\text{WL}x_2 : \quad -\frac{4}{3}R_{gg}z - 8(p - c_B q_3)z^3 + \sqrt{\frac{2}{3}}\,\phi'(z) + \frac{g'}{2g} - \frac{\nu+1}{\nu z}\,\Big|_{z=z_{DWx_2}} = 0$$

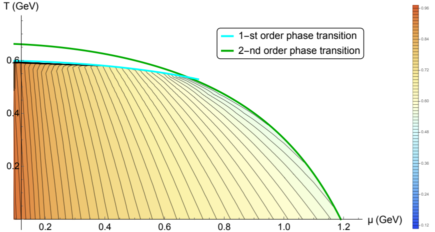
$$\text{WL}x_3 : \quad -\frac{4}{3}R_{gg}z - 8(p - c_B q_3)z^3 + \sqrt{\frac{2}{3}}\,\phi'(z) + \frac{g'}{2g} - \frac{\nu+1}{\nu z} + c_B z\,\Big|_{z=z_{DWx_3}} = 0$$

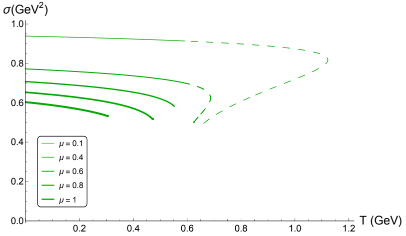
$$\sigma_{DW} = \frac{e^{-\frac{2R_{gg}z^2}{3} - 2(p - c_B q_3 z^4)}}{z^{1+\frac{1}{\nu}}}\; e^{\sqrt{\frac{2}{3}}\phi(z,z_0)}\;\sqrt{g(z)}, \qquad z_0 = e^{-z_h/4} + 0.1$$

Aref'eva, Hajilou, Slepov, Usova, PRD **111**, *046013 (2025) arXiv:2503.09444 [hep-th]*

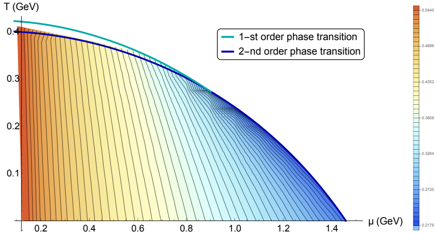
String tension $\sigma(\mu, T)$, no magnetic field $c_B = 0$

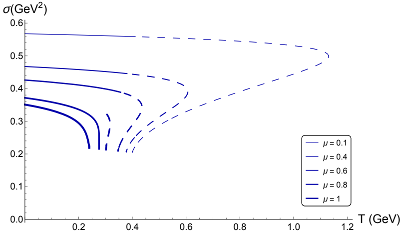
$\nu = 1$



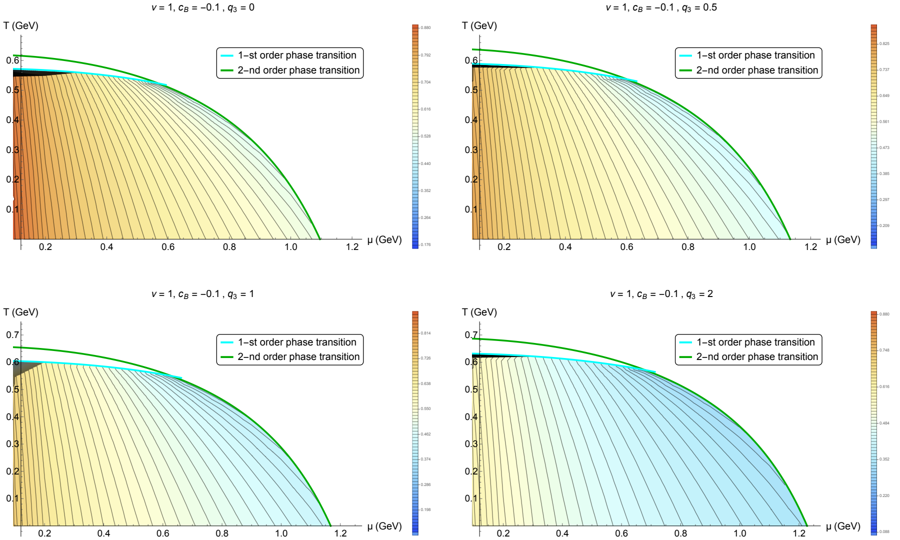


$\nu = 4.5$

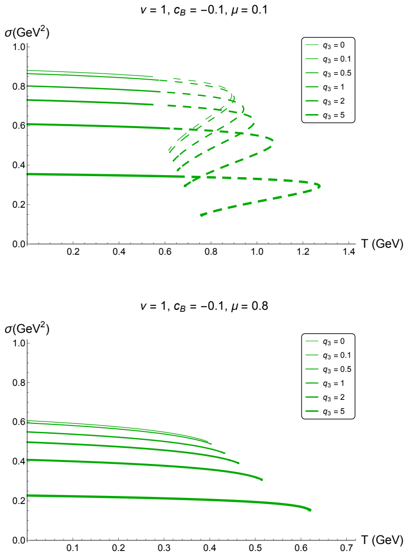




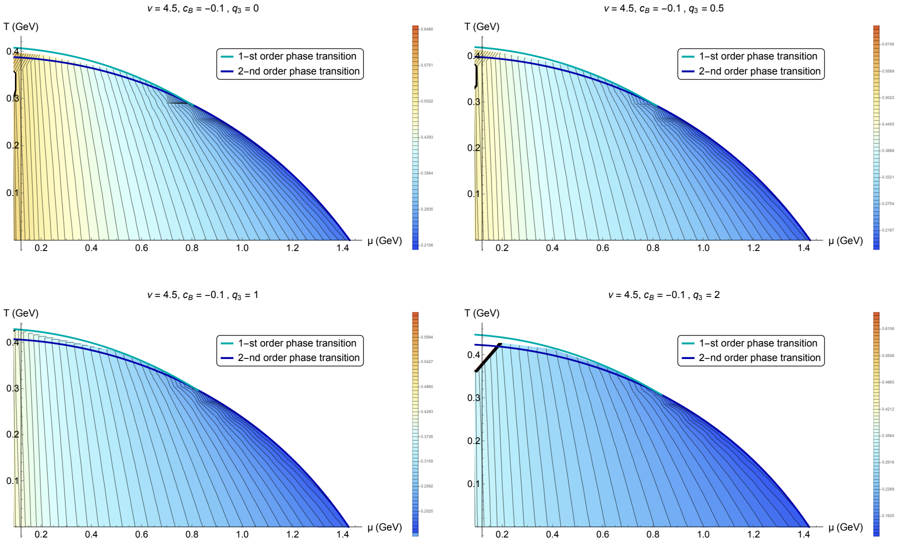
String tension $\sigma(\mu,T)$, $c_B = -0.1$, $\nu = 1$



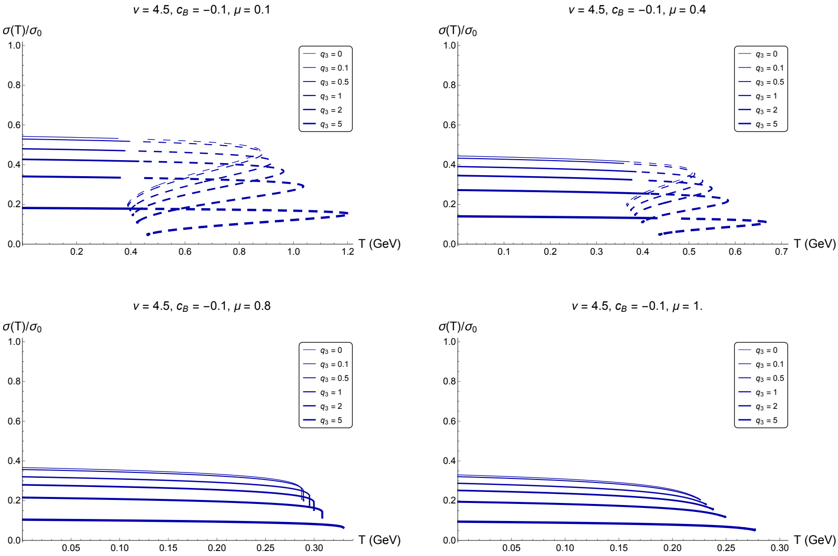
String tension $\sigma(T, \mu = \text{const})$ curves, $c_B = -0.1$



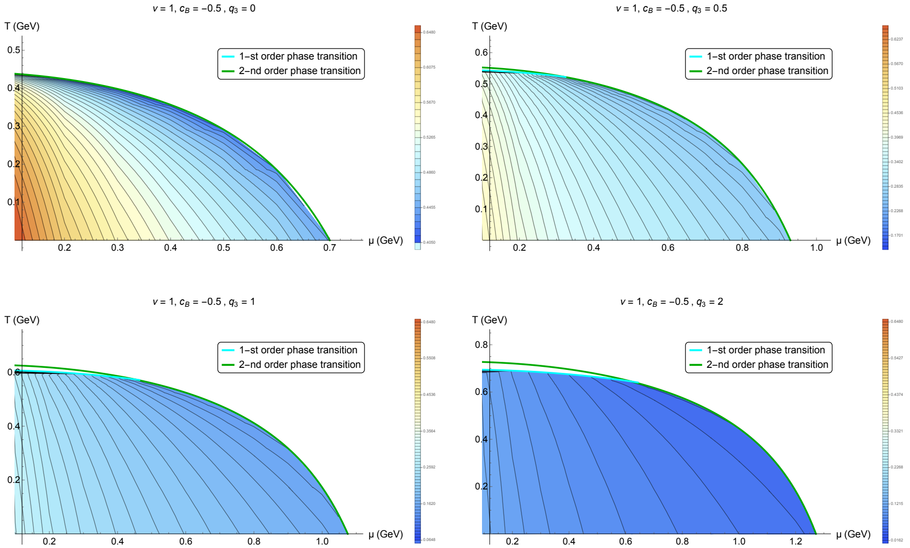
String tension $\sigma(\mu, T)$, $c_B = -0.1$, $\nu = 4.5$



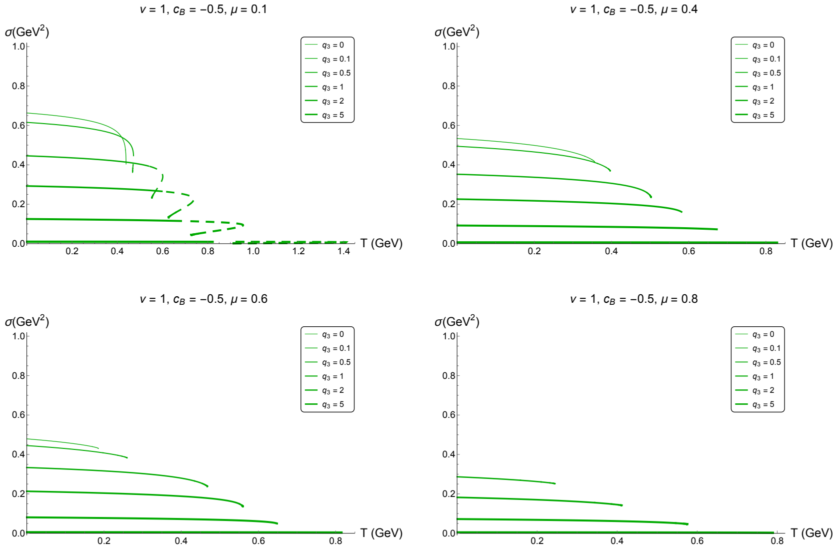
String tension $\sigma(T, \mu = \text{const})$ curves, $c_B = -0.1$



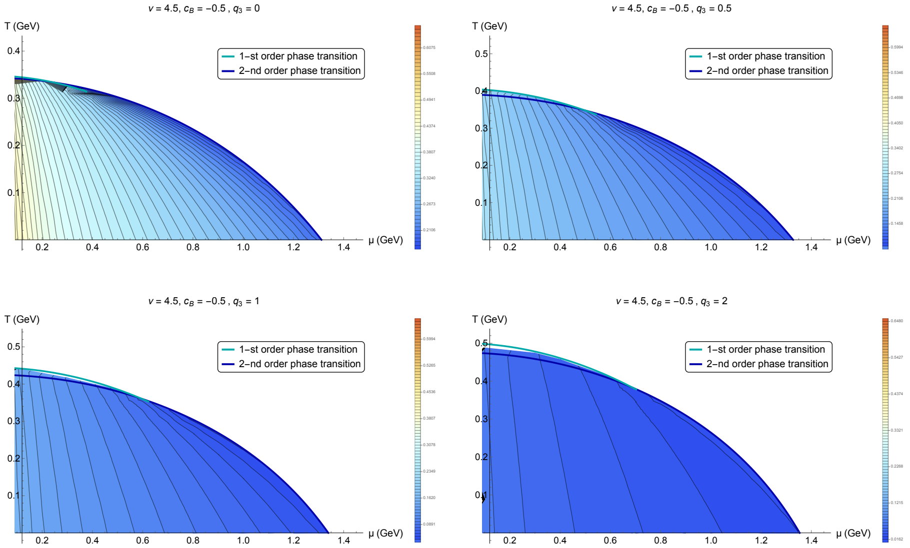
String tension $\sigma(\mu, T)$, $c_B = -0.5$, $\nu = 1$



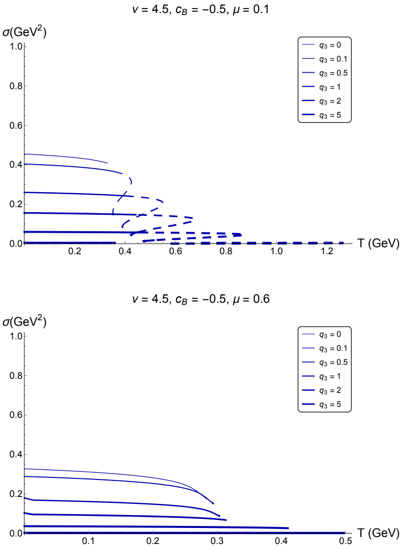
String tension $\sigma(T, \mu = \text{const})$ curves, $c_B = -0.5$



String tension $\sigma(\mu,T)$, $c_B = -0.5$, $\nu = 4.5$



String tension $\sigma(T, \mu = \text{const})$ curves, $c_B = -0.5$



Conclusion

String tension for temporal Wilson loop within HQCD model for heavy quarks with magnetic field exhibits the following properties:

- larger values of magnetic anisotropy parameters q_3 and $|c_B|$ weaken string tension;
- string tension becomes less sensitive to chemical potential and weakens with primary anisotropy ν;
- there exists an unstable branch with multivalued string tension behavior "above" the 1-st order phase transition.

Thank you
for your attention