22 LOMONOSOV CONFERENCE

ON ELEMENTARY PARTICLE PHYSICS

MOSCOW

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EINSTEIN-MAXWELL TETRAD GRAND UNIFICATION

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TETRADS IN GEOMETRODYNAMICS

$$f^{\mu\nu}{}_{;\nu} = 0$$

$$*f^{\mu\nu}{}_{;\nu} = 0$$

$$R_{\mu\nu} = f_{\mu\lambda}\, f_{\nu}{}^{\lambda} + *f_{\mu\lambda}\, *f_{\nu}{}^{\lambda}$$

$$f_{\mu\nu} = \left(G^{1/2} / c^{2} \right) F_{\mu\nu}$$ geometrized electromagnetic field.

$$*f_{\mu\nu} = \frac{1}{2}\, \epsilon_{\mu\nu\sigma\tau}\, f^{\sigma\tau}$$ dual tensor of $f_{\mu\nu}$.

WE MAKE THE DEFINITION

$$\xi_{\eta\nu} = \cos\alpha \; f_{\eta\nu} - {}^{*}f_{\eta\nu} \sin\alpha$$

THE FIELD HAS UNDERGONE A DUALITY ROTATION BY AN ANGLE $-\alpha$.

$$\xi_{\eta\nu} = e^{-{}^{*}\alpha} f_{\eta\nu} \qquad \text{(NOTATION)}$$

ASSUME THAT THE INVARIANTS

$f_{\eta\nu} f^{\eta\nu}$ AND $f_{\eta\nu} {}^{*}f^{\eta\nu}$ DO NOT VANISH.

- NON NULL FIELD -

CHOOSE THE ANGLE α SO THAT

$$\xi_{\eta\nu} {}^{*}\xi^{\eta\nu} = 0 \quad \text{(MISNER AND WHEELER)}$$

α : LOCAL SCALAR
THE COMPLEXION

$$\Rightarrow \quad \tan(2\alpha) = -\left(f_{\eta\nu} {}^{*}f^{\eta\nu}\right) / \left(f_{\sigma\tau} f^{\sigma\tau}\right)$$

STRESS-ENERGY TENSOR

$$T_{\eta\nu} = f_{\eta\lambda} f_{\nu}{}^{\lambda} + {}^*f_{\eta\lambda} {}^*f_{\nu}{}^{\lambda}$$

$$f_{\eta\nu} = \xi_{\eta\nu} \cos\alpha + {}^*\xi_{\eta\nu} \sin\alpha$$ DUALITY ROTATION

$$\Rightarrow T_{\eta\nu} = \xi_{\eta\lambda} \xi_{\nu}{}^{\lambda} + {}^*\xi_{\eta\lambda} {}^*\xi_{\nu}{}^{\lambda}$$

$\xi_{\eta\nu}$ EXTREMAL FIELD

WE WOULD LIKE TO FIND A TETRAD THAT DIAGONALIZES $T_{\eta\nu}$ COVARIANTLY.

WE CONSIDER THE NON-NULL ELECTROMAGNETIC FIELDS /

$$Q = \xi_{\eta\nu} \xi^{\eta\nu} = -\sqrt{T_{\eta\nu} T^{\eta\nu}} \neq 0.$$

THEN, WE FIND FOUR VECTORS THAT DIAGONALIZE $T_{\gamma\delta}$ IN GEOMETRODYNAMICS :

$$V_{(1)}^{\alpha} = \xi^{\alpha\lambda}\, \xi_{\rho\lambda}\, X^{\rho}$$

$$V_{(2)}^{\alpha} = \sqrt{\frac{-Q}{2}}\, \xi^{\alpha\lambda}\, X_{\lambda}$$

$$V_{(3)}^{\alpha} = \sqrt{\frac{-Q}{2}}\, {}^{*}\xi^{\alpha\lambda}\, Y_{\lambda}$$

$$V_{(4)}^{\alpha} = {}^{*}\xi^{\alpha\lambda}\, {}^{*}\xi_{\rho\lambda}\, Y^{\rho}$$

WE ARE FREE TO CHOOSE THE TWO VECTOR FIELDS X^{ρ} AND Y^{ρ} AS LONG AS THE FOUR VECTOR FIELDS ARE NOT TRIVIAL

X^{ρ} AND Y^{ρ} : GAUGE VECTORS.

IN A 4-DIM SPACE TIME TWO ANTISYMMETRIC FIELDS SATISFY THE RELATION:

$$A_{\mu\alpha} B^{\nu\alpha} - {}^{*}B_{\mu\alpha}\, {}^{*}A^{\nu\alpha} = \frac{1}{2}\, \delta_{\mu}^{\nu} \left(A_{\alpha\beta} B^{\alpha\beta}\right)$$

USING THIS RELATION AND THE EXTREMAL FIELD CONDITION

$\xi_{\mu\nu}\, {}^{*}\xi^{\mu\nu} = 0$ WE FIND:

$$\boxed{\xi_{\alpha\mu}\, {}^{*}\xi^{\mu\nu} = 0 \qquad (1)}$$

USING THE SAME RELATION:

$$\boxed{\xi_{\mu\alpha}\, \xi^{\nu\alpha} - {}^{*}\xi_{\mu\alpha}\, {}^{*}\xi^{\nu\alpha} = \frac{1}{2}\, \delta_{\mu}^{\nu}\, Q \qquad (2)}$$

$$Q = \xi_{\mu\nu}\, \xi^{\mu\nu} \neq 0$$

THEN, USING 1 AND 2 WE FIND

$$V_{(1)}^{\alpha} \; T_{\alpha}{}^{\beta} = \frac{a}{2} V_{(1)}^{\beta}$$

$$V_{(2)}^{\alpha} \; T_{\alpha}{}^{\beta} = \frac{a}{2} V_{(2)}^{\beta}$$

$$V_{(3)}^{\alpha} \; T_{\alpha}{}^{\beta} = - \frac{a}{2} V_{(3)}^{\beta}$$

$$V_{(4)}^{\alpha} \; T_{\alpha}{}^{\beta} = - \frac{a}{2} V_{(4)}^{\beta}$$

ELECTROMAGNETIC POTENTIALS IN GEOMETRODYNAMICS.

OUR GOAL: SIMPLIFY AS MUCH AS WE CAN THE EXPRESSION OF THE ELECTROMAGNETIC FIELD THROUGH THE USE OF AN ORTHONORMAL TETRAD

→ SO THAT ITS GEOMETRICAL PROPERTIES CAN BE UNDERSTOOD IN AN EASIER WAY.

IN GEOMETRODYNAMICS: $f^{\eta\nu}{}_{;\nu} = 0$

$$*f^{\eta\nu}{}_{;\nu} = 0$$

$\Rightarrow$ EXISTENCE OF TWO POTENTIAL VECTORS:

A_η AND $*A_\eta$ (NOTATION) NOT INDEPENDENT FROM EACH OTHER.

$$f_{\eta\nu} = A_{\nu;\eta} - A_{\eta;\nu}$$

$$*f_{\eta\nu} = *A_{\nu;\eta} - *A_{\eta;\nu}$$

WE CAN MAKE THE CHOICE

$$X^{\rho} = A^{\rho}$$

$$Y^{\rho} = *A^{\rho}$$

X^{ρ} AND Y^{ρ} : WE ARE FREE TO CHOOSE THE "GAUGE VECTORS"

THEN, THE FOUR VECTORS ARE :

$$V_{(1)}^{\alpha} = \xi^{\alpha\lambda} \, \xi_{\rho\lambda} \, A^{\rho}$$

$$V_{(2)}^{\alpha} = \sqrt{-\frac{Q}{2}} \; \xi^{\alpha\lambda} \, A_{\lambda}$$

$$V_{(3)}^{\alpha} = \sqrt{-\frac{Q}{2}} \; *\xi^{\alpha\lambda} \, *A_{\lambda}$$

$$V_{(4)}^{\alpha} = *\xi^{\alpha\lambda} \, *\xi_{\rho\lambda} \, *A^{\rho}$$

WE ASSUME FOR SIMPLICITY

$-V_{(1)}^{\alpha} \, V_{(1)\alpha} = V_{(2)}^{\alpha} \, V_{(2)\alpha} > 0 \; // \; V_{(3)}^{\alpha} \, V_{(3)\alpha} = V_{(4)}^{\alpha} \, V_{(4)\alpha} > 0$

GAUGE GEOMETRY

ONCE WE MAKE THE CHOICE

$$X^\rho = A^\rho \qquad Y^\rho = {}^*A^\rho$$

WHAT HAPPENS TO THE TETRAD VECTORS WHEN WE MAKE THE TRANSFORMATIONS:

NEW CHOICE OF GAUGE VECTORS:

$$A_\alpha \longrightarrow A_\alpha + \Lambda_{,\alpha}$$

$$^*A_\alpha \longrightarrow {}^*A_\alpha + {}^*\Lambda_{,\alpha}$$

NOTATION

$$\Lambda_{,\alpha} = \Lambda_\alpha \qquad {}^*\Lambda_{,\alpha} = {}^*\Lambda_\alpha$$

Λ AND $^*\Lambda$ ARE SCALARS.

SCHOUTEN DEFINED WHAT HE CALLED A TWO-BLADED STRUCTURE IN A SPACETIME.

BLADE ONE : $\left(V^{\alpha}_{(1)}, V^{\alpha}_{(2)} \right)$

BLADE TWO : $\left(V^{\alpha}_{(3)}, V^{\alpha}_{(4)} \right)$.

GAUGE TRANSFORMATIONS ON BLADE ONE

$$\begin{cases} \tilde{V}^{\alpha}_{(1)} = V^{\alpha}_{(1)} + \boxed{f^{\alpha\lambda} f_{\rho\lambda} \Lambda^{\rho}} \\ \tilde{V}^{\alpha}_{(2)} = V^{\alpha}_{(2)} + \boxed{\sqrt{\frac{-\mathcal{L}}{2}}\, f^{\alpha\lambda} \Lambda_{\lambda}} \end{cases}$$

$$f^{\alpha\lambda} f_{\rho\lambda} \Lambda^{\rho} V_{(3)\alpha} = f^{\alpha\lambda} f_{\rho\lambda} \Lambda^{\rho} V_{(4)\alpha} = 0$$

$$\sqrt{\frac{-\mathcal{L}}{2}}\, f^{\alpha\lambda} \Lambda_{\lambda} V_{(3)\alpha} = \sqrt{\frac{-\mathcal{L}}{2}}\, f^{\alpha\lambda} \Lambda_{\lambda} V_{(4)\alpha} = 0 .$$

WE WRITE THEN,

$$\begin{cases} \tilde{V}^{\alpha}_{(1)} = V^{\alpha}_{(1)} + C\, V^{\alpha}_{(1)} + D\, V^{\alpha}_{(2)} \\ \tilde{V}^{\alpha}_{(2)} = V^{\alpha}_{(2)} + E\, V^{\alpha}_{(1)} + F\, V^{\alpha}_{(2)} . \end{cases}$$

USING RELATIONS 1 AND 2

WE FIND: $E = D$

$F = C$

$$\begin{cases} C = \left(-\frac{Q}{2}\right) V_{(1)\sigma} \Lambda^{\sigma} \Big/ \left(V_{(2)\beta} V^{\beta}_{(2)}\right) \\ D = \left(-\frac{Q}{2}\right) V_{(2)\sigma} \Lambda^{\sigma} \Big/ \left(V_{(1)\beta} V^{\beta}_{(1)}\right) . \end{cases}$$

$$\begin{cases} \tilde{V}^{\alpha}_{(1)} \tilde{V}_{(1)\alpha} = \left[(1+C)^2 - D^2\right] V^{\alpha}_{(1)} V_{(1)\alpha} \\ \tilde{V}^{\alpha}_{(2)} \tilde{V}_{(2)\alpha} = \underbrace{\left[(1+C)^2 - D^2\right]}_{\text{SCALAR CAUSALITY FACTOR}} V^{\alpha}_{(2)} V_{(2)\alpha} \end{cases}$$

$$\boxed{V^{\alpha}_{(1)} V_{(1)\alpha} = \\ = - V^{\alpha}_{(2)} V_{(2)\alpha} .}$$

CASES: $\left[(1+c)^2 - D^2\right] > 0$

① $1+c > 0$

$$\frac{\tilde{V}_{(1)}^{\alpha}}{\sqrt{-\tilde{V}_{(1)}^{\beta}\,\tilde{V}_{(1)\beta}}} = \frac{(1+c)}{\sqrt{(1+c)^2 - D^2}}\,\frac{V_{(1)}^{\alpha}}{\sqrt{-V_{(1)}^{\beta}\,V_{(1)\beta}}} + \frac{D}{\sqrt{(1+c)^2 - D^2}}\,\frac{V_{(2)}^{\alpha}}{\sqrt{V_{(2)}^{\beta}\,V_{(2)\beta}}}$$

$$\frac{\tilde{V}_{(2)}^{\alpha}}{\sqrt{\tilde{V}_{(2)}^{\beta}\,\tilde{V}_{(2)\beta}}} = \frac{D}{\sqrt{(1+c)^2 - D^2}}\,\frac{V_{(1)}^{\alpha}}{\sqrt{-V_{(1)}^{\beta}\,V_{(1)\beta}}} + \frac{(1+c)}{\sqrt{(1+c)^2 - D^2}}\,\frac{V_{(2)}^{\alpha}}{\sqrt{V_{(2)}^{\beta}\,V_{(2)\beta}}}$$

AN ELECTROMAGNETIC GAUGE TRANSFORMATION GENERATES A BOOST TRANSFORMATION ON THE NORMALIZED $\left(\frac{V_{(1)}^{\alpha}}{\sqrt{-V_{(1)}^{\beta}\,V_{(1)\beta}}},\ \frac{V_{(2)}^{\alpha}}{\sqrt{V_{(2)}^{\beta}\,V_{(2)\beta}}}\right)$.

② $1+c<0$

$$\frac{\tilde{V}^{\alpha}_{(1)}}{\sqrt{-\tilde{V}^{\beta}_{(1)}\tilde{V}_{(1)\beta}}}=\frac{[-(1+c)]}{\sqrt{(1+c)^2-D^2}}\frac{(-V^{\alpha}_{(1)})}{\sqrt{-V^{\beta}_{(1)}V_{(1)\beta}}}+\frac{[-D]}{\sqrt{(1+c)^2-D^2}}\frac{(-V^{\alpha}_{(2)})}{\sqrt{V^{\beta}_{(2)}V_{(2)\beta}}}$$

$$\frac{\tilde{V}^{\alpha}_{(2)}}{\sqrt{\tilde{V}^{\beta}_{(2)}\tilde{V}_{(2)\beta}}}=\frac{[-D]}{\sqrt{(1+c)^2-D^2}}\frac{(-V^{\alpha}_{(1)})}{\sqrt{-V^{\beta}_{(1)}V_{(1)\beta}}}+\frac{[-(1+c)]}{\sqrt{(1+c)^2-D^2}}\frac{(-V^{\alpha}_{(2)})}{\sqrt{V^{\beta}_{(2)}V_{(2)\beta}}}$$

AN ELECTROMAGNETIC GAUGE TRANSFORMATION GENERATES THE COMPOSITION OF AN INVERSION AND A BOOST OF THE NORMALIZED

$$\left(\frac{V^{\alpha}_{(1)}}{\sqrt{-V^{\beta}_{(1)}V_{(1)\beta}}}\,,\ \frac{V^{\alpha}_{(2)}}{\sqrt{V^{\beta}_{(2)}V_{(2)\beta}}}\right).$$

$$[(1+c)^2 - D^2] < 0$$

SPECIAL IMPROPER TRANSFORMATIONS ON BLADE 1.

$$\tilde{V}^{\alpha}_{(1)} \tilde{V}_{(1)\alpha} = [-(1+c)^2 + D^2] (-V^{\alpha}_{(1)} V_{(1)\alpha})$$

$$(-\tilde{V}^{\alpha}_{(2)} \tilde{V}_{(2)\alpha}) = [-(1+c)^2 + D^2] (V^{\alpha}_{(2)} V_{(2)\alpha}).$$

3

$$1+c>0$$

$$\frac{\tilde{V}^{\alpha}_{(1)}}{\sqrt{\tilde{V}^{\beta}_{(1)} \tilde{V}_{(1)\beta}}} = \frac{(1+c)}{\sqrt{-(1+c)^2+D^2}} \frac{V^{\alpha}_{(1)}}{\sqrt{-V^{\beta}_{(1)} V_{(1)\beta}}} + \frac{D}{\sqrt{-(1+c)^2 + D^2}} \frac{V^{\alpha}_{(2)}}{\sqrt{V^{\beta}_{(2)} V_{(2)\beta}}}$$

$$\frac{\tilde{V}^{\alpha}_{(2)}}{\sqrt{-V^{\beta}_{(2)} V_{(2)\beta}}} = \frac{D}{\sqrt{-(1+c)^2+D^2}} \frac{V^{\alpha}_{(1)}}{\sqrt{-V^{\beta}_{(1)} V_{(1)\beta}}} + \frac{(1+c)}{\sqrt{-(1+c)^2+D^2}} \frac{V^{\alpha}_{(2)}}{\sqrt{V^{\beta}_{(2)} V_{(2)\beta}}}$$

BOOST $\begin{pmatrix} \frac{D}{\sqrt{-(1+c)^2+D^2}} & \frac{(1+c)}{\sqrt{-(1+c)^2+D^2}} \\ \frac{(1+c)}{\sqrt{-(1+c)^2+D^2}} & \frac{D}{\sqrt{-(1+c)^2+D^2}} \end{pmatrix}$ COMPOSED WITH

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ NOT A LORENTZ TRANSF.

REFLECTION

④ $1+c<0$

$$\frac{\tilde{V}^{\alpha}_{(1)}}{\sqrt{\tilde{V}^{\beta}_{(1)}\tilde{V}_{(1)\beta}}} = \frac{[-(1+c)]}{\sqrt{-(1+c)^2+D^2}}\frac{(-V^{\alpha}_{(1)})}{\sqrt{-V^{\beta}_{(1)}V_{(1)\beta}}} + \frac{[-D]}{\sqrt{-(1+c)^2+D^2}}\frac{(-V^{\alpha}_{(2)})}{\sqrt{V^{\beta}_{(2)}V_{(2)\beta}}}$$

$$\frac{\tilde{V}^{\alpha}_{(2)}}{\sqrt{-\tilde{V}^{\beta}_{(2)}\tilde{V}_{(2)\beta}}} = \frac{[-D]}{\sqrt{-(1+c)^2+D^2}}\frac{(-V^{\alpha}_{(1)})}{\sqrt{-V^{\beta}_{(1)}V_{(1)\beta}}} + \frac{[-(1+c)]}{\sqrt{-(1+c)^2+D^2}}\frac{(-V^{\alpha}_{(2)})}{\sqrt{V^{\beta}_{(2)}V_{(2)\beta}}}$$

$D = 1+C$ OR $D = -(1+c)$

MAP INTO THE INTERSECTION WITH THE LOCAL LIGHT CONE 4 SOLUTIONS.

BOOST $\begin{pmatrix} \frac{[-D]}{\sqrt{-(1+c)^2+D^2}} & \frac{[-(1+c)]}{\sqrt{-(1+c)^2+D^2}} \\ \frac{[-(1+c)]}{\sqrt{-(1+c)^2+D^2}} & \frac{[-D]}{\sqrt{-(1+c)^2+D^2}} \end{pmatrix}$

COMPOSED WITH $-\mathbb{1}_{2\times 2}$

AND COMPOSED WITH

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ ~~NOT A LORENTZ TRANSFORMATION~~

REFLECTION

GAUGE TRANSFORMATIONS ON BLADE TWO

$$\begin{cases} \tilde{V}^{\alpha}_{(3)} = V^{\alpha}_{(3)} + \sqrt{\frac{-\mathscr{l}}{2}}\ {}^{*}f^{\alpha\lambda}\ {}^{*}\Lambda_{\lambda} \\ \tilde{V}^{\alpha}_{(4)} = V^{\alpha}_{(4)} + {}^{*}f^{\alpha\lambda}\ {}^{*}f_{\rho\lambda}\ {}^{*}\Lambda^{\rho} \end{cases}$$

$$\begin{cases} \sqrt{\frac{-\mathscr{l}}{2}}\ {}^{*}f^{\alpha\lambda}\ {}^{*}\Lambda_{\lambda}\ V_{(1)\alpha} = \sqrt{\frac{-\mathscr{l}}{2}}\ {}^{*}f^{\alpha\lambda}\ {}^{*}\Lambda_{\lambda}\ V_{(2)\alpha} = 0 \\ {}^{*}f^{\alpha\lambda}\ {}^{*}f_{\rho\lambda}\ {}^{*}\Lambda^{\rho}\ V_{(1)\alpha} = {}^{*}f^{\alpha\lambda}\ {}^{*}f_{\rho\lambda}\ {}^{*}\Lambda^{\rho}\ V_{(2)\alpha} = 0 \end{cases}$$

$$\begin{cases} \tilde{V}^{\alpha}_{(3)} = V^{\alpha}_{(3)} + K\, V^{\alpha}_{(3)} + L\, V^{\alpha}_{(4)} \\ \tilde{V}^{\alpha}_{(4)} = V^{\alpha}_{(4)} + M\, V^{\alpha}_{(3)} + N\, V^{\alpha}_{(4)} \end{cases}$$

USING RELATIONS 1 AND 2 $\Longrightarrow$

$$K = N \qquad M = \left(-\frac{\mathscr{l}}{2}\right) V_{(3)\sigma}\ {}^{*}\Lambda^{\sigma} \Big/ \left(V_{(4)\beta}\, V^{\beta}_{(4)}\right)$$

$$L = -M \qquad N = \left(-\frac{\mathscr{l}}{2}\right) V_{(4)\sigma}\ {}^{*}\Lambda^{\sigma} \Big/ \left(V_{(3)\beta}\, V^{\beta}_{(3)}\right).$$

$$\begin{cases} \tilde{V}^{\beta}_{(3)} \tilde{V}_{(3)\beta} = \left[(1+N)^2 + M^2 \right] V^{\beta}_{(3)} V_{(3)\beta} \\ \tilde{V}^{\beta}_{(4)} \tilde{V}_{(4)\beta} = \left[(1+N)^2 + M^2 \right] V^{\beta}_{(4)} V_{(4)\beta} \end{cases}$$

$$\frac{\tilde{V}^{\alpha}_{(3)}}{\sqrt{\tilde{V}^{\beta}_{(3)} \tilde{V}_{(3)\beta}}} = \frac{(1+N)}{\sqrt{(1+N)^2 + M^2}} \frac{V^{\alpha}_{(3)}}{\sqrt{V^{\beta}_{(3)} V_{(3)\beta}}} - \frac{M}{\sqrt{(1+N)^2 + M^2}} \frac{V^{\alpha}_{(4)}}{\sqrt{V^{\beta}_{(4)} V_{(4)\beta}}}$$

$$\frac{\tilde{V}^{\alpha}_{(4)}}{\sqrt{\tilde{V}^{\beta}_{(4)} \tilde{V}_{(4)\beta}}} = \frac{M}{\sqrt{(1+N)^2 + M^2}} \frac{V^{\alpha}_{(3)}}{\sqrt{V^{\beta}_{(3)} V_{(3)\beta}}} + \frac{(1+N)}{\sqrt{(1+N)^2 + M^2}} \frac{V^{\alpha}_{(4)}}{\sqrt{V^{\beta}_{(4)} V_{(4)\beta}}}$$

FOR $(1+N)^2 + M^2 > 0$, THE $*A^{\alpha} \longrightarrow *A^{\alpha} + *\Lambda^{\alpha}$ GAUGE TRANSFORMATION, GENERATES A ROTATION TRANSFORMATION OF THE NORMALIZED VECTORS:

$$\left(\frac{V^{\alpha}_{(3)}}{\sqrt{V^{\beta}_{(3)} V_{(3)\beta}}} , \frac{V^{\alpha}_{(4)}}{\sqrt{V^{\beta}_{(4)} V_{(4)\beta}}} \right).$$

GROUP ISOMORPHISM

LB1 (LORENTZ BLADE ONE)

THE GROUP OF SO(1,1) BOOST TETRAD TRANSFORMATIONS ON BLADE ONE

PLUS $-\mathbb{1}_{2\times 2}$ PLUS $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ NOT A LORENTZ TRANSF.

PLUS 4 LIGHT CONE SOLUTIONS $\mathring{\Lambda}_0 = \dot{\Lambda}_1 = 0$ $\mathring{\Lambda}_1 = \dot{\Lambda}_0 = 1$

LB2 (LORENTZ BLADE TWO)

THE GROUP OF LORENTZ TETRAD TRANSFORMATIONS (ROTATIONS/SO(2)) ON BLADE TWO.

LB1-proper ISOMORPHIC TO LB2

LB1 $4 \longrightarrow 1$ LB2

ISOMORPHISM ON BLADE ONE

THEOREM: THE MAPPING BETWEEN THE LOCAL GAUGE GROUP AND THE GROUP LB1 HAS A KERNEL $\{\Lambda_\eta / \Lambda_\eta \in \text{plane } 2\}$

KERNEL OF MEASURE ZERO

MINUS THE KERNEL → ISOMORPHISM

ISOMORPHISM ON BLADE TWO

THEOREM: THE MAPPING BETWEEN THE LOCAL GAUGE GROUP AND THE GROUP LB2 HAS A KERNEL $\{\Lambda_\eta / \Lambda_\eta \in \text{plane } 1\}$ KERNEL OF MEASURE ZERO

THEOREM: THE MAPPING BETWEEN THE LOCAL GAUGE GROUP AND THE GROUP $LB1 \otimes LB2$ IS AN ISOMORPHISM PIECEWISE

EXAMPLE:

REISSNER-NORDSTRÖM

$$ds^2 = -\left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right) dt^2 + \left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right)^{-1} dr^2 + r^2\left(d\theta^2 + \sin^2\theta \, d\phi^2\right)$$

$$U^t = -\frac{(|q|/q)}{\sqrt{1 - \frac{2m}{r} + \frac{q^2}{r^2}}}$$

$$V^r = \sqrt{1 - \frac{2m}{r} + \frac{q^2}{r^2}}$$

$$Z^\theta = -\frac{|\cos\theta|}{r\cos\theta}$$

$$W^\phi = -\frac{|q|\,|\cos\theta|}{q\, r \sin\theta \cos\theta}$$

ENERGY MOMENTUM CURRENTS

$$T^{\alpha}{}_{\beta}\, \xi^{\beta}_{j} \qquad j = 1 \ldots 4$$

$$\xi_1 = (1,0,0,0) \qquad \xi_2 = (0, \sin\phi, \cos\phi \cot\theta, 0)$$

$$\xi_3 = (0, \cos\phi, -\sin\phi \cot\theta, 0) \qquad \xi_4 = (0,0,0,1)$$

ξ^{β}_{j} : KILLING VECTORS

$$J(\xi_1) = -\frac{q^2}{r^4}\, \xi_1 \qquad \Bigg| \qquad \underline{J(\xi_a) = \frac{q^2}{r^4}\, \xi_a}$$

$$a = 2, 3, 4$$

$J(\xi_1)$ BELONGS TO PLANE ONE SPANNED BY (U^{α}, V^{α})

$J(\xi_a)$ BELONGS TO PLANE TWO SPANNED BY (Z^{α}, W^{α}).

GENERAL CURRENTS FROM EINSTEIN-MAXWEL EQS.

$$\xi^{\eta\nu}{}_{;\nu} = - {}^{*}\xi^{\eta\nu}\alpha_{\nu}$$ ON PLANE TWO

$$^{*}\xi^{\eta\nu}{}_{;\nu} = \xi^{\eta\nu}\alpha_{\nu}$$ ON PLANE ONE

$$\alpha_{\nu} = \alpha_{,\nu} \neq 0$$ NON-TRIVIAL

THESE CURRENTS ARE CONSERVED

$$\xi^{\eta\nu}{}_{;\nu\eta} = -\left({}^{*}\xi^{\eta\nu}\alpha_{\nu}\right)_{;\eta} = 0$$

$$^{*}\xi^{\eta\nu}{}_{;\nu\eta} = \left(\xi^{\eta\nu}\alpha_{\nu}\right)_{;\eta} = 0$$

ORTHONORMAL TETRAD

$$U^\alpha = f^{\alpha\lambda} f_{\rho\lambda} A^\rho \Big/ \left(\sqrt{\frac{-Q}{2}} \sqrt{A_\eta f^{\eta\sigma} f_{\nu\sigma} A^\nu} \right)$$

$$V^\alpha = f^{\alpha\lambda} A_\lambda \Big/ \left(\sqrt{A_\eta f^{\eta\sigma} f_{\nu\sigma} A^\nu} \right)$$

$$Z^\alpha = {}^*f^{\alpha\lambda}\, {}^*A_\lambda \Big/ \left(\sqrt{{}^*A_\eta\, {}^*f^{\eta\sigma}\, {}^*f_{\nu\sigma}\, {}^*A^\nu} \right)$$

$$W^\alpha = {}^*f^{\alpha\lambda}\, {}^*f_{\rho\lambda}\, {}^*A^\rho \Big/ \left(\sqrt{\frac{-Q}{2}} \sqrt{{}^*A_\eta\, {}^*f^{\eta\sigma}\, {}^*f_{\nu\sigma}\, {}^*A^\nu} \right).$$

$$-U^\alpha U_\alpha = V^\alpha V_\alpha = Z^\alpha Z_\alpha = W^\alpha W_\alpha = 1.$$

$$g_{\alpha\beta} = -U_\alpha U_\beta + V_\alpha V_\beta + Z_\alpha Z_\beta + W_\alpha W_\beta$$

$$T_{\alpha\beta} = \left(\frac{Q}{2}\right)\left[-U_\alpha U_\beta + V_\alpha V_\beta - Z_\alpha Z_\beta - W_\alpha W_\beta\right]$$

$$f_{\alpha\beta} = -2\sqrt{\frac{-Q}{2}}\cos(\alpha)\ U_{[\alpha} V_{\beta]} +$$

$$+ 2\sqrt{\frac{-Q}{2}}\sin(\alpha)\ Z_{[\alpha} W_{\beta]} .$$

NULL TETRAD

$i^2 = -1$.

$$K_\alpha = \frac{1}{\sqrt{2}}\left(U_\alpha + V_\alpha\right) \qquad L_\alpha = \frac{1}{\sqrt{2}}\left(U_\alpha - V_\alpha\right)$$

$$T_\alpha = \frac{1}{\sqrt{2}}\left(Z_\alpha + i W_\alpha\right) \qquad \bar{T}_\alpha = \frac{1}{\sqrt{2}}\left(Z_\alpha - i W_\alpha\right)$$

BIVECTORS

$$U_{\alpha\beta} = \bar{T}_\alpha L_\beta - \bar{T}_\beta L_\alpha$$

$$V_{\alpha\beta} = K_\alpha \bar{T}_\beta - K_\beta \bar{T}_\alpha$$

$$W_{\alpha\beta} = T_\alpha \bar{T}_\beta - T_\beta \bar{T}_\alpha - K_\alpha L_\beta + K_\beta L_\alpha$$

SELF-DUAL ELECTROMAGNETIC BIVECTOR

$$\Phi_{\alpha\beta} = f_{\alpha\beta} + i\, {}^*f_{\alpha\beta} = \underbrace{\boxed{-\sqrt{\frac{-Q}{2}}\, e^{-i\alpha}}}_{=\phi_1} W_{\alpha\beta}$$

VACUUM EINSTEIN - MAXWELL EQUATIONS

(NEWMAN - PENROSE TETRAD FORMALISM)

$$D \phi_1 = 2 \rho \phi_1$$

$$\delta \phi_1 = -2 \sigma \phi_1$$

$$\bar{\delta} \phi_1 = -2 \pi \phi_1$$

$$\Delta \phi_1 = -2 \mu \phi_1 .$$

WE CONSIDER THE TETRAD VECTORS

$$V_{(1)}^{\alpha} = \zeta^{\alpha\lambda} \zeta_{\rho\lambda} X^{\rho}$$

$$V_{(2)}^{\alpha} = \zeta^{\alpha\lambda} X_{\lambda}$$

$$V_{(3)}^{\alpha} = {}^{*}\zeta^{\alpha\lambda} Y_{\lambda}$$

$$V_{(4)}^{\alpha} = {}^{*}\zeta^{\alpha\lambda}\, {}^{*}\zeta_{\rho\lambda} Y^{\rho}$$

IF WE CHOOSE $X^\rho = Y^\rho = A^\rho$

$$V^\alpha_{(1)} - V^\alpha_{(4)} = \xi^{\alpha\lambda}\xi_{\rho\lambda}A^\rho - {}^*\xi^{\alpha\lambda}\,{}^*\xi_{\rho\lambda}A^\rho =$$

$$= \frac{Q}{2} A^\alpha$$

USING IDENTITY 2

$$V^\alpha_{(1)} V_{(1)\alpha} = \left(-\frac{Q}{2}\right)\left(A_\eta \xi^{\eta\sigma}\xi_{\nu\sigma}A^\nu\right) =$$

$$= - V^\alpha_{(2)} V_{(2)\alpha} \left(\frac{Q}{2}\right)$$

WE CALL $$C = \frac{\left(V^\alpha_{(1)} V_{(1)\alpha}\right)}{\left(V^\beta_{(2)} V_{(2)\beta}\right)}$$

$$A^\alpha = -\left(V^\alpha_{(1)} - V^\alpha_{(4)}\right) / C$$

24-2

Int. J. Geom. Meth. Mod. Phys.
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$$\tan(2\alpha) = -f_{\mu\nu} {}^*f^{\mu\nu} / f_{\lambda\rho} f^{\lambda\rho}$$

$$\xi_{\alpha\beta} = -2\sqrt{\frac{-Q}{2}}\, U_{[\alpha} V_{\beta]}$$

$$^*\xi_{\alpha\beta} = 2\sqrt{\frac{-Q}{2}}\, Z_{[\alpha} W_{\beta]}$$

$$f_{\alpha\beta} = -2\sqrt{\frac{-Q}{2}} \cos\alpha\, U_{[\alpha} V_{\beta]}$$
$$+ 2\sqrt{\frac{-Q}{2}} \sin\alpha\, Z_{[\alpha} W_{\beta]}$$

COULOMB CASE $(- + + +)$

$$f_{tr} = \frac{e}{r^2} \qquad A_t = \frac{e}{r} \qquad A_r = 0$$

$$V_{(1)}^{t} = \xi^{tr}\,\xi_{tr}\,A^{t} = |\xi_{tr}|^2 A_t$$

$$V_{(1)}^{r} = \xi^{rt}\,\xi_{rt}\,A^{r} = 0$$

$$V_{(2)}^{t} = |\xi_{tr}|\,\xi^{tr} A_r = 0$$

$$V_{(2)}^{r} = |\xi_{tr}|\,\xi^{rt} A_t = |\xi_{tr}|\,\xi_{tr} A_t$$

$$Q = -2\,|\xi_{tr}|^2$$

$$V_{(1)}^{\alpha} V_{(1)\alpha} = V_{(1)}^{t} V_{(1)t} + V_{(1)}^{r} V_{(1)r} = -|\xi_{tr}|^4 (A_t)^2 + $$

$$+ |\xi_{tr}|^4 (A_r)^2 =$$

$$= -|\xi_{tr}|^4 (A_t)^2$$

$$V^{\alpha}_{(2)} V_{(2)\alpha} = V^{t}_{(2)} V_{(2)t} + V^{r}_{(2)} V_{(2)r} =$$

$$= -|\xi_{tr}|^4 (A_r)^2 + |\xi_{tr}|^4 (A_t)^2 =$$

$$= |\xi_{tr}|^4 (A_t)^2$$

$$V^{\alpha}_{(1)} V_{(1)\alpha} = -V^{\alpha}_{(2)} V_{(2)\alpha} \neq 0$$

THEN, WE TRANSFORM THE GAUGE

$$f_{tr} = \frac{e}{r^2} = \xi_{tr} \qquad A_t = \frac{e}{r} \qquad A^{new}_r = -\frac{e}{r}$$

GAUGE TRANSFORMATION: $\Lambda_t = 0 \qquad \Lambda_r = -\frac{e}{r}$

$\Lambda_{,\eta} = \Lambda_{\eta}$ NOTATION

$$V^{\alpha}_{(1)} V_{(1)\alpha} = |\xi_{tr}|^4 \left(-(A_t)^2 + (A^{new}_r)^2\right)$$

$$V^{\alpha}_{(2)} V_{(2)\alpha} = |\xi_{tr}|^4 \left((A_t)^2 - (A^{new}_r)^2\right)$$

$$\tilde{V}_{(1)}^{\alpha}\,\tilde{V}_{(1)\alpha} = \left[(1+C)^2 - D^2\right] V_{(1)}^{\alpha}\, V_{(1)\alpha}$$

$$\tilde{V}_{(2)}^{\alpha}\,\tilde{V}_{(2)\alpha} = \left[(1+C)^2 - D^2\right] V_{(2)}^{\alpha}\, V_{(2)\alpha}$$

$$C = \left(-\frac{a}{2}\right) \frac{V_{(1)\sigma}\,\Lambda^{\sigma}}{V_{(2)\beta}\,V_{(2)}^{\beta}}$$

$$D = \left(-\frac{a}{2}\right) \frac{V_{(2)\sigma}\,\Lambda^{\sigma}}{V_{(1)\beta}\,V_{(1)}^{\beta}}$$

$$\left[(1+C)^2 - D^2\right] = (1-0)^2 - \left(\frac{\Lambda r}{A t}\right)^2 = 0$$

REISS-NORDSTRÖM CASE

$$g_{tt} = -\left(1 - \frac{2M}{r} + \frac{e^2}{r^2}\right)$$

$$g_{rr} = \left(1 - \frac{2M}{r} + \frac{e^2}{r^2}\right)^{-1}$$

$$g_{\theta\theta} = r^2$$

$$g_{\phi\phi} = r^2 \sin^2\theta$$

$$V^t_{(1)} = \xi^{tr} \xi_{tr} A^t = -g^{tt} |\xi_{tr}|^2 A_t$$

$$V^r_{(1)} = \xi^{rt} \xi_{rt} A^r = -g^{rr} |\xi_{tr}|^2 A_r$$

$$V^t_{(2)} = |\xi_{tr}| \xi^{tr} A_r = -|\xi_{tr}| \xi_{tr} A_r$$

$$V^r_{(2)} = |\xi_{tr}| \xi^{rt} A_t = |\xi_{tr}| \xi_{tr} A_t$$

$$Q = -2 |\xi_{tr}|^2$$

GAUGE TRANSFORMATION Λ AND NULL CONDITION $\Rightarrow$

$$\Lambda_t - g_{tt}\Lambda_r = -A_t$$

$$\Lambda_H = \exp\left(-i\int_a^r \left(\frac{\omega}{g_{tt}} + \kappa\right) dr\right) \cdot$$

$$\cdot \exp[i(\kappa r - \omega t)]$$

a, κ, ω CONSTANTS.

$$\Lambda_{INH} = -\frac{e}{r\left(1 - \frac{2M}{r} + \frac{e^2}{r^2}\right)} = \frac{A_t}{g_{tt}}$$

$$D = \pm(1+c)$$

$D = -(1+c)$ $\quad \Lambda_r = g_{rr} A_t$ INH

FOR SU(2) LOCAL GAUGE TRS.
AN ANALOGOUS PROCEDURE.

EXAMPLE : RARE DECAYS

$b \to s \gamma$

LET US CONCENTRATE ON THE DIAGRAM WITH LOOP $c\bar{c}$
THERE WILL BE A CURRENT ASSOCIATED TO $c \to \bar{c} + \gamma$

$J^{\alpha}_{[c\bar{c}\gamma]}$

quarks have electric charge $\Rightarrow$

$E^{\rho}_{[c\bar{c}\gamma]\alpha}$ WE CAN ASSOCIATE TO THE VERTEX

$$X^{\rho}_{[c\bar{c}\gamma]} = Y^{\rho}_{[c\bar{c}\gamma]} = J^{\alpha}_{[c\bar{c}\gamma]} E^{\rho}_{[c\bar{c}\gamma]\alpha}$$

30

INITIAL GUGE FOR SOME SU(2) S

IN GENERAL

$$X^{\sigma} = Y^{\sigma} = \mathrm{Tr}\left[\Sigma^{\alpha\beta} E_{\alpha}^{\rho} E_{\beta}^{\lambda} * \int_{\rho}^{\sigma} * \int_{\lambda\zeta} A^{\zeta} \right]$$

A^{ζ} GAUGED BY $SU(2)$

FOR SOME $SU(2)$ S, INITIALLY WE KNOW $X^{\sigma} = Y^{\sigma}$

THEN WE PROCEED WITH A LOCAL $SU(2)$ GAUGE TRANSFORMATION AND MAKE THE VECTORS IN PLANE ONE, NULL.

DRELL-YAN PROCESS

LET US FOCUS ON THE DIAGRAM INCLUDING A VIRTUAL PHOTON WITH EMISSION OF A PAIR LEPTON-ANTILEPTON $\gamma^* \to e^+ + e^-$

THERE WILL BE A CURRENT

$$J^{\alpha}_{[\gamma e^+ e^-]}$$

e^+ AND e^- HAVE ELECTRIC CHARGE $\Rightarrow$

$$E^{\rho}_{[\gamma e^+ e^-]\alpha}$$

TO THE VERTEX $\gamma^* \to e^+ + e^-$

WE ASSOCIATE INITIAL GAUGE VECTORS

$$X^{\rho}_{[\gamma e^+ e^-]} = Y^{\rho}_{[\gamma e^+ e^-]} = J^{\alpha}_{[\gamma e^+ e^-]} \; E^{\rho}_{[\gamma e^+ e^-]\alpha}$$

UNDER A LOCAL $S \in SU(2)$ THE PLANE ONE VECTORS BECOME NULL.

SUMMARY

- NEW ORTHONORMAL TETRAD FOR NON-NULL ELECTROMAGNETIC FIELDS IN CURVED SPACETIMES.

- ISOMORPHISMS BETWEEN THE LOCAL GAUGE GROUP AND LOCAL LB1 AND LB2 GROUPS.
(GEOMETRIZATION OF GAUGE THEORIES).

- MAXIMUM SIMPLIFICATION OF RELEVANT TENSORS AND FIELD EQUATIONS.

- NEW TETRADS ENCODE GRAVITATIONAL AND GAUGE INFORMATION.

- EXPLICIT RELATIONSHIP BETWEEN "GAUGE" AND "GRAVITY".

- WE ARE INTRODUCING AND EXPLICIT "LINK" BETWEEN THE "INTERNAL" AND THE "SPACETIME", SO FAR DETACHED FROM EACH OTHER.

- FOR OTHER THEORIES
OTHER FIELD EQUATIONS
NON-ABELIAN

→ NEW CHOICES FOR THE "GAUGE VECTORS" ALLOW TO PROVE THEOREMS IN THE NON-ABELIN CASE.

SPACELIKE AND TIMELIKE VECTOR TRANSFORM INTO NULL VECTORS

A. Garat, Timelike and Spacelike vector transform into null vectors through local gauge transfs.

- EXTENSION OR GENERALIZATION TO NON-ABELIAN THEORIES

$SU(2) \times U(1)$

IS IT POSSIBLE TO BUILD THE NEW TETRADS WITH SKELETON - GAUGE VECTOR STRUCTURE IN HIGHER DIMENSIONAL SPACETIMES ?

IS IT POSSIBLE IN HIGHER DIMENSIONAL SPACETIMES TO PROVE NEW RESULTS IN GROUP THEORY FOR THE ASSOCIATED SYMMETRIES ?

AHARONOV - BOHM KIND OF EXPERIMENTS → FULL INVERSION

FULL SPACETIME INVERSION GENERATED BY EM ABEL GAUGE TRANSF. QUANTUM STUDIES: M&F 8 337–349 (2021)

- EXTENSION OR GENERALIZATION TO NON-ABELIAN THEORIES

$SU(3) \times SU(2) \times U(1)$.

Int. J. Geom. Meth. Mod. Phys. 15(3) (2018)

- FUTURE RESEARCH (FOLLOWING RESEARCH ONGOING)

1. CAN WE ASSOCIATE CLASSICAL SPACETIMES TO MICROPARTICLES?

2. TETRAD FIELDS CAN BE ASSOCIATED TO MICROPARTICLES?

3. ARE PARTICLE MULTIPLETS ASSOCIATED TO SYMMETRY PLANES IN 4-DIM LORENTZIAN SPACETIMES?

Tetrads in SU(N) Yang-Mills geometrodynamics

Int. J. Mod. Phys. A 34(29) (2019)

NULL FIELD

$$f_{\eta\nu} = K_\eta V_\nu - K_\nu V_\eta$$

$$V_\eta V^\eta = 1 \qquad K_\nu K^\nu = 0$$

$$K_\eta V^\eta = 0$$

$$R_{\eta\nu} = 2 K_\eta K_\nu$$

HOW TO GAUGE THE TETRAD WHEN THE MAXWELL EQUATIONS HAVE SOURCES J^α

$$f^{\mu\nu}{}_{;\nu} = J^\alpha$$

$$*f^{\mu\nu}{}_{;\nu} = 0$$

JUST ONE POTENTIAL $A^\alpha = X^\alpha$

FOR THE COULOMB CASE IN MINKOWSKY SPACETIME

$(-+++)$ (t, ρ, θ, z) $(-1, 1, \rho^2, 1)$

$$\sqrt{-g} = \rho$$

$K_t{}^\mu = (1, 0, 0, 0)$ $K_\rho{}^\mu = (0, 1, 0, 0)$

$K_\theta{}^\mu = (0, 0, \frac{1}{\rho}, 0)$ $K_z{}^\mu = (0, 0, 0, 1)$

WE CAN CHOOSE FOR SOME GEOMETRIES:

$$Y^\alpha = K_t^\alpha$$

OR EVEN $Y^\alpha = K_t^\alpha + A^\alpha$

FOR EXAMPLE FOR THE SOLENOID CASE WHERE THE GEOMETRY IS CYLINDRICAL AND

$$f_{\theta z} = B_\rho \qquad f_{\rho\theta} = B_z$$

$$\mathcal{J}_{\rho\theta} = f_{\rho\theta} \quad {}^*\mathcal{J}_{tz} = -f_{\theta\rho} \quad {}^*\mathcal{J}_{t\rho} = f_{\theta z}$$

$$V_{(3)}^\rho = {}^*\mathcal{J}^{\rho t} Y_t = -{}^*\mathcal{J}_{t\rho} Y^t$$

$$V_{(3)}^z = {}^*\mathcal{J}^{zt} Y_t = -{}^*\mathcal{J}_{tz} Y^t$$

$$V_{(4)}^t = {}^*\mathcal{J}^{t\rho}\,{}^*\mathcal{J}_{t\rho} Y^t + {}^*\mathcal{J}^{tz}\,{}^*\mathcal{J}_{tz} Y^t$$

TETRADS IN YANG-MILLS GEOMETRODYNAMICS

SU(2) X U(1) LOCAL GAUGE GROUP

$$R_{\mu\nu} = T^{(ym)}_{\mu\nu} + T^{(em)}_{\mu\nu}$$

$$f^{\mu\nu}{}_{;\nu} = 0$$

$$*f^{\mu\nu}{}_{;\nu} = 0$$

$$f^{k\mu\nu}{}_{|\nu} = 0$$

$$*f^{k\mu\nu}{}_{|\nu} = 0$$

Einstein-
-Maxwell-
-Yang-Mills
field equations

New Tetrads

new local gauge-invariant UNDER $SU(2) \times U(1)$ $\epsilon_{\mu\nu}$

extremal tensor $\epsilon_{\mu\nu} \cdot {*\epsilon}^{\mu\lambda} = 0$

X^ρ and Y^ρ local gauge-vectors

$$S^\mu_{(1)} = \epsilon^{\mu\lambda}\,\epsilon_{\rho\lambda}\,X^\rho$$

$$S^\mu_{(2)} = \sqrt{\frac{-Q_{ym}}{2}}\;\epsilon^{\mu\lambda}\,X_\lambda$$

$$S^\mu_{(3)} = \sqrt{\frac{-Q_{ym}}{2}}\;{*\epsilon}^{\mu\lambda}\,Y_\lambda$$

$$S^\mu_{(4)} = {*\epsilon}^{\mu\lambda}\,{*\epsilon}_{\rho\lambda}\,Y^\rho$$

$$Q_{ym} = \epsilon_{\mu\nu}\,\epsilon^{\mu\nu} \neq 0$$

gauge-vector choice:

$$X^\sigma = Y^\sigma = \mathrm{Tr}\Big[\sum^{\alpha,\beta} E_\alpha^\rho E_\beta^\lambda$$

$$* \xi_\rho{}^\sigma \; * \xi_{\lambda\tau} A^\tau \Big]$$ REAL OBJECTS

A^τ SU(2) CONNECTION.

E_α^ρ : electromagnetic tetrad.

$\xi_{\mu\nu}$: electromagnetic extremal field.

$$\xi_{\mu\lambda} * \xi^{\mu\sigma} = 0$$

$$\xi_{\eta\nu} = \cos\alpha \; f_{\eta\nu} - \sin\alpha \; {}^*f_{\eta\nu}$$

$$\tan(2\alpha) = - f_{\sigma\tau} \, {}^*f^{\sigma\tau} \Big/ f_{\rho\lambda} f^{\rho\lambda}$$

$f_{\eta\nu}$: electromagnetic field

α : local complexion.

$$E_0^{\eta} = \xi^{\eta\lambda} \xi_{\rho\lambda} A^{\rho} \Big/ \sqrt{\frac{-Q}{2}} \sqrt{A_\sigma \xi^{\sigma\tau} \xi_{\nu\tau} A^{\nu}}$$

$$E_1^{\eta} = \xi^{\eta\lambda} A_\lambda \Big/ \sqrt{A_\sigma \xi^{\sigma\tau} \xi_{\nu\tau} A^{\nu}}$$

$$E_2^{\eta} = {}^*\xi^{\eta\lambda} \, {}^*A_\lambda \Big/ \sqrt{{}^*A_\sigma \, {}^*\xi^{\sigma\tau} \, {}^*\xi_{\nu\tau} \, {}^*A^{\nu}}$$

$$E_3^{\eta} = {}^*\xi^{\eta\lambda} \, {}^*\xi_{\rho\lambda} \, {}^*A^{\rho} \Big/ \sqrt{\frac{-Q}{2}} \sqrt{{}^*A_\sigma \, {}^*\xi^{\sigma\tau} \, {}^*\xi_{\nu\tau} \, {}^*A^{\nu}}$$

electromagnetic tetrad.

$$f_{\mu\nu} = A_{\nu;\mu} - A_{\mu;\nu}$$

$$*f_{\mu\nu} = *A_{\nu;\mu} - *A_{\mu;\nu}$$

$*A_\mu$ NOT THE HODGE OPERATOR JUST A NAME.

$$\sigma_{\pm}^{\alpha} = (\mathbb{1}, \pm\sigma^i)$$

σ^i : PAULI MATRICES.

$$\sigma^{\alpha\beta} = \sigma_+^{\alpha}\sigma_-^{\beta} - \sigma_+^{\beta}\sigma_-^{\alpha}$$

$$\sigma^{\dagger\alpha\beta} = \sigma_-^{\alpha}\sigma_+^{\beta} - \sigma_-^{\beta}\sigma_+^{\alpha}$$

$$\Sigma^{\alpha\beta} = i\left(\sigma^{\alpha\beta} + \sigma^{\dagger\alpha\beta}\right) =$$

$$= \begin{cases} 0 & \text{if } \alpha = 0 \text{ and } \beta = i \\ 4\epsilon^{ijk}\sigma^k & \text{if } \alpha = i \text{ and } \beta = j \end{cases}$$

under $\left(\frac{1}{2}, 0\right)$ and $\left(0, \frac{1}{2}\right)$ spinor representations of the Lorentz group:

$$S^{-1}_{(1/2)} \, \sigma^{\alpha}_{\pm} \, S_{(1/2)} = \Lambda^{\alpha}{}_{\beta} \, \sigma^{\beta}_{\pm}$$

$\sigma^{\alpha}_{\pm}$ TRANSFORM AS VECTORS.

$$S^{-1} \, \Sigma^{\alpha\beta} \, S = \Lambda^{\alpha}{}_{\gamma} \, \Lambda^{\beta}{}_{\delta} \, \Sigma^{\gamma\delta}$$

$$E_{\alpha}^{[\rho} \, E_{\beta}^{\lambda]} \; {}^{*}\mathcal{J}_{\rho\sigma} \; {}^{*}\mathcal{J}_{\lambda\tau}$$

INVARIANT UNDER $U(1)$ local electromagnetic gauge transformations.

UNDER LOCAL SU(2) GAUGE TRANSFORMATIONS S :

$$A_\mu \longrightarrow S^{-1} A_\mu S + \frac{i}{g} S^{-1} \partial_\mu S$$

$$\Longrightarrow$$

$$\mathrm{Tr}\left[\Sigma^{\alpha\beta} E^\rho_\alpha E^\lambda_\beta \, {}^*\xi_\rho{}^\sigma \, {}^*\xi_{\lambda\zeta} \, A^\zeta\right]$$

$$\longrightarrow \mathrm{Tr}\left[\Sigma^{\alpha\beta} E^\rho_\alpha E^\lambda_\beta \, {}^*\xi_\rho{}^\sigma \, {}^*\xi_{\lambda\zeta} \, S^{-1} A^\zeta S\right] +$$

$$+ \frac{i}{g} \mathrm{Tr}\left[\Sigma^{\alpha\beta} E^\rho_\alpha E^\lambda_\beta \, {}^*\xi_{\rho\sigma} \, {}^*\xi_{\lambda\zeta} \, S^{-1} \partial^\zeta S\right]$$

$$f_{\mu\nu} \longrightarrow S^{-1} f_{\mu\nu} S.$$

LET US STUDY THE TRANSFORMATION OF $S^{\mu}_{(1)}$

$$\tilde{S}^{\mu}_{(1)} = \epsilon^{\mu\nu}\epsilon^{\sigma}{}_{\nu}\, \mathrm{Tr}\Big[\Sigma^{\alpha\beta} E^{\rho}_{\alpha} E^{\lambda}_{\beta}\; {*\zeta_{\rho\sigma}}\, {*\zeta_{\lambda\tau}}\, S^{-1} A^{\tau} S\Big] + \frac{i}{g}\, \epsilon^{\mu\nu}\epsilon^{\sigma}{}_{\nu}\, \mathrm{Tr}\Big[\Sigma^{\alpha\beta} E^{\rho}_{\alpha} E^{\lambda}_{\beta}\; {*\zeta_{\rho\sigma}}\, {*\zeta_{\lambda\tau}}\, S^{-1} \partial^{\tau} S\Big]$$

FOR $S^{\mu}_{(2)}$ WE OBTAIN A SIMILAR RESULT.

$$\tilde{S}^{\eta}_{(1)} = \epsilon^{\eta\nu}\,\epsilon^{\sigma}{}_{\nu}\ \mathrm{Tr}\Big[S\,\Sigma^{\alpha\beta}S^{-1}$$

$$E^{\rho}_{\alpha}\,E^{\lambda}_{\beta}\ {}^{*}\!\zeta_{\rho\sigma}\ {}^{*}\!\zeta_{\lambda\zeta}\ A^{\zeta}\Big]$$

$$+\frac{i}{g}\,\epsilon^{\eta\nu}\,\epsilon^{\sigma}{}_{\nu}\ \mathrm{Tr}\Big[S\,\Sigma^{\alpha\beta}S^{-1}$$

$$E^{\rho}_{\alpha}\,E^{\lambda}_{\beta}\ {}^{*}\!\zeta_{\rho\sigma}\ {}^{*}\!\zeta_{\lambda\zeta}\ \partial^{\zeta}(S)\ S^{-1}\Big] =$$

$$= \epsilon^{\eta\nu}\,\epsilon^{\sigma}{}_{\nu}\ \mathrm{Tr}\Big[\tilde{\Lambda}^{\alpha}{}_{\delta}\,\tilde{\Lambda}^{\beta}{}_{\gamma}\ \Sigma^{\delta\gamma}$$

$$E^{\rho}_{\alpha}\,E^{\lambda}_{\beta}\ {}^{*}\!\zeta_{\rho\sigma}\ {}^{*}\!\zeta_{\lambda\zeta}\ A^{\zeta}\Big] +$$

$$+\frac{i}{g}\,\epsilon^{\eta\nu}\,\epsilon^{\sigma}{}_{\nu}\ \mathrm{Tr}\Big[\tilde{\Lambda}^{\alpha}{}_{\delta}\,\tilde{\Lambda}^{\beta}{}_{\gamma}\ \Sigma^{\delta\gamma}$$

$$E^{\rho}_{\alpha}\,E^{\lambda}_{\beta}\ {}^{*}\!\zeta_{\rho\sigma}\ {}^{*}\!\zeta_{\lambda\zeta}\ \partial^{\zeta}(S)\,S^{-1}\Big]$$

NOTATION: $\Lambda^{(-1)\alpha}{}_{\delta} = \tilde{\Lambda}^{\alpha}{}_{\delta}$.

$$\tilde{S}^{\eta}_{(1)} = S'^{\eta}_{(1)} + C' S'^{\eta}_{(1)} + D' S'^{\eta}_{(2)}$$

$$\tilde{S}^{\eta}_{(2)} = S'^{\eta}_{(2)} + E' S'^{\eta}_{(1)} + F' S'^{\eta}_{(2)} .$$

WHERE

$$S'^{\eta}_{(1)} = \epsilon^{\eta\nu} \epsilon^{\sigma}{}_{\nu} \, \mathrm{Tr}\Big[\tilde{\Lambda}^{\alpha}{}_{\delta} \tilde{\Lambda}^{\beta}{}_{\gamma} \Sigma^{\delta\gamma} E^{\rho}_{\alpha} E^{\lambda}_{\beta} \, {}^{*}\!\int_{\rho\sigma} {}^{*}\!\int_{\lambda\zeta} A^{\zeta} \Big]$$

$$S'^{\eta}_{(2)} = \sqrt{\frac{-Q_{ym}}{2}} \, \epsilon^{\eta\sigma} \, \mathrm{Tr}\Big[\tilde{\Lambda}^{\alpha}{}_{\delta} \tilde{\Lambda}^{\beta}{}_{\gamma} \Sigma^{\delta\gamma} E^{\rho}_{\alpha} E^{\lambda}_{\beta} \, {}^{*}\!\int_{\rho\sigma} {}^{*}\!\int_{\lambda\zeta} A^{\zeta} \Big]$$

IT CAN BE PROVED THAT:

$$E' = D'$$

$$F' = C' \qquad \text{WHERE:}$$

$$C' = \frac{i}{g}\left(-\frac{a_{Jm}}{2}\right)\frac{S'^{\sigma}_{(1)}}{\left(S'_{(2)\eta}\, S'^{\eta}_{(2)}\right)} \times$$

$$\times \operatorname{Tr}\left[\tilde{\Lambda}^{\alpha}{}_{\delta}\, \tilde{\Lambda}^{\beta}{}_{\gamma}\, \Sigma^{\delta\gamma} E^{\rho}_{\alpha} E^{\lambda}_{\beta} * f_{\rho\sigma} * f_{\lambda\zeta}\, \partial^{\zeta}(S) S^{-1}\right]$$

$$D' = \frac{i}{g}\left(-\frac{a_{Jm}}{2}\right)\frac{S'^{\sigma}_{(2)}}{\left(S'_{(1)\eta}\, S'^{\eta}_{(1)}\right)} \times$$

$$\times \operatorname{Tr}\left[\tilde{\Lambda}^{\alpha}{}_{\delta}\, \tilde{\Lambda}^{\beta}{}_{\gamma}\, \Sigma^{\delta\gamma} E^{\rho}_{\alpha} E^{\lambda}_{\beta} * f_{\rho\sigma} * f_{\lambda\zeta}\, \partial^{\zeta}(S) S^{-1}\right].$$

$$[(1+c')^2 - D'^2] < 0 \qquad \text{IMPROPER}$$

$$\tilde{S}^{\mu}_{(1)}\, \tilde{S}_{(1)\mu} = [-(1+c')^2 + D'^2]\, (-S'^{\mu}_{(1)}\, S'_{(1)\mu})$$

$$(-\tilde{S}^{\mu}_{(2)}\, \tilde{S}_{(2)\mu}) = [-(1+c')^2 + D'^2]\, (S'^{\mu}_{(2)}\, S'_{(2)\mu}).$$

$$1+c' > 0$$

BOOST COMPOSED WITH

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$ NOT A LORENTZ TRANSFORMATION.

$$1+c' < 0$$

BOOST COMPOSED WITH $-\mathbb{1}_{2\times 2}$

AND COMPOSED WITH $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ NOT A LORENTZ TRANSFORMATION

BLADE 1

$$\tilde{S}^{\eta}_{(1)} \tilde{S}_{(1)\eta} = \left[(1+c')^2 - D'^2\right] S'^{\eta}_{(1)} S'_{(1)\eta}$$

$$\tilde{S}^{\eta}_{(2)} \tilde{S}_{(2)\eta} = \left[(1+c')^2 - D'^2\right] S'^{\eta}_{(2)} S'_{(2)\eta}$$

$$S'^{\eta}_{(1)} S'_{(1)\eta} = - S'^{\eta}_{(2)} S'_{(2)\eta} .$$

CASES: $\left[(1+c')^2 - D'^2\right] > 0$ PROPER

$1+c' > 0$ BOOST OF

$$\left(\frac{S^{\eta}_{(1)}}{\sqrt{-S^{\nu}_{(1)} S_{(1)\nu}}} , \frac{S^{\eta}_{(2)}}{\sqrt{S^{\nu}_{(2)} S_{(2)\nu}}} \right) .$$

$1+c' < 0$ INVERSION + BOOST

BLADE 2 : SPATIAL ROTATIONS SO(2).

SU(2) GENERATED LOCAL
GAUGE TETRAD TRANSF.
INDUCE LORENTZ + $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.
SO(1, 1) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ NOT LORENTZ
ON PLANE 1.
+ 4 LIGHT CONE SOLUTIONS

SU(2) GENERATED LOCAL
GAUGE TETRAD TRANSF.
INDUCE LORENTZ,
(SPATIAL ROTATIONS)
ON PLANE 2.

TWO CONSECUTIVE SU(2) S_1 AND S_2 LOCAL TETRAD GAUGE TRANSFORMATIONS.

$$\widetilde{\widetilde{S}}_{(1)} = \epsilon^{\mu\nu} \epsilon^{\sigma}{}_{\nu} \, Tr\Big[\tilde{\Lambda}_2{}^{\alpha}{}_{\kappa} \tilde{\Lambda}_2{}^{\beta}{}_{\epsilon} \tilde{\Lambda}_1{}^{\epsilon}{}_{\gamma} \tilde{\Lambda}_1{}^{\kappa}{}_{\delta} \Sigma^{\delta\gamma} E^{\rho}_{\alpha} E^{\lambda}_{\beta} \, {}^*f_{\rho\sigma} \, {}^*f_{\lambda\zeta} A^{\zeta} \Big]$$

$$+ \frac{i}{g} \epsilon^{\mu\nu} \epsilon^{\sigma}{}_{\nu} \, Tr\Big[\tilde{\Lambda}_2{}^{\alpha}{}_{\kappa} \tilde{\Lambda}_2{}^{\beta}{}_{\epsilon} \tilde{\Lambda}_1{}^{\kappa}{}_{\delta} \tilde{\Lambda}_1{}^{\epsilon}{}_{\gamma} \Sigma^{\delta\gamma} E^{\rho}_{\alpha} E^{\lambda}_{\beta} \, {}^*f_{\rho\sigma} \, {}^*f_{\lambda\zeta} \boxed{\partial^{\zeta}(S_1) S_1^{-1}} \Big] +$$

$$+ \frac{i}{g} \epsilon^{\mu\nu} \epsilon^{\sigma}{}_{\nu} \, Tr\Big[\tilde{\Lambda}_2{}^{\alpha}{}_{\delta} \tilde{\Lambda}_2{}^{\beta}{}_{\gamma} \Sigma^{\delta\gamma} E^{\rho}_{\alpha} E^{\lambda}_{\beta} \, {}^*f_{\rho\sigma} \, {}^*f_{\lambda\zeta} \boxed{\partial^{\zeta}(S_2) S_2^{-1}} \Big].$$

THEOREM: THE MAPPING BETWEEN THE LOCAL GAUGE GROUP SU(2) OF TRANSFORM. AND A TENSOR PRODUCT OF THREE LB1 GROUPS IS ISOMORPHIC.

(KERNEL OF MEASURE ZERO)

SU(2) ISOMORPHIC TO $(\otimes LB1)^3$.

SIMILARLY ON BLADE 2:

SU(2) ISOMORPHIC TO $(\otimes LB2)^3$.

(KERNEL OF MEASURE ZERO)

STRUCTURE COVARIANCE

$$S^{\mu}_{(1)} = \epsilon^{\mu\lambda} \epsilon_{\rho\lambda} X^{\rho}$$

$\epsilon^{\mu\lambda} \epsilon_{\rho\lambda}$: THE TETRAD VECTOR SKELETON

X^{ρ} : THE GAUGE VECTOR

LORENTZ LOCALLY TRANSFORMED TETRAD

$$\tilde{E}^{\rho}_{\delta} = \tilde{\Lambda}^{\alpha}{}_{\delta} E^{\rho}_{\alpha}$$

WE CAN REWRITE $\tilde{E}^{\rho}_{\delta}$ WITH THE SKELETON-
- GAUGE VECTOR STRUCTURE.

$$\tilde{\xi}^{\eta\nu} = -2\sqrt{\frac{-Q}{2}}\,\tilde{\lambda}^{\delta}{}_{0}\,\tilde{\lambda}^{\gamma}{}_{1}\,E^{\eta}_{[\delta}\,E^{\nu}_{\gamma]}$$

$$*\tilde{\xi}^{\eta\nu} = 2\sqrt{\frac{-Q}{2}}\,\tilde{\lambda}^{\delta}{}_{2}\,\tilde{\lambda}^{\gamma}{}_{3}\,E^{\eta}_{[\delta}\,E^{\nu}_{\gamma]}$$

$$Q = \xi_{\eta\nu}\,\xi^{\eta\nu} \neq 0 \quad \text{NON-NULL}$$

$$\tilde{\xi}^{\eta\lambda}\,*\tilde{\xi}_{\eta\nu} = 0$$

$$\tilde{E}_{0}^{\eta}\,*\tilde{\xi}_{\eta\nu} = 0 = \tilde{E}_{1}^{\eta}\,*\tilde{\xi}_{\eta\nu}$$

$$\tilde{E}_{0}^{\eta} = \text{NORM. FACTOR} \times \tilde{\xi}^{\eta\lambda}\,\tilde{\xi}_{\rho\lambda}\,\tilde{x}^{\rho}$$

$$\tilde{E}_{1}^{\eta} = \text{NORM. FACTOR} \times \tilde{\xi}^{\eta\rho}\,\tilde{x}_{\rho}$$

THEOREMS:

1) THE MAPPING BETWEEN THE LOCAL GAUGE GROUP SU(2) AND THE TENSOR PRODUCT $(\otimes LB1)^3$ IS ISOMORPHIC.

(KERNEL OF MEASURE ZERO)

---

2) THE MAPPING BETWEEN THE LOCAL GAUGE GROUP SU(2) AND THE TENSOR PRODUCT $(\otimes LB2)^3$ IS ISOMORPHIC.

(KERNEL OF MEASURE ZERO)

THE MAPPING BETWEEN THE LOCAL GAUGE GROUP SU(2) AND $(\otimes LB1)^3 \otimes (\otimes LB2)^3$ IS ISOMORPHIC

APPLICATION

GAUGE INVARIANTS

NORMALIZED VERSION OF

$S^{\mu}_{(i)}$ WE CALL $W^{\mu}_{(i)}$ $i = 0 \dots 3$

GAUGE TRANSFORMATION UNDER LOCAL $SU(2)$:

$$\tilde{W}^{\mu}_{(0)} = \mathrm{ch}\,\phi\, W^{\mu}_{(0)} + \mathrm{sh}\,\phi\, W^{\mu}_{(1)}$$

$$\tilde{W}^{\mu}_{(1)} = \mathrm{sh}\,\phi\, W^{\mu}_{(0)} + \mathrm{ch}\,\phi\, W^{\mu}_{(1)}$$

$$\tilde{W}^{\mu}_{(2)} = \cos\psi\, W^{\mu}_{(2)} - \sin\psi\, W^{\mu}_{(3)}$$

$$\tilde{W}^{\mu}_{(3)} = \sin\psi\, W^{\mu}_{(2)} + \cos\psi\, W^{\mu}_{(3)}$$

$$\left(W^{\mu}_{(0)} T_{\eta\nu} W^{\nu}_{(0)}\right) W^{\lambda}_{(0)} W^{\rho}_{(0)}$$

$$-\left(W^{\mu}_{(0)} T_{\eta\nu} W^{\nu}_{(1)}\right)\left[W^{\lambda}_{(0)} W^{\rho}_{(1)} + W^{\rho}_{(0)} W^{\lambda}_{(1)}\right]$$

$$+\left(W^{\mu}_{(1)} T_{\eta\nu} W^{\nu}_{(1)}\right) W^{\lambda}_{(1)} W^{\rho}_{(1)}$$

$$-\left(W^{\mu}_{(0)} T_{\eta\nu} W^{\nu}_{(2)}\right)\left[W^{\lambda}_{(0)} W^{\rho}_{(2)} + W^{\rho}_{(0)} W^{\lambda}_{(2)}\right]$$

$$-\left(W^{\mu}_{(0)} T_{\eta\nu} W^{\nu}_{(3)}\right)\left[W^{\lambda}_{(0)} W^{\rho}_{(3)} + W^{\rho}_{(0)} W^{\lambda}_{(3)}\right]$$

$$+\left(W^{\mu}_{(1)} T_{\eta\nu} W^{\nu}_{(2)}\right)\left[W^{\lambda}_{(1)} W^{\rho}_{(2)} + W^{\rho}_{(1)} W^{\lambda}_{(2)}\right]$$

$$+\left(W^{\mu}_{(1)} T_{\eta\nu} W^{\nu}_{(3)}\right)\left[W^{\lambda}_{(1)} W^{\rho}_{(3)} + W^{\rho}_{(1)} W^{\lambda}_{(3)}\right]$$

$$\left(W_{(2)}^{\eta}\, T_{\eta\nu}\, W_{(2)}^{\nu}\right) W_{(2)}^{\lambda}\, W_{(2)}^{\rho}$$

$$+ \left(W_{(2)}^{\eta}\, T_{\eta\nu}\, W_{(3)}^{\nu}\right) \left[W_{(2)}^{\lambda}\, W_{(3)}^{\rho} + W_{(2)}^{\rho}\, W_{(3)}^{\lambda}\right]$$

$$+ \left(W_{(3)}^{\eta}\, T_{\eta\nu}\, W_{(3)}^{\nu}\right) W_{(3)}^{\lambda}\, W_{(3)}^{\rho}$$

DIAGONALIZATION OF THE STRESS-ENERGY TENSOR

$$\mathcal{E}_{\eta\nu} = \mathrm{Tr}\left[\vec{n} \cdot f_{\mu\nu} - \vec{\ell} \cdot {}^{*}f_{\eta\nu}\right]$$

$$f_{\eta\nu} = f_{\eta\nu}^{a}\, \sigma^{a} \qquad \vec{n} = n^{a} \sigma^{a} \qquad \vec{\ell} = \ell^{a} \sigma^{a}$$

σ^{a} : PAULI MATRICES.

a : 1, 2, 3.

$$\vec{n} = \left(\cos\theta_1 \,,\; \cos\theta_2 \,,\; \cos\theta_3 \right)$$

$$\vec{\ell} = \left(\cos\beta_1 \,,\; \cos\beta_2 \,,\; \cos\beta_3 \right)$$

$$\sum_{a=1}^{3} \cos^2\theta_a = 1 \qquad \sum_{a=1}^{3} \cos^2\beta_a = 1$$

IN ISOSPACE UNDER A LOCAL SU(2) TRANSFORMATION $S^{-1}\vec{n}\,S$

AND $f_{\eta\nu} \longrightarrow S^{-1} f_{\eta\nu}\, S$.

SIMILAR FOR $\vec{\ell}$ AND $^{*}f_{\eta\nu}$.

$\Rightarrow$ $\Sigma_{\eta\nu}$ MANIFESTLY LOCAL GAUGE INVARIANT.

NEXT, ONE MORE DUALITY TRANSFORMATION:

$$\epsilon_{\mu\nu} = \cos\alpha_d \, \varepsilon_{\mu\nu} - \sin\alpha_d \, {}^*\varepsilon_{\mu\nu} .$$

$$\epsilon_{\mu\nu} \, {}^*\epsilon^{\mu\nu} = 0$$

$$\rightarrow \epsilon_{\mu\lambda} \, {}^*\epsilon^{\mu\nu} = 0$$

$$\tan(2\alpha_d) = - \epsilon_{\mu\nu} \, {}^*\epsilon^{\mu\nu} / \epsilon_{\lambda\rho} \epsilon^{\lambda\rho}$$

WE USE $\epsilon_{\mu\nu}$ TO CONSTRUCT A NEW ORTHONORMAL TETRAD.

$$T_\alpha{}^\mu$$

$$T_0{}^{\mu} = (\text{NORMALIZ. FACTOR})\ \epsilon^{\mu\lambda}\epsilon_{\rho\lambda} X_d{}^{\rho}$$

$$T_1{}^{\mu} = (\ \text{NF}\ )\ \epsilon^{\mu\rho} X_{d\rho}$$

$$T_2{}^{\mu} = (\ \text{NF}\ )\ {}^*\epsilon^{\mu\rho} Y_{d\rho}$$

$$T_3{}^{\mu} = (\ \text{NF}\ )\ {}^*\epsilon^{\mu\lambda}\, {}^*\epsilon_{\rho\lambda} Y_d{}^{\rho}.$$

UNDETERMINED VARIABLES:

$\theta_1 \quad \theta_2 \quad \beta_1 \quad \beta_2 \qquad \phi_d \quad \Psi_d$

DETERMINE THE SKELETONS

DETERMINE THE VECTORS ON BLADES 1 AND 2.

DIAGONALIZATION CONDITIONS.

$$T_{01} = T_0^{\mu} T_{\mu\nu} T_1^{\nu} = 0.$$

$$T_{02} = T_0^{\mu} T_{\mu\nu} T_2^{\nu} = 0$$

$$T_{03} = T_0^{\mu} T_{\mu\nu} T_3^{\nu} = 0$$

$$T_{12} = T_1^{\mu} T_{\mu\nu} T_2^{\nu} = 0$$

$$T_{13} = T_1^{\mu} T_{\mu\nu} T_3^{\nu} = 0$$

$$T_{23} = T_2^{\mu} T_{\mu\nu} T_3^{\nu} = 0.$$

SUMMARY

- INTERNAL LOCAL GROUP OF SYMMETRIES $SU(2) \times U(1)$ ISOMORPHIC TO LOCAL GROUP OF SPACETIME SYMMETRIES

- LOCAL GAUGE GROUP NON-ABELIAN ISOMORPHIC TO LOCAL GROUP OF INERTIAL FRAMES.

- NEW LOCAL GAUGE INVARIANTS ALLOW FOR BLOCK GAUGE INVARIANT DIAGONALIZ. STRESS-ENERGY TENSOR

SINCE INTERNAL LOCAL GROUP OF TRANSFORM. ARE ISOMORPHIC TO SPACETIME LOCAL GROUPS OF TRANSFORM.

IS IT POSSIBLE TO ASSOCIATE SPACETIMES TO MICROPARTICLES?

CAN WE ESTABLISH SIMILAR RESULTS FOR

$SU(3) \times SU(2) \times U(1)$?

EINSTEIN-MAXWELL-YANG-MILLS

SU(3)

$$R_{\mu\nu} = T_{\mu\nu}^{(SU(3))} + T_{\mu\nu}^{(SU(2))} + T_{\mu\nu}^{(em)}$$

$$f^{\mu\nu}{}_{;\nu} = 0$$

$$*f^{\mu\nu}{}_{;\nu} = 0$$

$$\left.\begin{array}{l} f^{k\mu\nu}{}_{|\nu} = 0 \\ *f^{k\mu\nu}{}_{|\nu} = 0 \end{array}\right\} \quad \begin{array}{l} k = 1 \ldots 3 \\ SU(2) \end{array}$$

$$\left.\begin{array}{l} f^{p\mu\nu}{}_{|\nu} = 0 \\ *f^{p\mu\nu}{}_{|\nu} = 0 \end{array}\right\} \quad \begin{array}{l} p = 1 \ldots 8 \\ SU(3) \end{array}$$

| gauge COVARIANT DERIVATIVE

$$T_{\eta\nu}^{SU(3)} = \sum_{\rho=1}^{8} \left[f^{\rho}_{\eta\lambda} \cdot f_{\nu}^{\rho\lambda} + {}^{*}f^{\rho}_{\eta\lambda} \, {}^{*}f_{\nu}^{\rho\lambda} \right] \tag{2}$$

$$T_{\eta\nu}^{SU(3)} = T_{\eta\nu}^{T} + T_{\eta\nu}^{V} + T_{\eta\nu}^{U}$$

$$f^{T}_{\eta\lambda} = \frac{1}{\sqrt{2}} f^{3}_{\eta\lambda}$$

$$f^{V}_{\eta\lambda} = \sqrt{2}\left(\frac{1}{2} f^{8}_{\eta\lambda} + \frac{1}{\sqrt{8}} f^{3}_{\eta\lambda}\right)$$

$$f^{U}_{\eta\lambda} = \sqrt{2}\left(\frac{1}{2} f^{8}_{\eta\lambda} - \frac{1}{\sqrt{8}} f^{3}_{\eta\lambda}\right)$$

$$T^{\mathrm{T}}_{\eta\nu} = \sum_{p=1}^{2} \left[f^{p}_{\eta\lambda} f^{p\,\lambda}_{\nu} + {}^{*}f^{p}_{\eta\lambda}\, {}^{*}f^{p\,\lambda}_{\nu} \right] + \left[f^{\mathrm{T}}_{\eta\lambda} f^{\mathrm{T}\,\lambda}_{\nu} + {}^{*}f^{\mathrm{T}}_{\eta\lambda}\, {}^{*}f^{\mathrm{T}\,\lambda}_{\nu} \right]$$

$$T^{\vee}_{\eta\nu} = \sum_{p=4}^{5} \left[f^{p}_{\eta\lambda} f^{p\,\lambda}_{\nu} + {}^{*}f^{p}_{\eta\lambda}\, {}^{*}f^{p\,\lambda}_{\nu} \right] + \left[f^{\vee}_{\eta\lambda} f^{\vee\,\lambda}_{\nu} + {}^{*}f^{\vee}_{\eta\lambda}\, {}^{*}f^{\vee\,\lambda}_{\nu} \right]$$

$$T^{\cup}_{\eta\nu} = \sum_{p=6}^{7} \left[f^{p}_{\eta\lambda} f^{p\,\lambda}_{\nu} + {}^{*}f^{p}_{\eta\lambda}\, {}^{*}f^{p\,\lambda}_{\nu} \right] + \left[f^{\cup}_{\eta\lambda} f^{\cup\,\lambda}_{\nu} + {}^{*}f^{\cup}_{\eta\lambda}\, {}^{*}f^{\cup\,\lambda}_{\nu} \right]$$

$$SO(3)/SO(2) \simeq S^2$$

$$SU(3)/SU(2) \simeq S^5$$

MANIFOLD NOT A GROUP

quotient space

WE CONSTRUCT THE TETRAD SKELETON FOLLOWING THE SU(2) EXAMPLE.

IF $S_{ROT} = \exp\left\{\frac{i}{2}\sum_{i=1}^{3}\theta_i X_i\right\}$

$X_i = \sigma^i$ "PAULI" MAT. in SU(3)

$$S = \exp\left\{\frac{i}{2}\sum_{i=4}^{8}\theta_i X_i\right\}$$

X_i, $i = 4 \dots 8$ generators SU(3)

COSET ELEMENT

$$S \cdot S_{ROT} \in SU(3)$$

FOLLOWING THE TETRAD FOR SU(2)

TETRAD FOR SU(3)

$$Q^{\mu}_{(1)} = \Omega^{\mu\lambda}\,\Omega_{\rho\lambda}\,X^{\rho}$$

$$Q^{\mu}_{(2)} = \sqrt{\frac{-Q_{YM}}{2}}\;\Omega^{\mu\lambda}\,X_{\lambda}$$

$$Q^{\mu}_{(3)} = \sqrt{\frac{-Q_{YM}}{2}}\;{*\Omega}^{\mu\lambda}\,Y_{\lambda}$$

$$Q^{\mu}_{(4)} = {*\Omega}^{\mu\lambda}\;{*\Omega}_{\rho\lambda}\,Y^{\rho}$$

$$Q_{YM} = \Omega_{\mu\nu}\,\Omega^{\mu\nu}$$

$$\Omega_{\mu\nu}\;{*\Omega}^{\mu\nu} = 0 \Rightarrow \Omega_{\mu\alpha}\;{*\Omega}^{\mu\beta} = 0$$

EXAMPLES INVARIANT SU(3)

SU(3) SKELETONS

$$\varepsilon_{\eta\nu} = [f^{a}_{\sigma\tau}\, f^{a}_{\eta\nu}]\, [f^{b\sigma\rho}{}_{;\rho}\, {}^{*}f^{b\tau\lambda}{}_{;\lambda}]$$

$$\varepsilon_{\eta\nu} = [{}^{*}f^{a}_{\sigma\tau}\, {}^{*}f^{a}_{\eta\nu}]\, [f^{b\sigma\rho}\, {}^{*}f^{b\tau\lambda} - {}^{*}f^{b\sigma\rho}\, f^{b\tau\lambda}]\, T_{\rho\lambda}$$

$$= [f^{a}_{\sigma\eta}\, f^{a}_{\tau\nu} - f^{a}_{\tau\eta}\, f^{a}_{\sigma\nu}] \cdot U^{\sigma\tau}$$

$\varepsilon_{\eta\nu} =$

$a = 1 \ldots 8$

SU(3) GAUGE VECTOR

$$X^\sigma = Y^\sigma = \mathrm{Tr}\left[\Sigma^{\alpha\beta} S_\alpha^{\ \rho} S_\beta^{\ \lambda} \cdot \right.$$

$$\left. \cdot {}^*\epsilon_\rho^{\ \sigma} \, {}^*\epsilon_{\lambda\zeta} A^\zeta \right]$$

$S_\alpha^{\ \rho}$: SU(2) TETRADS

$$S_\alpha^{\ \rho} S_\beta^{\ \lambda} \, {}^*\epsilon_\rho^{\ \sigma} \, {}^*\epsilon_{\lambda\zeta} =$$

$$= S_\alpha^{\ [\rho} S_\beta^{\ \lambda]} \, {}^*\epsilon_\rho^{\ \sigma} \, {}^*\epsilon_{\lambda\zeta}$$

CONSERVED UNDER SU(2) x U(1) LOCAL GAUGE TRANSFORMATIONS.

EQUIVALENT SUBALGEBRAS TO SU(3)

GENERATORS OF THE T-SUBALG.

$\sigma^{1}_{+T} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\sigma^{2}_{+T} = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\sigma^{3}_{+T} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

GENERATORS OF THE U-SUBALG.

$\sigma^{1}_{+U} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$	$\sigma^{2}_{+U} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$	$\sigma^{3}_{+U} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

GENERATORS OF THE V-SUBALG.

$\sigma^{1}_{+V} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$	$\sigma^{2}_{+V} = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$	$\sigma^{3}_{+V} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

EXCHANGE ALGEBRAS AMONG EACH OTHER

$S_{TU}=\begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix}$	$S_{VT}=\begin{pmatrix}-1&0&0\\0&0&-1\\0&-1&0\end{pmatrix}$	$S_{UV}=\begin{pmatrix}0&-1&0\\-1&0&0\\0&0&-1\end{pmatrix}$
$S_{UT}=\begin{pmatrix}0&1&0\\0&0&1\\1&0&0\end{pmatrix}$	$S_{TV}=\begin{pmatrix}-1&0&0\\0&0&-1\\0&-1&0\end{pmatrix}$	$S_{VU}=\begin{pmatrix}0&-1&0\\-1&0&0\\0&0&-1\end{pmatrix}$

$$S_{TU}\,\sigma^{1}_{+T}\,S_{UT}=\sigma^{1}_{+U}$$

$$S_{TU}\,\sigma^{2}_{+T}\,S_{UT}=\sigma^{2}_{+U}$$

EXAMPLE

$$S_{TU}\,\sigma^{3}_{+T}\,S_{UT}=\sigma^{3}_{+U}$$

$$\sigma_T^0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \sigma_U^0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \sigma_V^0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$S_{TU} \, \sigma_T^0 \, S_{UT} = \sigma_U^0$$

$$S_{VT} \, \sigma_V^0 \, S_{TV} = \sigma_T^0$$ EXAMPLES

$$S_{UV} \, \sigma_U^0 \, S_{VU} = \sigma_V^0$$

THE OBJECT $\Sigma^{\alpha\beta}$ WILL BE TRANSFORMED BY SU(3) DISCRETE UNITARY TRANSFOR-MATIONS BETWEEN SUBALGEBRAS

$$\Sigma^{\alpha\beta} = i\left(\sigma^{\alpha\beta} + \sigma^{\dagger\,\alpha\beta}\right)$$

$$\sigma^{\alpha\beta} = \sigma_+^\alpha \sigma_-^\beta - \sigma_+^\beta \sigma_-^\alpha$$

$$\sigma^{\dagger\,\alpha\beta} = \sigma_-^\alpha \sigma_+^\beta - \sigma_-^\beta \sigma_+^\alpha$$

$$\sigma_\pm^\alpha = (\mathbb{1}, \pm\sigma^i)$$

σ^i PAULI MATRICES $i = 1 \cdots 3$

$$i\left(\sigma^{\alpha\beta} + \sigma^{\dagger\,\alpha\beta}\right) = \begin{cases} 0 & \text{if } \alpha = 0, \beta = i \\ 4\epsilon^{ijk}\sigma^k & \text{if } \alpha = i, \beta = j \end{cases}$$

$\sigma^{\alpha\beta}$ AND $\sigma^{\dagger\alpha\beta}$

$$\Sigma^{\alpha\beta} = i\left(\sigma^{\alpha\beta} - \sigma^{\dagger\alpha\beta}\right)$$

$$\sigma^{\alpha\beta} - \sigma^{\dagger\alpha\beta} = \begin{cases} -i\,\sigma^{i} & \text{if } \alpha = 0,\ \beta = i \\ 0 & \text{if } \alpha = i,\ \beta = j. \end{cases}$$

$$S^{-1}_{(1/2)}\,\sigma^{\alpha}_{\pm}\,S_{(1/2)} = \Lambda^{\alpha}{}_{\gamma}\,\sigma^{\gamma}_{\pm}$$

UNDER THE $\left(\frac{1}{2}, 0\right)$ AND $\left(0, \frac{1}{2}\right)$ SPINOR REPRESENTATIONS OF THE LORENTZ GROUP.

CONCLUSIONS

1) CAN CONSTRUCT SU(3) INVARIANT TETRAD SKELETONS

2) CAN CONSTRUCT SU(3) GAUGE VECTORS USING NESTED SU(2) TETRAD VECTORS AND SU(3) $A^\mu = A^{a\mu} X^a$

$a = 1 \ldots 8$

3) CAN TRANSFORM OBJECT $\Sigma^{\alpha\beta}$ FROM ONE SUBALGEBRA (U, V, T) INTO ANOTHER USING UNITARY DISCRETE SU(3) TRANSFORMATION

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4) The key to understand the "link" between the quantum state multiplets and the local gauge transformations between each other as we move from one quantum state into the other inside a multiplet is to visualize that this is isomorphically reproduced by local gauge transfs. inside the local orthogonal planes of symmetry and stress-energy diagonal. one and two. (planes orthogonal)

5) mappings $SU(3) \rightarrow (\otimes LB1)^{8}$

$SU(3) \rightarrow (\otimes LB2)^{8}$

are isomorphic.

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