New results on gauge field decomposition in SU(3) gluodynamics

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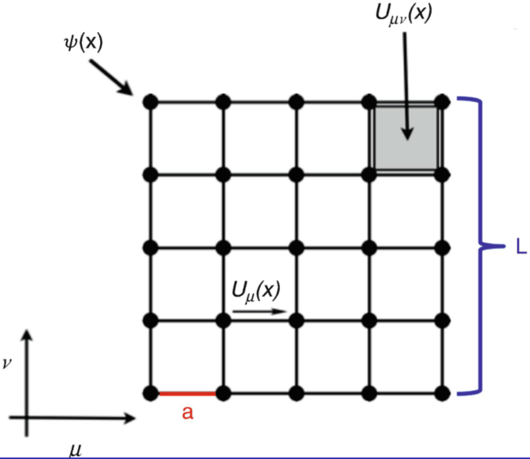
Plan

1. Introduction (Lattice, Confinement, MA gauge)
2. Decomposition of the gauge field
   $A_\mu(x) = A_\mu^{mon} + A_\mu^{mod}$
3. Decomposition of the static potential
   $V(r) = V_{mon}(r) + V_{mod}(r)$
4. Conclusions and Outlook

Lattice regularization

$$U_\mu(x) = P\exp\left(i\int_{C_{x,x+\hat{\mu}}} A_\mu(s)ds\right) \approx 1 + iaA_\mu(x) + O(a^2)$$

$$S_G = \beta\sum_{x,\mu<\nu}\left(1 - \frac{1}{N}\mathrm{ReTr}\,U_{\mu\nu}\right) = a^4\left(\frac{\beta}{2N_c}\sum_{x,\mu<\nu}\mathrm{Tr}[F_{\mu\nu}^2] + \mathcal{O}(a^2)\right), \quad \beta = \frac{2N_c}{g^2}$$

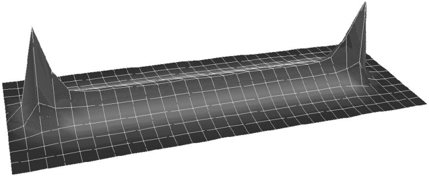


$$\langle O \rangle = \frac{1}{Z} \int \mathscr{D}U e^{-S_G(U)} O(U)$$

There is no problem of gauge fixing when $O(U)$ is gauge invariant

Examples of gauge non-invariant observables:
- Gluon, quark, ghost propagators
(needed to compare with, e.g. DSE approach)
- Various projected observables in MAG and center gauges
(needed to study respective scenarios of confinement)

Confinement problem



Quark confinement:

- is confirmed experimentally and in lattice calculations
- linear dependence of static quarks interaction potential on a distance between them
- hasn't been proven analytically so far
- one of the approaches - to describe QCD vacuum as a dual superconductor, t'Hooft, 1976, Mandelstam, 1976

Maximal Abelian gauge in SU(3) gluodynamics

Suggested by t'Hooft, 1981 to define color-magnetic monopoles

Gauge fixing functional (breaks $SU(3)$ to $U(1)^2$)

$$F_{MAG} = \frac{1}{12V} \int d^4x \sum_{\mu=1}^{4} \sum_{a\neq 3,8} (A_\mu^a(x))^2$$

$$f^a(A) = \sum_{b\neq 3,8} (\partial_\mu \delta^{ab} - g f^{ab3} A_\mu^3 - g f^{ab8} A_\mu^8) A_\mu^b = 0\,, \quad a \neq 3,8$$

Gauge fixing functional in lattice regularization:

$$F_{MAG}^{latt} = 1 - \frac{1}{12V} \sum_{x,\mu,a=3,8} \mathrm{Tr}\{U_\mu(x) \lambda_a U_\mu^\dagger(x) \lambda_a\} \approx a^2 F_{MAG}$$

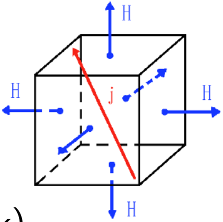
In the Higgs model the t'Hooft-Polyakov monopole has a form of a Dirac monopole in a unitary gauge.
In $SU(N_c)$ gluodynamics MA gauge plays this role.
We search for nonabelian color-magnetic monopoles making three steps (Kronfeld, Laursen, Schierholz, Wiese, 1987)

- to fix MA gauge (problem of Gribov copies)
- to make Abelian projection (decomposition)

$$A_\mu(x) = \sum_{a \neq 3,8} A^a_\mu(x)\lambda_a + A^3_\mu(x)\lambda_3 + A^8_\mu(x)\lambda_8 \equiv A^{offd}_\mu(x) + A^{abel}_\mu(x)$$

- to use the Abelian component $A^{abel}_\mu(x)$ for locatation of Dirac monopoles via procedure introduced for compact $U(1)$ in DeGrand, Toussaint, 1980

Abelian lattice gauge field is defined by diagonal matrix $u_\mu(x) \in U(1) \times U(1)$



$$u_\mu^{aa}(x) = e^{i\theta_\mu^a(x)}, \quad \sum_a \theta_\mu^a(x) = 2\pi n$$

$$\theta_{\mu\nu}^a(x) = \partial_\mu \theta_\nu^a(x) - \partial_\nu \theta_\mu^a(x)$$

$$\theta_{\mu\nu}^a(x) = \bar{\theta}_{\mu\nu}^a(x) + 2\pi m_{\mu\nu}^a(x), \quad \bar{\theta}_{\mu\nu}^a(x) \in (-\pi, \pi)$$

color-magnetic current :

$$k_\mu^a(x) = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}\,\partial_\nu m_{\alpha\beta}^a\,, \qquad \partial_\mu k_\mu^a(x) = 0$$

A decomposition of a gauge field in MAG

Abelian field can be further decomposed into 'monopole' and 'photon' components (names are borrowed from compact $U(1)$)

$$A_\mu^{abel}(x) = A_\mu^{mon}(x) + A_\mu^{phot}(x)$$

$$aA_\mu^{a,mon}(x) \equiv \theta_\mu^{a,mon} = \sum_y D(x-y)\partial_\nu m_{\mu,\nu}(y)$$

The following decomposition was introduced in
Bornyakov, Polikarpov, Schierholz, Suzuki, Syritsyn, 2006

$$A_\mu(x) \;=\; A_\mu^{mod}(x) + A_\mu^{mon}(x)$$

(non-confining) (confining)

$$A_\mu^{mod}(x) = A_\mu^{offd}(x) + A_\mu^{phot}(x)$$

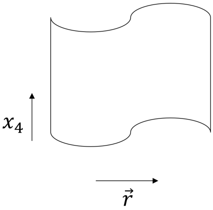
Static quark potential

$$W(r,t) = \frac{1}{3}\mathrm{Tr}P\exp\left(i\oint_C A_\mu(x)dx_\mu\right)$$

$$\to \frac{1}{3}\mathrm{ReTr}\prod_{l\in C} U_l$$

$$\langle W(r,t)\rangle = C_0 e^{-tV(r)} + C_1 e^{-tE_1(r)} + \ldots$$

$$aV(r) = \lim_{t\to\infty}\log\frac{\langle W(r,t)\rangle}{\langle W(r,t+a)\rangle}$$



We measure three types of $\langle W(r,t)\rangle$:

- for nonabelian gauge field $A_\mu(x)$,

- for monopole component $A_\mu^{mon}(x)$,

- for modified component $A^{mod}(x)$

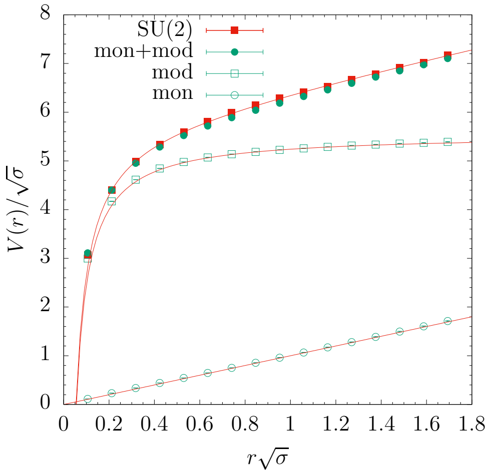
Decomposition of static potential $V(r)$ in $SU(2)$ gluodynamics and in $SU(2)$ QCD

First results on properties of this decomposition:
Bornyakov, Polikarpov, Schierhilz, Suzuki, Syritsyn, 2006
It was found that

$$V(r) \approx V_{mon}(r) + V_{mod}(r)$$

We demonstrated (Bornyakov, Kudrov, Rogalyov, 2021) that in $SU(2)$ gluodynamics the precision of this relation improves when lattice spacing a is decreasing (i.e. in the continuum limit)
Furthermore, we observed this decomposition in lattice $SU(2)$ gluodynamics with improved lattice action (universality) and in lattice $N_c = 2$ QCD.

Results in $SU(2)$ QCD



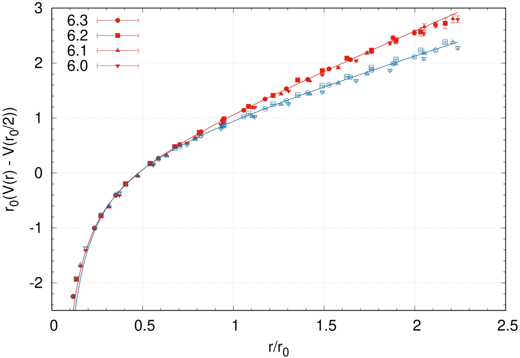
$V_{mon}(r) + V_{mod}(r)$ vs. physical static potential $V(r)$

Bornyakov, Kudrov, Rogalyov, 2021

Interpretation of this result for $V(r)$:

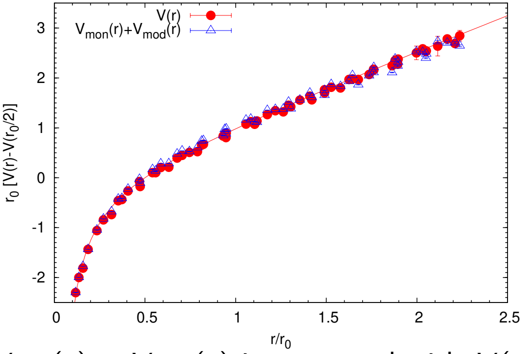
$A_\mu^{mon}(x)$ is responsible for the linear part of $V(r)$, i.e. it is a confining component,

$A_\mu^{mod}(x)$ is responsible for the perturbative part at small r and for hadron string fluctuations at large r, i.e. it is a non-confining component



$V_{mon}(r) + V_{mod}(r)$ is compared with $V(r)$,
results for a few values of lattice spacing $a \in [0.06, 0.09]$ fm
With **'global' minima** of F_{MAG} we find agreement at small r and disagreement at large r
Disagreement comes from low string tension in $V_{mon}(r)$

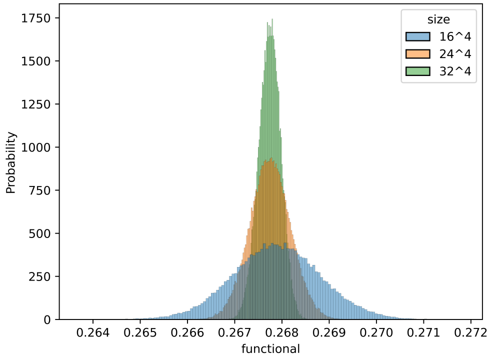
Decomposition of static potential in $SU(3)$ gluodynamics, cont.



$V_{mon}(r) + V_{mod}(r)$ is compared with $V(r)$,

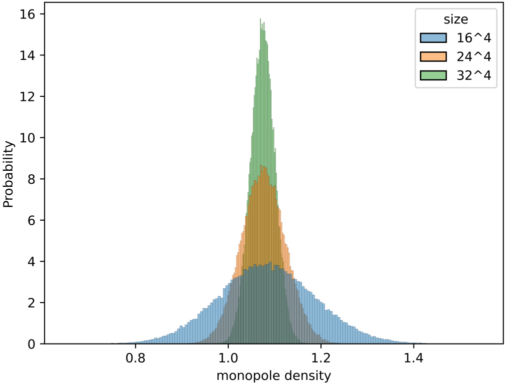
With **randomly chosen minima** (Gribov copies) we find agreement at all r

This is our main result



At one value of a we generated 100 copies fixed to MAG for every independent configuration $\{U_\mu(x)\}$. This figure shows probability to get particular value of the gauge fixing functional F_{MAG}^{latt}. The distribution is of the **gaussian form**, position of its maximum does not depend on the lattice size for $N_s > 16$. The result implies that **in the limit $N_s \to \infty$ all Gribiv copies have same value of F_{MAG}^{latt}.**

Gribov copies



The distribution of the monopole density. It is also of a gaussian form, position of its maximum does not depend on the lattice size for $N_s > 16$. The result gives further support to assumption that **in the limit $N_s \to \infty$ Gribiv copies effects disappear.**

Conclusions

1. In our study of the gauge field decomposition
$$A_\mu(x) = A_\mu^{mon}(x) + A_\mu^{mod}(x) \quad (1)$$
in MA gauge of $SU(3)$ gluodynamics we observed that Gribov copies exist which produce a numerically precise decomposition for the static potential
$$V(r) = V_{mon}(r) + V_{mod}(r) \quad (2)$$
2. Our results indicate that the Gribov copies effect disappears in the infinite volume limit if one averages over all Gribov coplies.
3. As a byproduct, we obtained that $\sigma_{abel} \approx \sigma$ (Abelian dominance, long standing problem) with high precision independent of the volume.

Future plans:

1. We found in $SU(2)$ gluodynamics that at $T > 0$ the decomposition works for the free energy $F_{q\bar{q}}(r)$ of static quark-antiquark pair. We will extend this study to $SU(3)$ case.
2. To compute the gauge field propagators for A^{mon} and A^{mod} including mixed propagator
3. To compute the quark propagator for A^{mon} and A^{mod}
4. to study properties of this gauge field decomposition in QCD (i.e. with quarks)
5. to study decomposition for other observables, in particular, for hadron spectrum