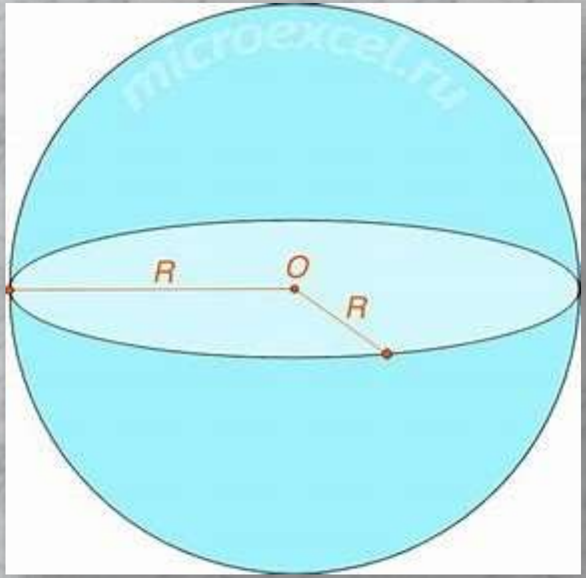
The 22nd Lomonosov Conference
on Elementary Particle Physics

Moscow State University, Moscow, Russia
(21 - 27 August, 2025).

On Radii of Hadrons



Vladimir A. Petrov
A.A. Logunov Institute for High Energy Physics
NRC "Kurchatov Institute", Protvino, RF

Classical proton radius

$$R_{\mathrm{CL}} = \frac{\alpha \hbar c}{M_p c^2} = \frac{\alpha}{M_p}$$

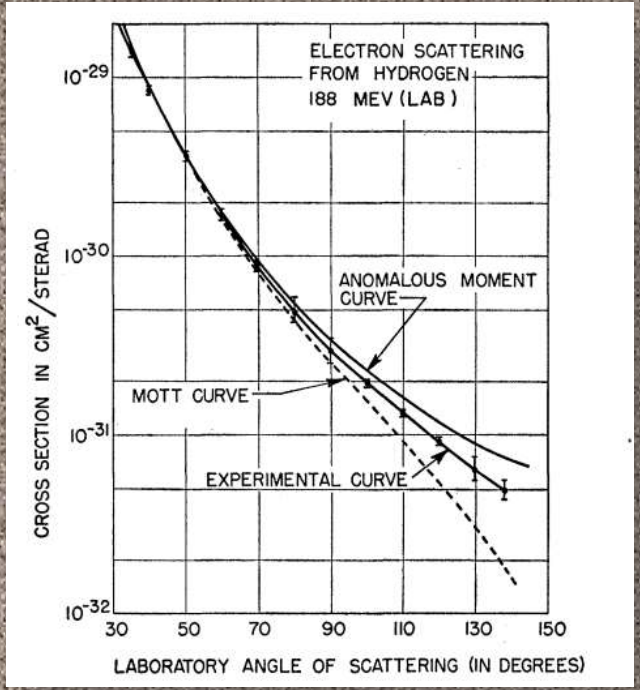
$$R_{CL} \approx 1,5 \cdot 10^{-3} fm$$

Electron Scattering from the Proton*†‡

ROBERT HOFSTADTER AND ROBERT W. MCALLISTER

Department of Physics and High-Energy Physics Laboratory, Stanford University, Stanford, California

(Received January 24, 1955)



- "If the proton were a spherical ball of charge, this rms radius would indicate a true radius of $\mathbf{9.5 \times 10^{-14}}$**cm…" (0.95 fm)**

MEAN-SQUARE CHARGE RADIUS

Citation: S. Navas *et al.* (Particle Data Group), Phys. Rev. D **110**, 030001 (2024)

p CHARGE RADIUS

This is the rms electric charge radius, $\sqrt{\langle r_E^2 \rangle}$.

There are three kinds of measurements of the proton radius: via transitions in atomic hydrogen; via electron scattering off hydrogen; and via muonic hydrogen Lamb shift. Most measurements of the radius of the proton involve electron-proton interactions, the most recent of which is the electron scattering measurement $r_p = 0.831(14)$ fm (XIONG 19), and the atomic-hydrogen value, $r_p = 0.833(10)$ fm (BEZGINOV 19). These agree well with another recent atomic-hydrogen value $r_p = 0.8335(95)$ fm

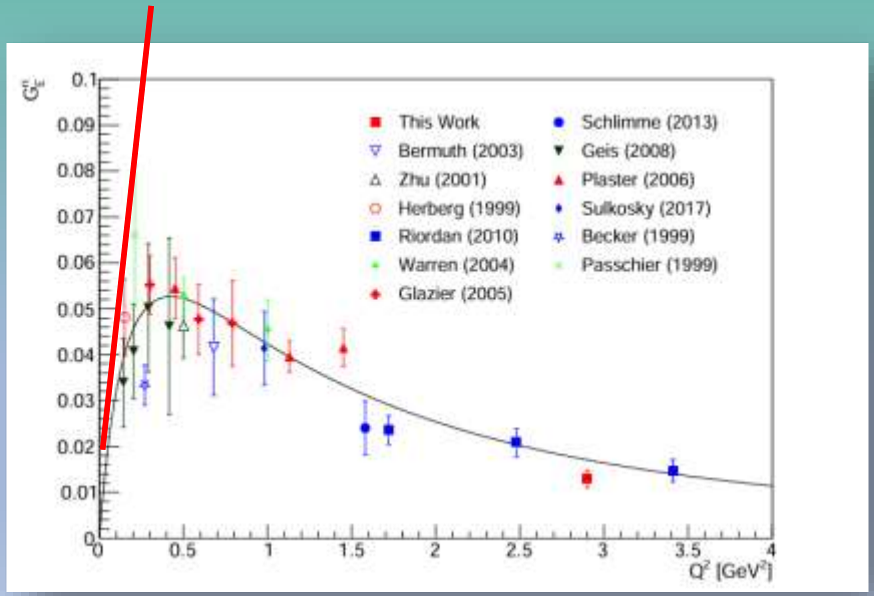
$$\langle r^2 \rangle_{ch,p} = 0.707 \pm \ldots (fm)^2$$

sults available at the time, was 0.8751(61) fm. This differs by 5.6 standard deviations from the muonic hydrogen value, leading to the so-called proton charge radius puzzle. See our 2018 edition (Physical Review **D98** 030001 (2018)) for a further discussion of interpretations of this puzzle. However, reflecting the new electronic measurements, the 2018 CODATA, TIESINGA 21, recommended value is 0.8414(19) fm, and the puzzle appears to be resolved.

$$\langle r^2 \rangle_{ch,n} = -0.115 \pm \ldots (fm)^2$$

$Why\ \langle r^2 \rangle_{ch,n} < 0?$

$$r_{ch}^2 = 6\frac{dF(t)}{dt}\Big|_{t=0}$$



From “charge radius" to physical radius of nucleon

$$F(t) = \sum_{i\in val} e_i \int_0^1 dx\, f^i(x,t)$$

$$r_{ch}^2 = 6\frac{dF(t)}{dt}|_{t=0}$$

$$f^i(x,t) = 2\pi \int_0^\infty db\, bJ_0(b\sqrt{-t})\,\tilde{f}^i(x,b),$$

$$f^i(x,t=0) = f^i(x), i = u,d$$

$$F_p(t) = \frac{2}{3}\int dx\, u_p(x,t) - \frac{1}{3}\int dx\, d_p(x,t),$$

$$F_n(t) = -\frac{1}{3}\int dx\, d_n(x,t) + \frac{2}{3}\int dx\, u_n(x,t).$$

$$\langle r_u^2\rangle = r_{\mathrm{ch},p}^2 + r_{\mathrm{ch},n}^2/2$$

$$\langle r_d^2\rangle = r_{\mathrm{ch},p}^2 + 2r_{\mathrm{ch},n}^2.$$

$$\langle r_u^2\rangle = (0.8056 \pm 0.0011\ \mathrm{fm})^2, \quad \langle r_d^2\rangle = (0.6891 \pm 0.0017\ \mathrm{fm})^2$$

$$\sqrt{r_{nucleon}^2} \approx 0.77 fm.$$

$$\sqrt{r_{ch,proton}^2} \approx 0.84 fm.$$

Physical radii

$$r_{proton}^2 = r_{neutron}^2 \equiv r_{nucleon}^2 = r_{ch,proton}^2 + r_{ch,neutron}^2.$$

MESONS

$$K^+ = u\bar{s}$$

Charge radius

$\langle r \rangle = 0.560 \pm 0.031$ fm

$$\langle r \rangle = \langle r^2 \rangle^{1/2}, \langle r^2 \rangle = 0.313 \pm 0.001\ fm^2$$

$$K^0 = d\bar{s}$$

Mean square charge radius

$\langle r^2 \rangle = -0.077 \pm 0.010$ fm^2

$$F_{K^+}(t) = \frac{2}{3}\int dx\, u_{K^+}(x,t) + \frac{1}{3}\int dx\, \bar{s}_{K^+}(x,t), \quad F_{K^0}(t) = -\frac{1}{3}\int dx\, d_{K^0}(x,t) + \frac{1}{3}\int dx\, \bar{s}_{K^0}(x,t)$$

$$SU_f(3): d_{K^0}(x,t) = u_{K^+}(x,t);\ \bar{s}_{K^0}(x,t) = \bar{s}_{K^+}(x,t)$$

$$\langle r^2 \rangle_{\langle K^+ \rangle} = \frac{2}{3}\bar{r}_u^2(K^+) + \frac{1}{3}\bar{r}_{\bar{s}}^2(K^+);\ \langle r^2 \rangle_{\langle K^0 \rangle} = -\frac{1}{3}\bar{r}_u^2(K^+) + \frac{1}{3}\bar{r}_{\bar{s}}^2(K^+)$$

$$\bar{r}_u^2(K^+) = 0.39 fm^2 \qquad \bar{r}_{\bar{s}}^2(K^+) = 0.16 fm^2$$

$$r_{Kaon}/r_{nucleon} = 0.68 \approx 2/3$$

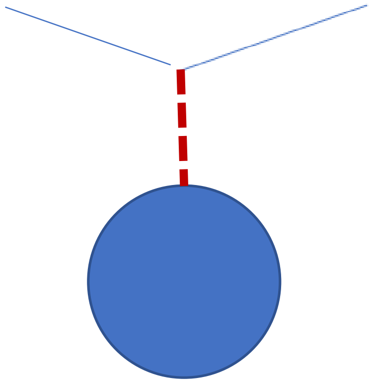
Physical radii:

$$\bar{r}_{K^+}^2 = \bar{r}_{K^0}^2 = 0.275 fm^2 = (0.525\ fm)^2$$

"Gravitational radius" of the proton.

- $$R_{grav}^{proton} = \frac{2Gm_{proton}}{c^2} \approx 2{,}5 \cdot 10^{-38}\ fm$$
- Interaction of gravity with matter

$$\int d^4\sqrt{-g}\, T_{\mu\nu} h^{\mu\nu}, \qquad g^{\mu\nu} \approx \gamma^{\mu\nu} + h^{\mu\nu}$$



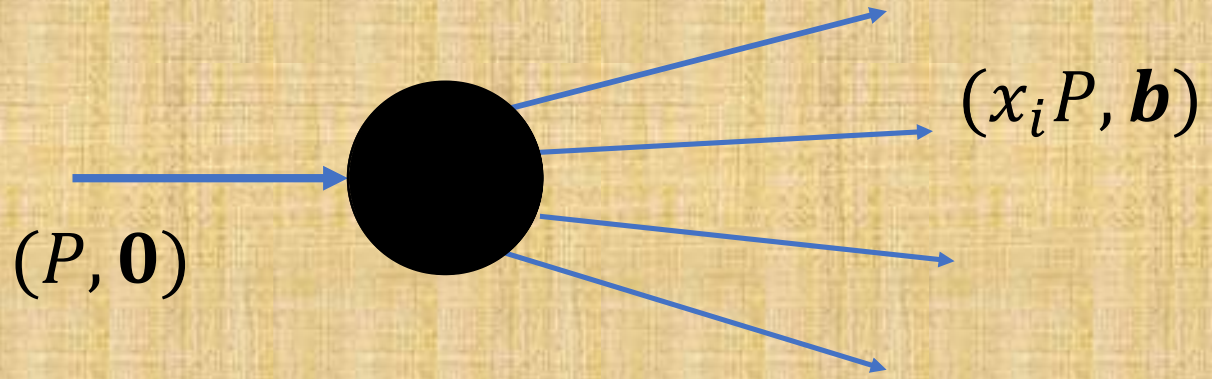
$$\langle p' | T_{\mu\nu} | p \rangle = \sum_j \Lambda_{\mu\nu}^{j}\,(p', p) G_j(t)$$

$$r_g^2 = 6 \frac{dG(t)}{dt}_{t=0} \approx (0.45\ fermi)^2 \to \sum_a \int dx x r_a^2(x) f_a(x, \mu^2)$$

"Mass structure and pressure forces inside the nucleon..."

Strength of materials inside a hadron?!

Parton picture (Bjorken-Feynman-Gribov)



$$\Delta t = \frac{2P}{\sum_i \frac{m_{T,i}^2}{x_i} - M^2}$$

$$\tilde{g}(x, \mathbf{b}) = cg(x, 0)\exp(-b^2/R^2(x))/\pi R^2(x).$$

$$R^2(x \ll 1) \approx 4\alpha'_{\mathcal{P}} ln(1/x) + b_0^2.$$

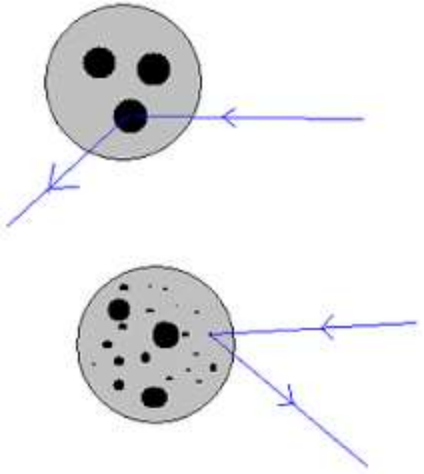
$$\boldsymbol{\alpha_{\mathcal{P}}(\mathbf{t}) \approx 1 + \Delta + \alpha'_{\mathcal{P}}(0)t + \cdots}$$

$$\langle R^2 \rangle = \int dx\, \tilde{g}(x) R^2(x) = \sum_n n \tilde{g}_n R_n^2$$

$$? \leftrightarrow \left(r_{nucleon}^{val}\right)^2$$

Important Digression: Missing elements of partonic description

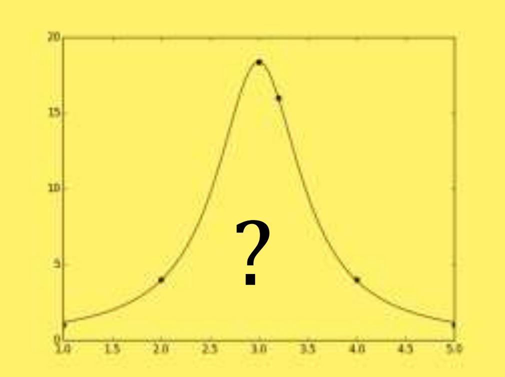
$$\tilde{g} = \sum_n n\tilde{g}_n$$



$$w = \sum_n \tilde{g}_n$$

Ersatz:

$$w \to \sum_n n\tilde{g}_n / \langle n \rangle$$



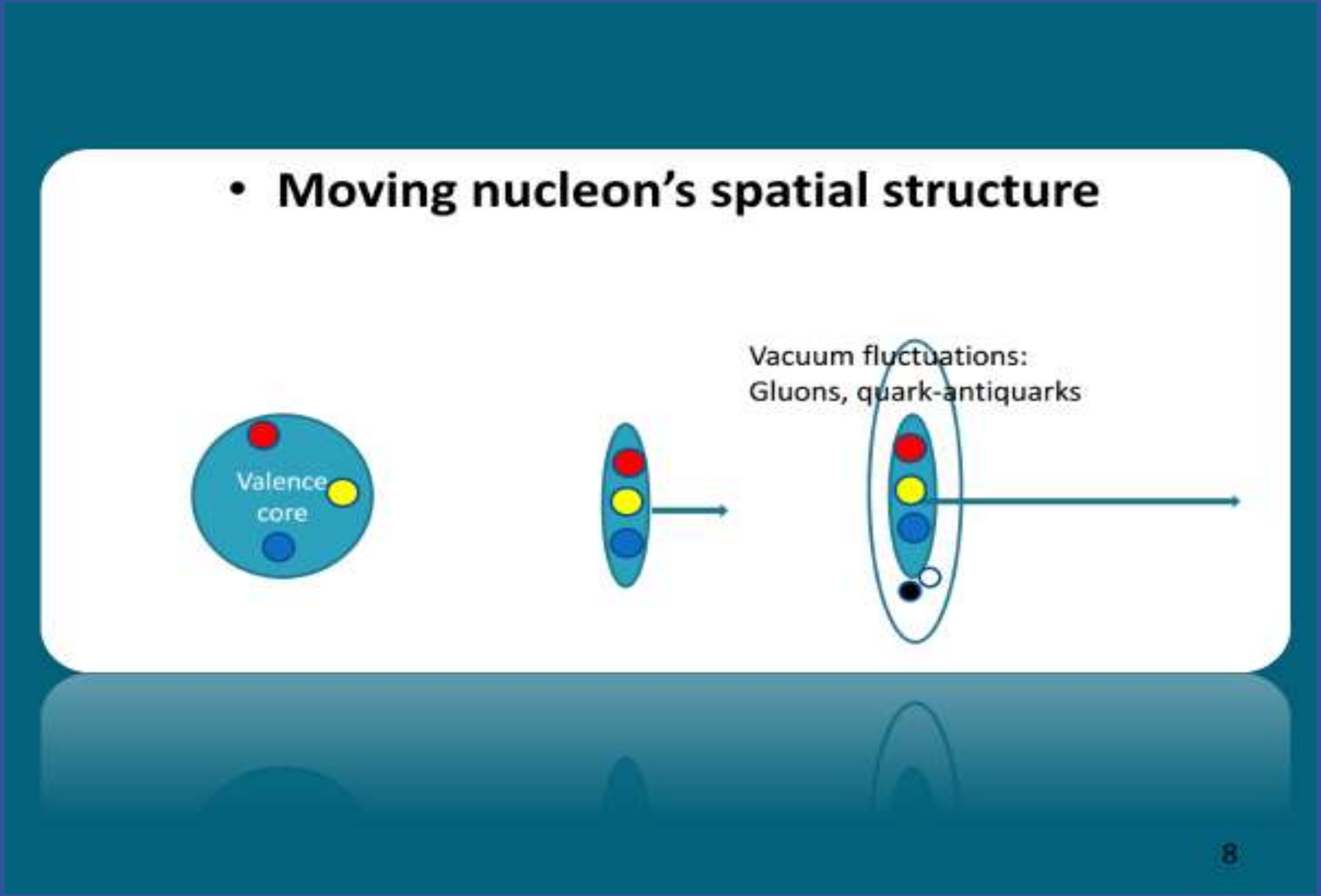
$$\tilde{g}_n \sim \delta(n - \langle n \rangle)$$

$$\langle R^2 \rangle = \int dx\, \tilde{g}(x) R^2(x) = \sum_n n\tilde{g}_n R_n^2$$

$$\langle R^2 \rangle_{true} = \sum_n \tilde{g}_n R_n^2$$

$$\langle R^2 \rangle_{ersatz} = \frac{\sum_n n\tilde{g}_n R_n^2}{\langle n \rangle}$$

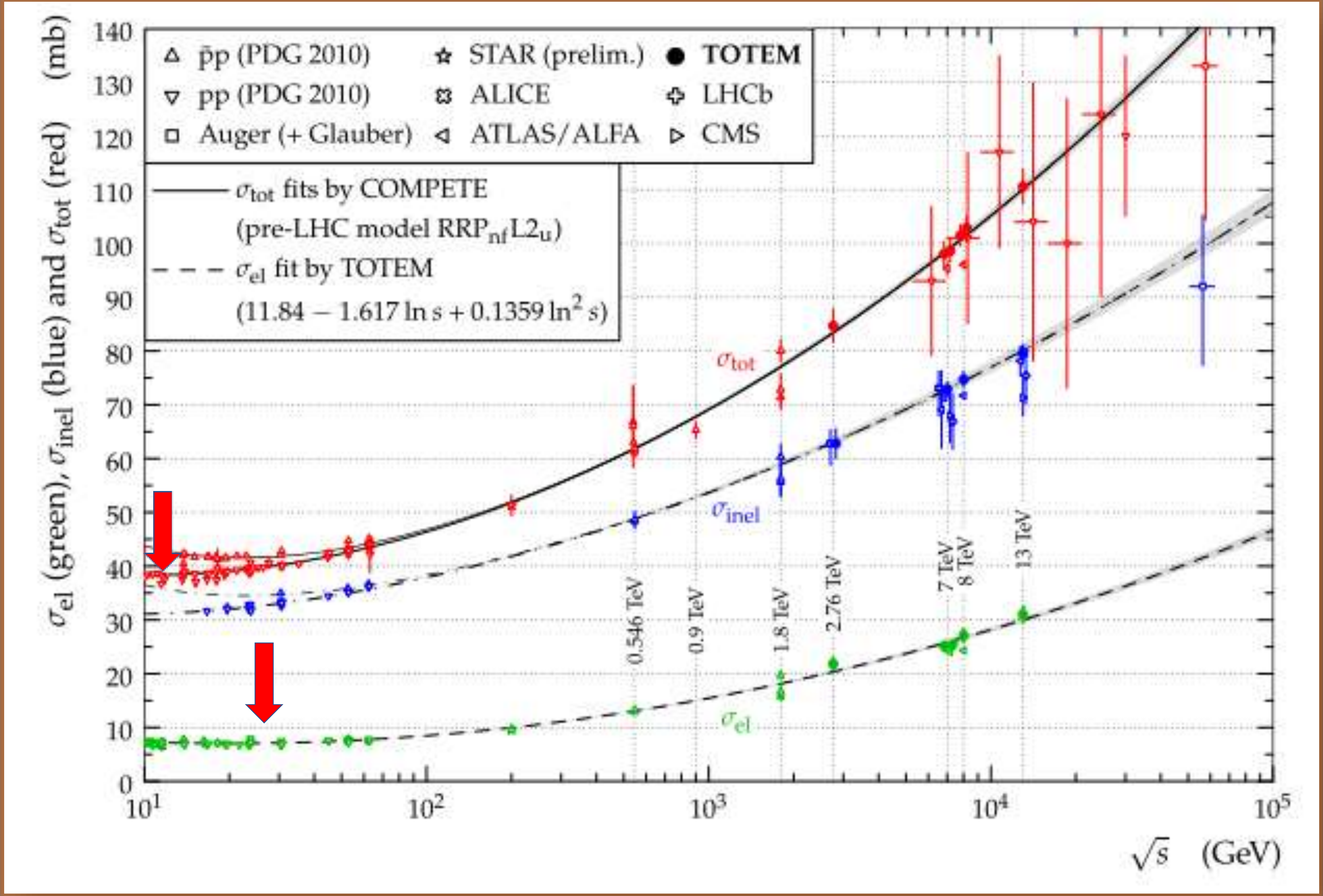
$$\langle n \rangle = \sum_n \tilde{g}_n n$$



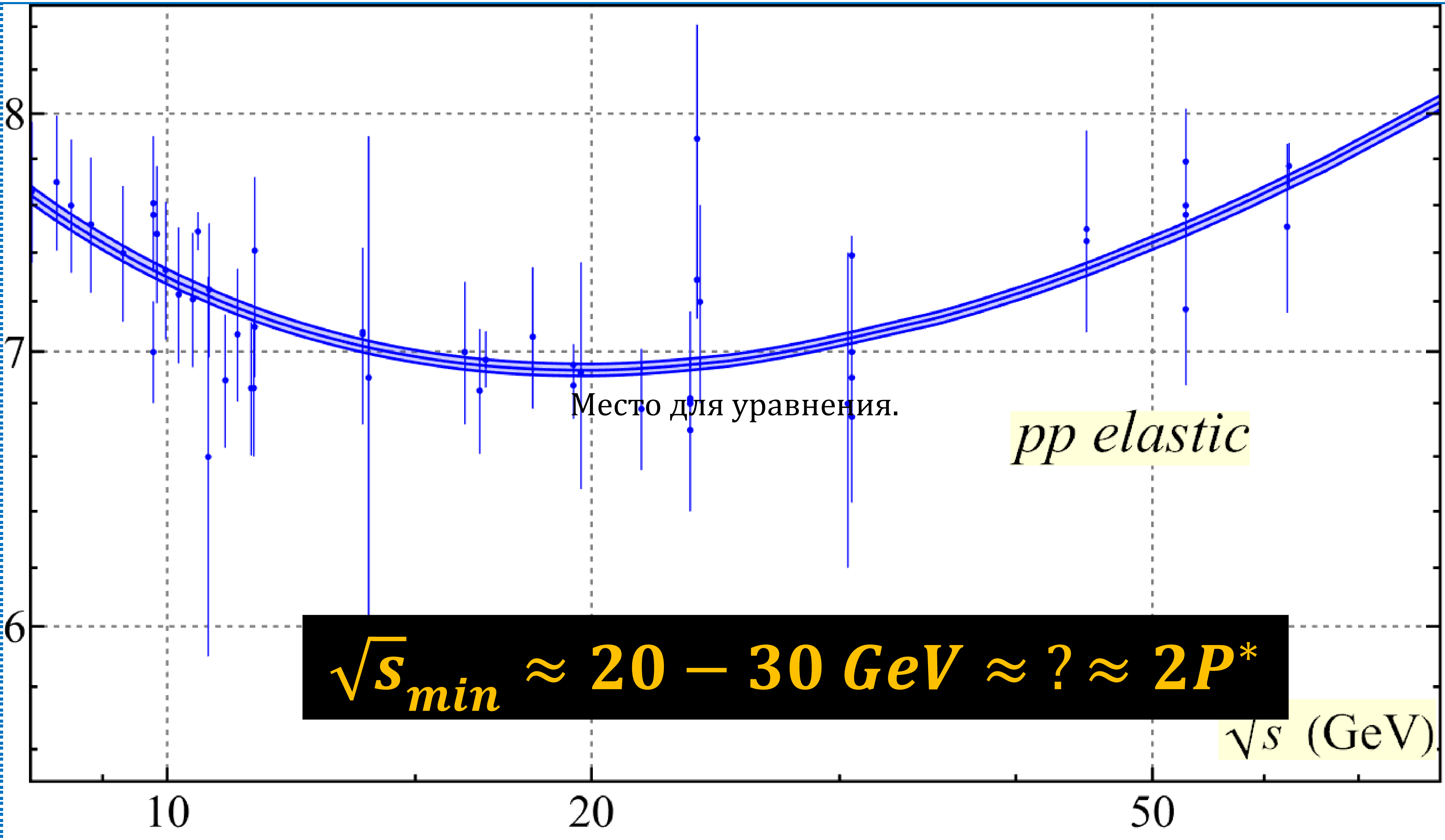
$$P = P^* \equiv \Lambda \exp\{(b_N^2 - b_0^2)/[4\alpha'_{\mathcal{P}}(0)\gamma(\Delta ln(P^*/\Lambda)]\}$$

$$\gamma(x) = \frac{e^x}{e^x - 1} - \frac{1}{x}.$$

$$\boldsymbol{P^* \approx O(10)\mathrm{GeV}}$$



p̄p (PDG 2010)
pp (PDG 2010)
Auger (+ Glauber)
STAR (prelim.)
ALICE
ATLAS/ALFA
TOTEM
LHCb
CMS
σ_{tot} fits by COMPETE
(pre-LHC model $RRP_{nf}L2_u$)
σ_{el} fit by TOTEM
$(11.84 - 1.617 \ln s + 0.1359 \ln^2 s)$
σ_{el} (green), σ_{inel} (blue) and σ_{tot} (red) (mb)
σ_{tot}
σ_{inel}
σ_{el}
0.546 TeV
0.9 TeV
1.8 TeV
2.76 TeV
7 TeV
8 TeV
13 TeV
$\sqrt{s}$ (GeV)



$$\sqrt{s}_{min} \approx 20-30\ GeV \approx ? \approx 2P^*$$

On the spatial size of bodies

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = c^2 d\tau^2 - dl^2$$

$$d\tau = \sqrt{g_{00}}dt + \frac{g_{0i}dx^i}{c\sqrt{g_{00}}}$$

$$dl^2 = (-g_{ik} + g_{0i}g_{0k}/g_{00})dx^i dx^k$$

$$dl^2 = -ds^2_{d\tau=0}$$

Distance and time: «a fly in the ointment»

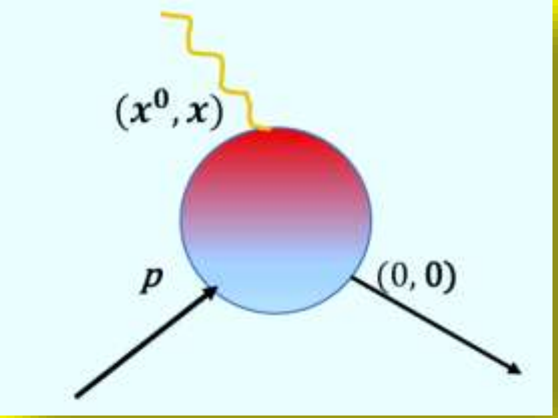
$$F(t) = \frac{2p_\mu}{4m^2 - t}\langle p'|J_\mu|p\rangle = \int d^4x\, e^{iqx}\left\langle \Omega\left|\frac{\delta J_\mu(x)}{\delta\varphi^+(0)}\right|p\right\rangle, t = q^2$$

$$\frac{\delta J_\mu(x)}{\delta\varphi^+(0)} = i\theta(-x^0)\left[J_\mu(x), I^+(0)\right],$$

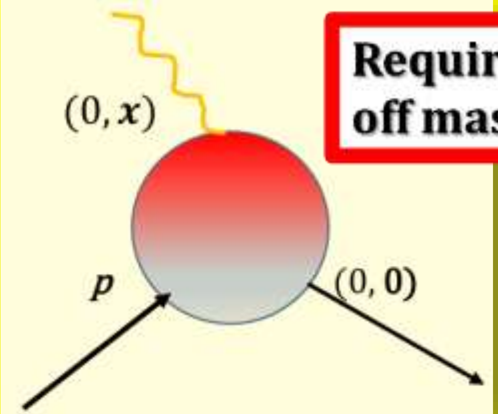
$$I^+(x) = (\partial^2 + m^2)\,\varphi^+(x)$$

$$r_{ch}^2 = \int d\,\boldsymbol{r}\,\boldsymbol{r}^2\,\rho_{ch}(\boldsymbol{r})$$

$$\rho_{ch}(\boldsymbol{r}) = \frac{1}{2m}\int d\,x^0\left\langle \Omega\left|\frac{\delta J_0(x^0,\boldsymbol{r})}{\delta\varphi^+(0)}\right|\boldsymbol{p}=\boldsymbol{0}\right\rangle$$



$$\left\langle \Omega\left|\frac{\delta J_0(x^0,\ \boldsymbol{r})}{\delta\varphi^+(0)}\right|\boldsymbol{p}=\boldsymbol{0}\right\rangle_{c\to\infty} \sim \delta(x^0)$$



Requires going off mass shell!

(Interim) conclusions

1. The physical (geometric) radius of a hadron is not determined directly but is derived from the “form factor radii”.

Two facts that seem to be related:

2. The elastic cross section in proton—proton scattering changes in the energy range **20÷30** GeV from decreasing to increasing.
3. The gluon cloud goes beyond the valence core when the proton reaches an energy of about 10 GeV.
4. All this modulo the problem with a missing partonic requisite.
5. **The main problem: non-simultaneity.**


THANK YOU
FOR YOUR
ATTENTION