TWENTY-SECOND LOMONOSOV CONFERENCE
ON ELEMENTARY PARTICLE PHYSICS

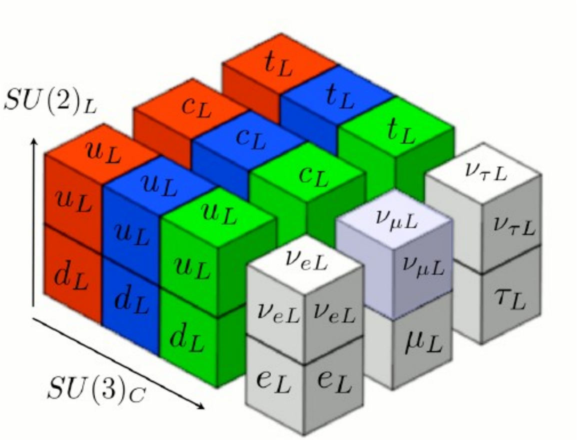
Moscow, August 21-27, 2025

Quark-lepton mixing and the leptonic CP violation

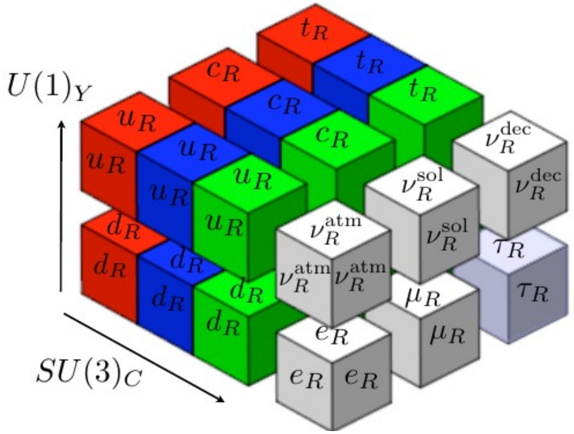
Davide Meloni
Dipartimento di Matematica e Fisica
Universita' Roma Tre

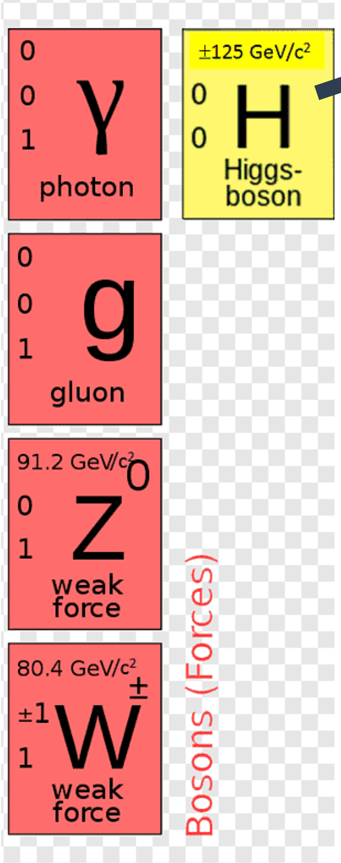
The Standard Model of Particle Physics

Left-handed



Right-handed





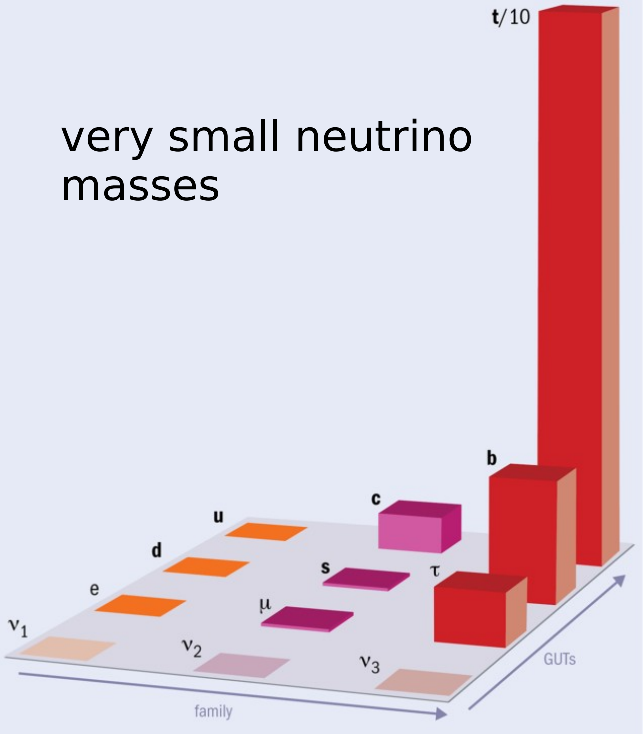
Scalar sector

Gauge boson sector

S.King, talk at Bethe Forum on Modular Flavor Symmetries

The Flavor Problem

Mass hierarchies



very small neutrino masses

$$m_d \ll m_s \ll m_b\,, \quad \frac{m_d}{m_s} = 5.02 \times 10^{-2}\,,$$

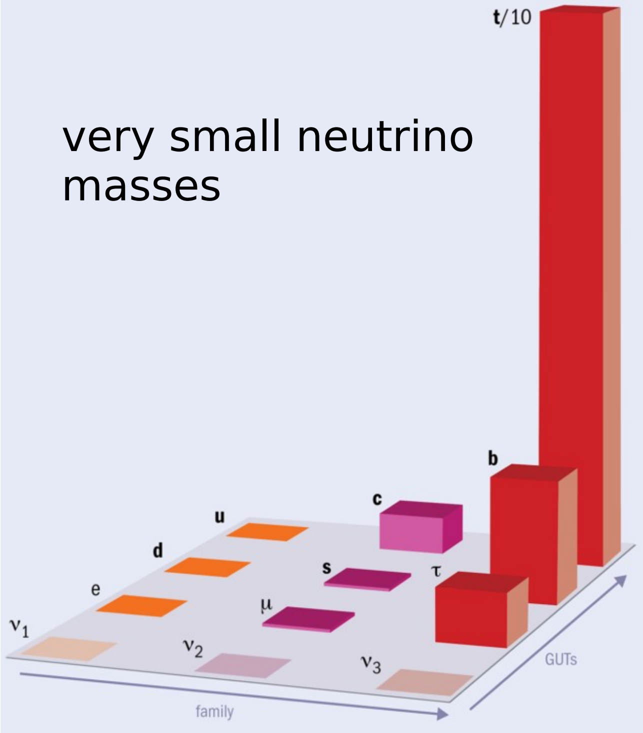
$$m_u \ll m_c \ll m_t\,, \quad \frac{m_u}{m_c} = 1.7 \times 10^{-3}\,,$$

$$\frac{m_s}{m_b} = 2.22 \times 10^{-2}\,, \quad m_b = 4.18 \ \text{GeV};$$

$$\frac{m_c}{m_t} = 7.3 \times 10^{-3}\,, \quad m_t = 172.9 \ \text{GeV};$$

The Flavor Problem

Mass hierarchies



very small neutrino masses

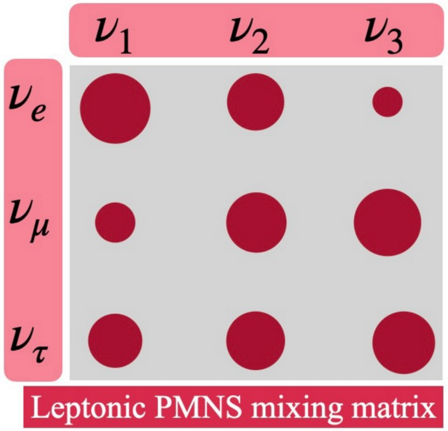
$$m_d \ll m_s \ll m_b\,, \quad \frac{m_d}{m_s} = 5.02 \times 10^{-2}\,,$$

$$m_u \ll m_c \ll m_t\,, \quad \frac{m_u}{m_c} = 1.7 \times 10^{-3}\,,$$

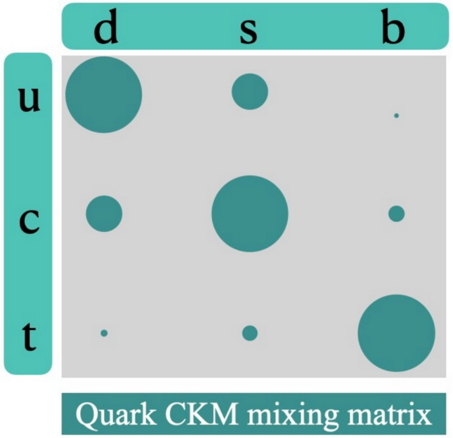
$$\frac{m_s}{m_b} = 2.22 \times 10^{-2}\,, \quad m_b = 4.18\ \text{GeV};$$

$$\frac{m_c}{m_t} = 7.3 \times 10^{-3}\,, \quad m_t = 172.9\ \text{GeV};$$

Fermion mixing



Leptonic PMNS mixing matrix



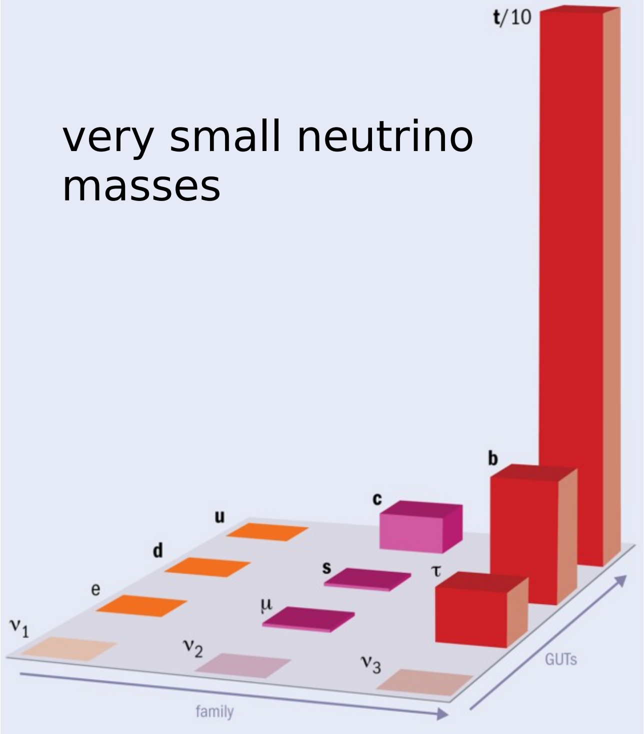
Quark CKM mixing matrix

all mixing are large but the 13 element

almost a diagonal matrix

The Flavor Problem

Mass hierarchies



very small neutrino masses

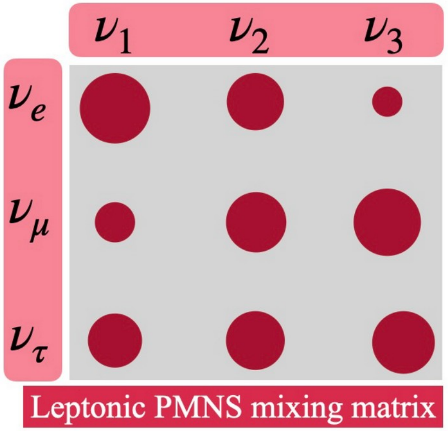
$$m_d \ll m_s \ll m_b\,, \quad \frac{m_d}{m_s} = 5.02 \times 10^{-2}\,,$$

$$m_u \ll m_c \ll m_t\,, \quad \frac{m_u}{m_c} = 1.7 \times 10^{-3}\,,$$

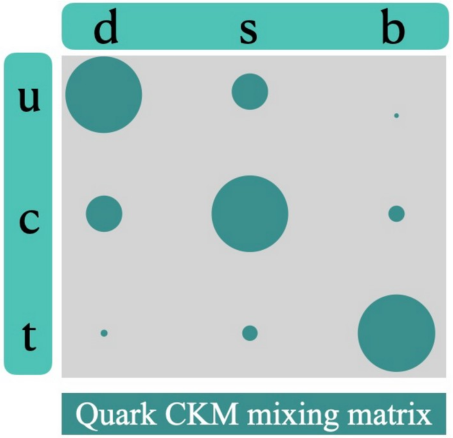
$$\frac{m_s}{m_b} = 2.22 \times 10^{-2}\,, \quad m_b = 4.18\ \text{GeV};$$

$$\frac{m_c}{m_t} = 7.3 \times 10^{-3}\,, \quad m_t = 172.9\ \text{GeV};$$

Fermion mixing



all mixing are large but the 13 element



almost a diagonal matrix

Why are they so different?

Suggested solutions

* Hierarchical Pattern

Froggatt-Nielsen mechanism

$$L \sim \overline{\Psi}_L H \Psi_R \left(\frac{\theta}{\Lambda} \right)^n$$

Too many O(1) coefficients

Works better for small mixing: good for quarks, no for neutrinos

Suggested solutions

* Hierarchical Pattern

Froggatt-Nielsen mechanism

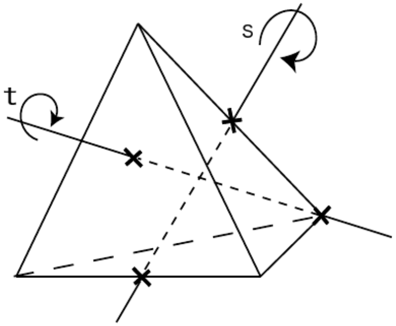
$$L \sim \overline{\Psi}_L H \Psi_R \left(\frac{\theta}{\Lambda}\right)^n$$

Too many O(1) coefficients

Works better for small mixing: good for quarks, no for neutrinos

* mixing angles

elegant explanation: non-Abelian discrete flavour symmetries



Complicated scalar sector. Good for neutrinos, not for quarks

Suggested solutions

* Hierarchical Pattern

Froggatt-Nielsen mechanism

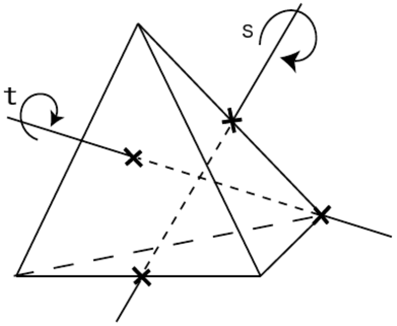
$$L \sim \overline{\Psi}_L H \Psi_R \left(\frac{\theta}{\Lambda}\right)^n$$

Too many O(1) coefficients

Works better for small mixing: good for quarks, no for neutrinos

* mixing angles

elegant explanation:
non-Abelian
discrete flavour symmetries



Complicated scalar sector. Good for neutrinos, not for quarks

Mixings in the lepton and hadron sector are unrelated?

Experimental facts

Quark-Lepton complementarity

$$\theta_{12}^{PMNS} + \theta_{12}^{CKM} \sim \pi/4$$

~33°

~13°

Experimental facts

Quark-Lepton complementarity

$$\theta_{12}^{PMNS} + \theta_{12}^{CKM} \sim \pi/4$$

$\sim 33^{o}$ $\sim 13^{o}$

$$\theta_{23}^{PMNS} + \theta_{23}^{CKM} \sim \pi/4$$

$\sim 45^{o}$ $\sim 2^{o}$

- appealing from a theoretical and phenomenological point of view

- no clue on which kind of symmetry could be responsible for them

Experimental facts

Quark-Lepton complementarity

$$\theta_{12}^{PMNS} + \theta_{12}^{CKM} \sim \pi/4$$

~33° ~13°

$$\theta_{23}^{PMNS} + \theta_{23}^{CKM} \sim \pi/4$$

~45° ~2°

- appealing from a theoretical and phenomenological point of view
- no clue on which kind of symmetry could be responsible for them

complete failure: $\theta_{13}^{PMNS} + \theta_{13}^{CKM} \sim 10^\circ$

~8.5° ~0.2°

Experimental facts

we need to replace the bad relation with a promising one:

$$\theta_{13}^{PMNS} = O(1) \cdot \theta_{12}^{CKM}$$

same order of magnitude

Experimental facts

we need to replace the bad relation with a promising one:

$$\theta_{13}^{PMNS} = O(1)\cdot\theta_{12}^{CKM}$$

same order of magnitude

Flavor symmetries

neutrino mass matrix:

$$\begin{pmatrix} x & y & y \\ y & z & x-z \\ y & x-z & z \end{pmatrix} \xrightarrow{\text{diagonalized by}} \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} = U_\nu$$

Bi-maximal mixing

$$\underbrace{\theta_{13}^{PMNS} = 0 \qquad \theta_{12}^{PMNS} = 45^o \qquad \theta_{23}^{PMNS} = 45^o}_{\text{good starting point}}$$

Experimental facts

Flavor symmetries

Altarelli et al.,
0903.1940

Corrections are needed from charged lepton diagonalization

$$U^{PMNS} = U_{cl}^{+} \cdot U_{\nu} \qquad U_{cl} \sim \begin{pmatrix} 1 & \lambda_C & \lambda_C \\ \lambda_C & 1 & 0 \\ \lambda_C & 0 & 1 \end{pmatrix}$$

introduced by *hand* *(me/mu ~* λ^{2}_{c} *)*

good results:

$$\begin{cases} \sin^2\theta_{12} = \frac{1}{2} - O(\lambda_C) \\ \sin^2\theta_{23} = \frac{1}{2} \\ \sin\theta_{13} = \frac{1}{\sqrt{2}} O(\lambda_C) \end{cases}$$

$$[\theta_{13}^{PMNS} = O(1) \cdot \theta_{12}^{CKM}]$$

Experimental facts

GUT: simple example from SU(5)

Let us take the electron and down quark relation:

$$m_e = m_D^T$$

$$U^{PMNS} = U_{cl}^{+} \cdot U_{\nu} \qquad V^{CKM} = U_u^{+} \cdot U_d$$

Experimental facts

GUT: simple example from SU(5)

Let us take the electron and down quark relation:

$$m_e = m_D^T$$

$$U^{PMNS} = U_{cl}^{+} \cdot U_{\nu} \qquad V^{CKM} = U_u^{+} \cdot U_d$$

Let us diagonalize the matrices:

$$U_{cl}\, m_e\, E_R^{+} = m_e^D \qquad U_d\, m_d\, D_R^{+} = m_d^D$$

this implies

$$U_{cl} = D_R^{*} \qquad U_d = E_R^{*}$$

relations involve <u>unobservable</u> right-handed rotations

Our approach

Our point of view:
assume a dependence of neutrino mixing on the CKM

$$\boxed{U^{PMNS} = V^*_{CKM} \cdot T^*}$$

$$T = U_{23}(\tilde{\theta}_{23}) U_{13}(\tilde{\theta}_{13}) U_{12}(\tilde{\theta}_{12})$$

Our approach

Our point of view:
assume a dependence of neutrino mixing on the CKM

$$\boxed{U^{PMNS} = V^*_{CKM} \cdot T^*}$$

$$T = U_{23}(\widetilde{\theta}_{23})\, U_{13}(\widetilde{\theta}_{13})\, U_{12}(\widetilde{\theta}_{12})$$

Strategy:

- take T as the well know leading order results
- correct them to match the experimental values of angles and phases
- check for neutrino mass predictions as well

Parameter	Best-fit value and 1σ range
$r \equiv \Delta m^2_{\mathrm{sol}}/\lvert\Delta m^2_{\mathrm{atm}}\rvert$	0.0295 ± 0.0008
$\tan(\theta_{12})$	0.666 ± 0.019
$\sin(\theta_{13})$	0.149 ± 0.002
$\tan(\theta_{23})$	0.912 ± 0.035
J_{CP}	-0.027 ± 0.010

$$J_{CP} = \mathrm{Im}\,\left[(U_{PMNS})_{11}(U_{PMNS})^*_{12}(U_{PMNS})^*_{21}(U_{PMNS})_{22}\right]$$

Is the ansatz successfull?

No CKM corrections

$$U^{PMNS} = V^{*}_{CKM} \cdot T^{*}$$

$$T = U_{23}(\widetilde{\theta}_{23})\, U_{13}(\widetilde{\theta}_{13})\, U_{12}(\widetilde{\theta}_{12})$$

$\tan\theta_{12} = 1/\phi$, with $\phi = (1+\sqrt{5})/2$

$$T_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Bi-maximal mixing

$$T_{TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Tri-Bi-maximal mixing

$$T_{GR} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ \frac{s_{12}}{\sqrt{2}} & -\frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s_{12}}{\sqrt{2}} & -\frac{c_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

Golden Ratio

No CKM corrections

$$U^{PMNS} = V^{*}_{CKM} \cdot T^{*}$$

$$T = U_{23}(\tilde{\theta}_{23}) U_{13}(\tilde{\theta}_{13}) U_{12}(\tilde{\theta}_{12})$$

$\tan\theta_{12} = 1/\phi$, with $\phi = (1+\sqrt{5})/2$

$$T_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Bi-maximal mixing

$$T_{TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Tri-Bi-maximal mixing

$$T_{GR} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ \frac{s_{12}}{\sqrt{2}} & -\frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s_{12}}{\sqrt{2}} & -\frac{c_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

Golden Ratio

<u>no</u> CKM corrections

$\sin(\theta_{13}) = 0$ $\sin(\theta_{13}) = 0$ $\sin(\theta_{13}) = 0$

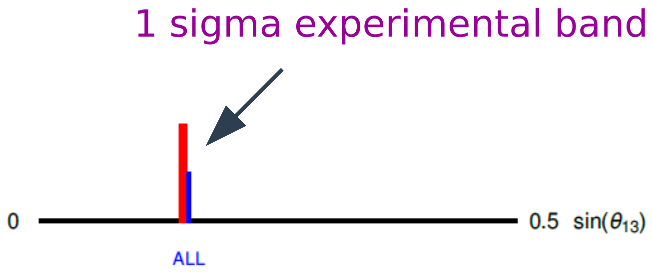
$\tan(\theta_{23}) = 1$ $\tan(\theta_{23}) = 1$ $\tan(\theta_{23}) = 1$

$\tan(\theta_{12}) = 1$ $\tan(\theta_{12}) = \frac{1}{\sqrt{2}}$ $\tan(\theta_{12}) = \frac{2\sqrt{5}}{\sqrt{5+\sqrt{5}}}$

no CP violation !

CKM corrected results

$$U^{PMNS} = V^{*}_{CKM} \cdot T^{*}$$



after CKM corrections

T	$\sin(\theta_{13})$	$\tan(\theta_{12})$	$\tan(\theta_{23})$	J_{CP}
U_{TBM}	$\frac{\lambda}{\sqrt{2}}$	$\frac{1}{\sqrt{2}} + \frac{3\lambda}{2\sqrt{2}}$	1	$-\frac{1}{6}A\eta\lambda^3$
U_{BM}	$\frac{\lambda}{\sqrt{2}}$	$1 - \sqrt{2}\lambda$	1	$\frac{1}{4\sqrt{2}}A\eta\lambda^3$
U_{GR}	$\frac{\lambda}{\sqrt{2}}$	$\frac{2\sqrt{5}}{5+\sqrt{5}} + \frac{5\sqrt{2}}{5+\sqrt{5}}\lambda$	1	$-\frac{1}{2\sqrt{10}}A\eta\lambda^3$

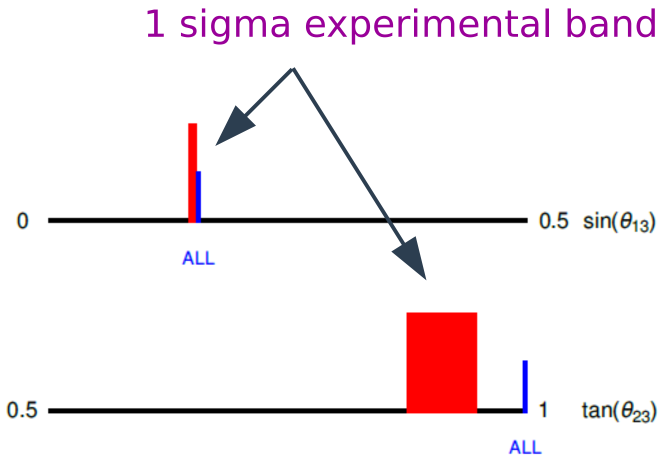
$$\boxed{\theta_{13}^{PMNS} = O(1) \cdot \theta_{12}^{CKM}}$$

CKM corrected results

$$U^{PMNS} = V^{*}_{CKM} \cdot T^{*}$$

after CKM corrections

T	$\sin(\theta_{13})$	$\tan(\theta_{12})$	$\tan(\theta_{23})$	$J_{\rm CP}$
U_{TBM}	$\frac{\lambda}{\sqrt{2}}$	$\frac{1}{\sqrt{2}} + \frac{3\lambda}{2\sqrt{2}}$	1	$-\frac{1}{6}A\eta\lambda^3$
U_{BM}	$\frac{\lambda}{\sqrt{2}}$	$1 - \sqrt{2}\lambda$	1	$\frac{1}{4\sqrt{2}}A\eta\lambda^3$
U_{GR}	$\frac{\lambda}{\sqrt{2}}$	$\frac{2\sqrt{5}}{5+\sqrt{5}} + \frac{5\sqrt{2}}{5+\sqrt{5}}\lambda$	1	$-\frac{1}{2\sqrt{10}}A\eta\lambda^3$


1 sigma experimental band

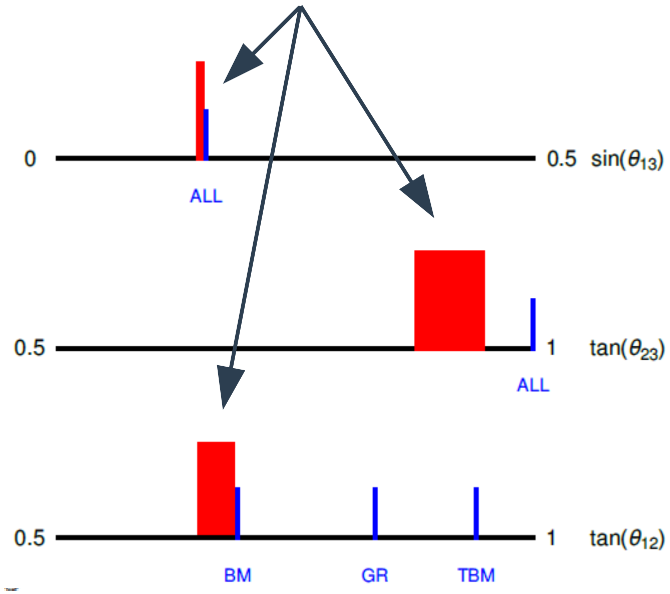
maximal atmospheric mixing

CKM corrected results

$$U^{PMNS} = V^{*}_{CKM} \cdot T^{*}$$

after CKM corrections

T	$\sin(\theta_{13})$	$\tan(\theta_{12})$	$\tan(\theta_{23})$	$J_{\rm CP}$
U_{TBM}	$\frac{\lambda}{\sqrt{2}}$	$\frac{1}{\sqrt{2}} + \frac{3\lambda}{2\sqrt{2}}$	1	$-\frac{1}{6}A\eta$
U_{BM}	$\frac{\lambda}{\sqrt{2}}$	$1 - \sqrt{2}\lambda$	1	$\frac{1}{4\sqrt{2}}A\eta$
U_{GR}	$\frac{\lambda}{\sqrt{2}}$	$\frac{2\sqrt{5}}{5+\sqrt{5}} + \frac{5\sqrt{2}}{5+\sqrt{5}}\lambda$	1	$-\frac{1}{2\sqrt{10}}A\eta\lambda$



1 sigma experimental band

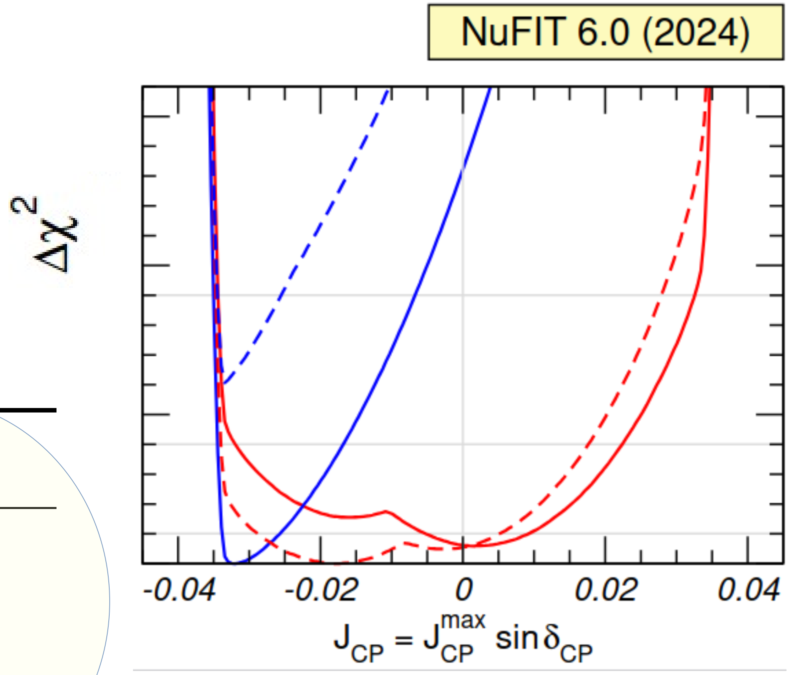
good for BM only

CKM corrected results

$$U^{PMNS} = V^*_{CKM} \cdot T^*$$

<u>after</u> CKM corrections

T	$\sin(\theta_{13})$	$\tan(\theta_{12})$	$\tan(\theta_{23})$	J_{CP}
U_{TBM}	$\frac{\lambda}{\sqrt{2}}$	$\frac{1}{\sqrt{2}} + \frac{3\lambda}{2\sqrt{2}}$	1	$-\frac{1}{6} A\eta\lambda^3$
U_{BM}	$\frac{\lambda}{\sqrt{2}}$	$1 - \sqrt{2}\lambda$	1	$\frac{1}{4\sqrt{2}} A\eta\lambda^3$
U_{GR}	$\frac{\lambda}{\sqrt{2}}$	$\frac{2\sqrt{5}}{5+\sqrt{5}} + \frac{5\sqrt{2}}{5+\sqrt{5}}\lambda$	1	$-\frac{1}{2\sqrt{10}} A\eta\lambda^3$



too small CP violation

Corrections

How to correct the wrong predictions?

T is a rotation, so: $T = U_{23}(\widetilde{\theta}_{23}) U_{13}(\widetilde{\theta}_{13}) U_{12}(\widetilde{\theta}_{12})$

can be BM, TBM or GR

maximal mixing

correction to 13 sector

Corrections

How to correct the wrong predictions?

T is a rotation, so: $T = U_{23}(\tilde{\theta}_{23}) U_{13}(\tilde{\theta}_{13}) U_{12}(\tilde{\theta}_{12})$

correction to 23 sector

correction to 13 sector

correction to 12 sector

1.

$$U_{13} = \begin{pmatrix} 1 - \frac{\lambda^2}{2}|u|^2 & 0 & u\lambda \\ 0 & 1 & 0 \\ -u^*\lambda & 0 & 1 - \frac{\lambda^2}{2}|u|^2 \end{pmatrix}$$

u = complex parameter

$$\boxed{J_{CP} \sim \lambda * \Im(u)}$$

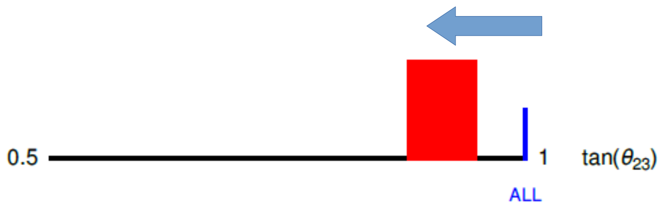
θ_{12} and θ_{13} not much affected

Corrections

How to correct the wrong predictions?

T is a rotation, so: $T = U_{23}(\widetilde{\theta}_{23}) U_{13}(\widetilde{\theta}_{13}) U_{12}(\widetilde{\theta}_{12})$

correction to 23 sector



2.

$$U_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} - \lambda\omega - \sqrt{2}\lambda^2\omega^2 - 2\lambda^3\omega^3 & \frac{1}{\sqrt{2}} + \omega\lambda \\ 0 & -\frac{1}{\sqrt{2}} - \omega\lambda & \frac{1}{\sqrt{2}} - \lambda\omega - \sqrt{2}\lambda^2\omega^2 - 2\lambda^3\omega^3 \end{pmatrix}$$

ω = real parameter
fixed by fit

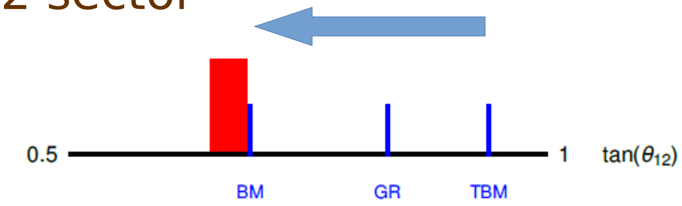
$$\boxed{\Delta \tan(\theta_{23}) \sim \lambda\, \omega}$$

Corrections

How to correct the wrong predictions?

T is a rotation, so: $T = U_{23}(\tilde{\theta}_{23}) U_{13}(\tilde{\theta}_{13}) U_{12}(\tilde{\theta}_{12})$

correction to 12 sector



3.

$$U_{12} = \begin{pmatrix} K & \tilde{s}_{12} + z\lambda & 0 \\ -\tilde{s}_{12} - z\lambda & K & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

z= real parameter
fixed by fit

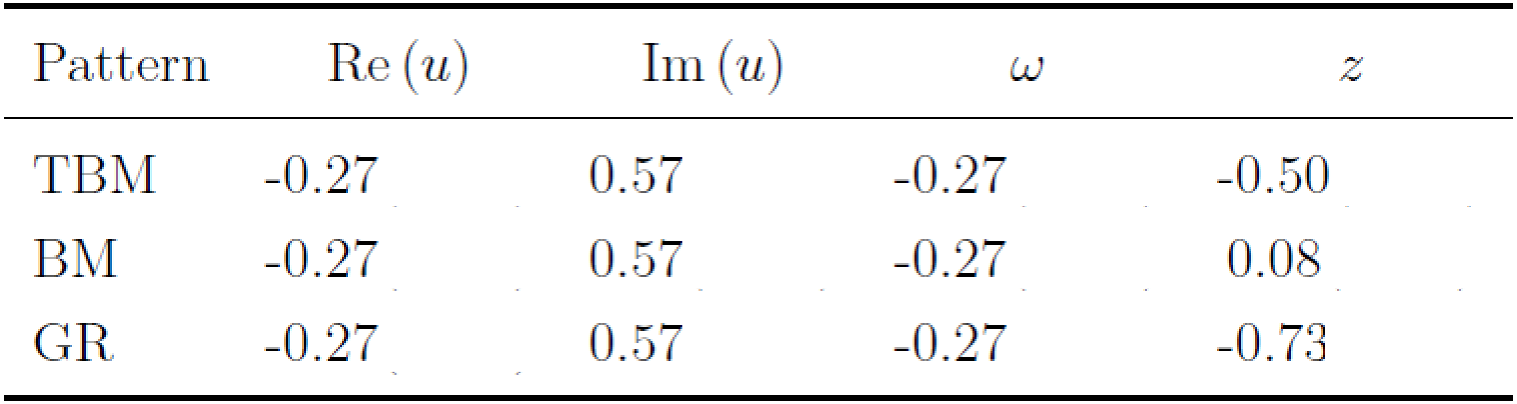
$$\boxed{\Delta \tan(\theta_{12}) \sim \lambda z}$$

Conclusions

χ^2 4 parameter-fit of u, ω and z: all patterns agree with experiments

→ our ansatz $U^{PMNS} = V^*_{CKM} \cdot T^*$ is *phenomenological viable*

$\chi^2 \sim 0$



Pattern	Re (u)	Im (u)	ω	z
TBM	-0.27	0.57	-0.27	-0.50
BM	-0.27	0.57	-0.27	0.08
GR	-0.27	0.57	-0.27	-0.73

CP violation

atmospheric angle

solar angle

Backup slides

Neutrino
masses