



Vadzim Haurysh¹

scientific adviser V. V. Andreev²

Sukhoi State Technical University of Gomel¹

Francisk Skorina Gomel State University²

Pionic decay of light mesons in the point form of Poincaré-invariant quantum mechanics

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Introduction

The pseudoscalar and vector mesons decays studying serves as a source of information on constituent quarks interaction. Unlike leptonic decays hadronic transitions provide information about the effects of non-perturbative dynamics and also allow one to indirectly estimate the universal strong coupling constant.

We present a method for calculating of hadronic decays effective constants in a model based on the point form of Poincaré-invariant quantum mechanics using the hypothesis of partial conservation of axial current (PCAC). It is shown that this approach allows reducing the calculation of amplitude involving three hadrons to simpler amplitudes $V \rightarrow P$ and $V \rightarrow V$ with the emission of a π -meson.

The aim of the work is

studying the pionic decays $V \rightarrow P\pi$ and $V \rightarrow V\pi$ in approach based on composite quark model and on the point form of Poincaré-invariant quantum mechanics (further PiQM) using PCAC

Main tasks

- 1 Calculate the model parameters (constituent quark masses and wave function β -parameters) using leptonic decay constants $P^\pm(q\bar{Q}) \rightarrow \ell^\pm \nu_{\ell^\pm}$, $V(q\bar{q}) \rightarrow \ell^\pm \ell^\mp$ together with Ward identity and pseudoscalar density constant.
- 2 Develop a methodology for integral representations calculating pionic decay constants $g_{VP\pi}$ and $g_{VV\pi}$ using PCAC.
- 3 Conduct a numerical assessment of decay form-factors $g_{\rho\pi\pi}$, $g_{K^*K\pi}$ and $g_{\omega\rho\pi}$ in comparison with modern experimental data and other models.

Model description

Meson state vector with M mass, spin J in point form of PiQM with 4-momentum $Q^\mu = \{\omega_M(\mathbf{Q}), \mathbf{Q}\}$, $Q^2 = M^2$ determined by the integral over the momentum of the relative motion of quarks \mathbf{k} with masses $m_q, m_{\bar{Q}}$ and momenta $\mathbf{p}_{1,2}$ respectively as

$$\begin{aligned}
 |\mathbf{Q}, J\mu, M\rangle = & \sum_{\lambda_1, \lambda_2} \sum_{\nu_1, \nu_2} \int d\mathbf{k} \Phi_{\ell S}^{J\mu}(\mathbf{k}, \beta_{q\bar{Q}}) \Omega\left(\begin{smallmatrix} \ell & S & J \\ \nu_1 & \nu_2 & \mu \end{smallmatrix}\right) (\theta_k, \phi_k) \times \\
 & \times \sqrt{\frac{\omega_{m_q}(\mathbf{p}_1) \omega_{m_{\bar{Q}}}(\mathbf{p}_2)}{\omega_{m_q}(\mathbf{k}) \omega_{m_{\bar{Q}}}(\mathbf{k}) \mathbb{V}_0}} D_{\lambda_1, \nu_1}^{1/2}(\mathbf{n}_{W_1}) D_{\lambda_2, \nu_2}^{1/2}(\mathbf{n}_{W_2}) |\mathbf{p}_1, \lambda_1, \mathbf{p}_2, \lambda_2\rangle.
 \end{aligned} \tag{1}$$

In (1) for brevity the notations are used

$$\begin{aligned}
 \mathbb{V}_0 = \omega_{M_0}(\mathbf{Q})/M_0, \quad \omega_m(\mathbf{Q}) = \sqrt{\mathbf{Q}^2 + m^2}, \\
 \Omega\left(\begin{smallmatrix} \ell & S & J \\ \nu_1 & \nu_2 & \mu \end{smallmatrix}\right) (\theta_k, \phi_k) = Y_{\ell m}(\theta_k, \phi_k) C\left(\begin{smallmatrix} s_1 & s_2 & S \\ \nu_1 & \nu_2 & \mu \end{smallmatrix}\right) C\left(\begin{smallmatrix} \ell & S & J \\ m & \lambda & \mu \end{smallmatrix}\right),
 \end{aligned} \tag{2}$$

Model description

where $Y_{\ell m}(\theta_k, \phi_k)$ is spherical functions defined by the angles of the \mathbf{k} – vector, $D_{\lambda, \nu}^{1/2}(\mathbf{n}_W)$ are Wigner rotation functions and $M_0 = \omega_{m_q}(\mathbf{k}) + \omega_{m_{\bar{q}}}(\mathbf{k})$ – invariant mass of constituent quarks. The wave function in (1) is normalized by the condition

$$\sum_{\ell, S} \int d\mathbf{k} k^2 \left| \Phi_{\ell S}^{J\mu}(\mathbf{k}, \beta_{q\bar{q}}) \right|^2 = 1. \quad (3)$$

Using the parametrization of pseudoscalar meson leptonic decay

$$\langle 0 | \hat{J}_P^\mu(0) | \mathbf{Q}, M_P \rangle = \frac{i}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\omega_{M_P}(\mathbf{P})}} P^\mu f_P \quad (4)$$

and vector meson leptonic decay

$$\langle 0 | \hat{J}_V^\mu(0) | \mathbf{Q}, 1\lambda_V, M_V \rangle = \frac{i}{(2\pi)^{3/2}} \frac{\varepsilon^\mu(\lambda_V)}{\sqrt{2\omega_{M_V}(\mathbf{P})}} M_V f_V \quad (5)$$

Model description

one can obtain integral representation of a pseudoscalar decay constants $P^\pm(q\bar{Q}) \rightarrow \ell^\pm \nu_{\ell^\pm}$ and $V(q\bar{q}) \rightarrow \ell^\pm \ell^\mp$ in point form of PiQM

$$f_I(m_q, m_{\bar{Q}}, \beta_{q\bar{Q}}^I) = \sqrt{\frac{3}{2}} \frac{1}{\pi} \int dk k^2 \Phi(k, \beta_{q\bar{Q}}^I) \sqrt{\frac{W_{m_q}^+(k) W_{m_{\bar{Q}}}^+(k)}{M_0 \omega_{m_q}(k) \omega_{m_{\bar{Q}}}(k)}} \times \\ \times \left(1 + a_I \frac{k^2}{W_{m_q}^+(k) W_{m_{\bar{Q}}}^+(k)} \right). \quad (6)$$

In relation (6) notations are used

$$W_m^\pm(k) = \omega_m(k) \pm m, \quad I = P, V, \quad a_P = -1, a_V = 1/3. \quad (7)$$

It should be noted that the expression (6) coincides with instant form dynamics calculations ¹.

¹Krutov, A. The radius of the ρ -meson determined from its decay constant/ A. Krutov, A. Polezhaev // Phys. Rev.-2016. – Vol. D93. – P. 036007.

Model description

For further determination the model parameters we will use the pseudoscalar density constant ²

$$g_P(m_q, m_{\bar{Q}}, \beta_{q\bar{Q}}^P) = \sqrt{\frac{3}{2}} \frac{1}{\pi} \int dk k^2 \Phi(k, \beta_{q\bar{Q}}^P) \sqrt{M_0} \times \quad (8)$$
$$\times \left(W_{m_q}^+ W_{m_{\bar{Q}}}^+ + W_{m_q}^- W_{m_{\bar{Q}}}^- \right).$$

Using Expressions (6), (8), as well as experimental ³ values $f_{\pi^\pm}^{(\text{exp.})}$ and $M_{\pi^\pm}^{(\text{exp.})}$ leads to a system of equations

$$\begin{cases} 1/2 (m_u + m_d) = (3.45 \pm 0.42) \text{ MeV} \\ f_P(m_u, m_{\bar{d}}, \beta_{u\bar{d}}^P) = f_{\pi^\pm}^{(\text{exp.})}, \\ (\hat{m}_u + \hat{m}_{\bar{d}}) g_P(m_u, m_{\bar{d}}, \beta_{u\bar{d}}^P) = f_{\pi^\pm}^{(\text{exp.})} \left(M_{\pi^\pm}^{(\text{exp.})} \right)^2. \end{cases} \quad (9)$$

² Jaus, W. Consistent treatment of spin 1 mesons in the light-front quark model/ W. Jaus// Phys.Rev. – 2003. – Vol. D67. – P. 094010.

³ Workman, R. L. and Particle Data Group Review of Particle Physics/ R. L. Workman and others // Progress of Theoretical and Experimental Physics – 2022. – Vol. 2022.

Basic parameters of the model

System (9) solution with an oscillatory wave function leads to the following values:

$$m_u = (217.64 \pm 3.60) \text{ MeV}, \quad m_d = (220.17 \pm 3.60) \text{ MeV}, \\ \beta_{u\bar{d}}^P = (371.81 \pm 3.51) \text{ MeV}. \quad (10)$$

For s -quark using similar system (9)

$$\begin{cases} f_P(m_u, m_{\bar{s}}, \beta_{u\bar{s}}^P) = f_{K^\pm}^{(\text{exp.})}, \\ (\hat{m}_u + \hat{m}_{\bar{s}}) g_P(m_u, m_{\bar{s}}, \beta_{u\bar{d}}^P) = f_{K^\pm}^{(\text{exp.})} \left(M_{K^\pm}^{(\text{exp.})} \right)^2. \end{cases} \quad (11)$$

and $K^\pm \rightarrow \ell^\pm \nu_{\ell^\pm}$ decay constant and K^\pm -meson mass values ⁴ leads to

$$m_s = (421.35 \pm 7.10) \text{ MeV} \quad \beta_{u\bar{s}}^P = (380.19 \pm 7.10) \text{ MeV}. \quad (12)$$

⁴ Workman, R. L. and Particle Data Group Review of Particle Physics/ R. L. Workman and others // Progress of Theoretical and Experimental Physics – 2022. – Vol. 2022.

Basic parameters of the model

For vector meson sector parameters calculation we use $\tau^\pm \rightarrow V^\pm \nu_{\tau^\pm}$ and $V \rightarrow \ell^\pm \ell^\mp$ decay constant

$$f_{\rho^\pm} = (209.30 \pm 1.50) \text{ MeV}, f_{K^{*\pm}} = (205.22 \pm 53.56) \text{ MeV} \quad (13)$$

and

$$f_{\rho^0} = (154.10 \pm 2.40) \text{ MeV}, f_\omega = (47.30 \pm 3.00) \text{ MeV}, \quad (14)$$

$$f_\phi = (75.20 \pm 1.50) \text{ MeV}.$$

Using relation (6) one can obtain following values

$$\beta_{u\bar{d}}^V = (310.95 \pm 2.22) \text{ MeV}, \beta_{u\bar{s}}^V = (313.87 \pm 77.71) \text{ MeV}, \quad (15)$$

$$\beta_{u\bar{u}}^V = (326.01 \pm 8.75) \text{ MeV}, \beta_{d\bar{d}}^V = (319.54 \pm 2.18) \text{ MeV}.$$

Pionic decay in point form of PiQM

Hadronic transition matrix element of vector V to pseudoscalar P meson in the case of π^0 -meson emission can be written as ⁵

$$\langle \mathbf{Q}', M_P | \hat{J}_{\pi^0} | \mathbf{Q}, \lambda_V, M_V \rangle = \frac{1}{(2\pi)^3} \frac{1}{\sqrt{2 V^0 M_V}} \frac{1}{\sqrt{2 V'^0 M_P}} \times \quad (16)$$
$$\times 2 g_{VP\pi^0} (\varepsilon(\lambda_V) \cdot \mathbf{Q}'),$$

where $g_{VP\pi^0}$ is pionic decay constant determined from the expression

$$\Gamma_{V \rightarrow P\pi^0} = |g_{VP\pi^0}|^2 \frac{|\mathbf{p}_{\text{out}}|^3}{6 \pi M_V^2}. \quad (17)$$

Matrix element (16) can be related to axial vector current by ⁶

⁵ Jaus, W. Consistent treatment of spin-1 mesons in the light-front quark model/ W. Jaus // Phys. Rev.-2003.-Vol. D67.-P. 094010.

⁶ Jaus, W. Semileptonic, radiative, and pionic decays of B, B^* and D, D^* mesons / W. Jaus // Phys. Rev.-1996.-Vol. D53.-P. 1349–1365.

Pionic decay in point form of PiQM

$$\langle \mathbf{Q}', M_P | \hat{A}^\mu | \mathbf{Q}, \lambda_V, M_V \rangle = \frac{1}{(2\pi)^3} \frac{1}{\sqrt{4 \mathbb{V}^{0'} \mathbb{V}^0}} \times \quad (18)$$

$$\times \left(f(q^2) \ell_f^\mu(\lambda) + a_+(q^2) \ell_{a_+}^\mu(\lambda) + a_-(q^2) \ell_{a_-}^\mu(\lambda) \right),$$

where $\hat{A}^\mu = e_q \bar{\psi} \gamma^\mu \gamma_5 \psi$. According to the developed method expression (18) is presented in terms of 4-velocity mesons with 4-vectors

$$\ell_f^\mu(\lambda) = \frac{\varepsilon_\lambda^\mu}{\sqrt{M_V M_P}}, \quad \ell_{a_+}^\mu(\lambda) = \sqrt{M_V M_P} (\varepsilon_\lambda \cdot \mathbb{V}') \left(\mathbb{V}^\mu + \frac{M_P}{M_V} \mathbb{V}'^\mu \right),$$

$$\ell_{a_-}^\mu(\lambda) = \sqrt{M_V M_P} (\varepsilon_\lambda \cdot \mathbb{V}') \left(\mathbb{V}^\mu - \frac{M_P}{M_V} \mathbb{V}'^\mu \right). \quad (19)$$

Partial conservation of axial vector current leads to the following relation between $V(q\bar{Q}) \rightarrow P(q\bar{Q})\pi^0$ and decay constant f_{π^\pm}

Pionic decay in point form of PiQM

$$4 |g_{VP\pi^0}| f_\pi = |f(0) - (M_V^2 - M_P^2) a_+(0)|. \quad (20)$$

Note, that for $\rho^\pm \rightarrow \pi^\pm \pi^0$ decay in (20) factor 4 must be replaced with 2^7 .

The following calculations will be carried out for generalized Breit system $\vec{V} + \vec{V}' = 0$. Below we introduce $\varpi = (\mathbf{V} \cdot \mathbf{V}')$; for soft pion case $Q_{\pi^0}^2 \equiv q^2 \rightarrow 0$ in our approach follows that $\varpi \rightarrow 1$. Taking into account relation (1) and (18), (19) one can obtain

$$f(0) = \int d\mathbf{k} k^2 \Phi(\mathbf{k}, \beta_{q\bar{Q}}^V) \Phi^*(\mathbf{k}, \beta_{q\bar{Q}}^P) \times \quad (21)$$

$$(e_q \eta^f(\mathbf{k}, m_q, m_{\bar{Q}}) - e_{\bar{Q}} \eta^f(\mathbf{k}, m_{\bar{Q}}, m_q)),$$

Pionic decay in point form of PiQM

$$a_+(0) = \int dk k^2 \Phi(k, \beta_{q\bar{Q}}^V) \Phi^*(k, \beta_{q\bar{Q}}^P) \times \quad (22)$$

$$(e_q \eta^{a+}(k, m_q, m_{\bar{Q}}) - e_{\bar{Q}} \eta^{a+}(k, m_{\bar{Q}}, m_q)).$$

In (21), (22) auxiliary functions are introduced

$$\eta^f(k, m_q, m_{\bar{Q}}) = \frac{2}{3} \frac{(2m_q + \omega_{m_q}(k))(\omega_{m_q}(k) + \omega_{m_{\bar{Q}}}(k))}{\omega_{m_q}(k)}, \quad (23)$$

$$\eta^{a+}(k, m_q, m_{\bar{Q}}) =$$

$$-\frac{\omega_{m_q}(k)(m_{\bar{Q}}^2 - 2m_q^2 + 2\omega_{m_q}^2(k)) + \omega_{m_{\bar{Q}}}(k)(3m_q^2 + 2\omega_{m_q}(k)(m_q + m_{\bar{Q}} - \omega_{m_q}(k)))}{6\omega_{m_q}^2(k)(m_{\bar{Q}}^2 - m_q^2 + \omega_{m_q}(k)(\omega_{m_q}(k) + \omega_{m_{\bar{Q}}}(k)))}. \quad (24)$$

Pionic decay in point form of PiQM

Hadronic transition matrix element of vector V to vector meson V' in the case of π^0 -meson emission can be written as ⁸

$$\langle \mathbf{Q}', \lambda_{V'}, M_{V'} | \hat{J}_{\pi^0} | \mathbf{Q}, \lambda_V, M_V \rangle = i \frac{1}{(2\pi)^{3/2}} \frac{1}{(2\pi)^{3/2}} \times \quad (25)$$
$$\frac{1}{\sqrt{2\mathbb{V}'_0 M_{V'}}} \frac{1}{\sqrt{2\mathbb{V}_0 M_V}} g_{VV'\pi} \epsilon^{\mu\nu\alpha\beta} Q_\mu Q'_\nu \varepsilon_\alpha(Q) \varepsilon_\beta^*(Q').$$

Matrix element (25) can be related to axial vector current by

$$\langle \mathbf{Q}', \lambda_{V'}, M_{V'} | \hat{A}^\mu | \mathbf{Q}, \lambda_V, M_V \rangle = i \frac{1}{(2\pi)^{3/2}} \frac{1}{(2\pi)^{3/2}} \times \quad (26)$$
$$\frac{1}{\sqrt{2\mathbb{V}'_0 M_{V'}}} \frac{1}{\sqrt{2\mathbb{V}_0 M_V}} \frac{h(q^2)}{2} \epsilon^{\mu\nu\alpha\beta} P_\nu \varepsilon_\alpha(Q) \varepsilon_\beta^*(Q'),$$

where $h(0) = g_{VV\pi} f_\pi$ and $P = Q + Q'$ and $q = Q - Q'$.

⁸ Melikhov, D., Beyer, M. Pionic coupling constants of heavy mesons in the quark model/ D. Melikhov, M. Beyer // Phys. Let.-1999.-Vol. B452.-P. 121-128.

Pionic decay in point form of PiQM

According to the developed method expression (25,26) with 4-velocities of initial and final mesons can be written as

$$\begin{aligned} \langle \mathbf{Q}', \lambda_{V'}, M_{V'} | \frac{(\hat{\mathbf{A}} \cdot \mathbf{K}^*(\lambda, \lambda'))}{(K(\lambda, \lambda') \cdot \mathbf{K}^*(\lambda, \lambda'))} | \mathbf{Q}, \lambda_V, M_V \rangle = \quad (27) \\ = i \frac{1}{(2\pi)^3} \frac{1}{\sqrt{V_0}} \frac{1}{\sqrt{V_0}} \frac{1}{2} \frac{h(q^2)}{2}, \end{aligned}$$

with 4-vector

$$K^\mu = \frac{1}{\sqrt{2M_{V'}}} \frac{1}{\sqrt{2M_V}} \epsilon^{\mu\nu\alpha\beta} (\mathbb{V} M_V + \mathbb{V}' M_{V'})_\nu \varepsilon_\alpha(Q) \varepsilon_\beta^*(Q'). \quad (28)$$

Similar calculations with axial quark current $A^\mu = e_q \bar{\psi} \gamma^\mu \gamma_5 \psi$ in generalized Breit system $\vec{\mathbb{V}} + \vec{\mathbb{V}}' = 0$ leads to

Pionic decay in point form of PiQM

$$g_{VV\pi} = \frac{h(0)}{f_\pi} = \int dk k^2 \Phi(k, \beta_{q\bar{Q}}^V) \Phi^*(k, \beta_{q\bar{Q}}^{V'}) \times \quad (29)$$
$$(e_q \eta^h(k, m_q) - e_{\bar{Q}} \eta^h(k, m_{\bar{Q}}))$$

with auxiliary function

$$\eta^h(k, m) = \frac{2}{3} \left(1 + \frac{2m}{\omega_m(k)} \right). \quad (30)$$

Below we will perform numerical calculations using the obtained expressions for decay constant $g_{VP\pi}$ and $g_{VV\pi}$ and model parameters (constituent quark masses m_u, m_d, m_s and β -parameters of wave function).

Numerical calculations and discussions

For $\rho^\pm \rightarrow \pi^\pm \pi^0$ decay relation

$$g_{\rho\pi\pi} = \frac{1}{2f_\pi} \left| f(0) - \left(M_{\rho^\pm}^2 - M_{\pi^\pm}^2 \right) a_+(0) \right|. \quad (31)$$

with model parameters (10),(15) leads to

$$g_{\rho\pi\pi} = (5.72 \pm 0.03) \text{ GeV}^0. \quad (32)$$

For $K^{*\pm} \rightarrow K^\pm \pi^0$ decay one can get

$$g_{K^*K\pi} = \frac{1}{4f_\pi} \left| f(0) - \left(M_{K^{*\pm}}^2 - M_{K^\pm}^2 \right) a_+(0) \right| = (5.89 \pm 0.40) \text{ GeV}^0. \quad (33)$$

Note, that value (33) where calculated using $g_{K^*K\pi} = \sqrt{3}g_{K^*K\pi^0}$ (see application B in ⁹).

⁹Jaus, W. Consistent treatment of spin-1 mesons in the light-front quark model/ W. Jaus // Phys. Rev.-2003.-Vol. D67.-P. 094010.

Numerical calculations and discussions

For $\omega \rightarrow \pi\pi\pi$ decay relation we use decay mechanism via $\omega \rightarrow \rho^*\pi \rightarrow \pi\pi\pi$. In such approximation relation (29) leads to

$$g_{\omega\rho\pi} = \frac{h(0)}{f_\pi} = (10.58 \pm 0.10) \text{ GeV}^{-1}. \quad (34)$$

Table: Comparison of decay constant $g_{VP\pi}$ and $g_{VV\pi}$ values with other approaches and experimental data

Approach	$g_{\rho\pi\pi}$	$g_{K^{*\pm}K^\pm\pi}$	$g_{\omega\rho\pi}$
Light-front calculation ¹⁰	6.10	5.43	—
Bethe-Salpeter based approach ¹¹	5.13 ± 0.25	5.10 ± 0.25	—
VMD-model ¹²	5.95 ± 0.08	—	11.31 ± 0.38
Experimental data	5.98 ± 0.02	5.49 ± 0.05	—
This work	5.72 ± 0.03	5.89 ± 0.40	10.58 ± 0.10

¹⁰ Jaus, W. Consistent treatment of spin-1 mesons in the light-front quark model/ W. Jaus // Phys. Rev.-2003.-Vol. D67.-P. 094010.

¹¹ da Silveira, R. Strong two-meson decays of light and charmed vector mesons/ R. da Silveira, F. Serna // Phys. Rev.-2023.-Vol. D107.-P. 034021.

¹² Gustavo, A. Role of the $\rho(1450)$ in low-energy observables from an analysis in the meson dominance approach/ A. Gustavo, A. Rojas // Phys. Rev.-2023.-Vol. D107.-P. 056006.

Conclusions and remarks

The work presents a method for calculating decay constants $g_{VP\pi}$ and $g_{VV\pi}$ in a model based on the compound quark model and the point form of the PiQM. The authors presented an original method for determining the parameters of the model and obtained integral representations of the $V \rightarrow P\pi$ and $V \rightarrow V\pi$ decays constants using the PCAC hypothesis. Numerical calculations have shown that the proposed model leads to comparable results in comparison with other approaches and models as well as experimental data.

As a result a self-consistent model that describes the leptonic and hadronic transitions of light pseudoscalar $\pi^{\pm-}$, $K^{\pm-}$ and vector $\rho^{\pm-}$, $\omega^{\pm-}$, K^{*-} mesons is proposed.

THANK YOU FOR YOUR ATTENTION!

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V. V. Andreev

V. Yu. Haurysh