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Neutrino oscillation inside supernovae: exact description

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Outlines

- Neutrino Oscillations: 2 nu vs 3 nu Scheme
- Neutrino Oscillations in 3 nu's Scheme
- Non-Adiabatic Resnonces
- Numerical Results
- Conclusion

Neutrino Oscillations: 2 nu vs 3 nu Scheme

The flavor basis $|\nu_\alpha\rangle$ $(\alpha = e, \mu)$ is related to mass basis $|\nu_i\rangle (i = 1, 2)$ as

$$| \nu_\alpha\rangle = \sum_{i=1}^{2} X_{\alpha i} \,| \nu_i\rangle, \text{ with } \quad X = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \text{ Then,}$$

$$E_i \approx p + \frac{m_i^2}{2p} \approx E + \frac{m_i^2}{2E}. \qquad H = \frac{1}{2E} \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix}. \qquad H_f = X H_m X^\dagger,$$

$$H_{vac} = \frac{\Delta m^2}{4E} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$$

Inside matter, this Hamiltonian is generalized into

$$H_{mat} = \frac{\Delta m^2}{4E} \begin{pmatrix} -\cos 2\theta + A & \sin 2\theta \\ \sin 2\theta & \cos 2\theta - A \end{pmatrix}, \qquad A \equiv \frac{2\sqrt{2} G_F N_e E}{\Delta m^2}.$$

that gives the eigenvalues $\lambda_\pm = \pm \frac{\Delta m_m^2}{4E}$, via the mixing angle $\tan 2\theta_m = \frac{\sin 2\theta}{\cos 2\theta - A}$,

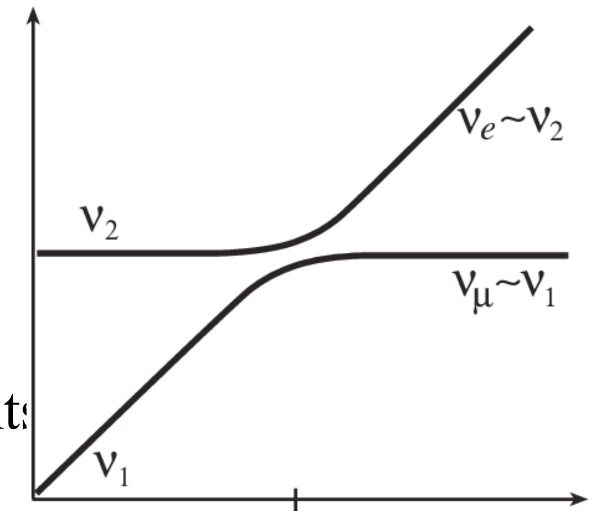
$$\Delta m_m^2 = \Delta m^2 \sqrt{(\cos 2\theta - A)^2 + \sin^2 2\theta}.$$

If the matter density is changing, the Schrodinger equation is written as

$$i\frac{d}{dr}\nu_i(r) = H^m_{ik}\nu_k(t) = U^\dagger_M H^f_{ik} U_M \nu_k(t) + i\left(\frac{d}{dr}U^\dagger_M(r)\right)U_M(r).$$

and the mass eigenstates behave as

$$\widetilde{H}_{mat} = \begin{pmatrix} \lambda_- & -i\frac{\partial\theta_m}{\partial r} \\ i\frac{\partial\theta_m}{\partial r} & \lambda_+ \end{pmatrix}$$



The adiabaticity is ensured if the off-diagonal elements are very small w.r.t. the diagonal ones, i.e., $\gamma_{ij} \gg 1$

$$\gamma(x) = \frac{\Delta m^2_m(x)}{2E}\left[\frac{d}{dx}\theta_m(x)\right]^{-1}$$

If not, a possible jump could happen between $\lambda_\pm$ at (around) the resonance region that defined by $(H_{mat})_{11} = (H_{mat})_{22}$ or by minimizing $|\lambda_+ - \lambda_-|$

At the resonance, we got $\gamma_{ij}(x) = \frac{\Delta m^2_{ij}}{2E}\frac{\sin^2 2\theta_{ij}}{\cos 2\theta_{ij}}\left|\frac{dlnN_e(x)}{dx}\right|^{-1}_{x_{res}}.$

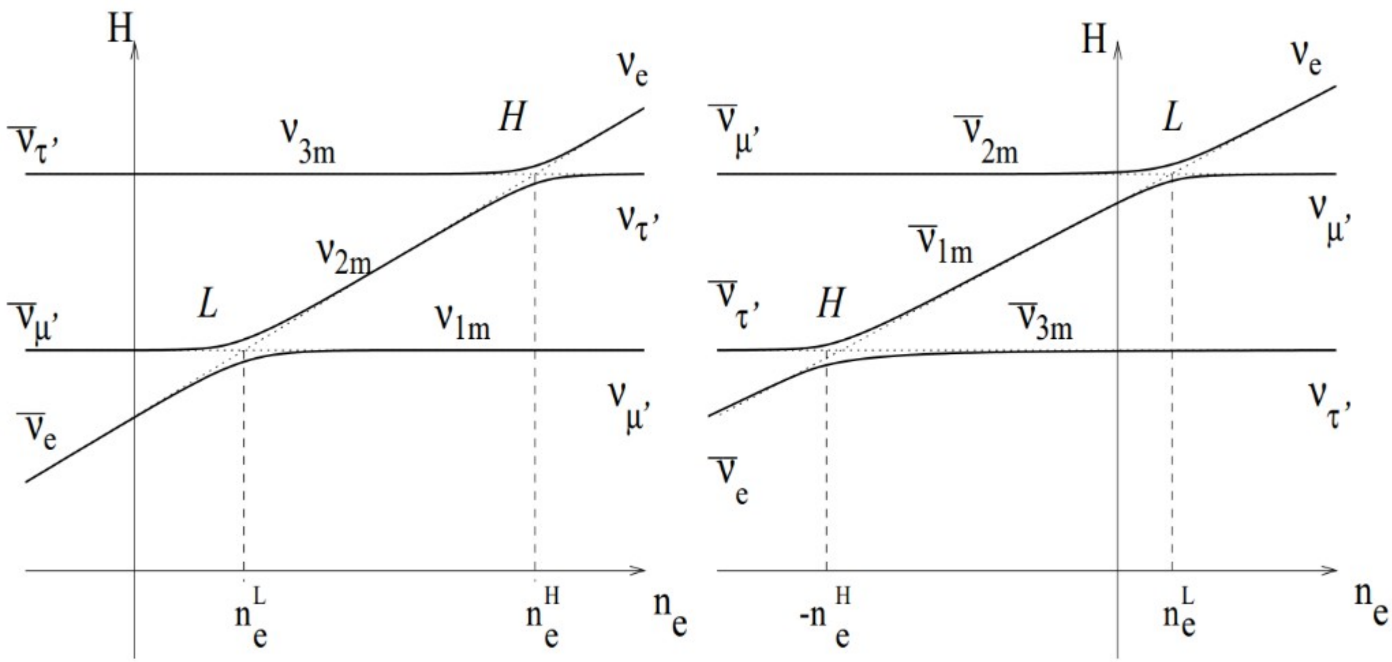
The transition probability is given by the LZ formula

$$P_{LZ} = \exp\left(-\frac{\pi}{2}\gamma\right) = \exp\left(-\frac{\pi}{4}\frac{(\Delta m^2_{ij})\sin^2 2\theta_{ij}}{E\cos 2\theta_{ij}\left|\frac{d\ln N_e}{dr}\right|_{\rm rcs}}\right).$$

In the 3nu scheme, the matter Hamiltonian can be written in flavor modified basis as

$$i\frac{d}{dx}\begin{pmatrix}\nu_e\\ \nu'_\mu\\ \nu'_\tau\end{pmatrix} = \frac{1}{2E}\begin{pmatrix}\#_{11}+A & \#_{12} & \#_{13}\\ \#_{12} & \#_{22} & 0\\ \#+13 & 0 & \#_{33}\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu'_\mu\\ \nu'_\tau\end{pmatrix}$$

which allows to use the 2nu scheme results. Thus in a dense object like SN we have



Are these results exact? Are these approximations reliable?

Neutrino Oscillations in 3 nu's Scheme

Starting by the Schrodinger equation in the flavor basis $|\nu_\alpha\rangle^T = \begin{pmatrix} \nu_e & \nu_\mu & \nu_\tau \end{pmatrix}$.

$i\frac{d}{dr}|\nu_\alpha\rangle = H_f|\nu_\alpha\rangle$, which can be written in the mass basis as

$$i\frac{d}{dr}|\nu_i\rangle = U_M^\dagger H_f U_M|\nu_i\rangle + i\frac{d}{dr}\left(U_M^\dagger\right)U_M|\nu_i\rangle$$

with $|\nu_\alpha\rangle = U_M|\nu_i\rangle$. then using these diemnionless quantities, one writes

$$i\frac{d}{d\xi}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix} = \left[U_M^\dagger U\begin{pmatrix}0&0&0\\0&\alpha&0\\0&0&\beta\end{pmatrix}U^\dagger U_M + \kappa U_M^\dagger\begin{pmatrix}f&0&0\\0&0&0\\0&0&0\end{pmatrix}U_M + \omega Q\right]\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix},$$

$\xi = r/r_0$ $\rho = \rho_0 f(\xi)$ $\omega = \frac{df}{d\xi}$ $Q = i\left(\frac{d}{df}U_M^\dagger\right)U_M$ $\kappa = 4.5416\times10^{12}$

$\alpha = 3.0205\times10^{-2}\left(\frac{E}{10\,MeV}\right)^{-1}$ $\beta_{NO} = 1.0219\left(\frac{E}{10\,MeV}\right)^{-1}$ $\beta_{IH} = -1.0122\left(\frac{E}{10\,MeV}\right)^{-1}$

Clearly, the Hamiltonian is not real but complex Hermitian; and for the case of vanishing Dirac CP phase in the PMNS matrix, it has the form

$$\begin{pmatrix}\epsilon_1 & i\eta_{12} & i\eta_{13}\\ -i\eta_{12} & \epsilon_2 & i\eta_{23}\\ -i\eta_{13} & -i\eta_{23} & \epsilon_3\end{pmatrix}$$

Here, the epsilon's are the dimensionless energy eigenvalues, and the eta's are anologous to the effective angles derivatives.

Howere, we know that $\delta_D \neq 0$, then the real part of the Hamiltonian is not diagonal!!

$$\mathcal{H}_{\Re}^{(0)} = diag\{\epsilon_1^{(0)}, \epsilon_2^{(0)}, \epsilon_3^{(0)}\} + \Re(Q^{(0)})$$

Then, it needs to be diagonalized again $diag\{\epsilon_1^{(1)}, \epsilon_2^{(1)}, \epsilon_3^{(1)}\} = U_M^{(1)\dagger}\mathcal{H}_{\Re}^{(0)}U_M^{(1)}$ and again the resulting Hamiltonian real part is not diagonal, so one diagonalizes it again! Then, the final form is

$$\mathcal{H}^{(\infty)} = \begin{pmatrix} \epsilon_1 & i\eta_{12} & i\eta_{13} \\ -i\eta_{12} & \epsilon_2 & i\eta_{23} \\ -i\eta_{13} & -i\eta_{23} & \epsilon_3 \end{pmatrix}$$

$$\tilde{\nu}_i = (\widetilde{U}_M)_{i\alpha}\nu_\alpha$$

$$\widetilde{U}_M = U_M^{(\infty)}...U_M^{(1)}U_M^{(0)}$$

Practically, all the unitary matrices $U_M^{(n>0)}$ are almost identical to the unit matrix for the majority of the SN density profiles, at least the power law one.

This form of $\mathcal{H}^{(\infty}$ is now suitable to study possible transitions.

Non-Adiabatic Resnonces

Assming that all possible resonances are separated, then 3nu system with the Hamiltion

$$\mathcal{H}^{(\infty)} = \begin{pmatrix} \epsilon_1 & i\eta_{12} & i\eta_{13} \\ -i\eta_{12} & \epsilon_2 & i\eta_{23} \\ -i\eta_{13} & -i\eta_{23} & \epsilon_3 \end{pmatrix}$$

can be reduced into 3 two nu subsystems $\{\nu_i, \nu_j\}$.

So, is it possible to have 3 (non)-adiabatic transitions with the adiabaticity parameter?

$$\gamma_{ij} = \left| \frac{\varepsilon_i - \varepsilon_j}{2\eta_{ij}} \right|_{res}.$$

If yes, how is the resoance region can be defined?
- by making the diagonal elments in the flavor Hamiltonian equals? (as in 2nu scheme).
- by minimzing the eigenvalues difference $|\varepsilon_i - \varepsilon_j|$?
- by minimizing the adiabaticity parameter γ_{ij} ? ==> This should be the most precise and meaningfull?

If the exist a resonance between $\tilde{\nu}_1 \longleftrightarrow \tilde{\nu}_3$ in the SN, its corresponding density should be between the L and H resonance densities, so we call it M (middle) resonance.

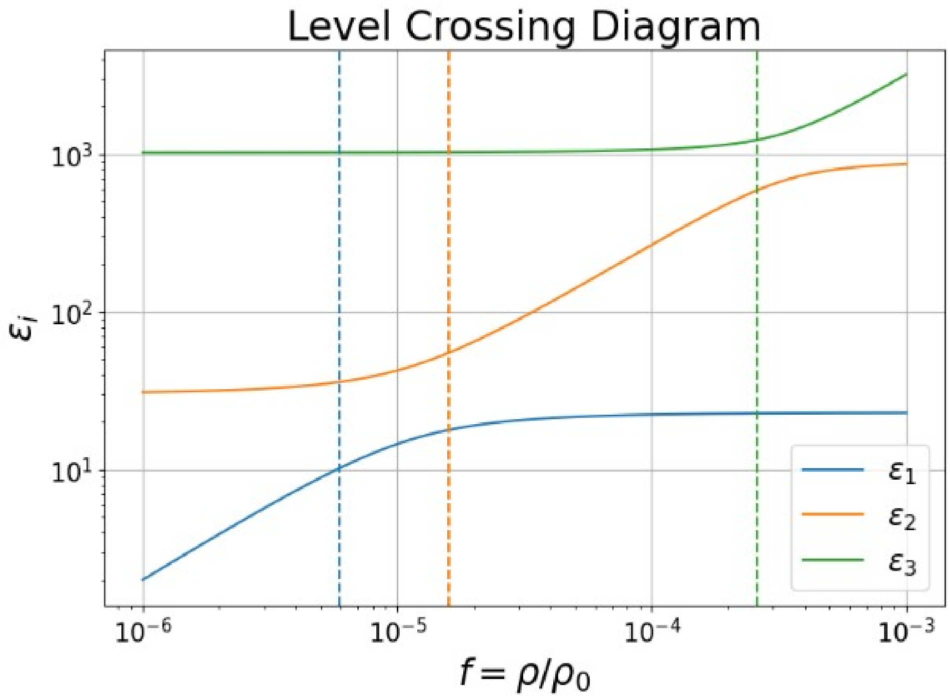
The neutrino final spectrum can be written as

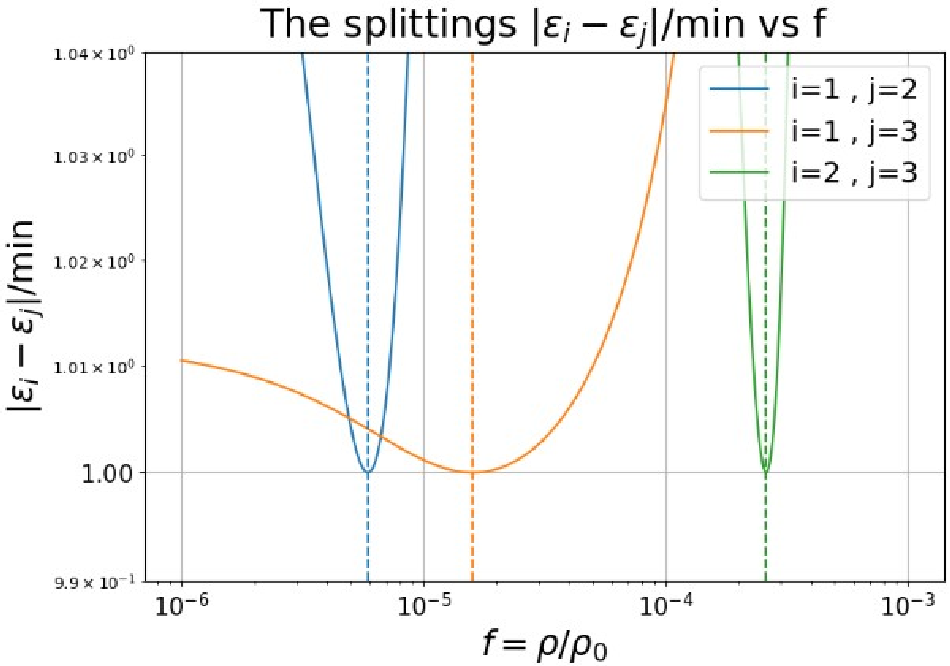
$$\begin{pmatrix} F_e \\ F_\mu \\ F_\tau \end{pmatrix} = \begin{pmatrix} |U_{e1}|^2 & |U_{e2}|^2 & |U_{e3}|^2 \\ |U_{\mu 1}|^2 & |U_{\mu 2}|^2 & |U_{\mu 3}|^2 \\ |U_{\tau 1}|^2 & |U_{\tau 2}|^2 & |U_{\tau 3}|^2 \end{pmatrix} \begin{pmatrix} 1-P_L & P_L & 0 \\ P_L & 1-P_L & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1-P_M & 0 & P_M \\ 0 & 1 & 0 \\ P_M & 0 & 1-P_M \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1-P_H & P_H \\ 0 & P_H & 1-P_H \end{pmatrix}] \begin{pmatrix} F_\mu^0 \\ F_\tau^0 \\ F_e^0 \end{pmatrix}$$

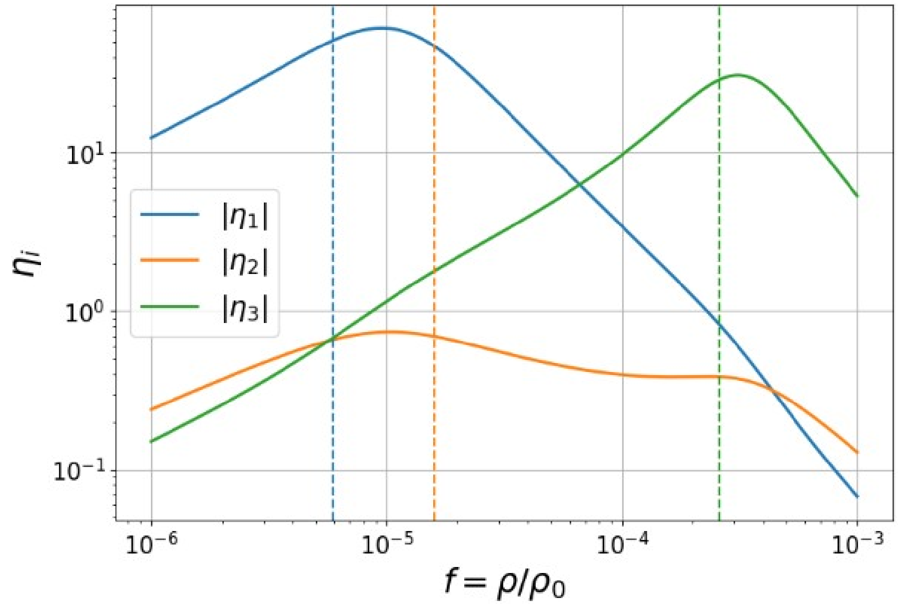
$$P_L = \exp\left(-\frac{\pi}{2}\gamma_{12}\right), P_M = \exp\left(-\frac{\pi}{2}\gamma_{13}\right), \; P_H = \exp\left(-\frac{\pi}{2}\gamma_{23}\right)$$

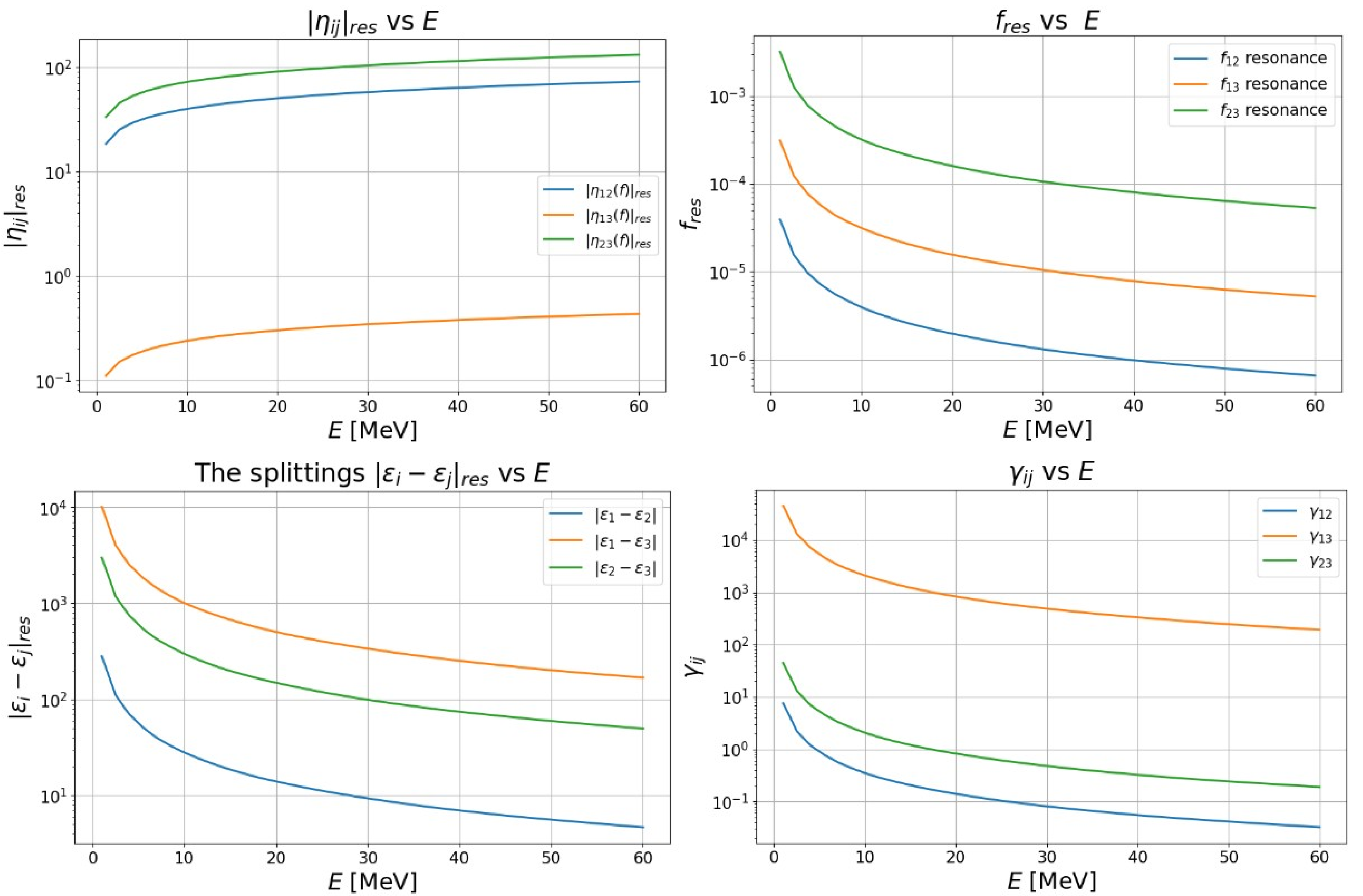
In the IH case, this definiton is different and it dpends on where the M resonance occurs? In the neutrino or antineutrino sector?

Numerical Results

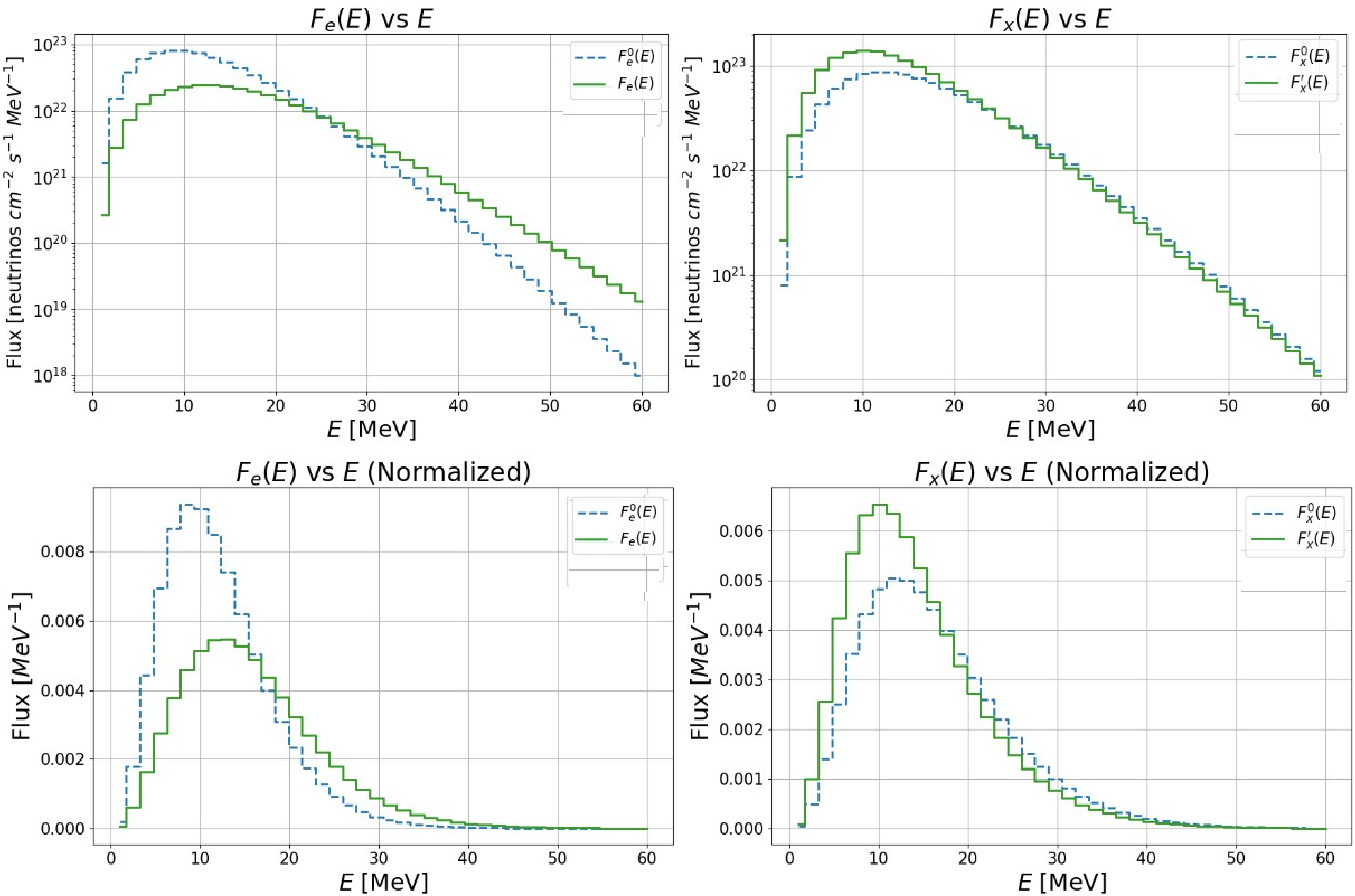








$|\eta_{ij}|_{res}$ vs E
$|\eta_{ij}|_{res}$
E [MeV]
$|\eta_{12}(f)|_{res}$
$|\eta_{13}(f)|_{res}$
$|\eta_{23}(f)|_{res}$
f_{res} vs E
f_{res}
E [MeV]
f_{12} resonance
f_{13} resonance
f_{23} resonance
The splittings $|\varepsilon_i - \varepsilon_j|_{res}$ vs E
$|\varepsilon_i - \varepsilon_j|_{res}$
E [MeV]
$|\varepsilon_1 - \varepsilon_2|$
$|\varepsilon_1 - \varepsilon_3|$
$|\varepsilon_2 - \varepsilon_3|$
γ_{ij} vs E
γ_{ij}
E [MeV]
γ_{12}
γ_{13}
γ_{23}



$F_e(E)$ vs E
Flux [neutrinos cm^{-2} s^{-1} MeV^{-1}]
10^{23}
10^{22}
10^{21}
10^{20}
10^{19}
10^{18}
$F_e^0(E)$
$F_e(E)$
0 10 20 30 40 50 60
E [MeV]
$F_X(E)$ vs E
Flux [neutrinos cm^{-2} s^{-1} MeV^{-1}]
10^{23}
10^{22}
10^{21}
10^{20}
$F_X^0(E)$
$F_X'(E)$
0 10 20 30 40 50 60
E [MeV]
$F_e(E)$ vs E (Normalized)
Flux [MeV^{-1}]
0.008
0.006
0.004
0.002
0.000
$F_e^0(E)$
$F_e(E)$
0 10 20 30 40 50 60
E [MeV]
$F_X(E)$ vs E (Normalized)
Flux [MeV^{-1}]
0.006
0.005
0.004
0.003
0.002
0.001
0.000
$F_X^0(E)$
$F_X'(E)$
0 10 20 30 40 50 60
E [MeV]

Conclusion

We numerically describe exactly the neutrino propagation inside a varying density medium (like SN), where we found:

- Possible new resoance (M, middle), purely adiabatic for a power law density profile, maybe not for realistic models.

- The resonance region should should correspond a minimal adiabaticity parameter.

- For realistic SN density profiles, the neurino final spectrum can help in probing the nu hierarchy, the SN density profile, and probably non-standard effect.

Thank You