





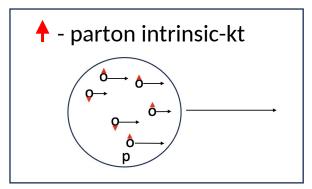
Nataša Raičević

On behalf of the CASCADE group

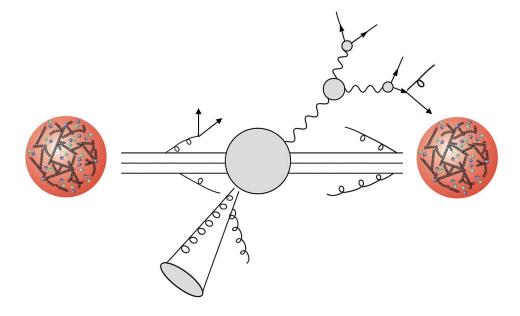
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> The largest component of an initial parton's momenta inside a hadron is its longitudinal momentum, received from the parent parton. However, in addition to this, partons also posses transverse momentum due to their internal (Fermi) motion - known as intrinsic k_T

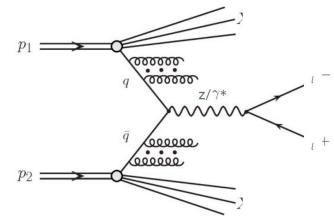


- Study of the internal parton motion with the Parton Branching (PB) method
- ☐ Recent studies of the internal parton motion with shower based Monte-Carlo event generators
- ☐ Validation studies of the new PDF2ISR approach PB framework implemented in PYTHIA event generator 1

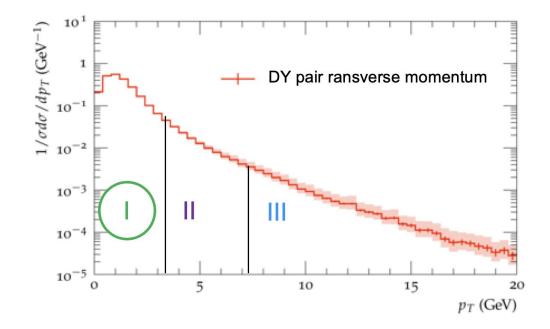
Non-perturbative processes in the parton evolution in the initial state

- ☐ Intrinsic motion of partons
- ☐ Multiple soft gluon emmissions

Direct impact on the transverse momentum distribution of Drell-Yan (DY) pairs produced in hadron-hadron collisions



☐ The production of DY lepton pairs in hadron collisions - excellent process to study various QCD effects

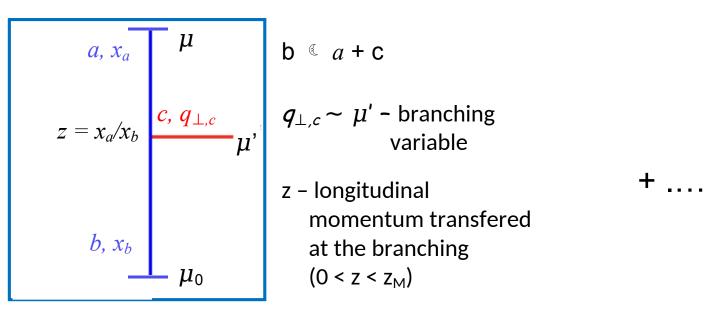


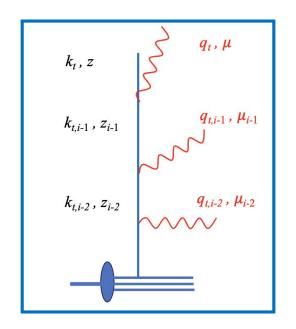
- I Non-perturbative region
 - Intrinsic-kT
 - resummation of multiple soft gluon emissions
- II Transition region
- III Perturbative higher-order contributions dominating

Why Parton Branching (PB) method?

- ☐ The main goal of all theoretical predictions in HEP nowdays is to reduce unceratinties at all levels
 - Testing the consistency of the Standard Model and identify potential deviations that could signal new physics
- ☐ However, the challenge of treating soft gluon emissions and their resummation in collinear parton shower generators still persists
 - [©] The development of the PB Method which takes a different approach by introducing the transverse degree of freedom already at the parton level (k_T parton transverse momentum)
- ☐ The PB Method describes partons from colliding hadron via Transverse Momentum Dependent (TMD) parton distribution functions (PDF) TMDs
 - $A_a(x,k_T,\mu^2)$ gives the probability of finding a parton **a** with a hadron momentum fraction x, transverse momentum k_T and at the evolution scale μ
 - TMDs for all flavors across a wide kinematic range obtained from the TMD evolution equation

Parton branching





One branching

More branchings

- \Box z_M soft gluon resolution parameter defining resolvable (z < z_M) and non-resolvable (z > z_M) parton branchings
- > PB method takes into account *angular ordering* based on colour coherence in QCD according to which the angles of partons with respect to an initial hadron increase in the subsequent branching

 $\mu' = |\mu'| = q_{\perp}/(1-z)$ – angular ordering is independent of the choice of the soft-gluon resolution scale when $z_{\rm M} \to 1$

TMDs

- \square Parton evolution is expressed in terms of resolvable, real emission DGLAP splitting functions, P_{ab} for parton splitting $b \to a$, and Sudakov form factors (Δ_a) which give the probability to evolve from one scale to another scale without resolvable branching
- The TMD for a parton a, with the longitudinal momentum fraction x of the proton and the transverse momentum k, evaluated at a scale μ :

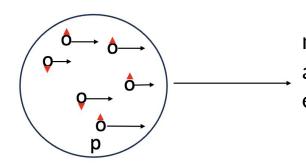
$$\mathcal{A}_{a}(x, \mathbf{k}, \mu^{2}) = \Delta_{a}(\mu^{2}) \, \mathcal{A}_{a}(x, \mathbf{k}, \mu_{0}^{2}) + \sum_{b} \int_{\mu_{0}}^{\mu} \frac{d^{2}\mu'}{\pi \mu'^{2}} \, \frac{\Delta_{a}(\mu^{2})}{\Delta_{a}(\mu'^{2})} \, \Theta(\mu^{2} - \mu'^{2}) \, \Theta(\mu'^{2} - \mu_{0}^{2})$$

$$\times \int_{x}^{z_{M}} \frac{dz}{z} \, P_{ab}^{(R)}(\alpha_{s}, z) \, \mathcal{A}_{b}\left(\frac{x}{z}, \mathbf{k} + (1 - z) \, \boldsymbol{\mu}, \mu'^{2}\right)$$

$$\Delta_{a}(z_{M}, \mu^{2}, \mu_{0}^{2}) = \exp\left(-\sum_{b} \int_{\mu_{0}^{2}}^{\mu^{2}} \frac{d\mu'^{2}}{\mu'^{2}} \int_{0}^{z_{M}} dz \, z \, P_{ba}^{(R)}(\alpha_{s}, z)\right)$$

- $\mathcal{A}_a(x,\mathbf{k},\mu_0^2)$ the TMD at the starting scale μ_0 is a nonperturbative boundary condition to the evolution equation and is determined from experimental data
- $ightharpoonup z_M
 ightharpoonup 1$ gives the exact solution of the DGLAP evolution
- \triangleright Integration of $\mathcal{A}_a(x,\mathbf{k},\mu^2)$ over all \mathbf{k} gives collinear PDFs $f_a(x,\mu^2)$
- \square $\alpha_s = \alpha_s(\mu'^2(1-z)^2) = \alpha_s(q_T^2)$ (the TMD set termed as PB-NLO-2018 set 2

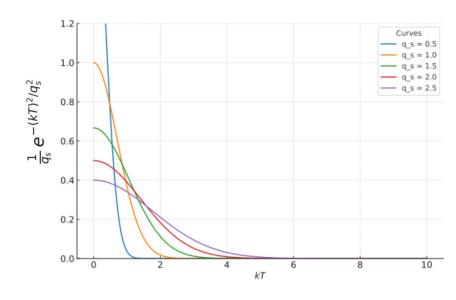
Intrinsic-k_T in TMDs

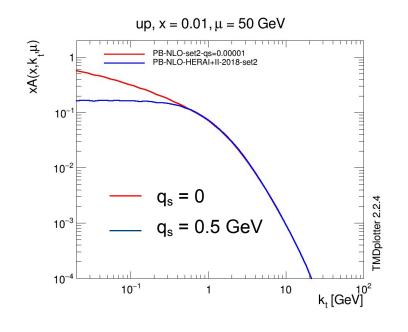


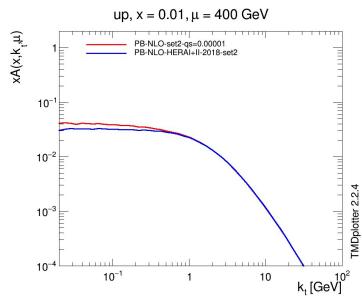
In the evolution, intrinsic k_T is introduced as a non-perturbative parameter and is generated from a Gaussian distribution with width σ which is expressed in the PB Model via the parameter \mathbf{q}_s :

$$\sigma^2 = q_s^2/2$$

$$A_a(x,k_0,\mu_0^2) = f_a(x,\mu_0^2) \cdot \exp(-|k_0^2|/q_s^2)/(\pi q_s^2)^{1/2}$$







Soft contributions and Sudakov form factor

Since $\alpha_s = \alpha_s$ (q_T) $\ ^{\ }$ $\ q_0$ where α_s is frozen leads to two different regions: a perturbative region, with $q_T > q_0$, and and non-perturbative region of $q_T < q_0$

$$z_{dyn} = 1 - q_0/\mu'$$

- Two regions of z:
 - a perturbative region, with $0 < z < z_{dyn} (q_T > q_0)$
 - a non-perturbative region with $z_{dyn} < z < z_M (q_T < q_0)$
- \triangleright define a perturbative (P) and non-perturbative (NP) ($z_{dyn} < z < z_M, z_M \rightarrow 1$) Sudakov form factors

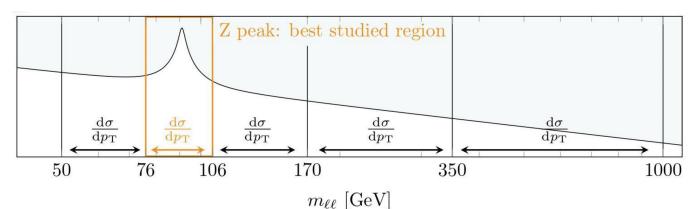
$$\Delta_{a}(\mu^{2}, \mu_{0}^{2}) = \exp\left(-\sum_{b} \int_{\mu_{0}^{2}}^{\mu^{2}} \frac{d\mathbf{q}'^{2}}{\mathbf{q}'^{2}} \int_{0}^{z_{\text{dyn}}} dz \ z \ P_{ba}^{(R)}(\alpha_{s}, z)\right)$$

$$\times \exp\left(-\sum_{b} \int_{\mu_{0}^{2}}^{\mu^{2}} \frac{d\mathbf{q}'^{2}}{\mathbf{q}'^{2}} \int_{z_{\text{dyn}}}^{z_{\text{M}} \approx 1} dz \ z \ P_{ba}^{(R)}(\alpha_{s}, z)\right)$$

$$= \Delta_{a}^{(P)}\left(\mu^{2}, \mu_{0}^{2}, q_{0}^{2}\right) \cdot \Delta_{a}^{(NP)}\left(\mu^{2}, \mu_{0}^{2}, q_{0}^{2}\right) .$$

Determination of the Gaussian width q_s

☐ The recent publication from CMS on transverse momentum distribution in a wide DY invariant mass [1] provides a detailed uncertainty breakdown



the basic data for the determination of the intrinsic-k_T parameter q_s

- \Box q_s parameter in PB-NLO-2018 Set 2 is varied and compared to the measurement
- $\succ \chi^2$ is calculated to quantify the model agreement to the measurement

$$\chi^2 = \sum_{i,k} (m_i - \mu_i) C_{ik}^{-1} (m_k - \mu_k)$$

The covariance matrix C_{ik} consists of a component describing the uncertainty in the measurement, $C_{ik}^{measurement}$, and the statistical (bin by bin stat. unc) and scale uncertainties in the prediction

$$C_{ik} = C_{ik}^{measurement} + C_{ik}^{model-stat.} + C_{ik}^{scale}$$

$$q_s = 1.04 \pm 0.08$$
 GeV

DY data at lower energies

- ☐ Most of the published experimental data do not include a detailed breakdown of uncertainties
- > The experimental uncertainties are treated as uncorrelated

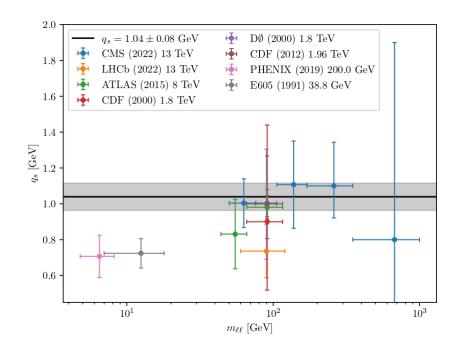
Analysis	√s	Collision type
CMS (2022) [1]	13 TeV	рр
LHCb (2022) [2]	13 TeV	рр
CMS (2021) [3]	8.1 TeV	pPb
ATLAS (2015) [4]	8 TeV	pp
CDF (2012) [5]	1.96 TeV	pp
CDF (2000) [6]	1.8 TeV	pp
D0 (2000) [7]	1.8 TeV	pp
PHENIX (2019) [8]	200 GeV	pp
E605 (1991) [9]	38.8 GeV	PN
E277 (1981) [10]	27.4 GeV	pΝ

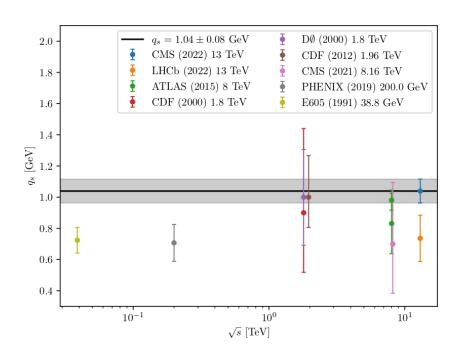
Intrinsic-kT width depending on √s and DY mass

 $q_0 = 10^{-2}$ GeV - minimal parton transverse momentum emitted at a branching

I. Bubanja et al, Eur.Phys.J.C 84 (2024) 2, 154 arXiv:2312.08655

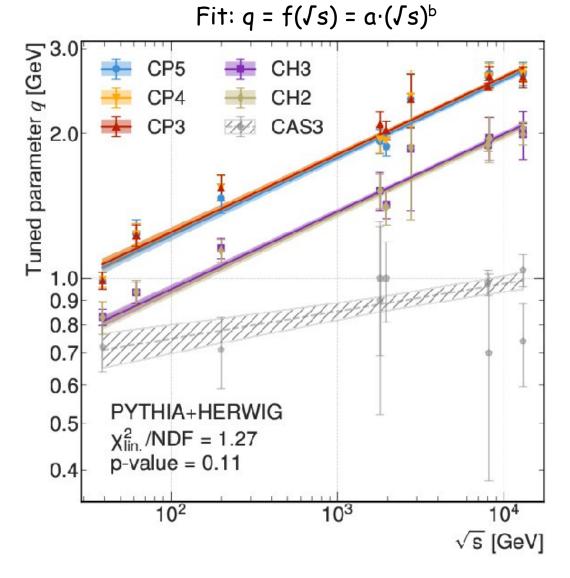
 $q_T > q_0$ soft contributions included





- Consistent values of q_s are observed across a wide range of DY pair invariant masses
- à No trend in centre-of-mass energy dependence of q_s is observed
- à The result stands in contrast to those from standard Monte Carlo event generators which require a strongly increasing intrinsic-k_⊤ width with √s

Center-of mass dependence of the intrinsic-k_T width in the shower-based generators



CMS Collaboration, *Phys.Rev.D* **111** (2025) 7, 072003 arXiv:2409.17770

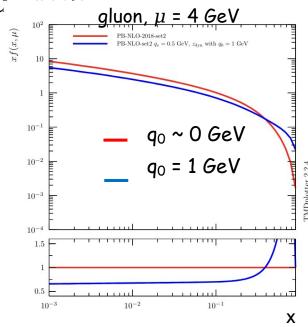
- ➤ In standard shower-based event generators such as **PYTHIA** and **HERWIG**, the intrinsic-k_T width increases with energy independently of the tune
- ➤ In contrast, the result from CASCADE3 (which is based on the PB Method), when fitted with the same function form, show only a very mild dependence
- ➤ A possible reason for this difference could be the exclusion of soft gluon emissions in PYTHIA and HERWIG, aimed at avoiding potential divergences
- ➤ The origin of this energy dependence has been studied using the CASCADE3 event generator by varying the contribution from soft gluon emissions

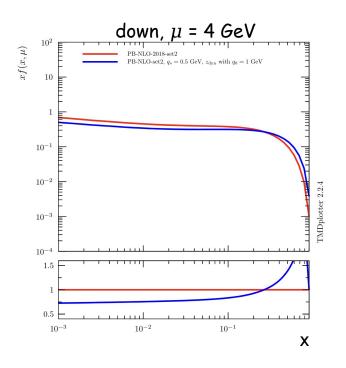
Try to introduce the energy dependence of the intrinsic-k_™ in PB

- \Box Try to mimic parton-shower event generators by demanding a minimal parton transverse momentum ($q_0 = 1$ and 2 GeV) \bigcirc $q_T > q_0$
 - \leq z_M constrained: z_M = z_{dyn} = 1 q₀/ μ' < 1

$$\dot{\mathbf{a}} \quad \Delta_a^{(\mathrm{NP})} \left(\mu^2, \mu_0^2, q_0^2 \right) = \\ \exp \left(- \sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_{z_{\mathrm{dyn}}}^{z_{\mathrm{M}}} dz \ z \ P_{ba}^{(R)} \left(\alpha_s, z \right) \right) - \\ \mathrm{neglected}$$

à Real emissions with $z > 1 - q_0/\mu'$

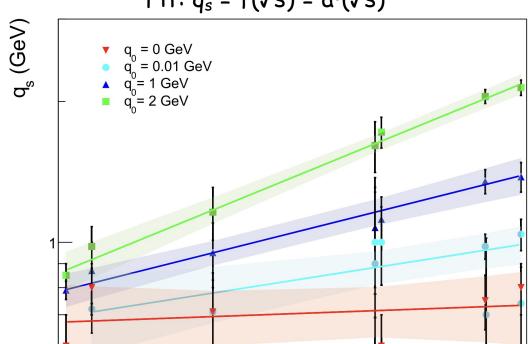




- ☐ Integrated parton distributions very different for the two cases
 - soft contributions important also for collinear distributions

q_s vs \sqrt{s} for different q_0 in the PB

Fit:
$$q_s = f(Js) = a \cdot (Js)^b$$



10³

√s (GeV)

I. Bubanja et al, Eur.Phys.J.C 85 (2025) 3, 278 arXiv:2404.04088

- ☐ The intrinsic-k_T width parameter increases with the collision energy
- \Box The slope od the dependence increases as q_0 increases
- \square Larger q_0 means that more soft contributions are excluded
 - Larger intrinsic-k_T needed to compensate missing contribution from soft gluons

 10^{2}

Dependence of the intrinsic-k_T width on the hard scale

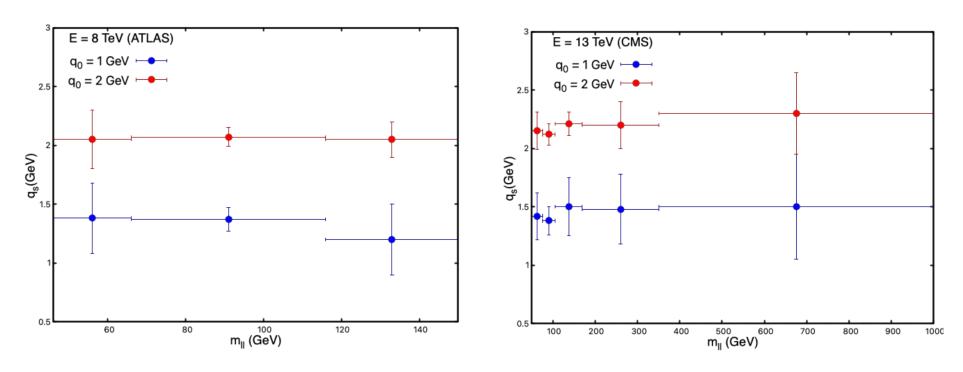
N. Raičević

Phys. Scr. **100 (2025)** 045306

arXiv:2412.00892

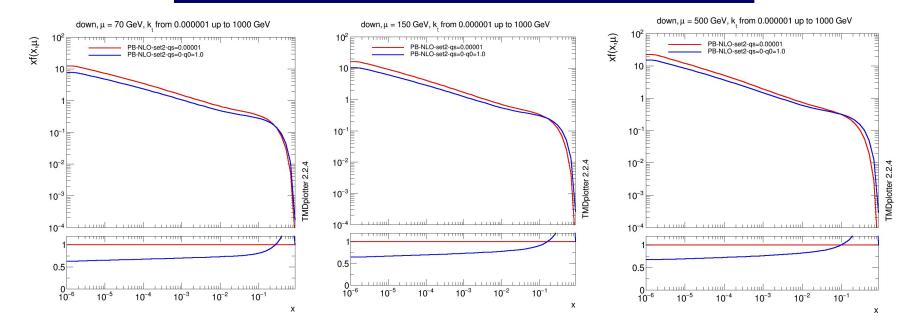
q_s as a function of DY pair invariant mass with a q₀ cut at LHC

☐ The evolution scale directly linked to the DY-pair invariant mass (m₁₁)



- ☐ From the available measurements and existing uncertainties: q_s remains independent of the DY pair invariant mass for $q_0 \lesssim 2 \text{ GeV}$
 - $^{\circ}$ the fraction of soft parton contributions with the transverse momentum ~1 GeV which populate the DY p_T region up to 2-3 GeV similar for different mass regions

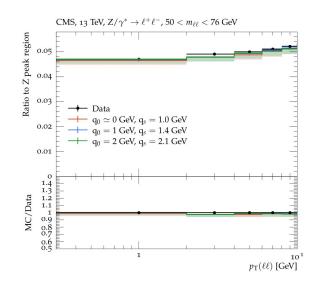
Integrated PDFs with a q₀ cut at high scales



The scale values relevant for available measurements from LHC

- \Box The change of the integrated PDFs by introducing a cut of $q_0 \sim 1$ GeV is similar for different scale values, μ , relevant for the available measurements at the LHC
 - © Consistent with the result of q_s vs m(II) from the available measurements from LHC
 - © Consistent with the non-perturbative Sudakov FF which is sensitive at small values of μ (the next slides) and changes slowly with μ in the region of μ ~ 100 GeV where measurements from the LHC have been performed

The ratio of the cross section to Z-peak region as a function of DY-p_T at different scales



CMS, 13 TeV, $Z/\gamma^* \to \ell^+\ell^-$, 170 < $m_{\ell\ell}$ < 350 GeV

 $q_0 \simeq 0$ GeV, $q_s = 1.0$ GeV

 $q_0 = 1 \text{ GeV}, q_s = 1.4 \text{ GeV}$

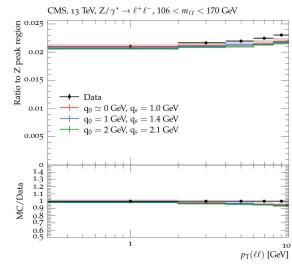
 $p_{\rm T}(\ell\ell)$ [GeV]

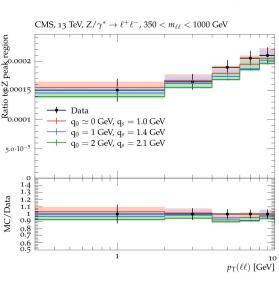
+- q₀ = 2 GeV, q_s = 2.1 GeV

₹0.0025

0.0005

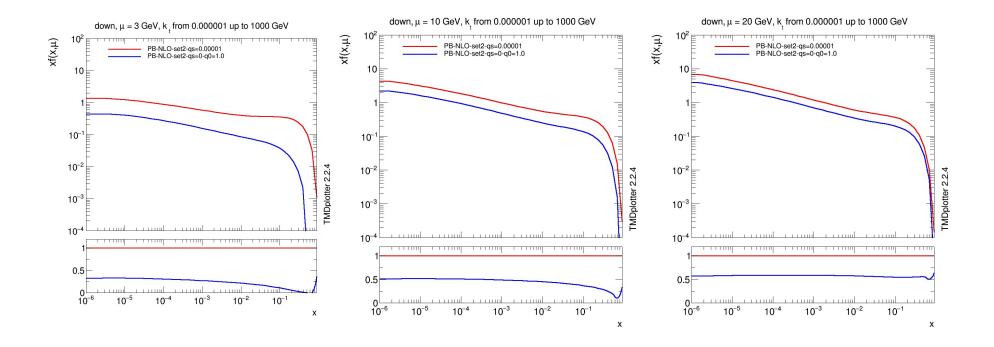
MC/Data





- -- q₀ = 0.01 GeV (q_s = 1.0 GeV)
- -- q₀ = 1 GeV (q_s = 1.4 GeV)
- --- q₀ = 2 GeV (q_s = 2.1 GeV)
- ☐ The ratio of the cross section to the Z peak similar for different DY pair invariant mass regions for $q_0 \lesssim 2 \text{ GeV}$
- ☐ The ratio is well described by the predictions, which use the same q_s width for each mass bin, and that the distributions obtained for different values of q_0 are similar.
 - The relative contributions of soft gluons are quite similar across different invariant mass bins, not only in the non-perturbative region but also in the transition region $(4 \le pT(ll) \le 15 \text{ GeV})$.

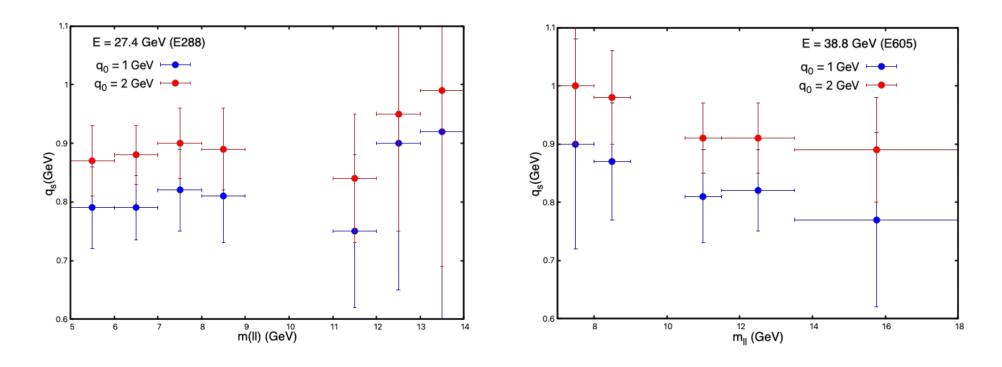
Integrated PDFs with q₀ cut at low scales



- \Box Different trend at low μ which corresponds to the measurements of the pair- p_T at low invariant masses not yet available at the LHC
- \triangleright The integrated PDFs by introducing the q₀ ~ 1 GeV cut varies rapidly with the μ scale relevant for DY pair masses ~10 GeV
 - $^{\circ}$ The relative amount of soft gluons removed by the cut changes significantly at low scales and the measurable changes in the value of q_s could be expected at low DY pair invariant masses for collision energies available at LHC

q_s as a function of DY pair invariant mass with a q_0 cut at small \sqrt{s}

- ☐ At the collision energies of 27.4 GeV (E288) and 38.8 GeV (E605), the DY pairs with the low inv. mass are mainly produced at high x from the valence contributions
- \triangleright One cannot expect measurable dependence of q_s on DY pair invariant mass



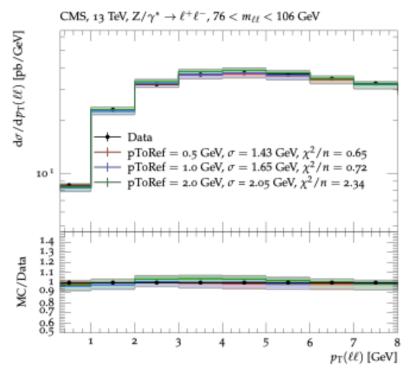
 $^{\circ}$ Although the errors are large and it is not possible to draw a firm conclusion, the trend of change of the q_s with m(II) is noticeable in the measurements obtained from the E605

Study of the non-perturbative effects with PYTHIA

I. Bubanja et al, Eur.Phys.J.C 85 (2025) 3, 363 arXiv:2412.05221

DY-pT distributions with different ISR cut-off parameter in PYTHIA

- \triangleright pt0ref minimum q_T (similar to q₀ in PB) ISR cut-off parameter in PYTHIA
- \blacktriangleright σ width of the Gaussian for intrinsic- k_T (similar to q_s)

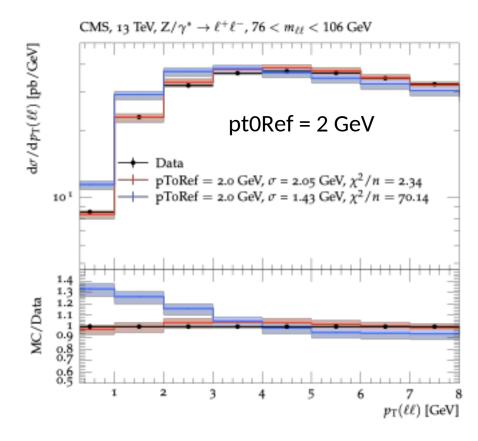


Optimal intrinsic-kT widths:

$$\sigma$$
 = 1.43 GeV for pt0Ref = 0.5 GeV

$$\sigma$$
 = 1.65 GeV for pt0Ref = 1.0 GeV

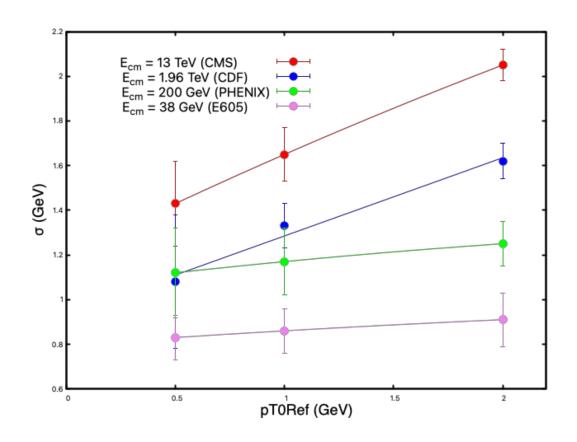
$$\sigma$$
 = 2.05 GeV for pt0Ref = 2.0 GeV



$$\sigma$$
 = 1.43 GeV

$$\sigma$$
 = 2.05 GeV

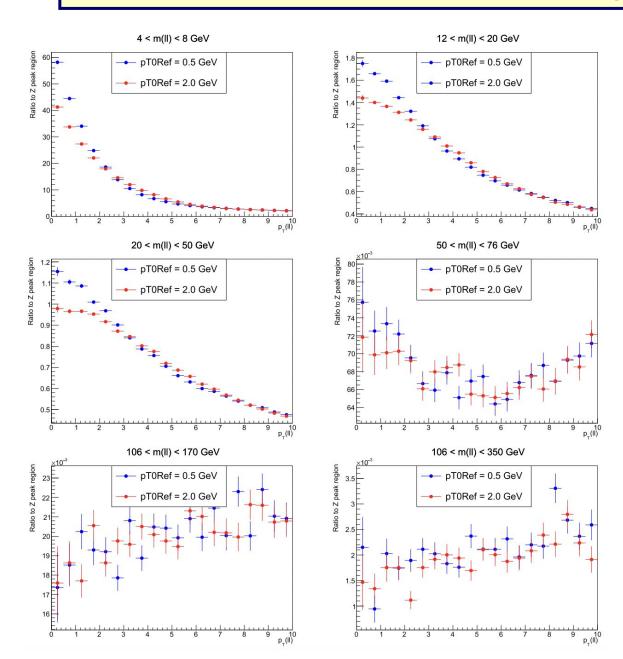
Dependence of the intrinsic-kT width on the ISR cut-off parameter



The intrinsic-kT width, reflecting the interplay between two non-perturbative processes (internal transverse motion inside the hadrons and soft gluon emissions), increases approximately linearly with the ISR cut-off parameter in the range 0.5 < pTORef < 2.0 GeV.

clear evidence that the energy dependence is related to the no-emission probability, Sudakov form factor, through its dependence on pT0Ref.

The ratio of the cross section to Z-peak region as a function of DY-p_T at different scales

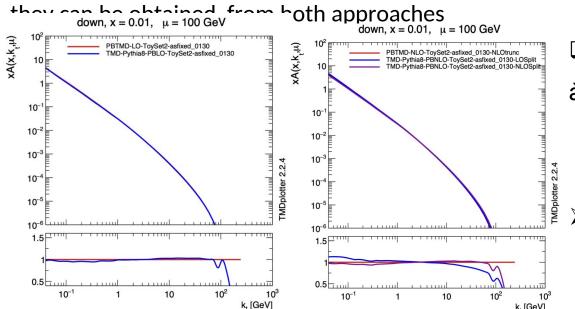


- ☐ The ratio of the cross section as a function of pT(II) to Z-peak region from PYTHIA predictions in different DY pair invariant mass bins, shown for the two values of the ISR cut-off parameter:
 - pT0Ref = 0.5 GeV and
 - pT0Ref = 2.0 GeV
- For high m_{DY} values as measured at the LHC no significant dependence of the width parameter σ , while extending the mass range to smaller values, such a dependence is clearly visible.
- ➤ The difference between the simulations with pTORef = 2 GeV and pTORef = 0.5 GeV increases as mDY decreases.
- © Consistent with the expectation that most of the contribution from the soft emissions arise at low scales

A new approach: **PDF2ISR** – implementation of the PB Method framework in PYTHIA

H. Jung et al, Eur. Phys. J. C (2025) 85:870, arXiv:2504.10243

- ☐ Implement the TMDs from PB Method in PYTHIA, and develop a procedure for obtaining an initial-state parton shower model in which the (backward) evolution is fully consistent with the (forward) evolution of the collinear parton density used
- > A parton shower consistent with parton densities at LO and NLO
- □ PDF2ISR approach constructs the initial-state radiation (ISR) simulated as a parton shower to exactly follow the evolution of the collinear parton density, using the PB Method
- > The PB-TMD distributions are, by construction, consistent with the collinear distributions upon integration over the transverse momentum
- à TMD parton densities are ideal for testing the consistency between the evolution and the parton shower, as



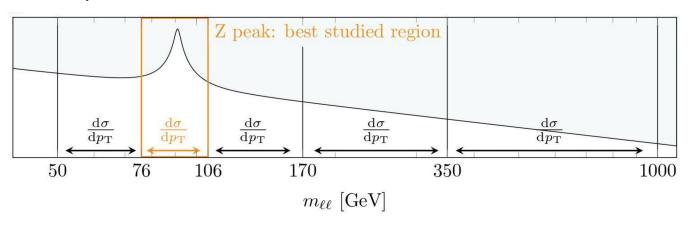
- ☐ By using dedicated Toy Model
- à Very good agreement between the TMDs from the PB and PYTHIA-PB at LO and NLO
 - [NLO splitting functions essential for consistent treatment with NLO PDF]
- Perform validation studies of the PDF2ISR implemented in PYTHIA event generator by applying on real DY pairs

Validation studies of the PDF2ISR approach

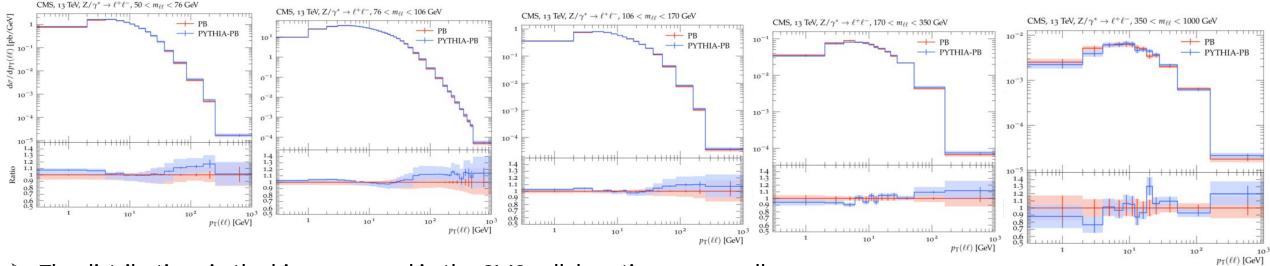
D. Subotić, H. Jung and N. Raičević 12th International Congress of the Balkan Physical Union (July, 2025)

Compare real DY p_T from PB and PDF2ISR (PYTHIA-PB) at NLO at 13 TeV

☐ Use phase space recently measured by the CMS at 13 TeV



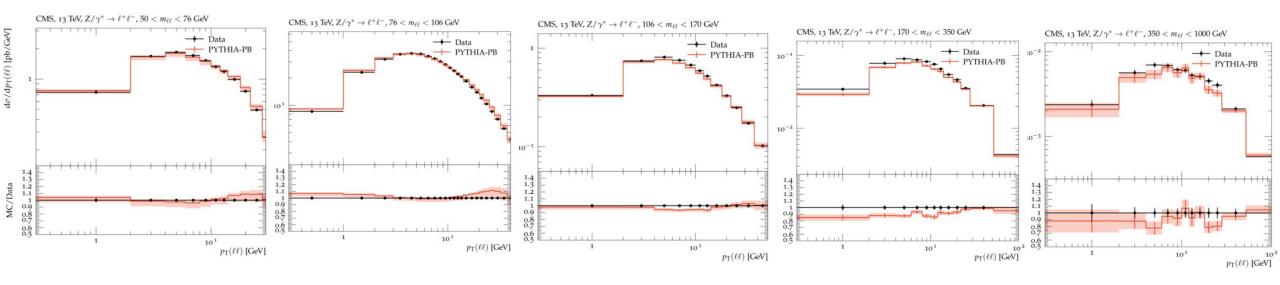
☐ All the processes switched-off but ISR



> The distributions in the bins measured in the CMS collaboration agree well

PYTHIA-PB at NLO at 13 TeV compared with the data

- ☐ Use the value of the intrinsic transverse momentum of 1 GeV
- ☐ Turn on FSR



☐ The data well described by the PDF2ISR approach



- ☐ A detailed study of non-perturbative processes and their interplay was performed using DY pair production with the PB Method
- ☐ The limitations of standard event generators in describing and distinguishing non-perturbative processes were identified
- > The intrinsic-kT width is found to increase with both the ISR cut-off parameter and the collision energy, due to the interplay between intrinsic transverse motion and multiple soft-gluon emissions in the non-perturbative region of DY production
- ➤ The intrinsic-k_T contribution can be disentangled from the non-perturbative Sudakov contribution only through a proper treatment of non-perturbative processes, which the PB Method enables due to its sensitivity to non-perturbative TMD contributions
- ☐ Using the recent results on DY production at the LHC, validation studies of a new approach, named **PDF2ISR**, were performed, in which the TMD framework from the PB Method was implemented into the PYTHIA event generator
- \triangleright very good agreement is observed between the DY-pair p_T distributions obtained using the PB Method and those from the PDF2ISR approach
- > Experimental data from the LHC are well described by the PDF2ISR approach

Thank you very much for your attention

References for the data used

- [1] https://arxiv.org/abs/2205.04897
- [2] https://arxiv.org/abs/2112.07458
- [3] https://arxiv.org/abs/2102.13648
- [4] https://arxiv.org/abs/1512.02192
- [5] https://arxiv.org/abs/1207.7138
- [6] https://arxiv.org/abs/hep-ex/0001021
- [7] https://arxiv.org/abs/hep-ex/9907009
- [8] https://arxiv.org/abs/1805.02448
- [9] https://journals.aps.org/prd/abstract/10.1103/PhysRevD.43.2815
- [10] https://journals.aps.org/prd/abstract/10.1103/PhysRevD.23.604