B-mesons oscillations: applications of two-particle wave functions

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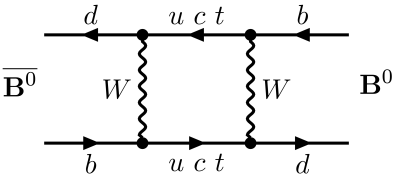
B-mesons oscillations overview

Feynman diagram of B^0-$\bar{B}^0$ oscillations



- Mixing:

$$B_L = pB^0 + q\bar{B}^0,$$
$$B_H = pB^0 - q\bar{B}^0,$$

where p, q are responsible for CPV.

1. Oscillations occur if the following value is nonzero:

$$R := \frac{\Gamma(B^0 \to \bar{B}^0 \to \ell^- \bar{\nu}_\ell X)}{\Gamma(B^0 \to \ell^+ \nu_\ell \bar{X})} = \frac{x^2}{2 + x^2}.$$

2. The number $x = \frac{m_H - m_L}{\Gamma} = \frac{\Delta m}{\Gamma}$ determines the value of oscillations.
3. The oscillations can be measured experimentally in $\Upsilon \to B\bar{B} \to \ell^\pm \ell^\pm X$:

$$r = \frac{N_{++} + N_{--}}{N_{+-} + N_{-+}},$$

where $N_{\pm\pm}$ are the numbers of dileptons of the required sign.

The way of $Br(B_s \to \mu^+\mu^-)$ precise measurement

1. Rare $B_s \to \mu^+\mu^-$ decay $\to$ possible New Physics:

$$Br = (3.63^{+0.15}_{-0.10}) \times 10^{-9} - \text{ SM;}$$
$$Br = (3.01 \pm 0.35) \times 10^{-9} - \text{ LHC.}$$

2. $f_s = (22.0^{+2.0}_{-2.1})\% \to$ it is important to decrease the error.
3. Our approach based on analysis of time-dependent numbers of dileptonic events allows to extract $Br\left(\Upsilon(5S) \to B_s^{(*)}\bar{B}_s^{(*)}\right)$ with 1% accuracy.
4. Determine $Br(B_s \to D_S\pi)/Br(B^0 \to D\pi)$ at Belle II $\to$ $B_s \to D_S\pi$ as normalization channel.

Normalization channel:

$$Br(B_q \to X) = Br(B_{q'} \to X')\frac{f_{q'}}{f_q}\frac{\epsilon_{X'}}{\epsilon_X}\frac{N_X}{N_{X'}}.$$

- f_q is a fragmentation function $b \to B_q$;
- ϵ_X is an efficiency to X final state;
- N_X is a total number of X events.

Extraction of ϵ_{SS}

- Two-particles wave functions allow to calculate the time-dependent numbers of dileptons $dN_{\pm\pm}/d\Delta t$ and $dN_{\pm\mp}/d\Delta t$ in $\Upsilon(5S) \to B\bar{B}X \to \ell^{\pm}\ell^{\pm}X'$:

$$\frac{d(N_{+-} + N_{-+} - N_{--} - N_{++})/d\Delta t}{d(N_{+-} + N_{-+} + N_{--} + N_{++})/d\Delta t};$$

$$\frac{d(N_{++} + N_{--})/d\Delta t}{d(N_{+-} + N_{-+} + N_{--} + N_{++})/d\Delta t}.$$

- The introduced ratios allow to decrease the uncertainty:

$$\frac{dN_{ij}}{d\Delta t} \sim L\sigma_{e^+e^- \to \Upsilon(5S)} Br(B^0 \to X\ell\nu)\Gamma.$$

C-parity of $B\bar{B}$ pair	Decay modes	Branching notation
	$B^0\bar{B}^0$ in the final state	
C-odd state	$B^0\bar{B}^0$ $B^{0*}\bar{B}^{0*}$ $B^0\bar{B}^0\pi^0$ $B^{0*}\bar{B}^{0*}\pi^0$	$(\epsilon_{00})^{\text{odd}}$
C-even state	$B^0\bar{B}^{0*}$ $B^{0*}\bar{B}^0$ $B^{0*}\bar{B}^0\pi^0$ $B^0\bar{B}^{0*}\pi^0$	$(\epsilon_{00})^{\text{even}}$
	B^+B^- in the final state	
C-odd and C-even states	B^+B^- $B^{+*}B^{-*}$ $B^+B^-\pi^0$ $B^{+*}B^{-*}\pi^0$ B^+B^{-*} $B^{+*}B^-$ $B^{+*}B^-\pi^0$ $B^+B^{-*}\pi^0$	ϵ_{+-}
	$B_S\bar{B}_S$ in the final state	
C-odd state	$B_S\bar{B}_S$ $B_S^*\bar{B}_S^*$	$(\epsilon_{SS})^{\text{odd}}$
C-even state	$B_S^*\bar{B}_S$ $B_S\bar{B}_S^*$	$(\epsilon_{SS})^{\text{even}}$
	$B^{\pm}B^0$ in the final state	
No definite C-parity	$B^+\bar{B}^0\pi^-$ $B^{+*}\bar{B}^0\pi^-$ $B^+\bar{B}^{0*}\pi^-$ $B^{+*}\bar{B}^{0*}\pi^-$ $B^-B^0\pi^+$ $B^{-*}B^0\pi^+$ $B^-B^{0*}\pi^+$ $B^{-*}B^{0*}\pi^+$	ϵ_+ ϵ_-

Two-particle wave function of the C-odd state

- Antisymmetric wave function:

$$\left|B^0\bar{B}^0\right\rangle_{odd}(t_1,t_2) = \left|B^0(t_1)\right\rangle\left|\bar{B}^0(t_2)\right\rangle - \left|B^0(t_2)\right\rangle\left|\bar{B}^0(t_1)\right\rangle .$$

- According to $N_{\pm\pm} = \frac{1}{2}|\left\langle B^0B^0\middle|B^0\bar{B}^0(t_1,t_2)\right\rangle_{odd}|^2$, $N_{\pm\mp} = \frac{1}{2}|\left\langle B^0\bar{B}^0\middle|B^0\bar{B}^0(t_1,t_2)\right\rangle_{odd}|^2$:

$$\frac{dN_{++}}{d\Delta t} = \frac{dN_{--}}{d\Delta t} = \frac{1}{2\Gamma}e^{-\Gamma\Delta t}\sin^2\frac{\Delta m\Delta t}{2},$$

  where $\Delta t = t_1 - t_2$.
- Normalization is as follows: $\sum\limits_{i,j=+,-} N_{ij} = 1/\Gamma^2$.
- For the R_{odd} we reproduce the well-known expression:

$$R_{odd} = \frac{N_{++}+N_{--}}{N_{+-}+N_{--}} = \frac{x^2}{2+x^2}.$$

Two-particle wave function of the C-even state

- C-even state is produced in $\Upsilon(5S)$ decay with further e/m decay: $\Upsilon \to B^{0*}\bar{B}^0 \to B^0\bar{B}^0\gamma$.
- Symmetric wave function:

$$\left|B^0\bar{B}^0\right\rangle_{even}(t_1, t_2) = \left|B^0(t_1)\right\rangle\left|\bar{B}^0(t_2)\right\rangle + \left|B^0(t_2)\right\rangle\left|\bar{B}^0(t_1)\right\rangle;$$

- Analogously to the C-odd state we obtain:

$$\frac{dN_{++}}{d\Delta t} = \frac{dN_{++}}{d\Delta t} = \frac{1}{4\Gamma(1+x^2)}e^{-\Gamma\Delta t}\left(1 + x^2 - \cos\Delta m\Delta t + x\sin\Delta m\Delta t\right);$$

$$\frac{dN_{+-}}{d\Delta t} = \frac{dN_{-+}}{d\Delta t} = \frac{1}{4\Gamma(1+x^2)}e^{-\Gamma\Delta t}\left(1 + x^2 + \cos\Delta m\Delta t - x\sin\Delta m\Delta t\right);$$

$$R_{even} = \frac{3x^2 + x^4}{2 + x^2 + x^4}.$$

Case of one charged B

- Neutral B oscillates:

$$\left|B^0(t)\right\rangle = e^{-iMt} e^{-(\Gamma/2)t}\left[\cos\left(\frac{\Delta m}{2}t\right)|B^0\rangle + i\frac{q}{p}\sin\left(\frac{\Delta m}{2}t\right)|\overline{B}^0\rangle\right]$$

- Charged B decays:

$$\left|B^+(t)\right\rangle = e^{-iM_+t} e^{-(\Gamma/2)t}.$$

- For time-dependent dilepton numbers we obtain:

$$\frac{dN_+}{d\Delta t} = \frac{1}{2}\frac{e^{-\Gamma\Delta t}}{\Gamma(4+x^2)}\left(2 + x^2 - 2\cos\Delta m\Delta t + x\sin\Delta m\Delta t\right).$$

$$\frac{dN_-}{d\Delta t} = \frac{1}{2}\frac{e^{-\Gamma\Delta t}}{\Gamma(4+x^2)}\left(6 + x^2 + 2\cos\Delta m\Delta t - x\sin\Delta m\Delta t\right).$$

Extraction of ϵ_{ss}

- Experimental fit of the ratios determines the relative probability of $\Upsilon(5S) \to B_s^{(*)} \bar{B}_s^{(*)}$:

$$\frac{d(N_{+-} + N_{-+} - N_{--} - N_{++})/d\Delta t}{d(N_{+-} + N_{-+} + N_{--} + N_{++})/d\Delta t} = C + A \sin(\Delta m \Delta t + \varphi),$$

$$\frac{d(N_{++} + N_{--})/d\Delta t}{d(N_{+-} + N_{-+} + N_{--} + N_{++})/d\Delta t} = C' - \frac{A}{2} \sin(\Delta m \Delta t + \varphi),$$

where

$$C = \epsilon_{+-} + \Big(\epsilon_{+} + \epsilon_{-}\Big) \frac{2}{4 + x^2},$$

$$C' = \frac{(\epsilon_{00})^{\text{odd}} + (\epsilon_{00})^{\text{even}} + (\epsilon_{SS})^{\text{odd}} + (\epsilon_{SS})^{\text{even}}}{2} + \Big(\epsilon_{+} + \epsilon_{-}\Big) \frac{1 + x^2/2}{4 + x^2}.$$

Extraction of ϵ_{ss}

- Using isotopic invariance we finally obtain:

$$2C' - C = \epsilon_{SS} + \left(\epsilon_+ + \epsilon_-\right)\frac{x^2}{4 + x^2}.$$

- The relative uncertainty from the value of the last term is about 1%.
- It allows to determine branching ratio of the normalization channel:

$$Br(B_s \to D_s^- \pi^+) = \frac{N(B_s \to D_s^- \pi^+)}{N(B^0 \to D^- \pi^+)} \frac{N(B^0)}{N(B_s)} Br(B^0 \to D^- \pi^+).$$

Experimental feasibility of the method

- Method is based on isotopic invariance. Possible violation can be overcome by direct measurement of $N(B^+)/N(B^0)$ with full reconstruction of B.
- Sufficiently high threshold on p_T of lepton in $b \to c\ell\nu$ leads to $b \to c \to \ell\nu$ contribution negligible.
- Statistic error in determination of f_s can be roughly estimated as $\pm 2\%$ for $L = 120 fb^{-1}$.
- B^+ and B^0 width difference $\approx 8\%$ can be taken into account: small correction for $\Delta t \leq 3/\Gamma_{B^0}$.
- Problem: relatively large difference in flight distance before first B-meson decay. Acceleration of $\Upsilon(5S)$ could do the job:

$$\frac{\Delta l}{l_0} = \frac{\gamma_2}{\gamma_1}\left(1 - \frac{v_2}{v_1}\right) = 0.2,$$

  for $V_\Upsilon = 0.7$. Another option is to restore close to maximum p_T events as it was done by BaBar.

$\epsilon_{+-}/\epsilon_{00}$ for $\Upsilon(4S)$

- According to the previous notation $\epsilon_{00} = Br(\Upsilon(4S) \to B^0\bar{B}^0)$ and $\epsilon_{+-} = Br(\Upsilon(4S) \to B^+B^-)$.
- Taking into account that for $\Upsilon(4S)$ $\epsilon_{00} + \epsilon_{+-} \approx 1$ for the $N_{\pm\pm}$, $N_{\pm\mp}$ ratios we obtain

$$\frac{d(N_{+-} + N_{-+} - N_{++} - N_{--})/d\Delta t}{d(N_{+-} + N_{-+} + N_{++} + N_{--})/d\Delta t} = \epsilon_{+-} + \epsilon_{00}\cos\Delta m\Delta t,$$

$$\frac{d(N_{++} + N_{--})/d\Delta t}{d(N_{+-} + N_{-+} + N_{++} + N_{--})/d\Delta t} = \frac{1}{2}\epsilon_{00}(1 - \cos\Delta m\Delta t),$$

- Experimental measurement of the left sides of the ratios above allow to extract the ratio $\epsilon_{+-}/\epsilon_{00}$. It can reduce current uncertainties $\to$ more accurate measurement of the probabilities of all exclusive B^+, B^0 decays in future Belle-II experiments.

Conclusions

- The approach based on two-particles wave functions describing oscillating $B\bar{B}$ system was presented.
- The method allowing to measure $Br(B_s \to \mu^+\mu^-)$ with 1% accuracy by means of model-independent determining $Br\left(\Upsilon(5S) \to B_s^{(*)}\bar{B}_s^{(*)}\right)$ was suggested.
- The same method allows to extract $\epsilon_{00}/\epsilon_{+-}$ ratio from $\Upsilon(4S)$ data.