Particle production via $\gamma\gamma$ fusion at hadron colliders with libepa

S. I. Godunov

in collaboration with

E. V. Zhemchugov, E. K. Karkaryan, V. A. Novikov,
A. N. Rozanov, M. I. Vysotsky

based on
CPC 305 (2024) 109347

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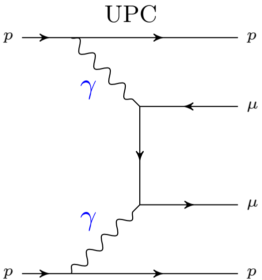
$\mu^+\mu^-$ production via $\gamma\gamma$ fusion



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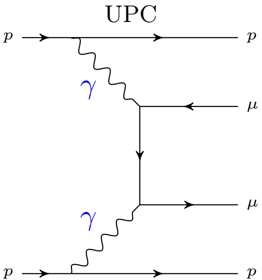
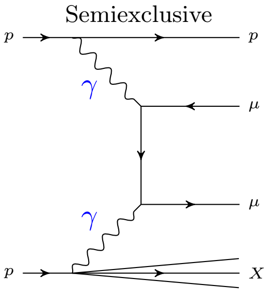
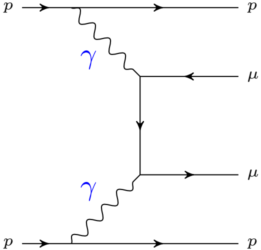


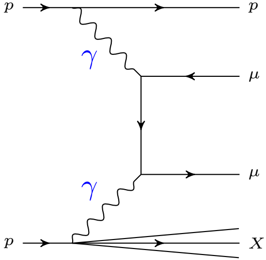


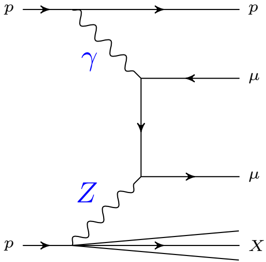
$\mu^+\mu^-$ production via $\gamma\gamma$ fusion

UPC



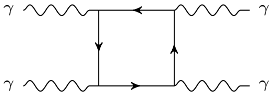
Semiexclusive





$\gamma\gamma$ fusion at the LHC

$\gamma\gamma \to \gamma\gamma$

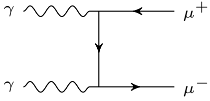


CMS, Phys.Lett. B797, 134826 (2019)

ATLAS, JHEP 03 243 (2021)

CMS, JHEP 08 (2025) 006

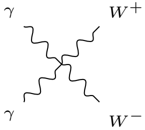
$\gamma\gamma \to \mu^+\mu^-$



ATLAS, Phys.Lett. B777, 303 (2018)

ATLAS, Phys. Rev. Lett. 125, 261801 (2020)

$\gamma\gamma \to W^+W^-$



ATLAS, Phys. Lett. B816, 136190 (2021)

CMS, JHEP 07 229 (2023)

Particle production in UPC

`libepa` approaches and code were developed while the authors were working on papers

- Phys. Usp. **62**, no.9, 910-919 (2019)
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Many of these results are included in the library documentation as examples.

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Many of these results are included in the library documentation as examples.

It was applied to semi-inclusive processes (where only one of the colliding particles remains intact, and the other disintegrates) in papers

- Eur. Phys. J. C **82**, no.11, 1055 (2022)
- Phys. Rev. D **108**, no.9, 093006 (2023)
- JETP Lett. **119**, no.1, 5-9 (2024)

Physical description and documentation: CPC **305** 109347 (2024)

Required features

- Proton form factors (including magnetic contribution)

New **CompHEP** functionality should allow this!

- Fiducial cross section
  For example, typical cuts for particle pair production are
  - $p_T > \hat{p}_T$ — transverse momentum of each particle.
  - $|\eta| < \hat{\eta}$ — pseudorapidity of each particle.
  - $\sqrt{s_{\min}} < \sqrt{s} < \sqrt{s_{\max}}$— invariant mass of produced pair.
  - $\hat{\omega}_{1,\min} < \omega_1 < \hat{\omega}_{1,\max},\ \hat{\omega}_{2,\min} < \omega_2 < \hat{\omega}_{2,\max}$ — bounds on photons energies due to forward detectors.

- Survival factor — distribution in the impact parameter space is needed

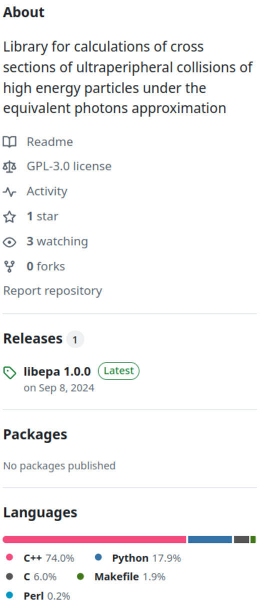
Notations!

The following notation is popular in the literature:

- $\sqrt{s}$ for the invariant mass of the colliding particles ($\Rightarrow 2E$ in what follows)
- W for the invariant mass of the produced particles, i.e. invariant mass of the colliding *photons* ($\Rightarrow \sqrt{s}$ in what follows).

libepa

- *Developer's repository link:* https://github.com/jini-zh/libepa.
- *Licensing provisions:* GNU General Public License 3 (GPL3).
- *Programming Language:* C++, Python.
- *Solution method:* Cross sections are expressed in terms of multiple integrals over the phase space parameters and numerically calculated through recurrent application of algorithms for one-dimensional integration. Functional programming approach is used to simplify the interface and optimize the calculations.
- *Physics description:* CPC 305 (2024) 109347
- *Programmer reference:* included in the repository, see also https://jini-zh.org/libepa/libepa.html



UPC cross section with EPA

Many references are not provided in this talk, see CPC 305 (2024) 109347 for details.
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$$\sigma(AB \to ABX) = \int\limits_0^\infty \mathrm{d}\omega_1 \int\limits_0^\infty \mathrm{d}\omega_2\, \sigma(\gamma\gamma \to X)\, n_A(\omega_1)\, n_B(\omega_2),$$

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It is convenient to change the integration variables from the photons energies ω_1, ω_2 to the invariant mass of the produced system $\sqrt{s} = \sqrt{4\omega_1\omega_2}$ and its rapidity $y = \frac{1}{2}\ln\frac{\omega_1}{\omega_2}$:

$$\frac{\mathrm{d}\sigma(AB \to ABX)}{\mathrm{d}\sqrt{s}} = \sigma(\gamma\gamma \to X) \cdot \frac{\mathrm{d}L_{AB}}{\mathrm{d}\sqrt{s}},$$

where L_{AB} is the photon-photon luminosity in the collision of particles A and B,

$$\frac{\mathrm{d}L_{AB}}{\mathrm{d}\sqrt{s}} = \frac{\sqrt{s}}{2} \int\limits_{-\infty}^{\infty} n_A\left(\frac{\sqrt{s}}{2}\mathrm{e}^{y}\right) n_B\left(\frac{\sqrt{s}}{2}\mathrm{e}^{-y}\right) \mathrm{d}y.$$

Fiducial cross section in UPC

$p_T > \hat{p_T}, \quad |\eta| < \hat{\eta}, \quad \hat{\omega}_{1,\min} < \omega_1 < \hat{\omega}_{1,\max}, \quad \hat{\omega}_{2,\min} < \omega_2 < \hat{\omega}_{2,\max}.$

$$\frac{\mathrm{d}\sigma_{\text{fid.}}(AB \to AB\chi^+\chi^-)}{\mathrm{d}\sqrt{s}} = \int\limits_{\max(\hat{p}_T,\tilde{p}_T)}^{\frac{\sqrt{s}}{2}\sqrt{1-\frac{4m_\chi^2}{s}}} \mathrm{d}p_T \, \frac{\mathrm{d}\sigma(\gamma\gamma \to \chi^+\chi^-)}{\mathrm{d}p_T} \, \frac{\mathrm{d}L_{AB}^{\text{fid.}}}{\mathrm{d}\sqrt{s}},$$

$$\frac{\mathrm{d}L_{AB}^{\text{fid.}}}{\mathrm{d}\sqrt{s}} = \frac{\sqrt{s}}{2} \int\limits_{\max(-\hat{y},\tilde{y})}^{\min(\hat{y},\tilde{Y})} \mathrm{d}y \, n_A\left(\frac{\sqrt{s}}{2}\mathrm{e}^{y}\right) n_B\left(\frac{\sqrt{s}}{2}\mathrm{e}^{-y}\right),$$

$$\hat{y} = \ln\left(\frac{2p_T}{\sqrt{s}} \cdot \frac{\sinh\hat{\eta} + \sqrt{\cosh^2\hat{\eta} + \frac{m_\chi^2}{p_T^2}}}{1 \mp \sqrt{1 - \frac{p_T^2 + m_\chi^2}{s/4}}}\right).$$

and $\tilde{y}$ and $\tilde{Y}$ are the constraints on rapidity coming from the constraints on photon energies,

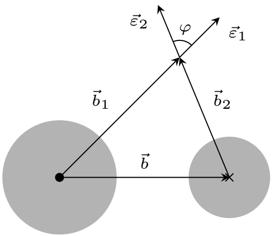
$$\tilde{y} = \max\left(\ln\frac{\hat{\omega}_{1,\min}}{\sqrt{s}/2}, \ln\frac{\sqrt{s}/2}{\hat{\omega}_{2,\max}}\right),$$

$$\tilde{Y} = \min\left(\ln\frac{\hat{\omega}_{1,\max}}{\sqrt{s}/2}, \ln\frac{\sqrt{s}/2}{\hat{\omega}_{2,\min}}\right),$$

and $\tilde{p}_T$ is an extra constraint on p_T that ensures that integrations are performed over physically meaningful domains:

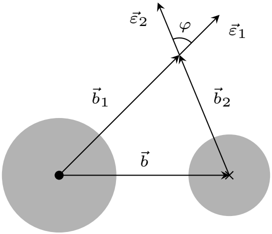
$$\hat{y} > 0, \; -\hat{y} < \tilde{Y}, \; \hat{y} > \tilde{y}.$$

Survival factor



$$\sigma(AB \to ABX) = \int_0^\infty \mathrm{d}\omega_1 \int_0^\infty \mathrm{d}\omega_2 \int \mathrm{d}^2 b_1 \int \mathrm{d}^2 b_2 \, \sigma(\gamma\gamma \to X) \, n_A(b_1, \omega_1) \, n_B(b_2, \omega_2) \, P_{AB}(b),$$

Survival factor



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$$\frac{\mathrm{d}\sigma(AB \to ABX)}{\mathrm{d}\sqrt{s}} = \sigma_\parallel(\gamma\gamma \to X)\frac{\mathrm{d}L_{AB}^{\parallel}}{\mathrm{d}\sqrt{s}} + \sigma_\perp(\gamma\gamma \to X)\frac{\mathrm{d}L_{AB}^{\perp}}{\mathrm{d}\sqrt{s}},$$

where

$$\frac{\mathrm{d}L_{AB}^{\parallel}}{\mathrm{d}\sqrt{s}} = \frac{\sqrt{s}}{2}\int \mathrm{d}^2 b_1 \int \mathrm{d}^2 b_2 \int_{-\infty}^{\infty} \mathrm{d}y\, n_A\left(b_1, \frac{\sqrt{s}}{2}\mathrm{e}^{y}\right)\, n_B\left(b_2, \frac{\sqrt{s}}{2}\mathrm{e}^{-y}\right)\, P_{AB}(b)\cos^2\varphi,$$

$$\frac{\mathrm{d}L_{AB}^{\perp}}{\mathrm{d}\sqrt{s}} = \frac{\sqrt{s}}{2}\int \mathrm{d}^2 b_1 \int \mathrm{d}^2 b_2 \int_{-\infty}^{\infty} \mathrm{d}y\, n_A\left(b_1, \frac{\sqrt{s}}{2}\mathrm{e}^{y}\right)\, n_B\left(b_2, \frac{\sqrt{s}}{2}\mathrm{e}^{-y}\right)\, P_{AB}(b)\sin^2\varphi.$$

$$\mathcal{J}_\mu = Ze \cdot \bar{\psi} \left[F_1(Q^2)\gamma_\mu - \frac{\sigma_{\mu\nu} q^\nu}{2m_\psi} F_2(Q^2) \right] \psi, \quad Q^2 \equiv -q^2, \quad \sigma_{\mu\nu} = \frac{\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu}{2},$$

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2), \qquad F_1(Q^2) = \frac{G_E(Q^2) + \frac{Q^2}{4m_\psi^2} G_M(Q^2)}{1 + \frac{Q^2}{4m_\psi^2}},$$

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4m_\psi^2} F_2(Q^2), \quad F_2(Q^2) = \frac{G_M(Q^2) - G_E(Q^2)}{1 + \frac{Q^2}{4m_\psi^2}}.$$

EPA spectra

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$$n(\omega) = \frac{2Z^2\alpha}{\pi\omega} \int_0^\infty \frac{D(q_\perp^2 + (\omega/\gamma)^2)}{(q_\perp^2 + (\omega/\gamma)^2)^2} \, q_\perp^3 \, \mathrm{d}q_\perp, \quad D(Q^2) = \frac{G_E^2(Q^2) + \frac{Q^2}{4m_\psi^2} G_M^2(Q^2)}{1 + \frac{Q^2}{4m_\psi^2}}$$

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We need distribution in impact parameter space to calculate survival factor:

$$n(\omega) = \int n(b,\omega) \, \mathrm{d}^2 b = 2\pi \int_0^\infty n(b,\omega) \, b \, \mathrm{d}b, \quad n(b,\omega) = ?$$

Dipole approximation

$$G_E(Q^2) = \frac{1}{\left(1+\frac{Q^2}{\Lambda^2}\right)^2},$$
$$G_M(Q^2) = \frac{\mu_\psi}{\left(1+\frac{Q^2}{\Lambda^2}\right)^2},$$

where μ_ψ is the fermion magnetic moment expressed in units of $e/2m_\psi$, and Λ is a parameter of the approximation related to the fermion charge radius R through

$$R^2 = -6 \lim_{Q^2\to 0} \frac{\mathrm{d}G_E(Q^2)}{\mathrm{d}Q^2} \quad \Rightarrow \quad \Lambda^2 = \frac{12}{R^2}.$$

Using the modern value of 0.8414 fm for the proton charge radius we get that for proton $\Lambda^2 = 0.66\ \mathrm{GeV}^2$.

Approximations for EPA spectra

For $F_2(Q^2)=0$ ($F_1(Q^2)=G_E(Q^2)=G_M(Q^2)$) we get

$$n_2(\omega)=\frac{Z^2\alpha}{\pi\omega}\left[(4a+1)\ln\left(1+\frac{1}{a}\right)-\frac{24a^2+42a+17}{6(a+1)^2}\right],\ a=\left(\frac{\omega}{\Lambda\gamma}\right)^2,$$

$$n_2(b,\omega)=\frac{Z^2\alpha}{\pi^2\omega}\left[\frac{\omega}{\gamma}K_1\left(\frac{b\omega}{\gamma}\right)-\sqrt{\Lambda^2+\left(\frac{\omega}{\gamma}\right)^2}K_1\left(b\sqrt{\Lambda^2+\left(\frac{\omega}{\gamma}\right)^2}\right)-\frac{b\Lambda^2}{2}K_0\left(b\sqrt{\Lambda^2+\left(\frac{\omega}{\gamma}\right)^2}\right)\right]^2.$$

If the Pauli form factor is neglected, i.e. $\mathcal{J}_\mu=ZeF_1(Q^2)\bar{\psi}\gamma_\mu\psi$, but the electric and magnetic form factors are not assumed to be equal ($G_E(Q^2)\neq G_M(Q^2)$), then we get $n_{2\mathrm{D}}(\omega)$ and $n_{2\mathrm{D}}(b,\omega)$.

EPA spectrum with all form factors

$$n_p(\omega)=\frac{Z^2\alpha}{\pi\omega}\left\{\left(1+4u-(\mu_\psi^2-1)\frac{u}{v}\right)\ln\left(1+\frac{1}{u}\right)-\frac{24u^2+42u+17}{6(u+1)^2}\right.$$
$$\left.-\frac{\mu_\psi^2-1}{(v-1)^3}\left[\frac{1+u/v}{v-1}\ln\frac{u+v}{u+1}-\frac{6u^2(v^2-3v+3)+3u(3v^2-9v+10)+2v^2-7v+11}{6(u+1)^2}\right]\right\},$$
$$u=\left(\frac{\omega}{\Lambda\gamma}\right)^2,\ v=\left(\frac{2m_\psi}{\Lambda}\right)^2.$$

This is the correct spectrum for proton, however its spatial counterpart $n_p(b,\omega)$ has not been derived yet (... but on its way!).

Available luminosities

Notation	$\tilde{L}$	$L_{2\text{D}}$	$\tilde{L}_{2\text{D}}$	L_2	$\tilde{L}_2$
Non-electromagnetic interactions	no	yes	no	yes	no
Pauli form factor	yes	no	no	no	no
Electric and magnetic form factors	distinct	distinct	distinct	equal	equal
Survival factor		$S_{2\text{D}} = \frac{\mathrm{d}L_{2\text{D}}/\mathrm{d}\sqrt{s}}{\mathrm{d}\tilde{L}_{2\text{D}}/\mathrm{d}\sqrt{s}}$		$S_2 = \frac{\mathrm{d}L_2/\mathrm{d}\sqrt{s}}{\mathrm{d}\tilde{L}_2/\mathrm{d}\sqrt{s}}$	

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Since the spatial equivalent photon spectrum for proton $n_p(b,\omega)$ is unavailable, we calculate three cross sections $\tilde{\sigma}$, $\tilde{\sigma}_{2D}$, σ_{2D} corresponding to the luminosities $\tilde{L}$, $\tilde{L}_{2D}$, L_{2D} and then obtain an estimation for the cross section taking into account non-electromagnetic interactions *and* the Pauli form factor as $\sigma = \tilde{\sigma} \cdot (\sigma_{2D}/\tilde{\sigma}_{2D})$.

Results

Experimental value:

$$\sigma_{\text{exp}} = 3.12 \pm 0.07 \text{ (stat.)} \pm 0.10 \text{ (syst.) pb.}$$

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`libepa` (with required accuracy for differential cross sections):

$$\begin{aligned} \tilde{\sigma}_{2\text{D}} &= 3.46 \text{ pb} \quad (10^{-3}), \\ \sigma_{2\text{D}} &= 3.31 \text{ pb} \quad (10^{-2}), \quad \Rightarrow \quad \sigma = \tilde{\sigma} \cdot \frac{\sigma_{2\text{D}}}{\tilde{\sigma}_{2\text{D}}} = (3.44 \pm 0.04) \text{ pb.} \\ \tilde{\sigma} &= 3.57 \text{ pb} \quad (10^{-3}) \end{aligned}$$

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`SuperChic2`:

$$\sigma = 3.45 \pm 0.05\ \text{pb}.$$

`SuperChic2` uses the dipole form factor approximation with $\Lambda^2 = 0.71\ \text{GeV}^2$.
`libepa` cross section σ with this Λ is 3.50 pb.

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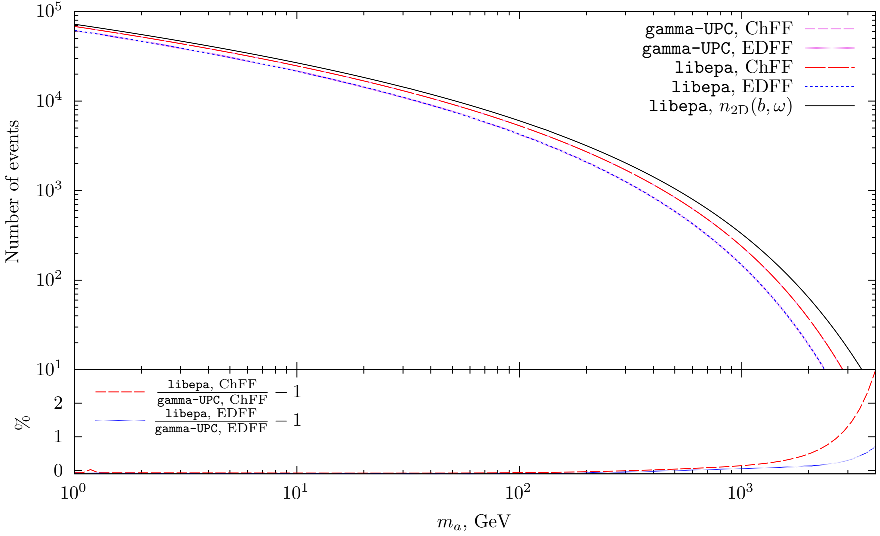
`HERWIG`:

$\tilde{\sigma} = 3.56 \pm 0.05$ pb,

$\sigma = 3.06 \pm 0.05$ pb with the help of corrections from PLB 741, 66 (2015)

ALP searches (comparison with gammaUPC)



pp, 14 TeV, 3 ab^{-1}, $g_{a\gamma} = 0.1$ TeV^{-1}

Conclusions

- UPC are a great source of events for studying physics in $\gamma\gamma$ fusion, and `libepa` provides tools for it.
- `libepa` takes into account survival factor and allows to impose experimental cuts. These features are necessary for comparison with experimental data.
- Results are consistent with existing Monte Carlo codes.

`libepa` is quite different from other programs used to calculate UPC cross sections:

- `libepa` is a library rather than a standalone program.
- `libepa` relies on deterministic one-dimensional integration rather than the Monte Carlo approach.
- `libepa` is designed in the functional programming paradigm.

Large room for improvements!

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Large room for improvements!

Thank you for your attention!

Backup slides

Conclusions

- `libepa` is a library rather than a standalone program.

  It provides a set of tools for the user to create their own computation rather than a set of pre-programmed computations with variable numerical parameters. At the same time common computations are kept simple, and cross sections for proton-proton collisions can be obtained by a single call to `libepa`.

- `libepa` relies on deterministic one-dimensional integration rather than the Monte Carlo approach.

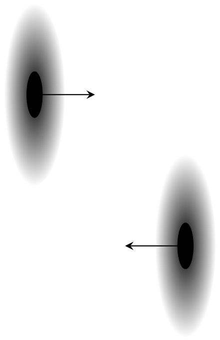
  The fact that `libepa` uses deterministic integration rather than Monte Carlo may be an advantage or a disadvantage depending on the problem at hand and the approach to solve it. An explicit representation of the computation function in terms of mathematical expressions possibly involving recurring one-dimensional integrals over well-defined domains is required.

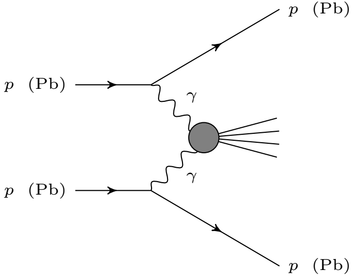
- `libepa` is designed in the functional programming paradigm.

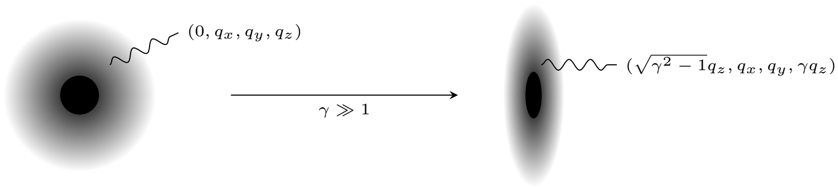
  The functional programming approach allowed for the interface when the user can replace part of a common computation with their own function, e.g., by changing the spectrum of a colliding particle, tweak the integration algorithm, or build a computation for a function not explicitly supported by the library.

  When combined with CFFI bindings to a language that features a read-evaluate-print loop (REPL), it gives the user a powerful calculator that can quickly evaluate various values of interest to the research at hand.

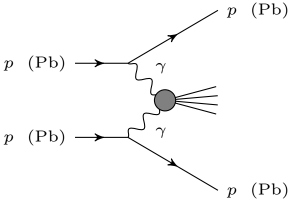
UPC







Photon virtuality: $Q^2 \equiv -q^2 = q_x^2 + q_y^2 + q_z^2 \ll (\gamma q_z)^2 \equiv \omega^2$

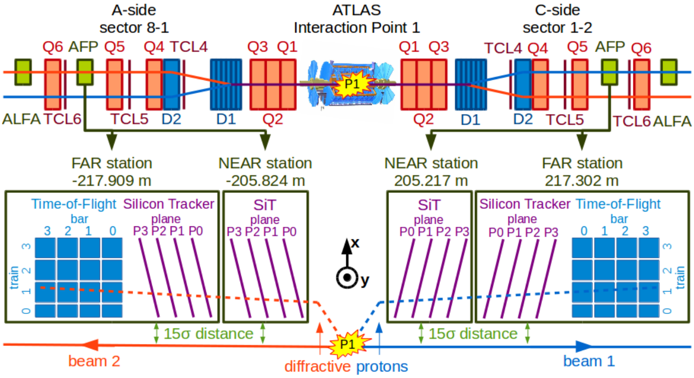


$$\sigma \sim Z^4$$

	pp	Pb Pb
Energy	13 TeV	5.02 TeV/(nucleon pair)
Z	1	82
Z^4	1	$4.5 \cdot 10^7$
Luminosity	147 fb^{-1}	2.3 nb^{-1}
	ratio:	$6.4 \cdot 10^7$
Duration	21 months (Run 2)	2 months (2015, 2018)
Q	$\lesssim$ 200 MeV	$\lesssim$ 50 MeV
$\sqrt{s_{\gamma\gamma}}$	$\lesssim$ 2.6 TeV	$\lesssim$ 160 GeV

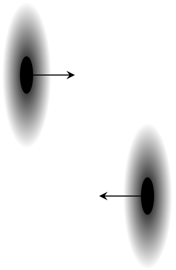
Forward detectors

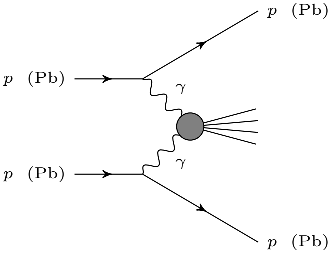
[1909.10827]



Distance from the IP, m	200	420
ξ range	0.015–0.15	0.002–0.02
6.5 TeV p energy loss, GeV	97.5–975	13–130
in the center-of-mass frame, MeV	14–141	1.9–19
0.5 PeV ^{208}Pb energy loss, TeV	7.8–78	1.0–10
in the center-of-mass frame, GeV	2.9–29	0.37–3.7

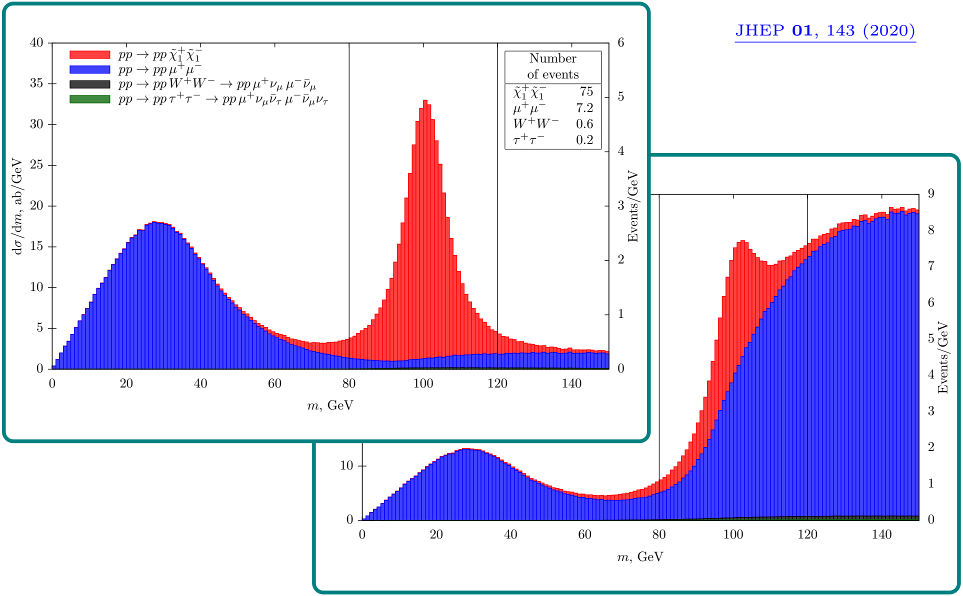
Ultraperipheral collisions (UPC) at the LHC





- It is possible to detect protons in forward detectors to reconstruct full kinematics.
- Accessible analytically with equivalent photons approximation (EPA).
- Formulae can be easily adopted for new particles (γ couples to electric charge).

JHEP **01**, 143 (2020)



Survival factor: pp case

$$P_{pp}(b) = \left(1 - \mathrm{e}^{-\frac{b^2}{2B}}\right)^2,$$

where B is an empirical parameter depending on the collision energy E.

$$\frac{\mathrm{d}L_{pp}^{\parallel}}{\mathrm{d}\sqrt{s}} = \pi^2\sqrt{s}\int\limits_0^\infty b_1\,\mathrm{d}b_1 \int\limits_0^\infty b_2\,\mathrm{d}b_2 \int\limits_{-\infty}^\infty \mathrm{d}y\, n_p\left(b_1, \tfrac{\sqrt{s}}{2}\,\mathrm{e}^{y}\right)\, n_p\left(b_2, \tfrac{\sqrt{s}}{2}\,\mathrm{e}^{-y}\right)$$
$$\times\left\{1 - 2\mathrm{e}^{-\frac{b_1^2+b_2^2}{2B}}\left[I_0\left(\frac{b_1 b_2}{B}\right) + I_2\left(\frac{b_1 b_2}{B}\right)\right] + \mathrm{e}^{-\frac{b_1^2+b_2^2}{B}}\left[I_0\left(\frac{2b_1 b_2}{B}\right) + I_2\left(\frac{2b_1 b_2}{B}\right)\right]\right\},$$

$$\frac{\mathrm{d}L_{pp}^{\perp}}{\mathrm{d}\sqrt{s}} = \pi^2\sqrt{s}\int\limits_0^\infty b_1\,\mathrm{d}b_1 \int\limits_0^\infty b_2\,\mathrm{d}b_2 \int\limits_{-\infty}^\infty \mathrm{d}y\, n_p\left(b_1, \tfrac{\sqrt{s}}{2}\,\mathrm{e}^{y}\right)\, n_p\left(b_2, \tfrac{\sqrt{s}}{2}\,\mathrm{e}^{-y}\right)$$
$$\times\left\{1 - 2\mathrm{e}^{-\frac{b_1^2+b_2^2}{2B}}\left[I_0\left(\frac{b_1 b_2}{B}\right) - I_2\left(\frac{b_1 b_2}{B}\right)\right] + \mathrm{e}^{-\frac{b_1^2+b_2^2}{B}}\left[I_0\left(\frac{2b_1 b_2}{B}\right) - I_2\left(\frac{2b_1 b_2}{B}\right)\right]\right\}.$$

n_{2D}

If the Pauli form factor is neglected, i.e. $\mathcal{J}_\mu = ZeF_1(Q^2)\bar{\psi}\gamma_\mu\psi$, but the electric and magnetic form factors are not assumed to be equal ($G_E(Q^2) \neq G_M(Q^2)$), then

$$
\begin{aligned}
n_{\text{2D}}(\omega) = \frac{Z^2\alpha}{\pi\omega} \Bigg\{ & \left(1 + 4u - 2(\mu_\psi - 1)\frac{u}{v}\right) \ln\left(1 + \frac{1}{u}\right) \\
& + \frac{\mu_\psi - 1}{(v-1)^4}\left[\frac{\mu_\psi - 1}{v - 1}(1 + 4u + 3v) - 2\left(1 + \frac{u}{v}\right)\right] \ln\frac{u+v}{u+1} - \frac{24u^2 + 42u + 17}{6(u+1)^2} \\
& + (\mu_\psi - 1)\,\frac{6u^2(v^2 - 3v + 3) + 3u(3v^2 - 9v + 10) + 2v^2 - 7v + 11}{3(u+1)^2(v-1)^3} \\
& - (\mu_\psi - 1)^2\,\frac{24u^2 + 6u(v+7) - v^2 + 8v + 17}{6(u+1)^2(v-1)^4}\Bigg\}, \quad u = \left(\frac{\omega}{\Lambda\gamma}\right)^2, \; v = \left(\frac{2m_\psi}{\Lambda}\right)^2,
\end{aligned}
$$

$$
\begin{aligned}
n_{\text{2D}}(b,\omega) = \frac{Z^2\alpha}{\pi^2\omega} \Bigg[& \frac{\omega}{\gamma}K_1\left(\frac{b\omega}{\gamma}\right) - \left(1 + \frac{(\mu_\psi - 1)\frac{\Lambda^4}{16m_\psi^4}}{\left(1 - \frac{\Lambda^2}{4m_\psi^2}\right)^2}\right)\sqrt{\Lambda^2 + \frac{\omega^2}{\gamma^2}}\,K_1\left(b\sqrt{\Lambda^2 + \frac{\omega^2}{\gamma^2}}\right) \\
& + \frac{(\mu_\psi - 1)\frac{\Lambda^4}{16m_\psi^4}}{\left(1 - \frac{\Lambda^2}{4m_\psi^2}\right)^2}\sqrt{4m_\psi^2 + \frac{\omega^2}{\gamma^2}}\,K_1\left(b\sqrt{4m_\psi^2 + \frac{\omega^2}{\gamma^2}}\right) \\
& - \frac{1 - \frac{\mu_\psi\Lambda^2}{4m_\psi^2}}{1 - \frac{\Lambda^2}{4m_\psi^2}} \cdot \frac{b\Lambda^2}{2}\,K_0\left(b\sqrt{\Lambda^2 + \frac{\omega^2}{\gamma^2}}\right)\Bigg]^2.
\end{aligned}
$$

Muon pair production at the LHC

ATLAS, PLB 777, 303 (2018)

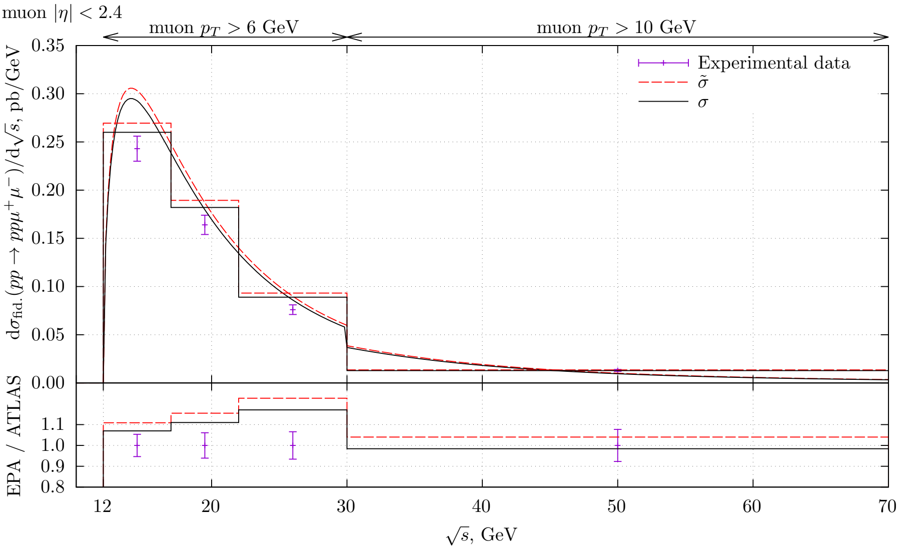
The measured value is the fiducial cross section for the $pp \to pp\mu^+\mu^-$ reaction with the following constraints:

- for 12 GeV $< \sqrt{s} <$ 30 GeV, $p_T >$ 6 GeV,
- for 30 GeV $< \sqrt{s} <$ 70 GeV, $p_T >$ 10 GeV,
- $|\eta| < 2.4$.

Experimental value:

$$\sigma_{\text{exp}} = 3.12 \pm 0.07\ (\text{stat.}) \pm 0.10\ (\text{syst.})\ \text{pb}.$$

libepa and experimental data



Simple cases

The differential cross section for the production of a pair of muons with the invariant mass 100 GeV in collisions of protons with the energy 13 TeV (C++):

```
#include <epa/proton.hpp>

int main(void)
{
  const double muon_mass = 105.6583745e-3; // GeV
  const double collision_energy = 13e3; // GeV
  const double invariant_mass = 100; // GeV

  auto luminosity = epa::pp_luminosity(collision_energy);
  auto photons_to_muons = epa::photons_to_fermions(muon_mass);
  auto cross_section = epa::xsection(photons_to_muons, luminosity);

  double result = cross_section(invariant_mass); // barn/GeV
  printf("%e\n", result);

  return 0;
}
```

Cross section for the production of a pair of fermions in *pp* collisions with the energy $E = 13$ TeV for the fermion mass range from 90 to 250 GeV (Python interface):

```
import epa

integrate = epa.default_integrator(0)
luminosity = epa.pp_luminosity(13e3, integrator = epa.default_integrator(1))
def xsection(mass):
    return integrate(
            epa.xsection(epa.photons_to_fermions(mass), luminosity),
            2 * mass,
            6.5e3
    )
for mass in range(90, 251, 5):
    print(f'{mass:3d} {xsection(mass):19.12e}')
```

Byproduct: convenient GSL wrappings

$$I(a) \equiv \frac{15}{a} \int_0^a \mathrm{d}x\, x \int_0^{\sqrt{1-\left(\frac{x}{a}\right)^2}} \mathrm{d}y\, y \int_0^{\sqrt{1-\left(\frac{x}{a}\right)^2 - y^2}} \mathrm{d}z\, \frac{z}{\sqrt{x^2+y^2+z^2}} = \frac{a(a+\frac{1}{2})}{(a+1)^2}.$$

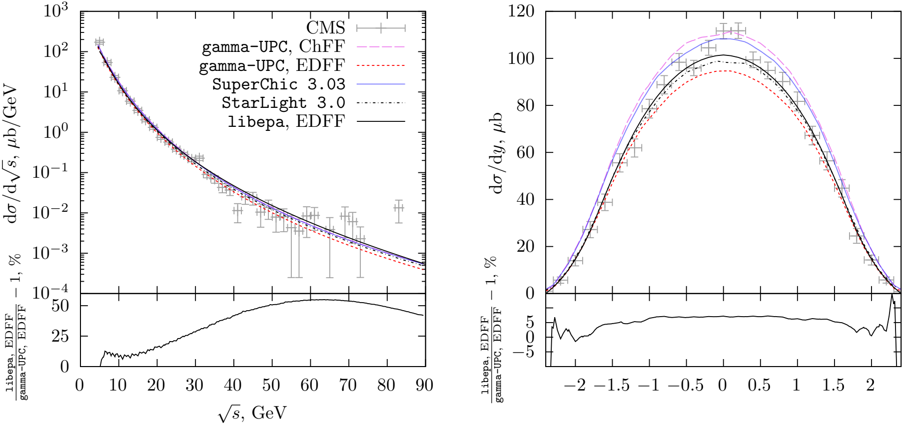
```
#include <epa/epa.hpp>

int main(void) {
  // build the computation function
  auto I = [
    // precompute integrators
    integrate_x = epa::default_integrator(0),
    integrate_y = epa::default_integrator(1),
    integrate_z = epa::default_integrator(2)
  ](double a) -> double {
    return 15.0 / a * integrate_x([&](double x) -> double {
        return x * integrate_y([&](double y) -> double {
            return y * integrate_z([&](double z) -> double {
                return z / sqrt(x*x + y*y + z*z);},
                0, sqrt(1 - pow(x / a, 2) - y * y));},
            0, sqrt(1 - pow(x / a, 2)));},
        0, a);
  };

  for (double a = 1; a <= 100; a += 1)
    printf("%3.0f\t%.5f\t%.5f\n", a, I(a), a * (a + 0.5) / pow(a + 1, 2));
  return 0;
};
```

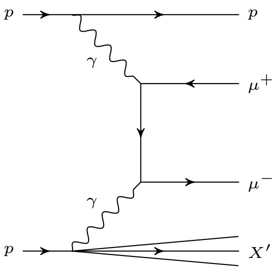
Here `I` is a closure computing $I(a)$. It captures variables **integrate_x**, **integrate_y** and **integrate_z** which are integrators used to calculate the integrals with respect to x, y and z.

e^+e^- production in PbPb collisions



Semi-inclusive processes with proton(s) in forward detector

ATLAS [PRL 125, 261801 (2020)]



Experimental selections:

- $p_T > 15$ GeV.
- $|\eta| < 2.4$.
- $p_T^{\mu\mu} < 5$ GeV.
- 20 GeV $< m_{\mu\mu} <$ 70 GeV or $m_{\mu\mu} > 105$ GeV.
- At least one proton hits a forward detector.

$\gamma\gamma$ and γZ fusion are not the only diagrams! (bremsstrahlung-like: production of real Z)

See our recent results (Eur. Phys. J. C82, 1055 (2022));
cut on transverse momentum allows to calculate inelastic part with the help of EPA

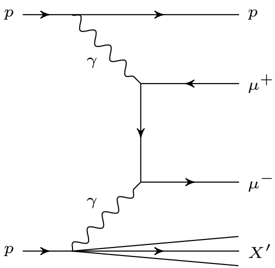
- Exclusive process ($pp \to p + \mu^+\mu^- + p$): 8.6 fb.
- Inclusive process ($pp \to p + \mu^+\mu^- + X$): 9.2 fb.

- Experiment: 7.2 ± 1.6 (stat.) ± 0.9 (syst.) ± 0.2 (lumi.) fb.

Survival factor should reduce the cross section by up to $\sim 30\%$
(10% for the elastic cross section;
$\sim 50\%$ for the inelastic cross section according to MC simulations).

Semi-inclusive processes with proton(s) in forward detector

ATLAS [PRL 125, 261801 (2020)]



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$\gamma\gamma$ and γZ fusion are not the only diagrams! (bremsstrahlung-like: production of real Z)

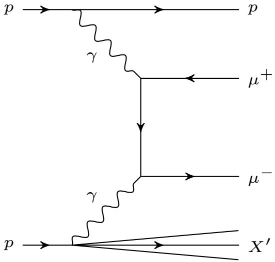
For $(p_T^{\mu\mu})^2 \gg 1$ GeV^2 we have $Q_2^2 \approx (p_T^{\mu\mu})^2$ and

$$\frac{Q^2}{M_Z^2 + Q^2} \approx \frac{(p_T^{\mu\mu})^2}{M_Z^2 + (p_T^{\mu\mu})^2} \sim 10^{-3}$$

Weak interaction contribution is negligible

Semi-inclusive processes with proton(s) in forward detector

ATLAS [PRL 125, 261801 (2020)]



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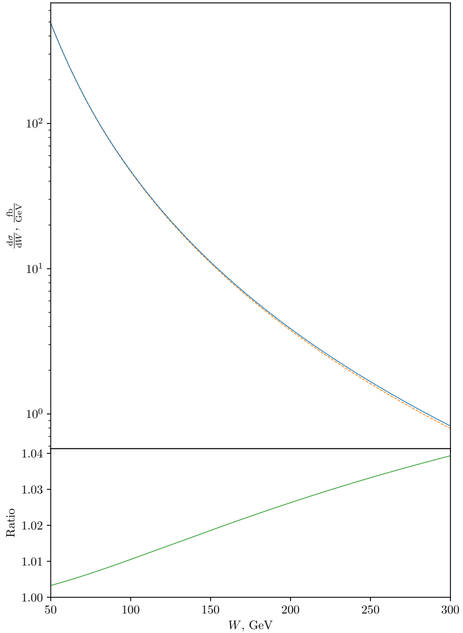
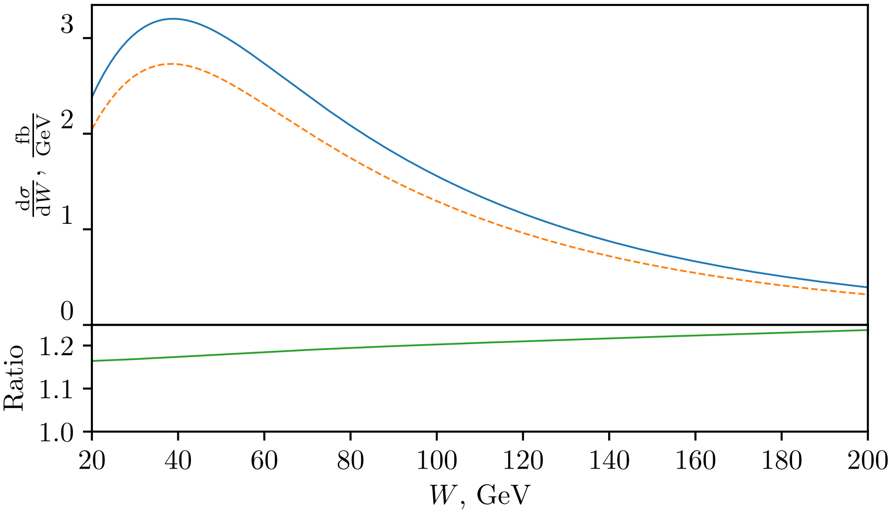
But what if we remove cuts?

Weak interaction contribution (no cuts)



$\hat{Q}_2 = 70$ GeV



3
2
1
0
$\frac{d\sigma}{dW}$, $\frac{\text{fb}}{\text{GeV}}$
Ratio
1.2
1.1
1.0
20
40
60
80
100
120
140
160
180
200
W, GeV