Symmetry arguments for solving important problems in Physics

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A REVIEW ABOUT SYMMETRIES

- What is a symmetry?

Any transformation leaving a system invariant

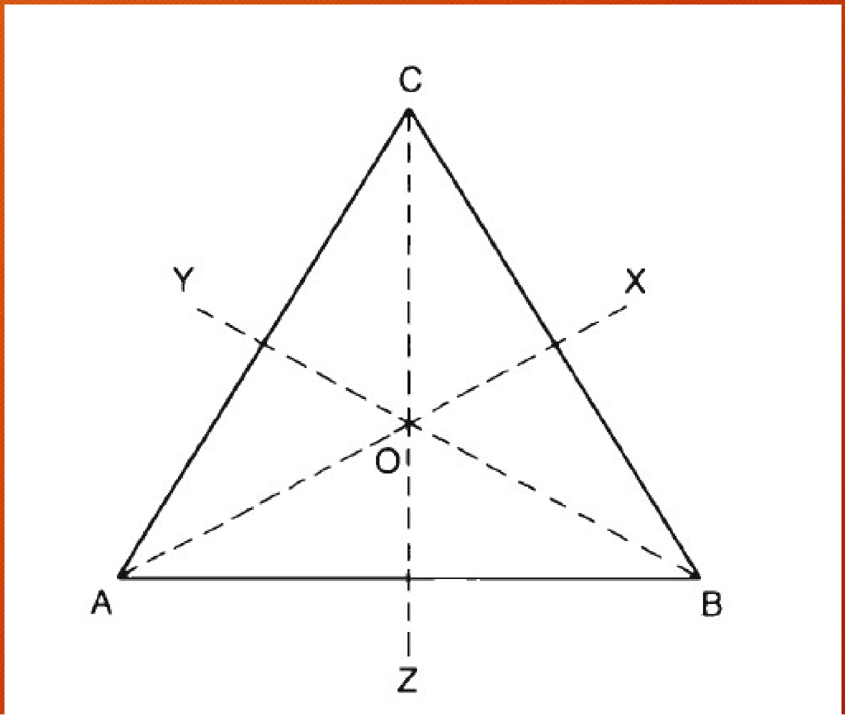
Then for example, if we rotate the ball, its shape does not change. This could be considered as a symmetry.

This is a global symmetry.

Local symmetry would deform the ball, unless a gauge field emerges.

What is a symmetry mathematically?

- Symmetry corresponds to a quantity that does not change under some group of transformations.



Given the representation of the group, defined as $U(g)$

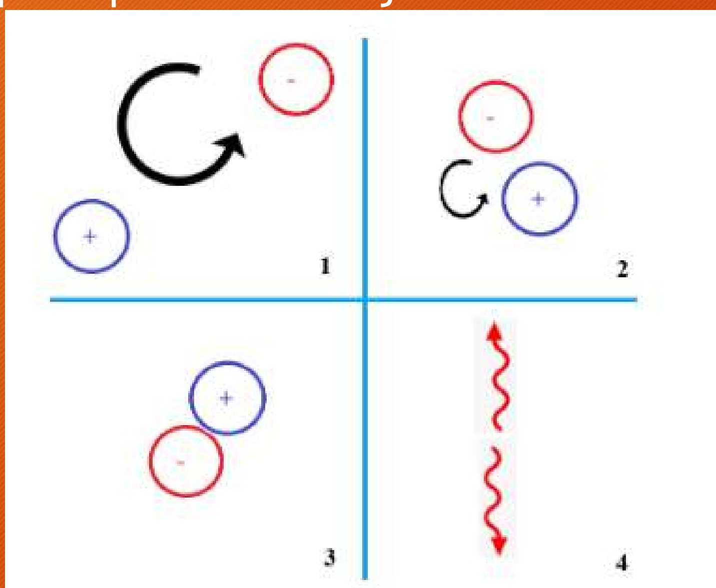
$U(g)\,S = S$. Then since it is related to a symmetry of the action, we have naturally an invariant action up to some total derivative. This is what in physics people call symmetry.

Why are the symmetries in physics so important?

- 1). Symmetries govern the interactions.
- 2). Symmetries tell us which quantities are conserved.
- 3). Symmetries explain the behavior of a system.
- 4). Symmetries, when broken, give us information about phase transitions and/or general changes on the configuration of a system.

Symmetry arguments solving the Dark Energy problem

- The observed cosmological constant is small because the UV vacuum modes are strongly suppressed by the angular symmetry (Angular momentum conservation).
- The symmetry conatraint over the background metric plus the uncertainty principle make the job.



$$E_0 \approx \frac{1}{(2\pi)^3}\int_0^{k_{max}}\int_0^{\phi_{max}}\int_{-\pi}^{\pi} \hbar k^3 sin\theta dk d\phi d\theta.$$

$$\phi_{max} \sim \frac{\beta\lambda}{Ck^2}.$$

$$E_0 \sim \frac{\hbar}{(2\pi)^3}\frac{\beta\lambda}{C}k_{max}^2 = \frac{\hbar}{(2\pi)^3}\left(\frac{\lambda}{3C\beta}\right)k_{max}^2\Lambda.$$

This result was obtained from the static coordinates

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Symmetries in galactic dynamics: Modifications on the rotational symmetry reproduce the Dark Matter effects.

- The effects attributed to Dark Matter can emerge from symmetry arguments. From the proposed vacuum metric

$$ds^2 = -e^{2\alpha(r)}dt^2 + e^{2\beta(r)}dr^2 + \zeta r d\Omega^2.$$

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MPLA, 2550125
(The metric fails to be a vacuum metric)

A new conserved quantity emerges, related to the angular component (conservation of the tangential velocity) is obtained

$$ds^2 = -\left(\frac{4C_0}{\zeta} + \frac{C_1}{\sqrt{r}}\right)dt^2 + \left(\frac{4C_0}{\zeta} + \frac{C_1}{\sqrt{r}}\right)^{-1}\frac{C_0}{r}dr^2 + r\zeta d\Omega^2,$$

→ This is the teal vacuum metric

$$\frac{1}{2}\left(\frac{dr}{d\lambda}\right)^2 + V(r) = E,$$

→

$$V(r) = \frac{C_1\eta^2}{2\zeta C_0\sqrt{r}} + \frac{C_1}{2C_0}\sqrt{r}.$$

DARK MATTER EFFECTS A LA MOND

- The previous potential, reproduces naturally the TF law and the DM scales

$$v^4 = 4GMa_0$$

Tully-Fisher law

$$r_2 = \sqrt{\frac{GM}{a_0}} = \sqrt{GMr_\Lambda}, \quad a_0 \sim \frac{1}{r_\Lambda}.$$

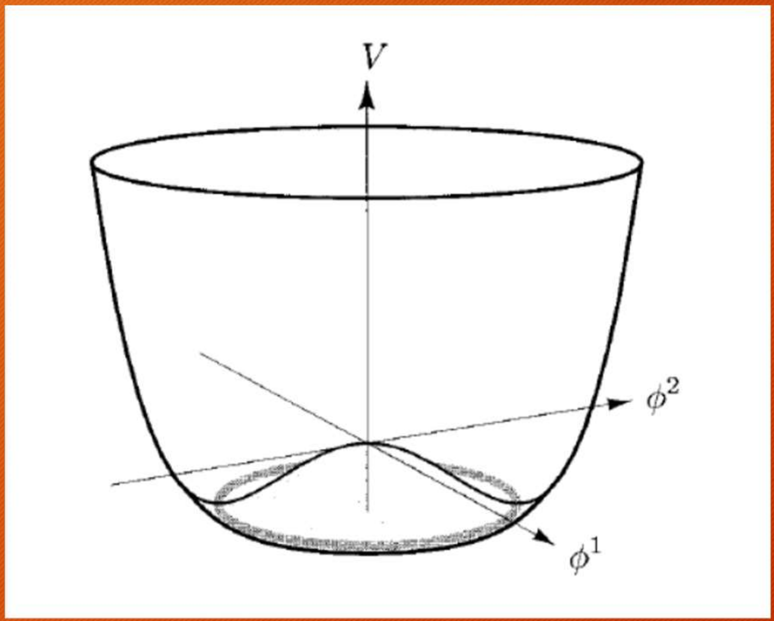
Dark Matter scales

- The scales are obtained naturally from the extreme condition

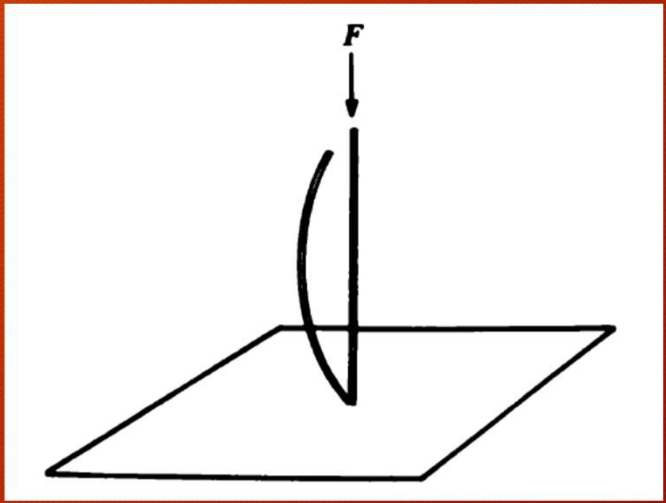
$$\frac{dV}{dr} = 0$$

Which marks the equilibrium condition

SPONTANEOUS SYMMETRY BREAKING



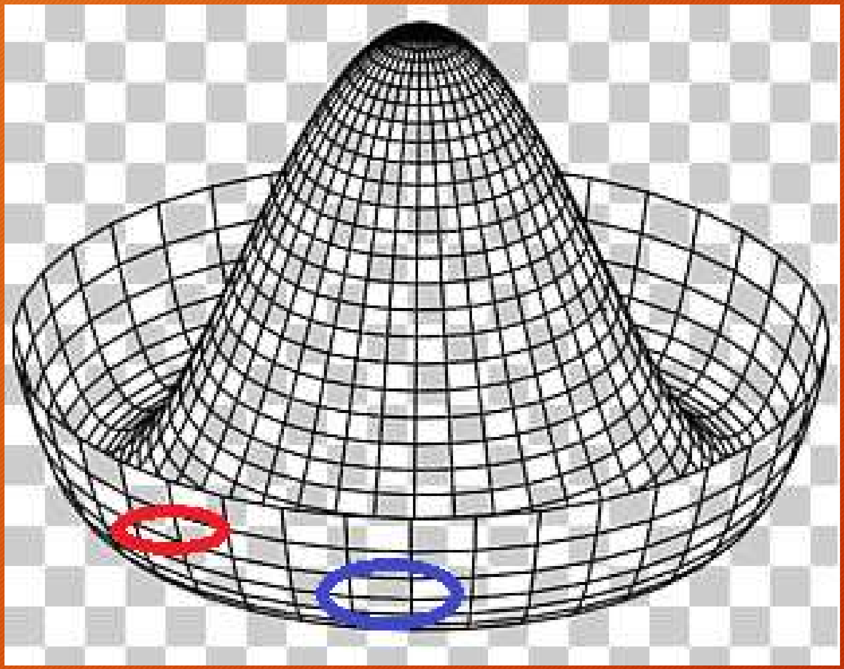
Apply a force over some plastic bar



When the symmetry is always respected by the Lagrangian (Hamiltonian) but not by the ground state.

BROKEN GENERATORS

- A generator of a symmetry of the Lagrangian which is not a symmetry of the ground state. What is the effect of this operator?

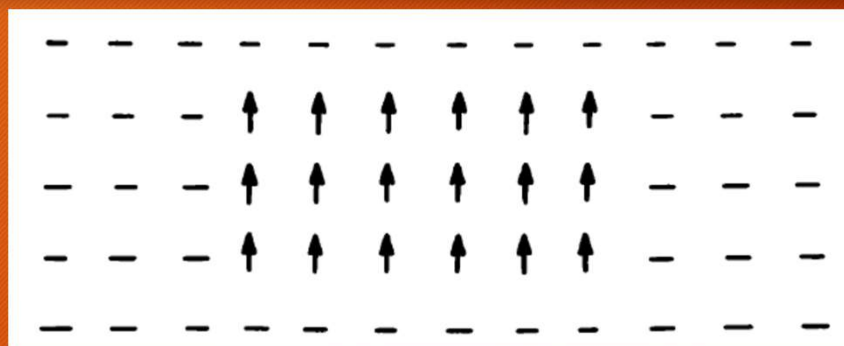


It maps one vacuum state towards another one. Then for example, it could map the red vacuum towards the blue one.

Nambu, Y. and Jona-Lasinio, G. Phys. Rev. 124, (1961), 246.

J. Goldstone, A. Salam and S. Weinberg, Phys. Rev. 127 (1962) 965; S.A. Bludman and A. Klein, Phys. Rev. 131 (1963) 2364; J. Goldstone, Nuovo Cim., 19, 154 (1961); Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961).

EXAMPLE

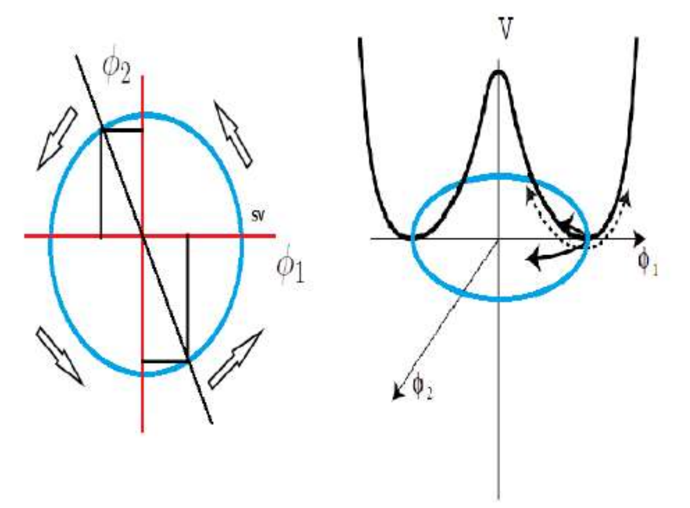


$$H = -\sum_{i,j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

The alignment of spins breaks the symmetry under rotations spontaneously

- By the non-vanishing value of the vacuum expectation of the order parameter of the system. $< 0_{SV}|\phi_1|0_{SV} >$
- Note in particular that:

$$\sum_0 < 0_{DV}|\phi_1|0_{DV} >= \bar{\phi}_1 - \bar{\phi}_1 + \bar{\phi}_2 - \bar{\phi}_2 + ... + \bar{\phi}_n - \bar{\phi}_n = \sum_{n=1}^{N} \bar{\phi}_i = 0.$$

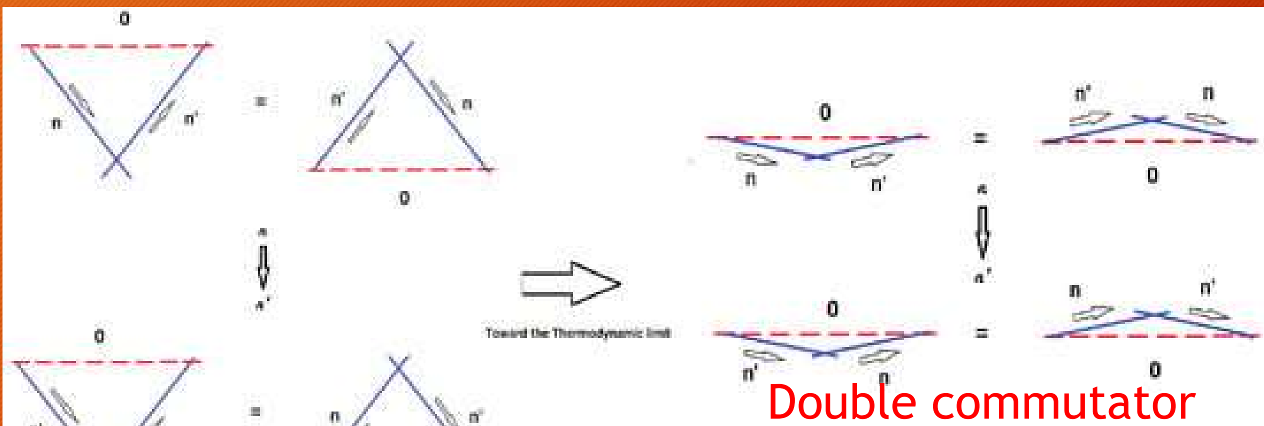


The superposition of all the possible order parameters vanishes

I. Arraut, *Symmetry* 11 (2019) 803.

THE STANDARD APPROACH DOES NOT ALWAYS WORKS

- There are situations where the Nambu-Goldstone bosons, even being massless, have a quadratic dispersion relation. There are different explanations for this issue, but the one which I proposed was based on the QYBE.



$$< 0_{SV}| \left[\phi_{b,a}(x), [Q_{p,k}(y), Q_{l,m}(z)]\right] |0_{SV} >\neq 0.$$

- *Int.J.Mod.Phys.A* 33 (2018) 07, 1850041

  Symmetry 11 (2019) 803

  Int.J.Mod.Phys.A 32 (2017) 1750127

GROUP THEORY AND SSB

- We have a group of symmetries, which do not leave the vacuum invariant

$$G: \quad \phi_0' = U(g)\phi_0 \neq \phi_0;$$

$$H: \quad \phi_0' = U(h)\phi_0 = \phi_0$$

$$U(h) = e^{iT_3\alpha_3}.$$

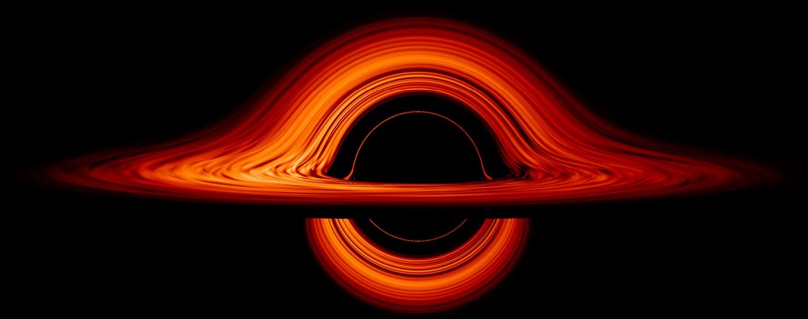
→ Subgroup of symmetries leaving the ground state invariant

$$G/H$$

↓

The dimension of the coset tells us about
The number of broken generators
(number) of Nambu-Goldstone bosons

SSB CAN EXPLAIN THE HAWKING RADIATION AND IT SOLVES THE INFORMATION PARADOX



Spontaneous symmetry breaking

$$U(p) < 0|\hat{n}^a_{\mathbf{p}}|0 >=< 0|\hat{n}^a_{\mathbf{p}}|0 >= 0.$$

Before the formation of the Black Hole

$$U(p) < 0|\hat{n}^a_{\mathbf{p}}|0 > \neq < 0|\hat{n}^a_{\mathbf{p}}|0 > .$$

After the formation of the Black Hole

$$\mathcal{L} = \frac{1}{2}\partial^\mu \hat{n}^a_{\mathbf{p}}(\omega)\partial_\mu \hat{n}^a_{\mathbf{p}}(\omega) - V(\hat{n}^a(\omega)),$$

Lagrangian reproducing the Hawking radiation

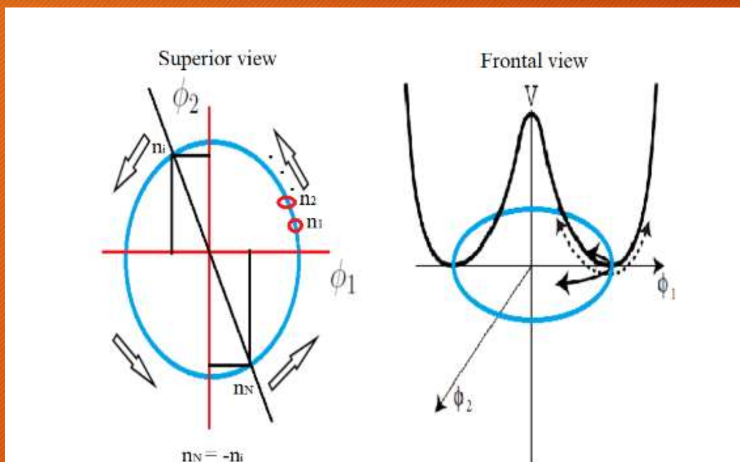
$$V(\hat{n}_{\mathbf{p}}) = \frac{1}{2}m^2\hat{n}^2_{\mathbf{p}} + \frac{\beta}{3}\hat{n}^3_{\mathbf{p}} + \frac{\lambda}{4}\hat{n}^4_{\mathbf{p}}.$$

For some combination of parameters, $m^2 < 0$ we get

$$\hat{n}^a = \frac{A}{e^{-\gamma\omega} \pm 1}, \quad \rightarrow \quad \gamma = -\frac{2\pi}{\kappa}.$$

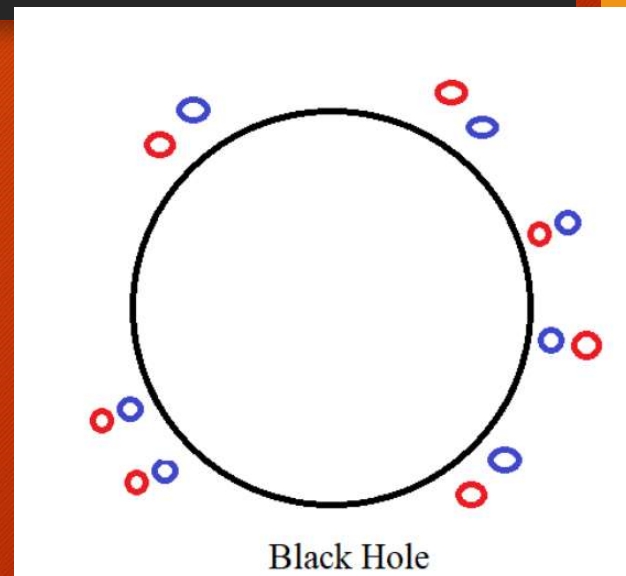
Ivan Arraut, Symmetry 2024, 16(5), 519

Symmetry arguments might solve the information paradox in black holes



Picture of Hawking radiation:
Spontaneous Symmetry Breaking

$$< 0|\hat{n}_1|0 >_1 + < 0|\hat{n}_2|0 >_2 + ... + < 0|\hat{n}_N|0 >_N = 0.$$



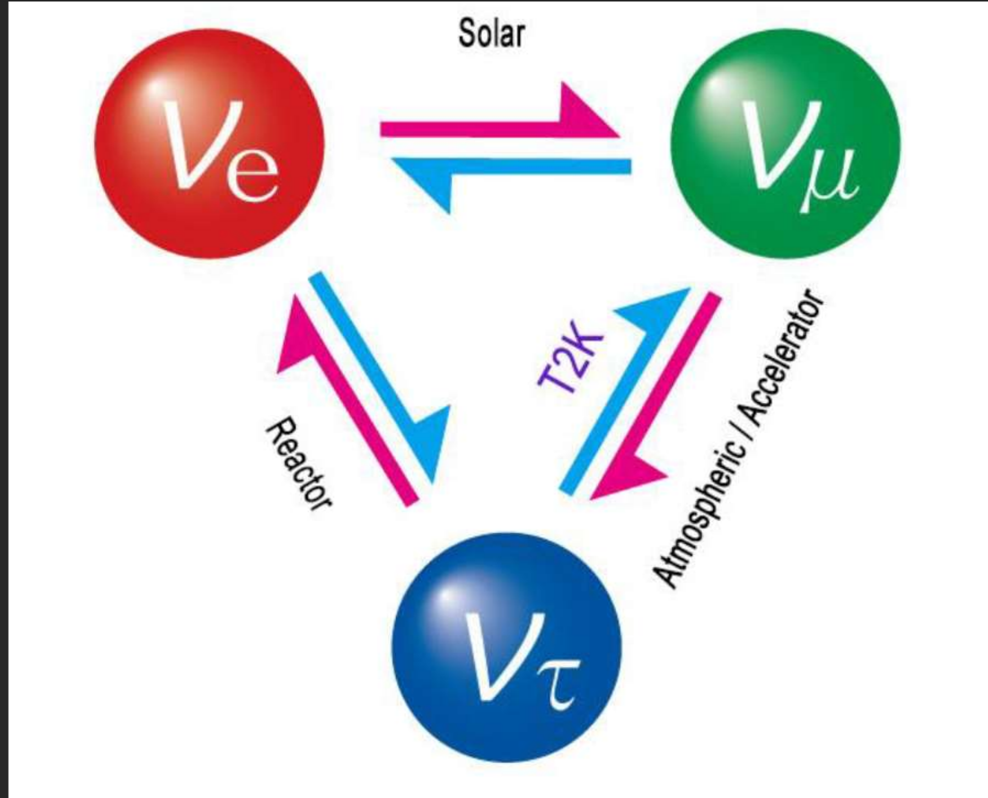
Picture of Hawking radiation:
Pair production

The black holes must emit partricles and antiparticles at different instants for solving the information paradox.

Ivan Arraut, AppliedMath 2025, 5(1), 4

Symmetry arguments solving the neutrino oscillation problem

Yang Baxter and the neutrino mass problem



Neutrino oscillation between three generations

SUPERPOSITION OF VACUUMS

$$\nu_\alpha = \sum_i U_{\alpha i}\nu_i.$$

$$\begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{bmatrix}$$

$$\hat{a}_e^+|0_e>=|\nu_e>, \quad \hat{a}_e|0_e>=0.$$

$$\hat{a}_i|0>_i=0. \qquad |0>_i=\sum_\alpha a_\alpha|0>_\alpha,$$

DEGENERATE VACUUM

arXiv:2409.00560 **[hep-ph]**

$$\begin{aligned}
|0\rangle_e &= c_{12}c_{13}|0\rangle\rangle_1 + s_{12}c_{13}|0\rangle\rangle_2 + s_{13}|0\rangle\rangle_3, \\
|0\rangle_\mu &= (-s_{12}c_{23} - c_{12}s_{23}s_{13})|0\rangle_1 + \\
&\quad (c_{12}c_{23} - s_{12}s_{23}s_{13})|0\rangle_2 + s_{23}c_{13}|0\rangle_3, \\
|0\rangle_\tau &= (s_{12}s_{23} - c_{12}c_{23}s_{13})|0\rangle_1 + \\
&\quad (-c_{12}s_{23} - s_{12}c_{23}s_{13})|0\rangle_2 + c_{23}c_{13}|0\rangle_3.
\end{aligned}$$

By using the QYBE

$$|0\rangle_e + |0\rangle_\mu + |0\rangle_\tau = 0.$$

This convert into the following expressions

$$\begin{aligned}
c_{12}c_{13} - s_{12}c_{23} - c_{12}s_{23}s_{13} + s_{12}s_{23} - c_{12}c_{23}s_{13} &= 0, \\
s_{12}c_{13} + c_{12}c_{23} - s_{12}s_{23}s_{13} - c_{12}s_{23} - s_{12}c_{23}s_{13} &= 0, \\
s_{13} + s_{23}c_{13} + c_{23}c_{13} &= 0.
\end{aligned}$$

$$\bar{c}_{12} = cos\left(\theta_{12} - \frac{\pi}{2}\right)$$

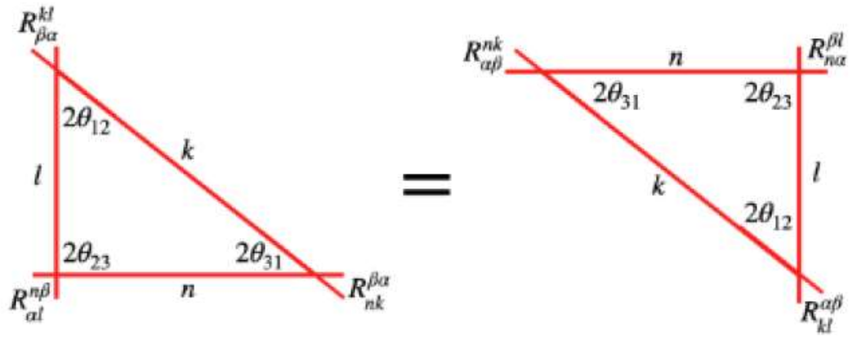
$$\bar{s}_{12} = sin\left(\theta_{12} - \frac{\pi}{2}\right).$$

The first two equations are redundant

$$\bar{c}_{12}c_{13} - \bar{s}_{12}c_{23} - \bar{c}_{12}s_{23}s_{13} + \bar{s}_{12}s_{23} - \bar{c}_{12}c_{23}s_{13} = 0.$$

The results match with the observations!!!

$$tan(\theta_{13}) = -sin(\theta_{23}) - cos(\theta_{23}).$$



arXiv:2406.02641

CONCLUSIONS

- 1). The origin of the neutrino mass comes from symmetry arguments.
- 2). QYBE have a correspondence with the Nambu-Goldstone theorem.
- 3). The Black Hole information paradox might be solved through symmetry arguments.
- 4). The observed cosmological constant values is obtained via symmetry constraints.
- 5). The effects attributed to Dark Matter emerge from deviations of the spherical symmetry.