Plasmon-polariton modes on a single electron wave packet

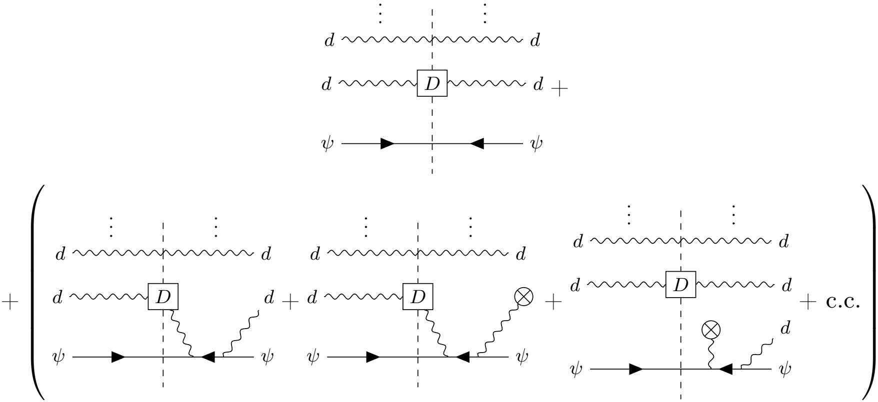
Akimov I.M., Kazinski P.O., Sokolov A.A.

Tomsk State University,
e-mail:ima8908@mail.ru

Moscow State University, Moscow, Russia.
22nd Lomonosov Conference on Elementary Particle Physics
22 August 2025.

Based on the paper Phys. Rev. D 111, 036028 (2025).
The talk was supported by a grant from the Government of the Russian Federation (Agreement No. 075-15-2025-009 of 28 February 2025)

Introduction



Diagrams illustrating the contribution to the inclusive probability of photon detection in stimulated radiation by a single electron in the presence of an external electromagnetic field.

P.O. Kazinski, T.V. Solovyev, Eur. Phys. J. C **82**, 790 (2022).

Brief summary of the work

1. In this paper, the polarization operator of a photon in the presence of a wave packet of single electron is obtained using methods of *in-in* perturbation theory.
2. Explicit solutions of the effective Maxwell equation are obtained when the typical scale of variation of the electron wave packet in coordinate space are much larger than the wavelength of the external field.
3. In this limit demonstrated that are exist additional degrees of freedom stemming from the pole of the polarization operator
4. In opposite, infrared limit, it is shown that the additional degrees of freedom are reduced to the dynamic dipole moment of the electron.

Notation

In the interaction representation, the state of the electron at the time $t = 0$ is given as

$$|\overline{in}\rangle = \sqrt{\frac{V}{(2\pi)^3}} \sum_s \int d\mathbf{p} \varphi_s(\mathbf{p}) \hat{a}_s^\dagger(\mathbf{p})|0\rangle. \quad (1)$$

The normalization condition has the form

$$\sum_s \int d\mathbf{p} |\varphi_s(\mathbf{p})|^2 = 1. \quad (2)$$

In $in - in$ perturbation theory, using the Schwinger representation [1], the interaction vertex has the form

$$S_{int} = -e \sum_{a=\pm} a \int d^4x \bar{\psi}^a \gamma^\mu A_\mu^a \psi^a. \quad (3)$$

[1]J. Schwinger, J. Math. Phys. **2**, 407 (1961).

Polarization operator

The polarization operator is defined in a standard way

$$\Pi^{ab}_{\mu\nu}(x,y) := \frac{\delta^2 \bar{\Gamma}[A_a, \bar{\psi}_a, \psi_a]}{\delta A^\mu_a(x) \delta A^\nu_b(y)}\Big|_{A^\mu_a(x)=0, \psi_a(x)=\bar{\psi}_a(x)=0}, \quad (4)$$

where $\bar{\Gamma}[A_a, \bar{\psi}_a, \psi_a]$ denotes quantum corrections to the effective action in *in-in* perturbation theory.

The polarization operator is written as

$$\Pi^{\mu\nu}_{ab}(x,y) = \overset{0}{\Pi}{}^{\mu\nu}_{ab}(x,y) + \overset{\psi}{\Pi}{}^{\mu\nu}_{ab}(x,y), \quad (5)$$

where $\overset{0}{\Pi}{}^{\mu\nu}_{ab}$ is the part of the polarization operator that is independent of the shape of the wave packet. Further, we will call such terms vacuum.

Polarization operator

The transition from the Schwinegr representation to the Keldysh representation is carried out using the transformation

$$A_\mu^\pm = A_\mu^c \pm \frac{1}{2} A_\mu^q. \tag{6}$$

In Keldysh's representation, the vacuum polarization operator takes the usual form after renormalization

$$\begin{gathered}\overset{0}{\Pi}{}^{\mu\nu}_{qc}(k) = (k^2\eta^{\mu\nu} - k^\mu k^\nu)\overset{0}{\Pi}(k_+^2), \\ \overset{0}{\Pi}{}^{\mu\nu}_{qq}(k) = -\frac{4i\alpha}{3}\sqrt{1-\frac{4m^2}{k^2}}(1+\frac{2m^2}{k^2})\theta(k^2-4m^2)\times \\ \times(k^2\eta^{\mu\nu} - k^\mu k^\nu),\end{gathered} \tag{7}$$

$$\text{where } \overset{0}{\Pi}(k^2) = \frac{2\alpha}{\pi}\int_0^1 dx x(1-x)\ln(1-x(1-x)\frac{k^2}{m^2})$$

L. V. Keldysh,Zh. Eksp. Teor. Fiz. **47**, 1515 (1964) [Sov. Phys. JETP **20**, 1018 (1965)].
M. E. Peskin, D. V. Schroeder, *An Introduction to Quantum Field Theory* (Addison-Wesley, Reading, 1995).

Polarization operator

The part of the polarization operator that depends on the wave packet has the form

$$\overset{\psi}{\Pi}{}^{\mu\nu}_{qc}(x,y) = -e^2\bar{\psi}(x)\gamma^\mu S_-(x,y)\gamma^\nu\psi(y) - e^2\bar{\psi}(y)\gamma^\nu S_+(y,x)\gamma^\mu\psi(x), \tag{8}$$

where

$$\psi(x) = \langle 0|\hat{\psi}(x)|\overline{in}\rangle = \sum_s \int d\mathbf{p}\sqrt{\frac{m}{(2\pi)^3 p_0}}u_s(\mathbf{p})\varphi_s(\mathbf{p})e^{-ip_\mu x^\mu}. \tag{9}$$

To describe the mixed states of electron, it is useful to introduce a relativistic density matrix

$$\psi(x)\bar{\psi}(y) \to \rho(x,y). \tag{10}$$

Effective Maxwell equations

The effective Maxwell equations in the momentum representation become

$$-(1-\overset{0}{\Pi}(k_{+}^{\prime 2}))(k^{\prime 2}\eta^{\mu\nu}-k^{\prime\mu}k^{\prime\nu})A_{\nu}(k^{\prime})+ \\ +\int\frac{d^4k}{(2\pi)^4}\overset{\psi}{\Pi}{}^{\mu\nu}_{qc}(k^{\prime},k)A_{\nu}(k)=0. \tag{11}$$

$$\overset{\psi}{\Pi}{}^{\mu\nu}_{qc}(k^{\prime},k)=-2\pi e^2 m\sum_{s,s^{\prime}}\int d\mathbf{p}_c d\mathbf{q}\,\delta(k^{\prime}-k-q)\frac{\rho_{ss^{\prime}}(\mathbf{p},\mathbf{p}^{\prime})}{\sqrt{p_0 p_0^{\prime}}}\times \\ \times\bar{u}_{s^{\prime}}(\mathbf{p}^{\prime})\Big[\frac{\gamma^{\mu}(\hat{p}_c+\hat{k}_c+m)\gamma^{\nu}}{(p^c+k_{+}^c)^2-m^2}+\frac{\gamma^{\nu}(\hat{p}_c-\hat{k}_c+m)\gamma^{\mu}}{(p^c-k_{+}^c)^2-m^2}\Big]u_s(\mathbf{p}), \tag{12}$$

where $q_{\mu}=k_{\mu}^{\prime}-k_{\mu}=p_{\mu}-p_{\mu}^{\prime}$, $p_{\mu}^{c}:=(p_{\mu}+p_{\mu}^{\prime})/2$, и $k_{\mu}^{c}:=(k_{\mu}+k_{\mu}^{\prime})/2$.

Short-wavelength limit

We assume that the electron wave packet is rather narrow in the momentum space, i.e.

$$|\mathbf{q}| \ll p_0^c, \qquad |\mathbf{q}| \ll |\mathbf{p}_0^c|, \qquad |\mathbf{q}| \ll |k_0^c|, \qquad |\mathbf{q}| \ll |\mathbf{k}_c|, \tag{13}$$

where p_0^c is the typical value of momenta in the wave packet

In this case, the polarization operator can be cast into the form

$$\overset{\psi}{\Pi}{}^{\mu\nu}_{qc}(x, k_c) = \frac{\omega_p^2(x)}{(k_c p_c)^2 - k_c^4/4}\big[(k_c p_c)^2 \eta^{\mu\nu} - (k_c p_c) k_c^{(\mu} p_c^{\nu)} + \\ + k_c^2 p_c^\mu p_c^\nu - \frac{imk_c^2}{2}\varepsilon^{\mu\nu\rho\sigma} k_\rho^c s_\sigma(x, \mathbf{p}_c)\big], \tag{14}$$

where s_μ is the electron spin vector, $\omega_p^2(x) := e^2\rho(x)/m$ is corresponding plasma frequency, where

$$\rho(x) := m \int \frac{d\mathbf{p}_c}{p_c^0} \rho(x, \mathbf{p}_c). \tag{15}$$

Plasmon-polariton solutions

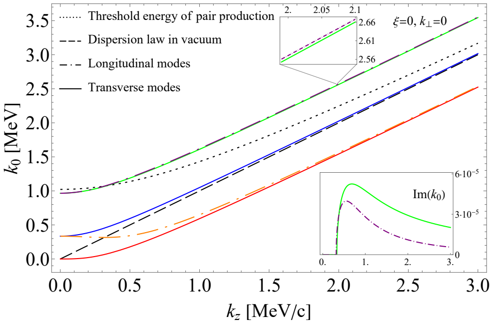
We will solve the equations in the electrons rest frame

$$p_c^\mu = (m, 0). \tag{16}$$

We assume that the plasma frequency and spin are independent of the spatial coordinates. In this case, the Maxwell equation can be written in the momentum space as

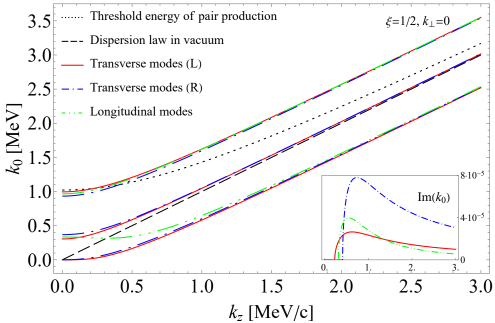
$$\Big\{ - k^2 \eta^{\mu\nu} + k^\mu k^\nu + \frac{\omega_p^2(k)}{(kp)^2 - k^4/4} \big[(kp)^2 \eta^{\mu\nu} - (kp) k^{(\mu} p^{\nu)} + \\ + k^2 p^\mu p^\nu - \frac{imk^2}{2} \varepsilon^{\mu\nu\rho\sigma} k_\rho s_\sigma \big] \Big\} A_\nu(k) = 0. \tag{17}$$

Plasmon-polariton solutions



The figure shows the laws of dispersion of plasmon-polariton modes for a single unpolarized electron in the electron rest frame. The law of dispersion in vacuum and the threshold for the formation of an electron-positron pair are also shown. The energies of plasmon polaritons exceeding the pair formation threshold have positive imaginary parts shown in the inset.

Plasmon-polariton solutions



The same as in the previous figure, but for a partially polarized electron with a degree of polarization of $\xi = 1/2$ ($\xi = 1$ corresponds to a completely polarized electron state). In this case, the modes are split in comparison with unpolarized state.

Infrared limit

Let us consider another limit for the polarization operator. We assume that the wavelength of the external field is much larger than the typical scale of variation of the electron wave packet in the coordinate and momentum space

$$|k^\mu| \ll |p^\mu|, \qquad |q^\mu| \ll |p^\mu|, \tag{18}$$

and

$$\rho_{ss'}(\mathbf{p}, \mathbf{p}') \approx \rho_{ss'}(\mathbf{p}_c, \mathbf{p}_c) e^{-i\mathbf{q}\mathbf{x}_0}, \tag{19}$$

where x_0 is the position of the center of the electron wave packet

In this approximation the polarization operator has the form

$$\begin{aligned} \overset{\psi}{\Pi}{}^{\mu\nu}_{qc}(k, k') \approx & -\frac{1}{m}\Big[\eta^{\mu\nu} - \frac{k^\mu p_c^\nu}{(kp_c)} - \frac{p_c^\mu k'^\nu}{(k'p_c)} + \frac{(k'k) p_c^\mu p_c^\nu}{(k'p_c)(kp_c)}\Big] \\ =: & -\frac{1}{m}\pi^{\mu\nu}(k', k), \end{aligned} \tag{20}$$

Infrared limit

The effective Maxwell equations without sources are given by

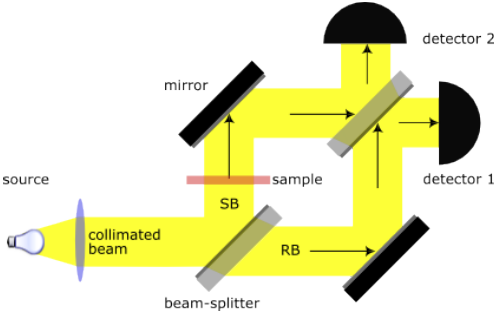
$$(\Box\eta^{\mu\nu} - \partial^\mu\partial^\nu)A_\nu(x) + \frac{e^2}{m}\int d\tau\pi^{\mu\nu}\left(i\frac{\partial}{\partial x}, i\frac{\partial}{\partial x}\Big|_A\right)\delta(x - x(\tau))A_\nu(x) = 0. \quad (21)$$

It is easy to verify that the action,

$$\begin{aligned} S[A_\mu(x), d_\mu(\tau)] := -\frac{1}{4}\int d^4x F_{\mu\nu}(x)F^{\mu\nu}(x) + \\ + \int d\tau\left[-\frac{1}{8\pi r_0}\dot{d}_\perp^\mu\eta_{\mu\nu}\dot{d}_\perp^\nu + \dot{x}^\mu F_{\mu\nu}(x(\tau))d^\nu\right], \end{aligned} \quad (22)$$

where $r_0 := \alpha/m$ is the classical electron radius, d^μ is the dynamic dipole moment, $d_\perp^\mu := d^\mu - (\dot{x}d)\dot{x}^\mu$, $p_c^\mu = m\dot{x}^\mu$, and τ is a natural parameter on the electron worldline, reproduces effective equations (21) on excluding d^μ.

Proposal for an experiment



The scheme of the Mach—Zehnder interferometer. A solitary electron is supposed to be placed as a sample in one of the arms of the interferometer. For this experiment, it is necessary to calculate the phase shift of an electromagnetic wave when passing through an electron wave packet.

Conclusion

1. The work shows that in coherent processes, the wave packet of one electron carries additional degrees of freedom – plasmons.
2. There are eight plasmon-polariton modes on a single electron that are confined to the electron wave packet.
3. In the infrared limit, these additional degrees of freedom are reduced to the vector of the dipole moment.

See for more arXiv:2412.00750.
Phys. Rev. D 111, 036028 (2025)