# Sterile neutrinos and dark matter in models with left-right chiral symmetry

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Twenty-second Lomonosov conference on elementary particle physics

arXiv: 2206.05186 (JETP 2023),
2212.11310 (Phys.Rev.D 2024),
2303.06680 (Symmetry 2023),
2308.02240 (JETP Lett 2023),
Phys.Part.Nucl.Lett. 20 (2023) 456,
Moscow Univ.Phys.Bull. 79 (2024) Suppl 1, 408,
Phys.Part.Nucl. 56 (2025) 871,
2506.04035 (Phys.Part.Nucl.Lett. 22 (2025) 966

# Outline

**Minimal left-right model (MLRM)** $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

**MLRM sterile neutrino warm dark matter** $-$ $\mathrm{N}_1$

$\nu$**MSM model. 3 gen. seesaw type I mechanism (minimal seesaw).**

**Seesaw type II mechanism in the framework of MLRM**

$\mathrm{N}_2$ **and** $\mathrm{N}_3$ **decays. Feynman rules for Majorana fermions.**

**Alternatives. Inverse seesaw mechnism**

**Summary**

# $SO(10)$ **GUT** $\to SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ $\to SU(2)_L \times U(1)_Y$

LRSM gauge group may appear in the sequence of SO(10) Grand Unification symmetry breaking steps down to SM. $\mathcal{G}_{3221}$ corresponds to *minimai left-right model* (MLRM in the literature).

**Обозначения:**

$$\mathcal{G}_{51} = SU(5) \times U(1)$$

$$\mathcal{G}_{5} = SU(5)$$

$$\mathcal{G}_{PS} = SU(4)_C \times SU(2)_L \times SU(2)_R$$

$$\mathcal{G}_{421} = SU(4)_C \times SU(2)_L \times U(1)_{B-L}$$

$$\mathcal{G}_{3221} = SU(\mathbf{3})_C \times SU(\mathbf{2})_L \times SU(\mathbf{2})_R \times U(\mathbf{1})_{B-L}$$

$$\mathcal{G}_{3211} = SU(\mathbf{3})_C \times SU(\mathbf{2})_L \times U(\mathbf{1})_R \times U(\mathbf{1})_{B-L}$$

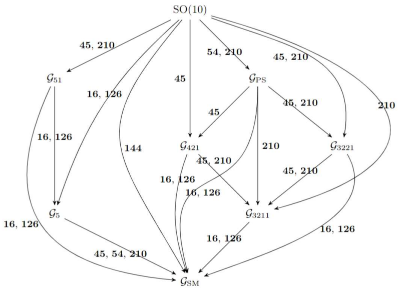



**Figure:** **Возможные цепочки нарушения SO(10) симметрии до** $\mathcal{G}_{SM}$. **[Fig. from M.Pernow,"Models of SO(10) Grand Unified Theories", 2021.]**

# Neutrino mass problem

- Neutrinos have extremely small non-zero masses. Neutrino oscillation data:

$$m_{\text{light}} = \boxed{?} \; \sqrt{|\Delta m^2_{21}|} \simeq 0.009 \text{ eV} \quad \sqrt{|\Delta m^2_{31}|} = 0.049 \text{ eV} \quad \sqrt{|\Delta m^2_{32|}} = 0.050 \text{ eV}$$

- Neutrino mixing matrix $U_{\text{PMNS}}$

$$\nu_i = \sum_{\alpha} (U_{\text{PMNS}})_{\alpha i} \nu_{\alpha}$$

- In the Standard Model right singlet Dirac neutrino is sterile and does not mix
- More natural framework - Majorana neutrinos, mixing, **seesaw mechanism** (and its variations)
  1. Seesaw I (minimal seesaw)
  2. Seesaw II (MLRM seesaw)
  3. Inverse seesaw (ISS)
- They can be used as models with fermionic warm dark matter (WDM) candidate $m_{DM} \sim \mathcal{O}(\text{keV})$

# MLRM: Spinor and vector fields

**MLRM gauge group**

$$G_{LR} = SU(3)_c \times SU(2)_R \times SU(2)_L \times U(1)_{B-L}$$

| Fermions | $SU(3)_c$ | $SU(2)_L$ | $SU(2)_R$ | $U(1)_{B-L}$ | New |
|---|---|---|---|---|---|
| $L_{\alpha_L} = \begin{pmatrix} \nu_{\alpha_L} \\ l_{\alpha_L} \end{pmatrix}$ | **1** | **2** | **1** | $-1$ | |
| $L_{\alpha_R} = \begin{pmatrix} \nu_{\alpha_R} \\ l_{\alpha_R} \end{pmatrix}$ | **1** | **1** | **2** | $-1$ | $N_1, N_2, N_3$ |
| $Q_{a_L} = \begin{pmatrix} u_{a_L} \\ d_{a_L} \end{pmatrix}$ | **3** | **2** | **1** | $\frac{1}{3}$ | |
| $Q_{a_R} = \begin{pmatrix} u_{a_R} \\ d_{a_R} \end{pmatrix}$ | **3** | **1** | **2** | $\frac{1}{3}$ | |
| $W_L = \{W_L^+, W_L^-, W_L^3\}$<br>$W_R = \{W_R^+, W_R^-, W_R^3\}$<br>$B$ | **1**<br>**1**<br>**1** | **3**<br>**1**<br>**1** | **1**<br>**3**<br>**1** | **0**<br>**0**<br>**0** | $W_2^\pm, Z_2$ |

**Table:** Fermions and gauge fields

# MLRM scalars

| Higgs fields | $SU(3)_c$ | $SU(2)_L$ | $SU(2)_R$ | $U(1)_{B-L}$ | New |
|---|---|---|---|---|---|
| $\Delta_L = \begin{pmatrix} \frac{\delta_L^+}{\sqrt{2}} & \delta_L^{++} \\ \delta_L^0 & -\frac{\delta_L^+}{\sqrt{2}} \end{pmatrix}$ | **1** | **3** | **1** | 2 | $\begin{pmatrix} A_1^0, & A_2^0, \\ H_1^\pm, & H_2^\pm \end{pmatrix}$ |
| $\Delta_R = \begin{pmatrix} \frac{\delta_R^+}{\sqrt{2}} & \delta_R^{++} \\ \delta_R^0 & -\frac{\delta_L^+}{\sqrt{2}} \end{pmatrix}$ | **1** | **1** | **3** | 2 | $H_1^{\pm\pm}, H_2^{\pm\pm}$ |
| $\Phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}$ | **1** | **2** | **2** | 0 | $\begin{pmatrix} H_{125}, & H_1^0, \\ H_2^0, & H_3^0 \end{pmatrix}$ |

**Table:** MLRM scalar multiplets

$$\mathcal{L}_{Higgs} = tr|D_\mu \Phi|^2 + tr|D_\mu \Delta_R|^2 + tr|D_\mu \Delta_L|^2 - V(\Phi, \Delta_L, \Delta_R)$$

# Higgs potential

$$
\begin{aligned}
V(\phi,\Delta_L,\Delta_R) \;=\;& -\mu_1^2\left(Tr[\phi^\dagger\phi]\right)-\mu_2^2\left(Tr[\tilde{\phi}\phi^\dagger]+\left(Tr[\tilde{\phi}^\dagger\phi]\right)\right)-\mu_3^2\left(Tr[\Delta_L\Delta_L^\dagger]+Tr[\Delta_R\Delta_R^\dagger]\right)\\
&+\lambda_1\left(\left(Tr[\phi\phi^\dagger]\right)^2\right)+\lambda_2\left(\left(Tr[\tilde{\phi}\phi^\dagger]\right)^2+\left(Tr[\tilde{\phi}^\dagger\phi]\right)^2\right)+\lambda_3\left(Tr[\tilde{\phi}\phi^\dagger]Tr[\tilde{\phi}^\dagger\phi]\right)\\
&+\lambda_4\left(Tr[\phi\phi^\dagger]\left(Tr[\tilde{\phi}\phi^\dagger]+Tr[\tilde{\phi}^\dagger\phi]\right)\right)\\
&+\rho_1\left(\left(Tr[\Delta_L\Delta_L^\dagger]\right)^2+\left(Tr[\Delta_R\Delta_R^\dagger]\right)^2\right)\\
&+\rho_2\left(Tr[\Delta_L\Delta_L]Tr[\Delta_L^\dagger\Delta_L^\dagger]+Tr[\Delta_R\Delta_R]Tr[\Delta_R^\dagger\Delta_R^\dagger]\right)\\
&+\rho_3\left(Tr[\Delta_L\Delta_L^\dagger]Tr[\Delta_R\Delta_R^\dagger]\right)\\
&+\rho_4\left(Tr[\Delta_L\Delta_L]Tr[\Delta_R^\dagger\Delta_R^\dagger]+Tr[\Delta_L^\dagger\Delta_L^\dagger]Tr[\Delta_R\Delta_R]\right)\\
&+\alpha_1\left(Tr[\phi\phi^\dagger]\left(Tr[\Delta_L\Delta_L^\dagger]+Tr[\Delta_R\Delta_R^\dagger]\right)\right)\\
&+\alpha_2\left(Tr[\phi\tilde{\phi}^\dagger]Tr[\Delta_R\Delta_R^\dagger]+Tr[\phi^\dagger\tilde{\phi}]Tr[\Delta_L\Delta_L^\dagger]\right)\\
&+\alpha_2^*\left(Tr[\phi^\dagger\tilde{\phi}]Tr[\Delta_R\Delta_R^\dagger]+Tr[\tilde{\phi}^\dagger\phi]Tr[\Delta_L\Delta_L^\dagger]\right)\\
&+\alpha_3\left(Tr[\phi\phi^\dagger\Delta_L\Delta_L^\dagger]+Tr[\phi^\dagger\phi\Delta_R\Delta_R^\dagger]\right)\\
&+\beta_1\left(Tr[\phi\Delta_R\phi^\dagger\Delta_L^\dagger]+Tr[\phi^\dagger\Delta_L\phi\Delta_R^\dagger]\right)+\beta_2\left(Tr[\tilde{\phi}\Delta_R\phi^\dagger\Delta_L^\dagger]+Tr[\tilde{\phi}^\dagger\Delta_L\phi\Delta_R^\dagger]\right)\\
&+\beta_3\left(Tr[\phi\Delta_R\tilde{\phi}^\dagger\Delta_L^\dagger]+Tr[\phi^\dagger\Delta_L\tilde{\phi}\Delta_R^\dagger]\right),
\end{aligned}
\tag{1}
$$

# Symmetry breaking

$$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

1. The initial LR symmetry is spontaneously broken

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \xrightarrow{\langle\Delta_R\rangle} SU(2)_L \times U(1)_Y, \quad (2)$$

$$\langle\Delta_R\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}. \quad (3)$$

2. The bidoublet and the left handed triplet acquire VEVs as a result of spontaneous symmetry breaking

$$SU(2)_L \times U(1)_Y \xrightarrow{\langle\phi\rangle,\langle\Delta_L\rangle} U(1)_Q, \quad (4)$$

$$\langle\phi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix}, \qquad \langle\Delta_L\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_L & 0 \end{pmatrix}, \quad (5)$$

where $\sqrt{k_1^2 + k_2^2} = 246$ GeV,

# Seesaw relation for Higgs triplet VEVs

**$\beta$-terms of the Higgs potential:**

$$V_\beta(\Phi,\Delta_L,\Delta_R) = \beta_1\Big(Tr[\phi\Delta_R\phi^\dagger\Delta_L^\dagger]+Tr[\phi^\dagger\Delta_L\phi\Delta_R^\dagger]\Big)+\beta_2\Big(Tr[\tilde\phi\Delta_R\phi^\dagger\Delta_L^\dagger]+Tr[\tilde\phi^\dagger\Delta_L\phi\Delta_R^\dagger]\Big)+ \beta_3\Big(Tr[\phi\Delta_R\tilde\phi^\dagger\Delta_L^\dagger]+Tr[\phi^\dagger\Delta_L\tilde\phi\Delta_R^\dagger]\Big),$$

**Additional assumptions (GUT motovated): zero or marginal $\beta$: $\rightarrow$** $\beta_i = 0$ or $\beta_i \simeq 0$

**"VEV seesaw" relation for $v_L$ and $v_R$**

$$\boxed{v_L = \gamma\,\frac{(246\ \text{GeV})^2}{v_R},} \qquad \text{where}\ \ \gamma \equiv \frac{\beta_2 k_1^2 + \beta_1 k_1 k_2 + \beta_3 k_2^2}{(2\rho_1 - \rho_3)(246\ \text{GeV})^2}\ ,$$

$\beta_i = 0:\quad (2\rho_1 - \rho_3)v_R v_L = 0$

$\boxed{v_L = 0}$

General LR-condition:

$\rightarrow v_R \neq 0$

Vacuum stability

$\rightarrow (2\rho_1 - \rho_3) \neq 0$

$\beta_i \rightarrow 0$

$$\boxed{v_L \simeq \frac{(246\ \text{GeV})^2}{v_R}}$$

$(v_R \gg 246$ GeV $\quad$ or $\gamma \ll 1)$

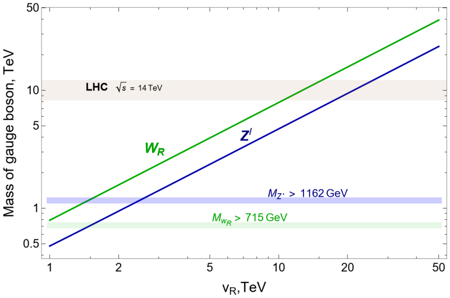



**Figure:** Masses of new vector gauge bosons $Z_2$ and $W_2$

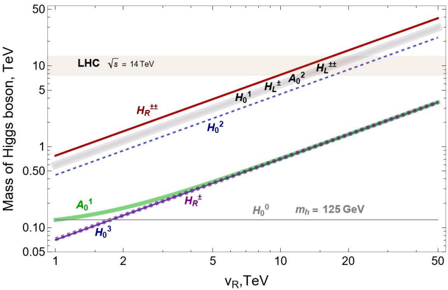



**Figure:** Masses of all 14 Higgs bosons in LRSM with tuning of self-interaction constants: $\alpha_3 = 0.01$ $\rho_1 = 0.1$, $\rho_2 = 0.3$, $\rho_3 = 0.9$, $\lambda_1 = \lambda_{SM} = 0.118$, $\lambda_2 = 0.01$, $\lambda_3 = 0.1$

# keV sterile neutrino $N_1$ as warm DM, simplified

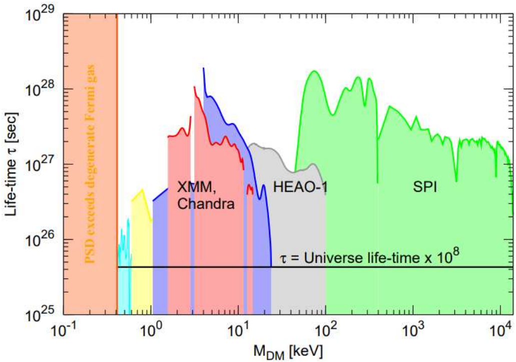
**DM candidate:** the lightest sterile neutrino with mass $\sim 1 - 10$ keV.

- **$N_1$ lifetime restriction:** age of the Universe

$$\tau_{\nu_s} = 10^{19} \left(\frac{m_s}{1\ \text{keV}}\right)^{-5} \frac{1}{\sin^2(2\theta)}\ \text{sec}\ >\ H_0^{-1} \simeq 10^{17}\ \text{sec}$$

- **non-observation of radiative one-loop decay**

[Aliev, Vysotsky, Sov. Phys. Usp. 24 (1981)]




**X-ray bound:**
$N_1 \to \gamma, \nu$ with $E_\gamma \simeq M_1/2$ leads to stronger lifetime limit $\tau_{\nu_s} > 10^{25}$ sec, so

$$\sin^2(2\theta) < 10^{-6} \left(\frac{m_s}{1\ \text{keV}}\right)^{-5}$$

[Boyarsky et al, arXiv:0811.2385v1]

# DM relic density: production via oscillations

simplified: $1\ \nu_\alpha + 1\ \nu_s$ states. Single mixing parameter is $\sin(\theta)$

- Boltzmann equation for DM production via $\nu_a \leftrightarrow \nu_s$ oscillations

$$\frac{\partial}{\partial t} f_s(p,t) \quad - \quad H\,p\,\frac{\partial}{\partial p} f_{N_1}(p,t) = C_{\rm oscl}(p,t,T)$$

$$\text{where } C_{oscl} \quad \approx \quad \frac{\Gamma_\alpha(p)}{2}\sin^2(2\theta_{\rm eff})\left[1+\left(\frac{\Gamma_\alpha(p) l_m}{2}\right)^2\right]^{-1}[f_{\nu_\alpha}(p,t)-f_s(p,t)]$$

$$\sin^2(\theta_{\rm eff}) \quad = \quad \frac{\Delta^2(p)\sin^2 2\theta}{\Delta^2(p)\sin^2 2\theta+\left[\Delta(p)\cos 2\theta - V^D - V^T(p)\right]^2} \qquad \Delta=(m_2^2-m_1^2)/2p$$

[Abazajian et al, Phys.Rev. D64 (2001) 023501]

- Rough estimate for relic density of sterile neutrino (non-resonant case)

$$\Omega_{\nu_s} h^2 \quad = \quad K_\alpha(m_s)\left(\frac{\sin^2(2\theta)}{10^{-8}}\right)\left(\frac{m_s}{1\ \text{keV}}\right)^2$$

where $K_\alpha \sim 0.3$ with weak dependence on $M_1$ within the considered limits, $\alpha = e, \mu, \tau$

Sterile neutrino mass of about 10 keV and a mixing angle with SM left handed neutrino of about $10^{-5}$ are compatible with the Universe structure formation. However, mixing is significantly reduced by **interaction with the primary plasma** ( [D. Notzold and G. Raffelt, Nucl. Phys. B307 (1988) 924] ) The reduction factor depends on neutrino momentum and in **the case of lepton asymmetry** can disappear for a particular value of momentum, leading to resonance production of of sterile neutrino or anti-neutrino depending on the sign of asymmetry. Such production in turn cancels the asymmetry and generation of sterile DM particles may stop.

A calculation of relic density of sterile neutrino was realized in **micrOMEGAs 6** package following the framework of T. Venumadhav, F. Cyr-Racine, K. Abazajian and C. Hirata, Phys. Rev. D94 (2016) 043515 Input parameters are the $\nu$-$N$ mixing angle, lepton asymmetry normalized to entropy density $L/s$ and the mass of sterile neutrino. Our code calculates the relic density and produces plots for final momentum distributions of sterile neutrino and for temperature evolution of $L/s$.

# MicrOMEGAs 6 **calculation:** $L/s$, $f_E$

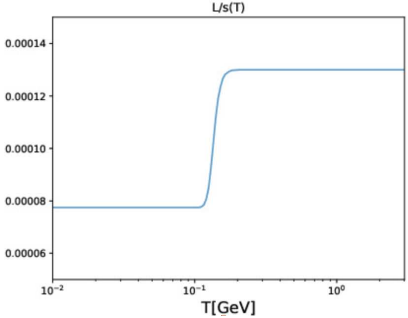



**Figure:**

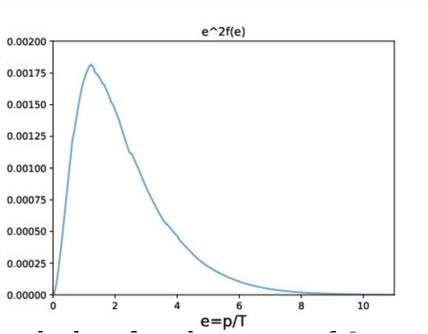



**Figure:**

For a parameter set $m$=7.1keV, $\sin^2\theta = 8\cdot 10^{-12}$, $L/s = 1.3\cdot 10^{-3}$ and in the case of mixing with muon neutrino the relic density is approaching $\Omega h^2 = 0.12$.

# Seesaw type I mechanism. Three generations, MLRM with $v_R \to \infty$, $v_L = 0$: $\nu$MSM model.

**Additional fields:** $SU(2)_L \times U(1)_Y$ - singlets $\nu_{R,k}$, $k = \overline{1,3}$ (flavour basis) with heavy Majorana mass term $\sim M_R$. Mass states $N_J$, $J = \overline{1,3}$ are heavy neutral leptons (HNL).

**Lagrangian:** $\mathcal{L} = \mathcal{L}_{SM} + i\overline{\nu}_R \partial_\mu \gamma^\mu \nu_R - \left( Y\, \bar{l}_L \widetilde{\phi} \nu_R + \frac{1}{2} \overline{\nu^c}_R M_R \nu_R + h.c \right)$,

**After SSB:** $\mathcal{L} \supset (\overline{\nu_L},\ \overline{\nu^c_R}) \begin{pmatrix} \mathbb{O} & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix}$, $\quad \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} = P_L\, \mathrm{U} \begin{pmatrix} \nu \\ N \end{pmatrix}$

Casas-Ibarra diagonalization: [J. Casas, A. Ibarra, Nucl.Phys.B 618 (2001) 171]

$$\mathrm{U} = \mathcal{W}\mathrm{V} \qquad \mathcal{W} = \exp \begin{pmatrix} \mathbb{O} & -\theta \\ \theta^\dagger & \mathbb{O} \end{pmatrix} \simeq \begin{pmatrix} I - \frac{1}{2}\theta\theta^\dagger & -\theta \\ \theta^\dagger & I - \frac{1}{2}\theta^\dagger\theta \end{pmatrix} + \mathcal{O}(\theta^3)$$

$$\mathrm{V} = \begin{pmatrix} U_\nu & 0 \\ 0 & U_N \end{pmatrix} \qquad m_\nu \equiv U_\nu \hat{m} U_\nu^T \qquad M_N \equiv U_N \hat{M} U_N^T$$

$$\mathcal{W}^\dagger \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix} \mathcal{W}^* = \begin{pmatrix} U_\nu \hat{m} U_\nu^T & 0 \\ 0^T & U_N^* \hat{M} U_N^\dagger \end{pmatrix}$$

# Heavy neutrino mixing and $\nu$MSM

$$(\nu - N)\text{-mixing:} \quad \Theta \equiv \theta U_N$$

$$\text{PMNS:} \quad U_{\rm PMNS} = \left(I - \frac{1}{2}\theta^\dagger\theta\right) U_\nu$$

**A system of matrix equations for diagonalizing transformation** U: ***(leading order*** $\theta$***-accuracy)***

$$\begin{cases} \theta \simeq m_D M_R^{-1}, \\ m_\nu = -\theta M_R \theta^T, \\ M_N \simeq M_R \end{cases} \Rightarrow$$

seesaw I equation

$$m_\nu = -m_D M_N^{-1} m_D^T$$

Seesaw I equation can be rewritten

$$\begin{aligned} I = \Omega\Omega^T &= \left[i\sqrt{\tilde{m}^{-1}} U_\nu^\dagger m_D U_N \sqrt{\hat{M}^{-1}}\right]^T \left[-i\sqrt{\tilde{m}^{-1}} U_\nu^\dagger m_D U_N \sqrt{\hat{M}^{-1}}\right], \\ m_D &= iU_{\rm PMNS}^\dagger \sqrt{\hat{m}}\Omega\sqrt{\hat{M}^{-1}} \quad \rightarrow \quad \Theta = iU_{\rm PMNS}^\dagger \sqrt{\hat{m}}\Omega\sqrt{\hat{M}^{-1}} \end{aligned}$$

$\nu$**MSM - model** [T.Asaka, M.Shaposhnikov,Phys.Lett.B 620, 17(2005)]

3 sterile neutrino:
$N_1$ - WDM with $M_1 \sim \mathcal{O}(\text{keV})$
$N_2$ and $N_3$ heavy neutrinos with masses $M_2 \simeq M_3 \sim \Lambda_{EW}$, $\Delta = |M_2 - M_3| \ll M_{2,3}$ need for Resonant leptogenesis (lepton asym. $\rightarrow$ baryon asym.)

Факторы смешивания, измеряемые в экспериментах:

$$U_{\alpha I}^2 = |\Theta_{\alpha I}|^2, \qquad U_i^2 = \sum_\alpha U_{\alpha I}^2, \qquad U^2 = \sum_i U_i^2$$

$$M_1 \sum_\alpha |\Theta_{\alpha 1}|^2 \equiv m_D^{dm} = \sum_\alpha |U_{\alpha i}(\sqrt{\tilde{m}})_{ij}\Omega_{j1}|^2 = |\sqrt{\tilde{m}}|_{kn}^2 \Omega_{n1}\Omega_{k1}^*.$$

# Neutrino mixing in MLRM: Seesaw type II

$$-\sum_{i,j}\{\bar{L}_{iL}[(h_L)_{ij}\phi + (\tilde{h}_L)_{ij}\tilde{\phi}]L_{jR} - \overline{(L_{iR})^c}\,\Sigma_R(h_M)_{ij}L_{jR} - \overline{(L_{iL})^c}\,\Sigma_L(h_M)_{ij}L_{jL}\} + \text{h.c.},$$

где $\tilde{\phi} \equiv \tau_2\phi^*\tau_2$, $\Sigma_{L,R} = i\tau_2\Delta_{L,R}$ и $h_L$, $\tilde{h}_L$, $h_M$ – $3\times3$ матрицы Юкавы в калибровочном базисе. Массовая матрица в калибровочном базисе

$$\begin{pmatrix} M_L & m_D \\ m_D^T & M_R \end{pmatrix} \qquad \begin{matrix} M_D = \frac{1}{\sqrt{2}}(h_L k_1 + \tilde{h}_L k_2), \\ M_L = \sqrt{2}h_M v_L, \quad M_R = \sqrt{2}h_M v_R, \end{matrix}$$

here $h_L$, $\tilde{h}_L$, $h_M$ are **Yukawa couplings** with left triplet $\Delta_L$ and bi-doublet $\Phi$

## Mixing matrix in Casas-Ibarra parametrization

**seesaw II equation:** $\boxed{m_\nu = M_L - M_D M_N^{-1} M_D^T}$

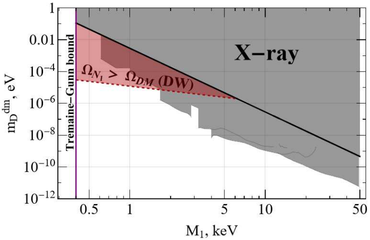
$$\Theta = iU_\nu\left(\sqrt{\tilde{m}}\right)\Omega\sqrt{\hat{M}^{-1}}, \qquad \sqrt{\tilde{m}} = \sqrt{\hat{m} - U^\dagger_{\mathrm{PMNS}} M_L U^*_{\mathrm{PMNS}}}$$

**Assumption:** $U_N = I,\ \theta^2 \ll 1$

$$h_M \simeq \frac{\hat{M}}{\sqrt{2}v_R} \Rightarrow \boxed{\tilde{m} = \hat{m} - \frac{v_L}{v_R}U^\dagger_{\mathrm{PMNS}}\hat{M}U^*_{\mathrm{PMNS}}}$$

# DM mixing: seesaw I and seesaw II

$$m_D^{dm} = |\sqrt{\tilde{m}}|^2_{kn}\Omega_{n1}\Omega^*_{k1} \quad |\Theta_{DM}|^2 = \frac{m_D^{dm}}{M_1}$$



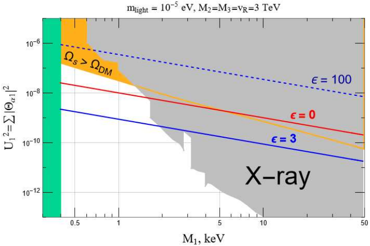

## $\nu$MSM limit: pure seesaw I

$v_L = 0 \to \sqrt{\tilde{m}} = \sqrt{\hat{m}}$. Strong DM constraints and $\nu$-oscillation data lead to a fine-tuned (FT) form of $\Omega$

$\Omega_{NH}$ : $\Omega_{j1} \to \delta_{j1}$

$\Omega_{IH}$ : $\Omega_{j3} \to \delta_{j3}$

$$\boxed{m_D^{dm}(v_L = 0, \Omega \to \text{FT}) = m_{\text{light}}}$$




## MLRM with $v_L \neq 0$: seesaw II

- $m_{\text{light}} \gg v_L \frac{\max M_J}{v_R}$ – $\nu MSM$-limit;
- $m_{\text{light}} \ll v_L \frac{\max M_J}{v_R}$ – **seesaw II dominance** $\to$ strong increase in mixing, inconsistent with DM constraints;
- $\hat{m} \simeq U^\dagger_{\text{PMNS}} M_L U^*_{\text{PMNS}} \to$ DM mixing decreases by 1-2 order due to **seesaw I – seesaw II** cancellation for some entries of $|\sqrt{\tilde{m}}|^2_{kn}$

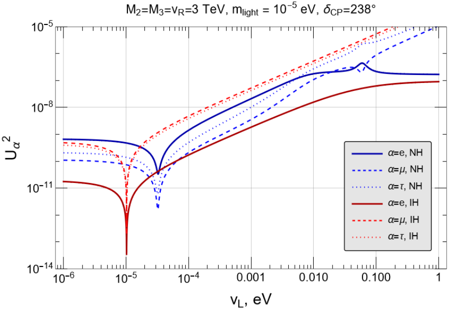
# Cancellation effect in the seesaw II mixing

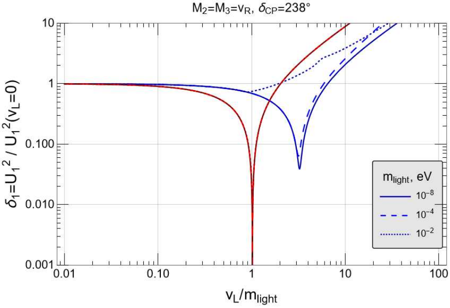
$\nu$**MSM-benchmark for** $\Omega$ :  $m_D^{dm} = \left| \sqrt{\hat{m} - \frac{v_L}{v_R} U_{\mathrm{PMNS}}^{\dagger} \hat{M} U_{\mathrm{PMNS}}^{*}} \right|^2_{11\ \text{(NH) or } 33\ \text{(}}$

$$\Omega_{\mathrm{NH}}^{(FT)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \multicolumn{2}{c}{\Omega_{2\times 2}} \\ 0 & & \end{pmatrix}, \qquad \Omega_{\mathrm{IH}}^{(FT)} = \begin{pmatrix} 0 & \multicolumn{2}{c}{\Omega_{2\times 2}} \\ 0 & & \\ 1 & 0 & 0 \end{pmatrix}.$$

$$U_\alpha^2 = \sum_{I=1}^{3} |\Theta_{\alpha I}|^2$$

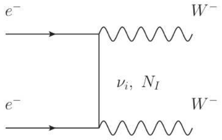
$$U_I^2 = \sum_{\alpha=e,\mu,\tau} |\Theta_{\alpha I}|^2, \quad |\Theta_{\mathrm{DM}}|^2 = U_1^2$$

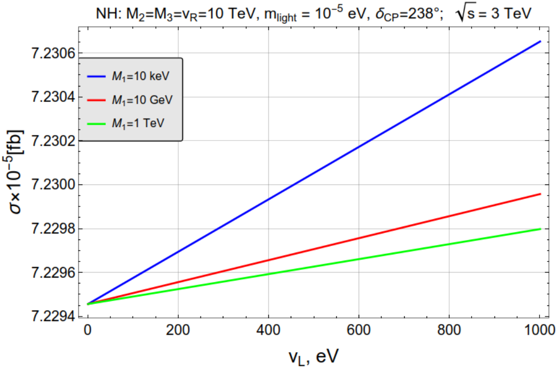


# Inverse neutrinoless double beta decay, $i0\nu\beta\beta$

$$e^- e^- \to W^- W^-$$



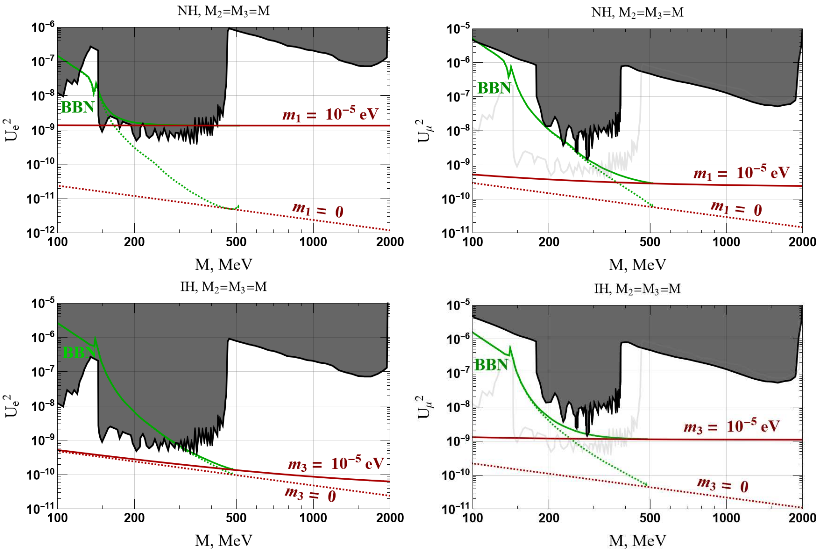
# Constraints for $N_2, N_3$ in the $\nu$MSM model

- There are two types of accelerator constraints from above: experiments with determining *missing energy* (PIENU, TRIUMPH, KEK, NA62, E949) and experiments on determining *displaced vertices* (PS-191, CHARM, NuTeV, DELPHI). The combination of these constraints gives upper bounds for

$$U_\alpha^2 = \sum_{I=1}^{3} |\Theta_{\alpha I}|^2 = \begin{cases} \frac{m_1}{M_1}|U_{\alpha 1}|^2 + |\Theta_{\alpha 2}^{(NH)}|^2 + |\Theta_{\alpha 3}^{(NH)}|^2, & \text{NH} \\ \frac{m_3}{M_1}|U_{\alpha 3}|^2 + |\Theta_{\alpha 2}^{(IH)}|^2 + |\Theta_{\alpha 3}^{(IH)}|^2, & \text{IH} \end{cases}$$

- Inequality for the lifetime of $N_2$ and $N_3$, $\tau_N < 0.02$ seconds, at which there is no overproduction of light elements ($He^4, He^2$) in primary plasma (primary nucleosynthesis or Big Bang nucleosynthesis, **BBN**). Gives a lower limit on the parameters $U_\alpha^2$.

# Constraints for $U_e^2$ and $U_\mu^2$ mixings in the $\nu$MSM



# Alternative: inverse seesaw, generic $ISS(\mathrm{p},\mathrm{q})$

## Field content of the $ISS(\mathrm{p},\mathrm{q})$

Consider **three types** of neutral lepton fields:

– left-handed flavour neutrino $\nu_{L\ \alpha}$, $\alpha = e, \mu, \tau$,

– right-handed neutrino $N_{R\ a}$ $a = \overline{1, \mathrm{p}}$

– right-handed sterile fermions $S_{R\ b}$ $b = \overline{1, \mathrm{q}}$

**Lagrangian after SSB:** **(Naturalness condition:** $\mu \ll m_D < M_R$)

$$\mathcal{L}_{ISS} = \frac{1}{2}(\overline{\nu}_L,\ \ \overline{\nu_R^c},\ \ \overline{\Sigma_R^c}) \begin{pmatrix} \mathbb{O}_{3\times 3} & m_{D\ 3\times p} & \mathbb{O}_{3\times q} \\ m^T_{D\ p\times 3} & \mathbb{O}_{p\times p} & M_{R\ p\times q} \\ \mathbb{O}_{q\times 3} & M^T_{R\ q\times p} & \mu_{\ q\times q} \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \\ \Sigma_R \end{pmatrix}$$

## Diagonalization: step 1

Rewrite mass matrix to the **seesaw I-like** form $\tilde{m}_D \equiv (m_D,\ 0)$

$$\mathrm{M}_{9\times 9} = \begin{pmatrix} \mathbb{O}_{\ 3\times 3} & \widetilde{m}_{D\ 3\times(p+q)} \\ \widetilde{m}^T_{D\ (p+q)\times 3} & \mathcal{X}_{\ q\times q} \end{pmatrix} \quad \text{where} \quad \mathcal{X} = \begin{pmatrix} \mathbb{O}_{\ p\times p} & M_{R\ p\times q} \\ M^T_{R\ q\times p} & \mu_{\ q\times q} \end{pmatrix}$$

$$\mathrm{U}^T \mathrm{M} \mathrm{U}, \quad \mathrm{U} = \mathrm{W} \begin{pmatrix} U_{\nu\ 3\times 3} & \mathbb{O}_{3\times(p+q)} \\ \mathbb{O}_{(p+q)\times 3} & \mathcal{U}_{\ (p+q)\times(p+q)} \end{pmatrix} \quad \mathrm{W} = \exp(\omega) \simeq \mathrm{I} + \omega + \ldots$$

# Inverse seesaw: ISS(p,q) diagonalization

**Technical comment: Inverse matrix (Frobenius)**

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}^{-1} = \begin{pmatrix} A^{-1} + A^{-1}B(M|A)^{-1}CA^{-1} & -A^{-1}B(M|A)^{-1} \\ -(M|A)^{-1}CA^{-1} & (M|A)^{-1} \end{pmatrix}$$

where for a matrix $M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$ Shur complement is $(M|A) \equiv D - CA^{-1}B$

## Neutrino effective mass operator

$$m_\nu = (\mathrm{M}|\mathcal{X}) \equiv -\tilde{m}_D \mathcal{X}^{-1} \tilde{m}_D^T = -(m_D,\ \mathbb{O}) \begin{pmatrix} (\mathcal{X}|\mu)^{-1} & \star \\ \star & \star \end{pmatrix} \begin{pmatrix} m_D^T \\ \mathbb{O} \end{pmatrix} = m_D \left( (M_R)\mu^{-1}(M_R)^T \right)^{-1} m_D^T$$

**Only if** $p = q$ :

$$\boxed{m_\nu = m_D (M_R^T)^{-1} \mu (M_R)^{-1} m_D^T \sim \frac{\mu m_D^2}{M^2}}$$

## Diagonalization: step 2 (Sterile block)

**Case 1:** $\boxed{p = q}$ All sterile fields form pseudo-Dirac pairs

$$\mathcal{X}' = \mathcal{U} \begin{pmatrix} \mathbb{O}_{p\times p} & M_{R\ p\times q} \\ M^T_{R\ q\times p} & \mu \end{pmatrix} \mathcal{U}^T = \begin{pmatrix} \sim -M_R + \mu & \mathcal{O}(\mu) \\ \mathcal{O}(\mu) & \sim M_R + \mu \end{pmatrix}$$

# Inverse seesaw: Toy-model ISS(1,1)

**Toy-model ISS(1,1):** $m_{\pm} \equiv \frac{\mu \pm \sqrt{\mu^2 + 4M^2}}{2}$

$$\mathcal{X} = \begin{pmatrix} 0 & M \\ M & \mu \end{pmatrix} \to \begin{pmatrix} m_- & 0 \\ 0 & m_+ \end{pmatrix} \simeq \begin{pmatrix} \mu - M & 0 \\ 0 & \mu + M \end{pmatrix}$$

$$U = \begin{pmatrix} \frac{1}{\sqrt{1+\frac{m_-^2}{M^2}}} & \frac{1}{\sqrt{1+\frac{m_+^2}{M^2}}} \\ \frac{m_-}{M\sqrt{1+\frac{m_-^2}{M^2}}} & \frac{m_+}{M\sqrt{1+\frac{m_+^2}{M^2}}} \end{pmatrix} \simeq \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

$S'', N''$ are mass-states, $N', S'$ - "middle" basis states (after the 1st step of diagonalization) **Pseudo-dirac** HEAVY ($\sim M$) states with small mass splitting $\sim \mu$.

$$N'' \equiv N_- \simeq \frac{1}{\sqrt{2}}(N' - S')$$
$$S'' \equiv N_+ \simeq \frac{1}{\sqrt{2}}(N' + S')$$

Warm dark matter candidate is absent in such a model

# Inverse seesaw: model ISS(1,2)

**ISS(1,2):** $$m_{\pm}^0 \simeq \pm\sqrt{M_1^2 + M_2^2} + \mathcal{O}(\mu) \qquad m_3 \simeq \mathcal{O}(\mu)$$

$$\mathcal{X} = \begin{pmatrix} 0 & M_1 & M_2 \\ M_1 & \mu_1 & 0 \\ M_2 & 0 & \mu_2 \end{pmatrix} = \mathcal{X}_0 + \delta\mathcal{X}(\mu) \qquad \mathcal{X}_I \to \begin{pmatrix} \boxed{\sim 0} & 0 & 0 \\ 0 & m_- & 0 \\ 0 & 0 & m_+ \end{pmatrix}$$

**Mass of light sterile state:** $$m_3 \approx \frac{M_2^2\mu_1 + M_1^2\mu_2}{M_1^2 + M_2^2} \sim \mu \sim \mathcal{O}(\text{keV})$$

$$\begin{pmatrix} \frac{M_1}{\sqrt{2(M_1^2+M_2^2)}} & \frac{M_1}{\sqrt{2(M_1^2+M_2^2)}} & \frac{M_2}{\sqrt{M_1^2+M_2^2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{M_2}{\sqrt{2(M_1^2+M_2^2)}} & \frac{M_2}{\sqrt{2(M_1^2+M_2^2)}} & -\frac{M_1}{\sqrt{M_1^2+M_2^2}} \end{pmatrix} \qquad \begin{pmatrix} N' \\ S_1' \\ S_2' \end{pmatrix} = U \underbrace{\begin{pmatrix} N_+ \\ N_- \\ \boxed{\mathcal{S}_3} \end{pmatrix}}_{phys.states}$$

Light sterile fermions ($q - p$ states with masses at the scale $\mu$) are embedded in $ISS(p, q)$ model when $p < q$.

# Summary

- Minimal Left-Right Model (MLRM) Higgs potential demonstrates a number of phenomenologically acceptable regimes with splitting of 16 new states which are not observed at the LHC. $\beta$-terms of the potential are important for active neutrino and sterile neutrino mixing scenarios. The $\nu$**MSM** (neutrino minimal standard model) can be embedded into the MLRM as a limiting scenario $v_R \to \infty$, $k_2 = 0$, seesaw type I.
- *MicrOMEGAs* 6 calculations for the oscillation term in the r.h.s. of the Boltzmann equation give an adequate relic density results both for resonant and nonresonant cases of the freeze-in DM scenario.
- Comparisons with data are possible in the limiting scenarios and are model dependent. Mixing parameters are not mass independent.
- Modification of seesaw type II for MLRM yields mixing in the lepton sector defined by VEVs. Four VEVs participate in the neutrino mass and mixing matrices with untrivial (orders of magntude) cancellations dependent on $v_L$, $v_R$ of scalar triplets and $U_{PMNS}$

$$\Theta = iU_{\text{PMNS}}\sqrt{\tilde{m}}\Omega\sqrt{\hat{M}^{-1}}, \qquad \sqrt{\tilde{m}} = \sqrt{\hat{m} - \frac{v_L}{v_R}U^{\dagger}_{\text{PMNS}}M_L U^{*}_{\text{PMNS}}}$$

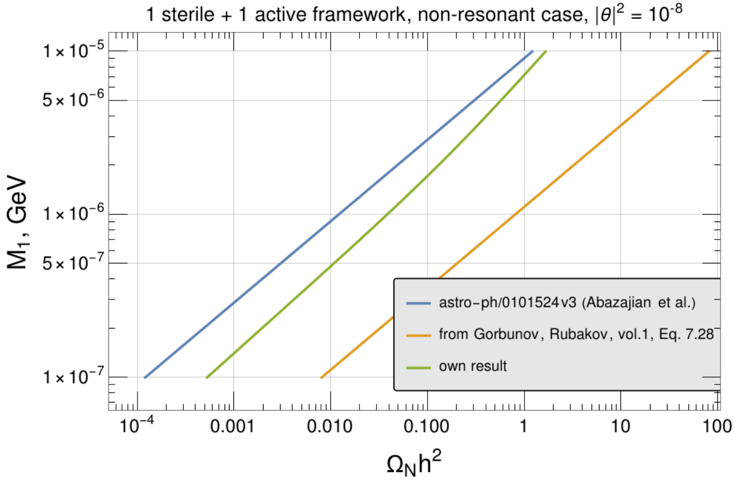
- Other mass neutrino and HNL mass hierarchies are possible within the inverse seesaw ISS(2,3) or ISS$(p, p+s)$, $p, s \in \mathbb{N}$. All seesaw follow the form of Shur complement which can be considered as a useful instrument for definition of the mass states and mass scales.

# Acknowledgments

*Thank you for attention*

The research was carried out within the framework of the scientific program of the National Center for Physics and Mathematics, project *"Particle Physics and Cosmology"*

# Backup slides


1 sterile + 1 active framework, non-resonant case, |θ|^2 = 10^-8
M_1, GeV
1 × 10^-5
5 × 10^-6
1 × 10^-6
5 × 10^-7
1 × 10^-7
10^-4
0.001
0.010
0.100
1
10
100
Ω_N h^2
astro-ph/0101524v3 (Abazajian et al.)
from Gorbunov, Rubakov, vol.1, Eq. 7.28
own result

# Charged and neutral currents, generic

$$\mathcal{L}_{NC}^{\nu} = \frac{1}{2}\sum_{X=Z_1,Z_2,A}\overline{\nu}\hat{X}\left(U_{\nu\nu}^{L}a_X^L P_L + U_{\nu\nu}^{R}a_X^R P_R\right)\nu,$$

$$\mathcal{L}_{NC}^{N} = \frac{1}{2}\sum_{X=Z_1,Z_2,A}\overline{N}\hat{X}(U_{NN}^{L}a_X^L P_L + a_X^R P_R)N + \frac{1}{2}\left(\sum_{X=Z_1,Z_2,A}\overline{\nu}\hat{X}(U_{\nu N}^{L}a_X^L P_L - U_{\nu N}^{R}a_X^R P_R)N + h.c.\right),$$

$$\mathcal{L}_{CC}^{\nu} = \frac{1}{\sqrt{2}}\bar{l}\hat{W}_1^-(U_{l\nu}^L g_L c_\xi P_L + U_{l\nu}^R g_R s_\xi P_R)\nu + \frac{1}{\sqrt{2}}\bar{l}\hat{W}_2^-(U_{l\nu}^L g_L s_\xi P_L - U_{l\nu}^R g_R c_\xi P_R)\nu + h.c.,$$

$$\mathcal{L}_{CC}^{N} = \frac{1}{\sqrt{2}}\bar{l}\hat{W}_1^-(U_{lN}^L g_L c_\xi P_L - U_{lN}^R g_R s_\xi P_R)N + \frac{1}{\sqrt{2}}\bar{l}\hat{W}_2^-(U_{lN}^L g_L s_\xi P_L + U_{lN}^R g_R c_\xi P_R)N + h.c.,$$

где

$$U_{\nu\nu}^{L} = U_{\mathrm{PMNS}}^{\dagger}U_{\mathrm{PMNS}}, \qquad U_{\nu\nu}^{R} = U_{\nu}^{T}\theta^{*}\theta^{T}U_{\nu}^{*},$$

$$U_{NN}^{L} = \Theta^{\dagger}\Theta, \qquad U_{\nu N}^{L} = U_{\mathrm{PMNS}}^{\dagger}\Theta, \qquad U_{\nu N}^{R} = U_{\nu}^{T}\theta^{*}U_{N},$$

$$U_{l\nu}^{L} = (V_L^l)^{\dagger}U_{\mathrm{PMNS}}, \qquad U_{l\nu}^{R} = (V_R^l)^{\dagger}\theta^{T}U_{\nu}^{*},$$

$$U_{lN}^{L} = (V_L^l)^{\dagger}\Theta, \qquad U_{lN}^{R} = (V_R^l)^{\dagger}U_{N},$$

$$a_{Z_1}^{L} = g_L S_{11} - g' S_{31}, \qquad a_{Z_1}^{R} = g_R S_{21} - g' S_{31},$$

$$a_{Z_2}^{L} = g_L S_{12} - g' S_{32}, \qquad a_{Z_2}^{R} = g_R S_{22} - g' S_{32},$$

$$a_{A}^{L} = g_L S_{13} - g' S_{33}, \qquad a_{A}^{R} = g_R S_{23} - g' S_{33}.$$

# Charged and neutral currents, simplified

Обычно используются. Предположим, что секторы заряженных лептонов и стерильных нейтрино диагональны

$$V^l_{L,R} = I, \qquad U_N = I, \qquad g_L = g_R \equiv g, \qquad \theta\theta^\dagger \ll I, \text{ тогда } \quad \Theta \simeq \theta U_N^* = \theta, \quad U_{\rm PMNS} \simeq U_\nu, \tag{6}$$

то есть (здесь и ниже $U = U_{\rm PMNS}$)

$$\begin{aligned}
&U^L_{\nu\nu} = U^\dagger U, \quad U^R_{\nu\nu} = (U^T\Theta^*)(\Theta^T U^*),\\
&U^L_{NN} = \Theta^\dagger\Theta, \quad U^L_{\nu N} = U^\dagger\Theta, \quad U^R_{\nu N} = U^T\Theta^*,\\
&U^L_{l\nu} = U, \quad U^R_{l\nu} = \Theta^T U^*,\\
&U^L_{lN} = \Theta, \quad U^R_{lN} = I.
\end{aligned}$$

тогда

$$\begin{aligned}
\mathcal{L}^\nu_{NC} &= \frac{1}{2}\sum_{X=Z_{\mathbf{1}},Z_{\mathbf{2}},A} \overline{\nu}\gamma^\mu X_\mu \left[(U^\dagger U)a^L_X P_L + (U^T\Theta^*)(\Theta^T U^*)a^R_X P_R\right]\nu,\\
\mathcal{L}^N_{NC} &= \frac{1}{2}\sum_{X=Z_{\mathbf{1}},Z_{\mathbf{2}},A} \overline{N}\gamma^\mu X_\mu \left[(\Theta^\dagger\Theta)a^L_X P_L + a^R_X P_R\right] N\\
&+ \frac{1}{2}\left(\sum_{X=Z_{\mathbf{1}},Z_{\mathbf{2}},A} \overline{\nu}\gamma^\mu X_\mu \left[(U^\dagger\Theta)a^L_X P_L - (U^T\Theta^*)a^R_X P_R\right] N + h.c.\right),\\
\mathcal{L}^\nu_{CC} &= \frac{g}{\sqrt{2}}\bar{l}\gamma^\mu W^-_{\mathbf{1}\mu}\left[U c_\xi P_L + (\Theta^T U^*)s_\xi P_R\right]\nu + \frac{g}{\sqrt{2}}\bar{l}\gamma^\mu W^-_{\mathbf{2}\mu}\left[U s_\xi P_L - (\Theta^T U^*)c_\xi P_R\right]\nu + h.c.,\\
\mathcal{L}^N_{CC} &= \frac{g}{\sqrt{2}}\bar{l}\gamma^\mu W^-_{\mathbf{1}\mu}(\Theta c_\xi P_L - s_\xi P_R)N + \frac{g}{\sqrt{2}}\bar{l}\gamma^\mu W^-_{\mathbf{2}\mu}(\Theta s_\xi P_L + c_\xi P_R)N + h.c.,
\end{aligned}$$

где

$$a^L_{Z_1} = g c_W c - g'(-s_W c_M c + s_M s), \qquad a^R_{Z_1} = g(-s_W s_M c - c_M s) - g'(-s_W c_M c + s_M s),$$
$$a^L_{Z_2} = g c_W s - g'(-s_W c_M s - s_M c), \qquad a^R_{Z_2} = g(-s_W s_M s + c_M c) - g'(-s_W c_M s - s_M c),$$
$$a^L_A = g s_W - g' c_W c_M, \qquad a^R_A = g c_W s_M - g' c_W c_M,$$

$$c_W = \cos\theta_W, \qquad s_W = \sin\theta_W,$$
$$c_M = \frac{\sqrt{\cos 2\theta_W}}{\cos\theta_W}, \qquad s_M = \tan\theta_W,$$
$$s = \sin\phi, \qquad c = \cos\phi,$$
$$g = \frac{e}{\sin\theta_W}, \qquad g' = \frac{e}{\sqrt{\cos 2\theta_W}}.$$

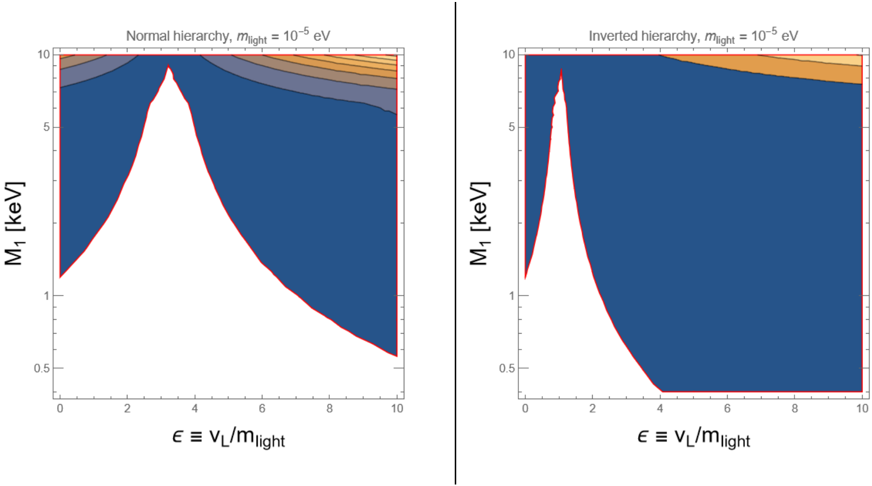
**Table:** Массы дополнительных бозонов MLRM для используемого параметрического набора и $k_2$=0

| $v_R$, ТэВ | Массы, ГэВ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $W_R$ | $Z_R$ | $H_1^0$ | $H_2^0$ | $H_3^0$ | $A_1^0$ | $A_2^0$ | $H_1^{\pm}$ | $H_2^{\pm}$ | $H_1^{\pm\pm}$ |
| 3 | 1412 | 2360 | 129 | 1342 | 1775 | 234 | 1775 | 1775 | 212 | 1775 |
| 12 | 5638 | 9437 | 849 | 5367 | 7099 | 854 | 7099 | 7099 | 849 | 7099 |

Ср. с

Bambhaniya G. et al. Left-right symmetry and the charged Higgs bosons at the LHC //JHEP.— 2014.— V. 33— arXiv:1311.4144 [hep-ph].

# Эффект сокращения seesaw I и II



## Inverse Seesaw Formula

$$m_\nu = m_D (M_R^T)^{-1} \mu (M_R)^{-1} m_D^T \qquad \text{(ISS with } p = q\text{)}$$

$$\tilde{\theta} = \tilde{m}_D \mathcal{X}^{-1} = (m_D, \ \ \mathbb{O}) \begin{pmatrix} (\mathcal{X}|\mu)^{-1} & (\mathcal{X}|\mu)^{-1} M_R \mu^{-1} \\ \star & \star \end{pmatrix}$$

## Casas-Ibarra Parametrization

Through matrix decomposition:

$$\Omega = \sqrt{\hat{m}}^{\,-1} U_\nu m_D (M_R^T)^{-1} \sqrt{\mu} \qquad\qquad m_D = U_\nu^\dagger \sqrt{\hat{m}}\, \Omega \sqrt{\mu}^{\,-1} M_R^T$$

where $\Omega$ is orthogonal ($\Omega\Omega^T = I$).

**$\nu - N$ Mixing**

$$\theta_1 = U_\nu^\dagger \sqrt{\hat{m}}\, \Omega \sqrt{\mu} M_R^{-1} \sim \mathcal{O}\left(\frac{\sqrt{m_\nu \mu}}{M}\right)$$

**$\nu - S$ Mixing**

$$\theta_2 = U_\nu^\dagger \sqrt{\hat{m}}\, \Omega \sqrt{\mu}^{\,-1} \sim \mathcal{O}\left(\sqrt{\frac{m_\nu}{\mu}}\right)$$

# Реализации MLRSM

*Реализация FeynRules*
Roitgrund A., Eilam G., Bar-Shalom S. Implementation of the left-right symmetric model in FeynRules, CPC 2016
*Реализация LanHEP*
*in progress*. Наблюдаются множественные несоответствия.

*FeynRules:* Adam Alloul et al, FeynRules 2.0 - A complete toolbox for tree-level phenomenology, Comput.Phys.Commun. 185 (2014) 2250-2300 (arXiv:1310.1921[hep-ph])
*LanHEP:* A. Semenov, LanHEP - A package for automatic generation of Feynman rules from the Lagrangian. Version 3.2, Comput.Phys.Commun. 201 (2016) 167-170
*Реализация SARAH отсутствует.*
*CalcHEP:* A. Belyaev, N. Christensen, A. Pukhov, CalcHEP 3.4 for collider physics within and beyond the Standard Model, Comput.Phys.Commun. 184 (2013) 1729 - (arXiv: 11207.6082 [hep-ph])
*CompHEP:* E. Boos, V. Bunichev, M. Dubinin, L. Dudko, V. Edneral, V. Ilyin, A. Kryukov, V. Savrin, A. Semenov, and A. Sherstnev [CompHEP Collaboration], CompHEP 4.4: Automatic computations from Lagrangians to events, Nucl. Instrum. Meth. A534 (2004), 250