Anomalous magnetic moment of muon

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On behalf of Muon G-2 and CMD-3 Collaborations

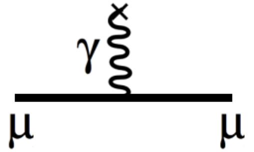
22 Lomonosov Conference on Particle Physics

August 21, 2025
Moscow

The basics

Gyromagnetic ratio g connects magnetic moment μ and spin s

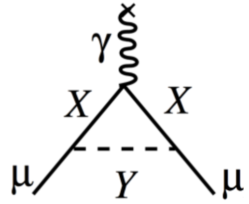
$$\vec{\mu}_S = g\frac{e}{2m}\vec{S}$$



For point-like particle $\boldsymbol{g = 2}$

Anomalous magnetic moment a arises in higher-orders

$$a = (g-2)/2$$



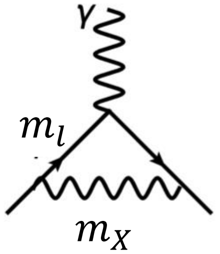
$$a_e \approx a_\mu \approx \frac{\alpha}{2\pi} \approx 10^{-3}$$ (QED dominated)

Idea of experiment: by comparing measured value of a with the theory prediction we probe extra contributions to a beyond theory expectations

$$a_\mu(strong)/a_\mu(QED) \approx 6\times10^{-5} \qquad a_\mu(weak)/a_\mu(QED) \approx 10^{-6}$$

Why muon? For massive fields there is natural scaling, which enhances contribution to a_μ by $\left(m_\mu/m_e\right)^2 \sim 43000$ compared to a_e

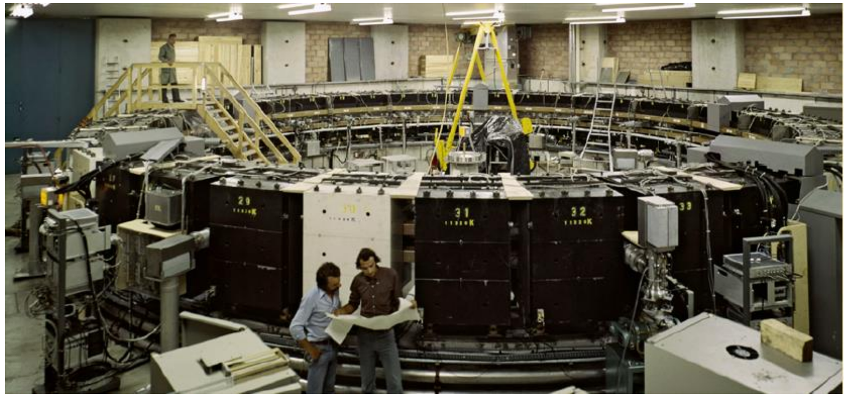
$$\Delta a \sim \left(\frac{m_l}{m_X}\right)^2$$



Generations of a_μ measurements before FNAL

A view from 2020

CERN 3



BNL

3.6 σ !!!

$$a_\mu(exp) \quad = 1.001\,165\,920\,89\,(63)$$

$$a_\mu(theory) = 1.001\,165\,918\,10\,(43)$$

WP2020

QED

Strong

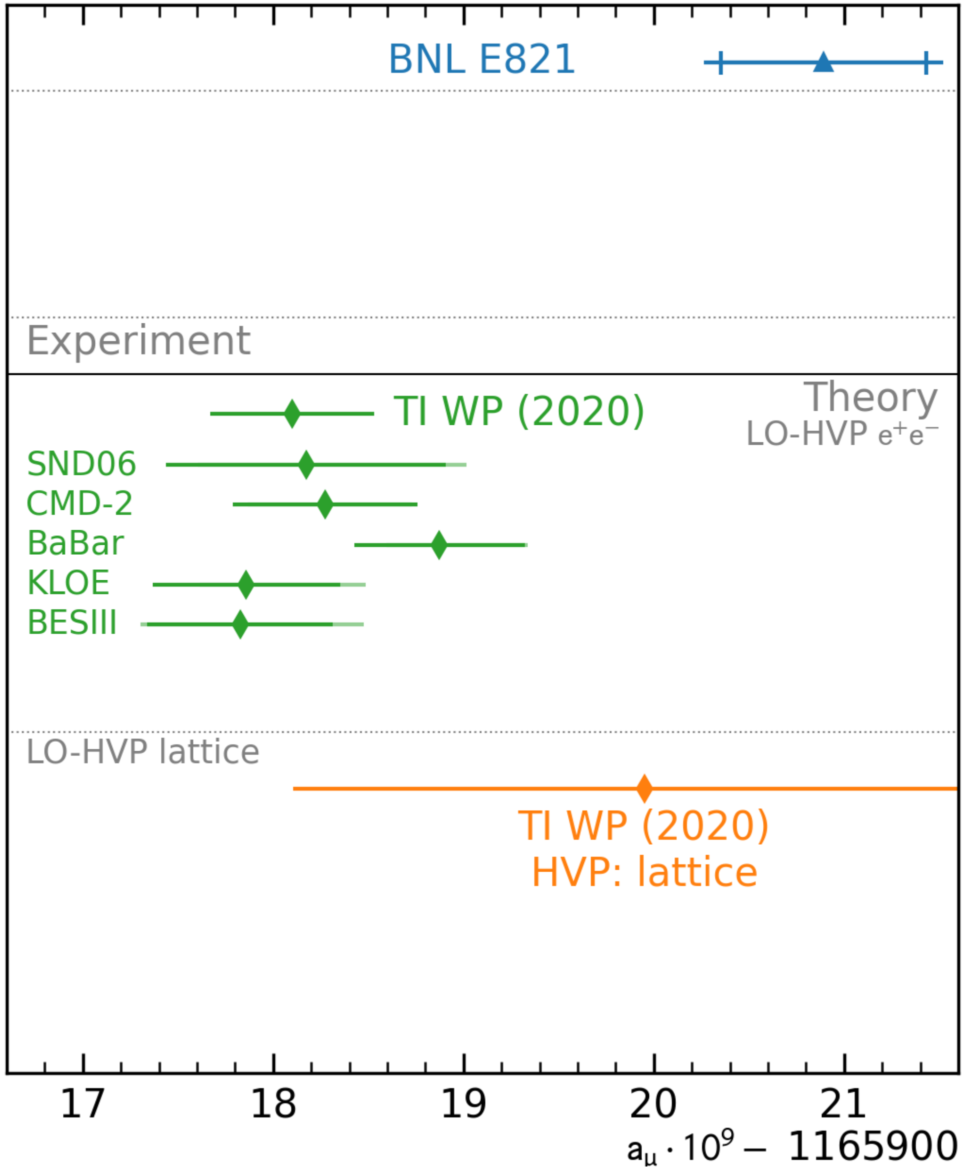
Weak

Contributions of known interactions

Muon g-2 Theory initiative

A view from 2020

- **TI White Paper 2020**
  Physics Reports 887 (2020) 1-166
- 100+ theorist compile the theoretical input and provide recommendations (*muon-gm2-theory.illinois.edu*)
- WP2020 recommended value based on dispersive approach for HVP-LO
- Lattice-QCD calculations for HVP-LO available at that time but large uncertainties



Muon G-2 collaboration

USA
- Boston
- Cornell
- Illinois
- James Madison
- Kentucky
- Massachusetts
- Michigan
- Michigan State
- Mississippi
- North Central
- Northern Illinois
- Regis
- Virginia
- Washington

USA National Labs
- Argonne
- Brookhaven
- Fermilab

176 collaborators
34 Institutions
7 countries

China
- Shanghai Jiao Tong

Germany
- Dresden
- Mainz

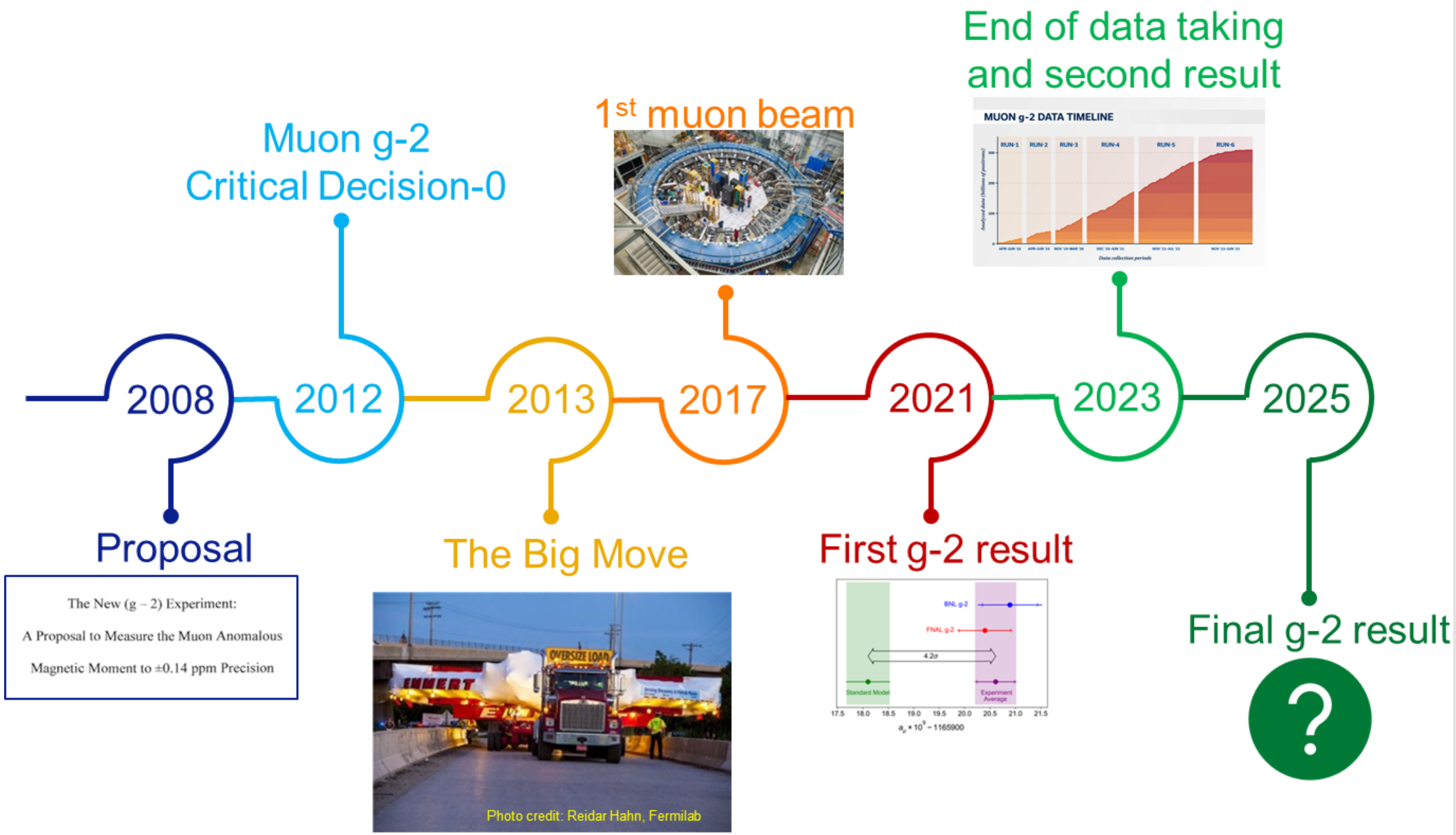
Italy
- Frascati
- Molise
- Naples
- Pisa
- Roma Tor Vergata
- Trieste
- Udine

Korea
- CAPP/IBS
- KAIST

Russia
- Budker/Novosibirsk
- JINR Dubna

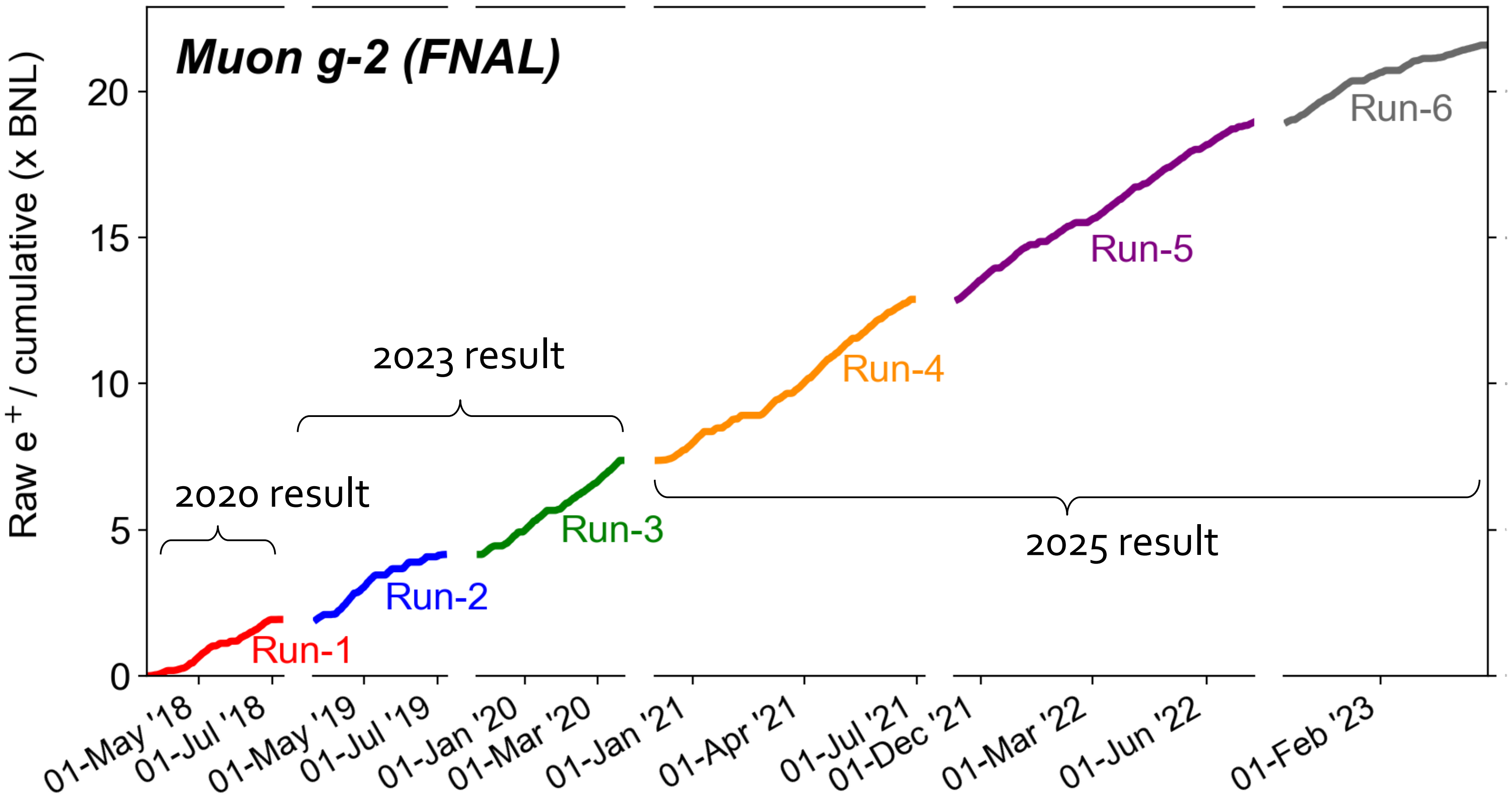
United Kingdom
- Lancaster/Cockcroft
- Liverpool
- Manchester
- University College London

Milestones for Fermilab Muon G-2 experiment



P.Winter, CERN seminar, 2025

Total collected statistics



21.9 BNL datasets have been collected in FNAL (proposal – 21 BNL)

Run 4/5/6 statistics is x3 Run-1/2/3

Principles of CERN-III type measurement

1. Spin precesses relative to momentum with frequency ω_a proportional directly to a_μ

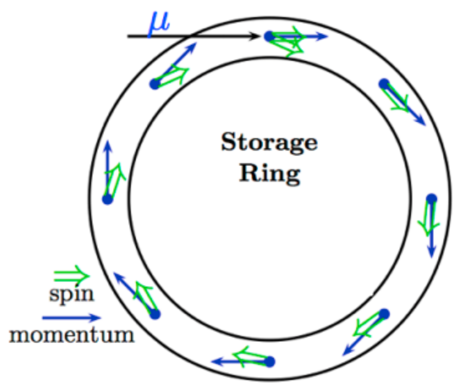
$$\omega_a = \omega_s - \omega_c = a_\mu eB/mc$$

$$a_\mu = \frac{mc}{e}\frac{\omega_a}{B}$$

2. Effect of electric field is cancels out for muons of "magic" momentum

$$\vec{\omega}_a = -\frac{e}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]$$

zero for $\gamma_\mu = 29.3$



Muons are stored in a storage ring
ω_a and B are measured

Need focusing!

Muons with $p = 3.09$ GeV/c are used

Focusing with electrostatic quadrupoles

Extracting a_μ

$$\boldsymbol{\omega_a = a_\mu \frac{e}{m} B} \rightarrow \boldsymbol{a_\mu = \omega_a / B \cdot \frac{m}{e}}$$

by expressing B in terms of the (shielded) proton precession frequency: $(B = \hbar\omega_p / 2\mu_p)$:

$$a_\mu = \frac{\omega_a}{\tilde{\omega}'_p} \cdot \frac{\mu_p}{\mu_B} \frac{m_\mu}{m_e}$$

External data (±22 ppb)

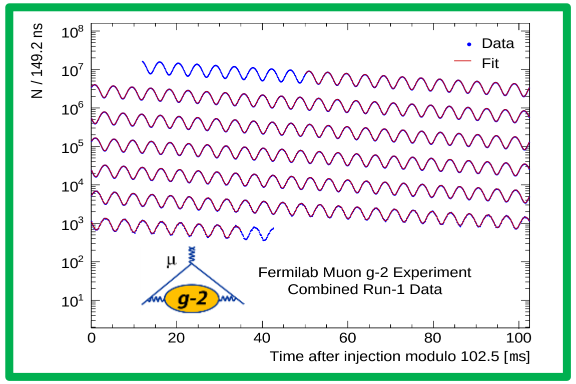
$$R_\mu = \frac{\omega_a}{\tilde{\omega}'_p}$$

ratio of muon to proton precessions in the same magnetic field

$\tilde{\omega}'_p$ = (shielded) Proton angular velocity **weighted for the muon distribution**

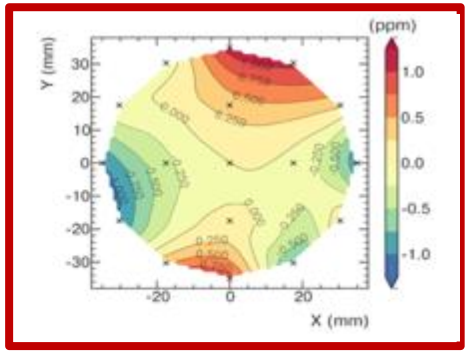
The basic ingredients of a_μ measurement

$$R'_\mu = \frac{\omega_a}{\widetilde{\omega'}_p} \sim$$

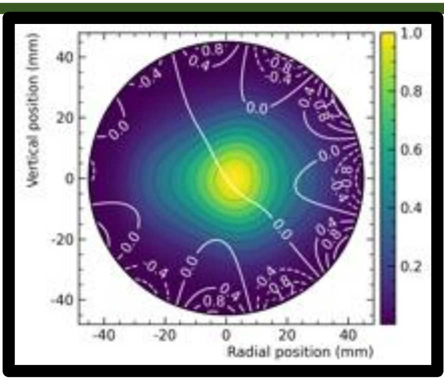


$\boldsymbol{\omega_a}$

muon precession



$\otimes$



$\boldsymbol{\omega'_p}$

proton precession

$\boldsymbol{M(x,y,\varphi)}$

muon distribution

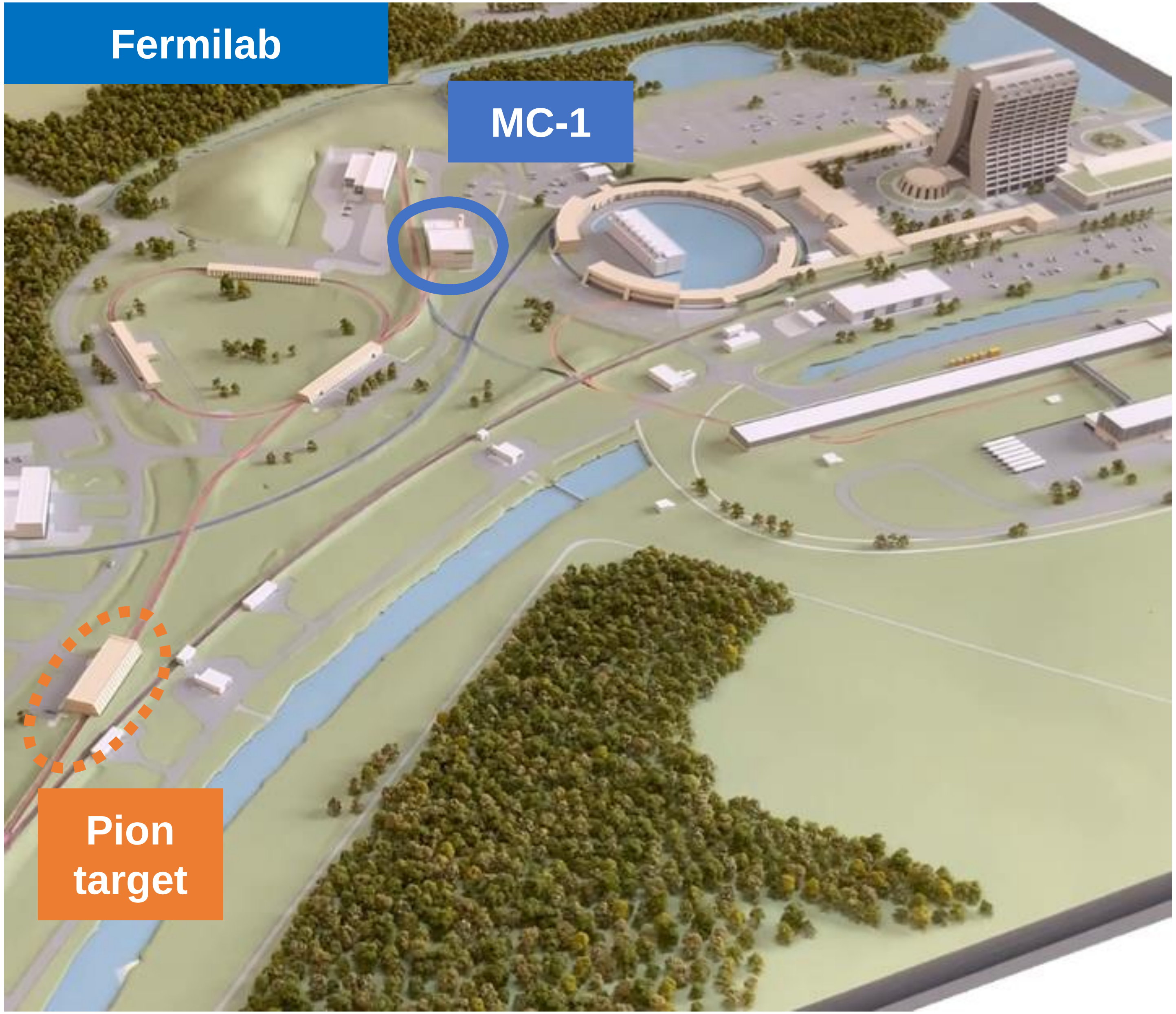
$\widetilde{\boldsymbol{\omega}'}_{\boldsymbol{p}} = \boldsymbol{\omega'_p} \otimes \ \boldsymbol{M(x,y,\varphi)}$ - magnetic field weighted by the muon distribution in the storage ring

Generating muons

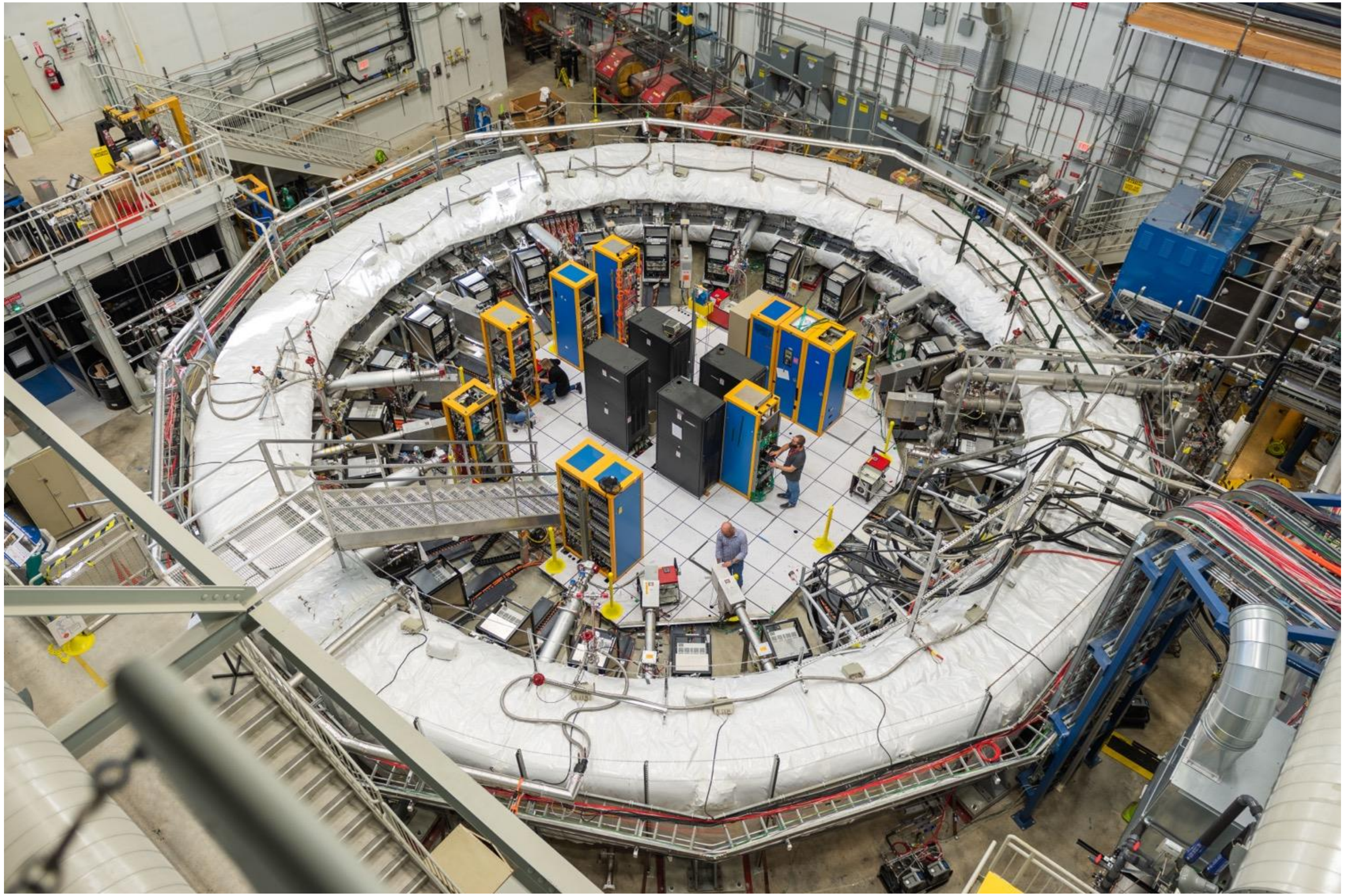
2 km pion decay line

95% polarization of muons

8 GeV p

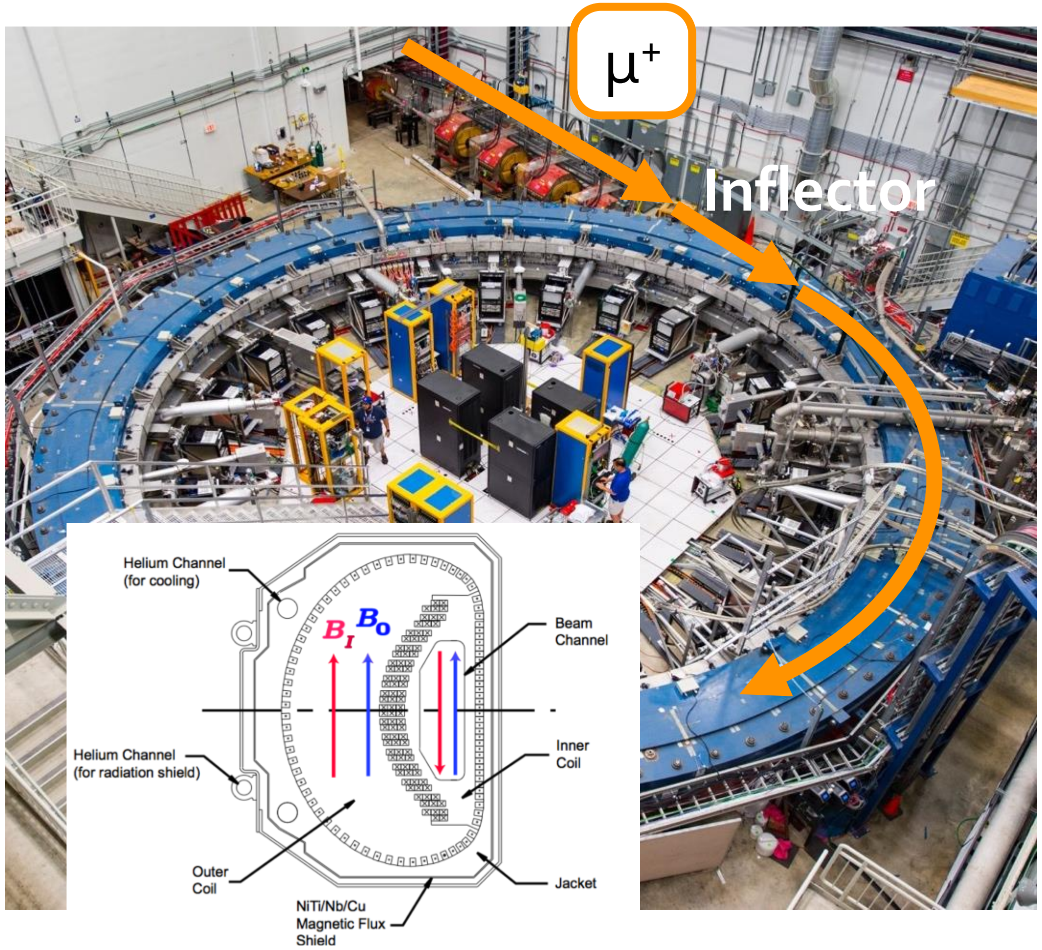


Muon G-2 Ring @FNAL

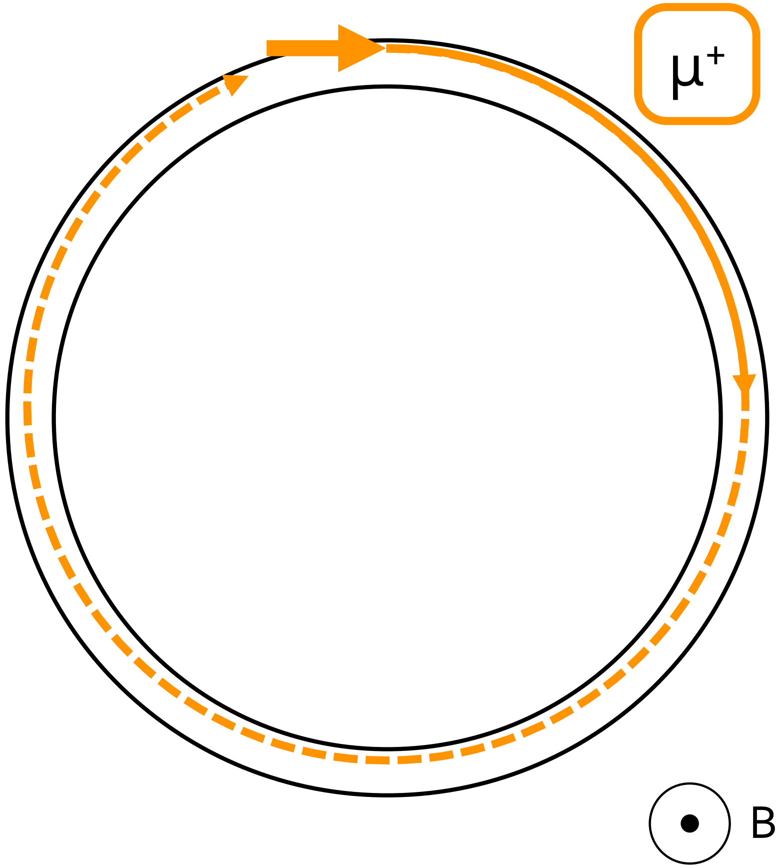


Injection of muons

Muons are injected into the storage ring with uniform field. After one turn they hit the wall, unless…

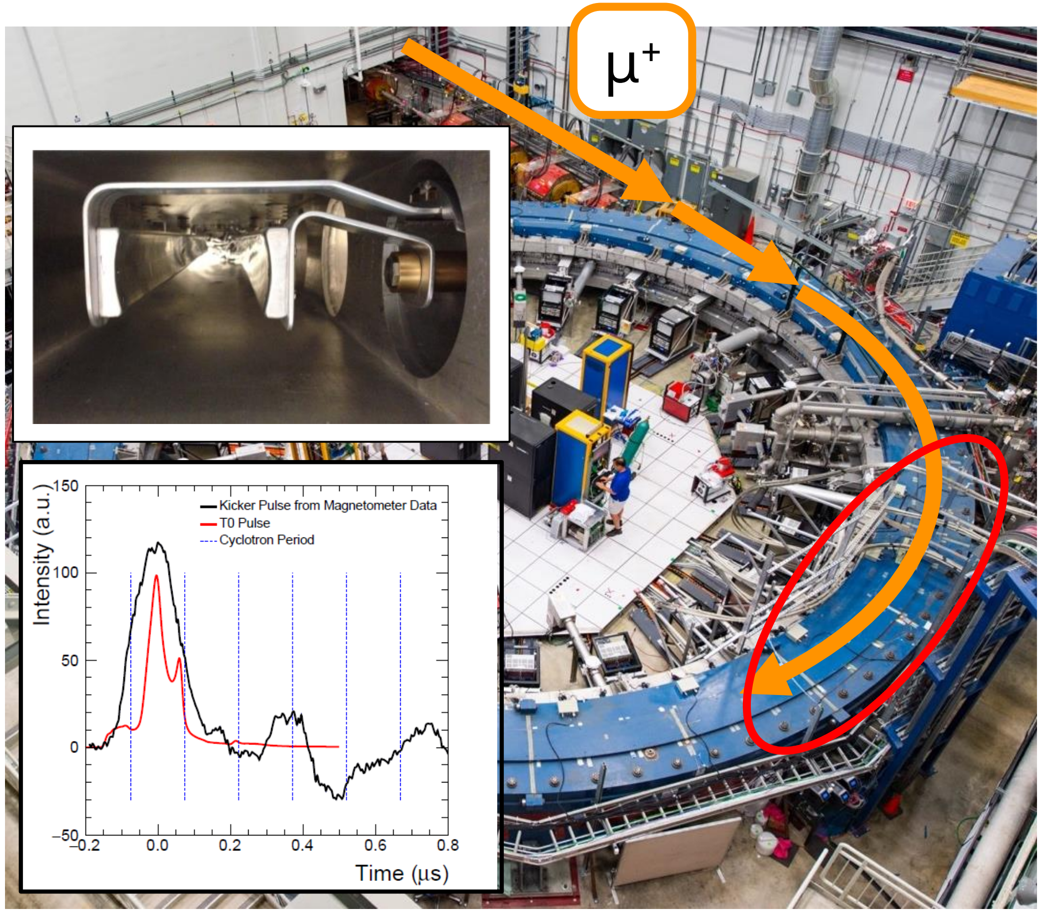


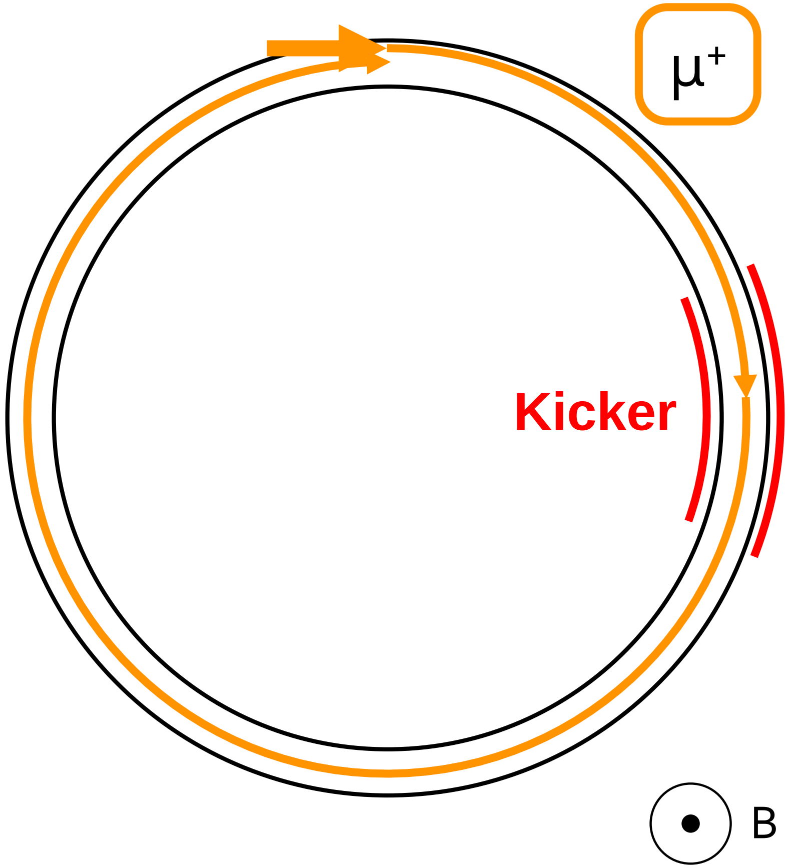
Inflector cross section



Kicker

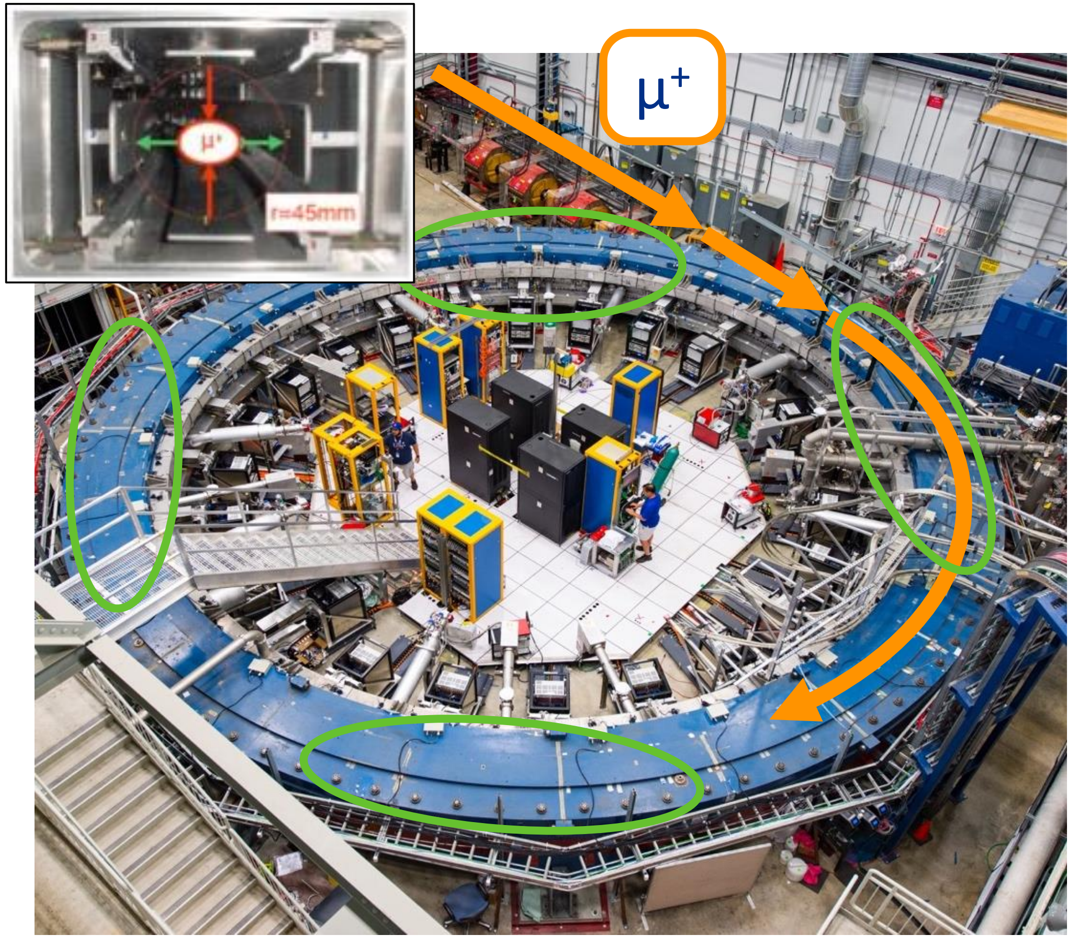
Fast kicker magnet briefly reduces field at 90° and puts beam to standard orbit

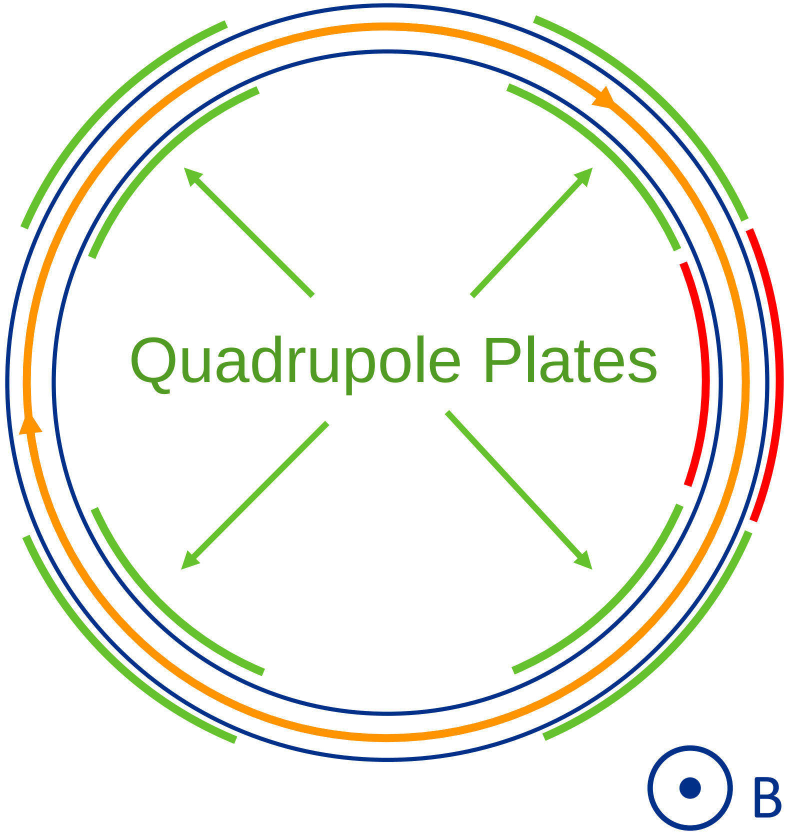




Quads

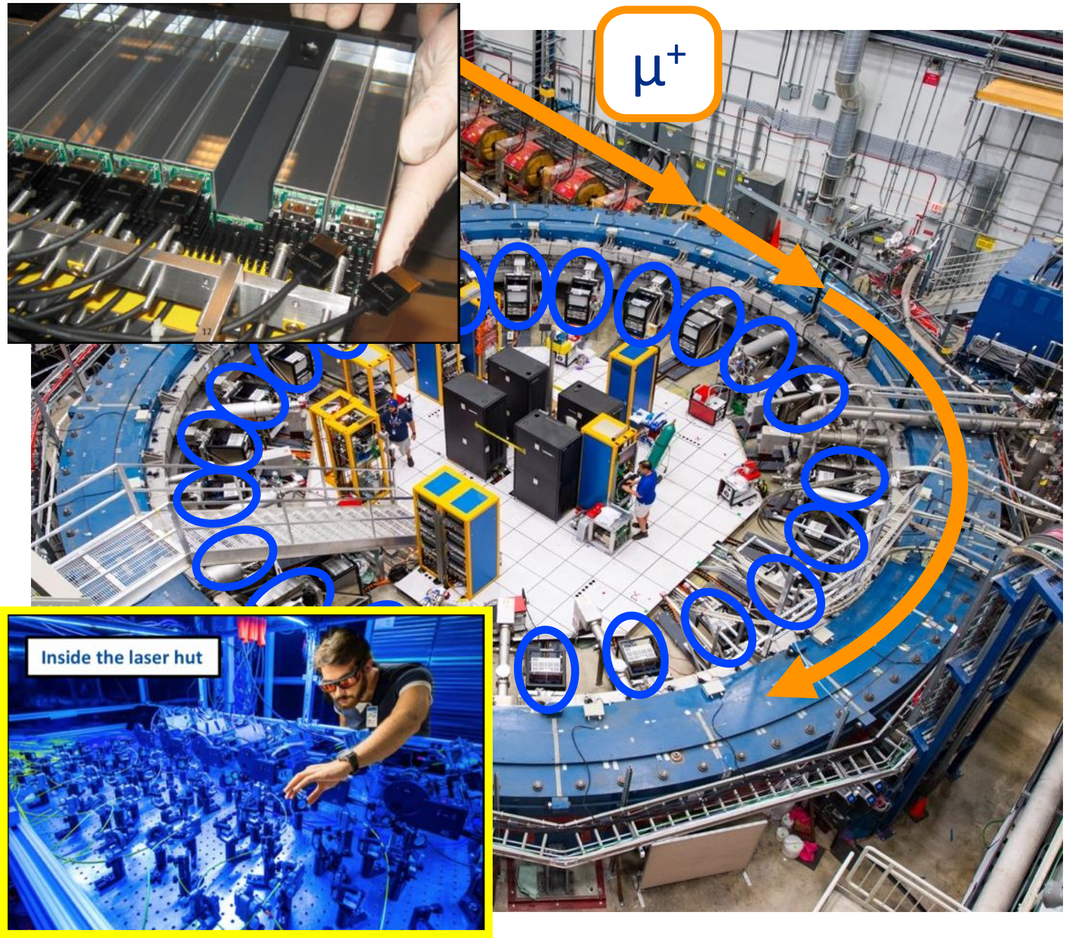
- Electrostatic quadrupoles vertically contain the beam

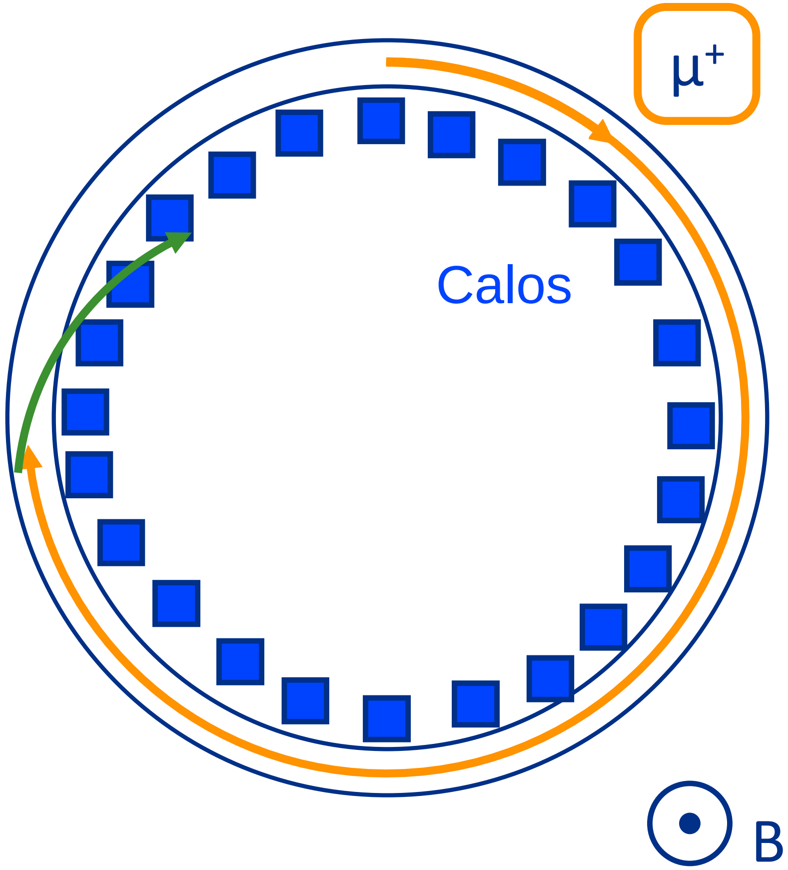




Calorimeters

- Time & energy of decay e^+ are measured by **24 calorimeters**

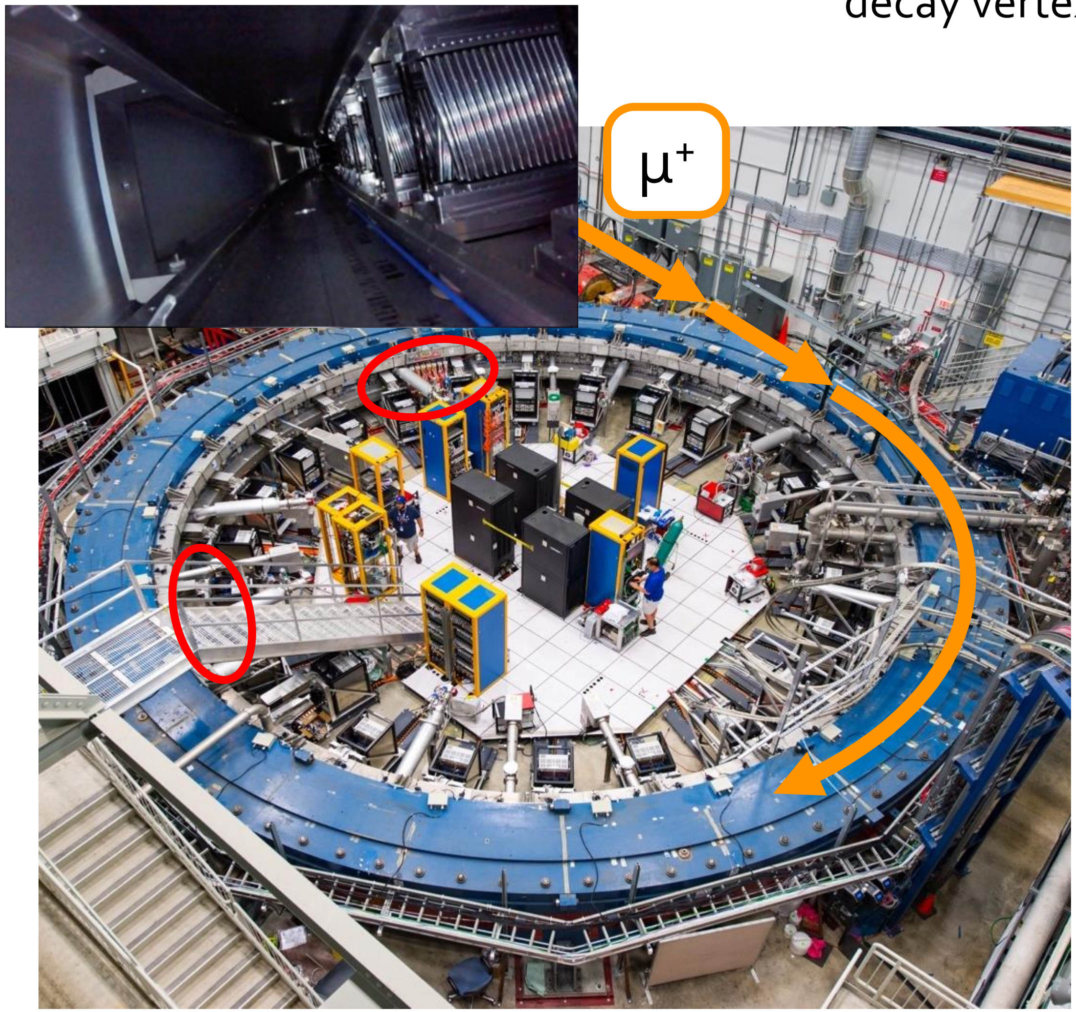


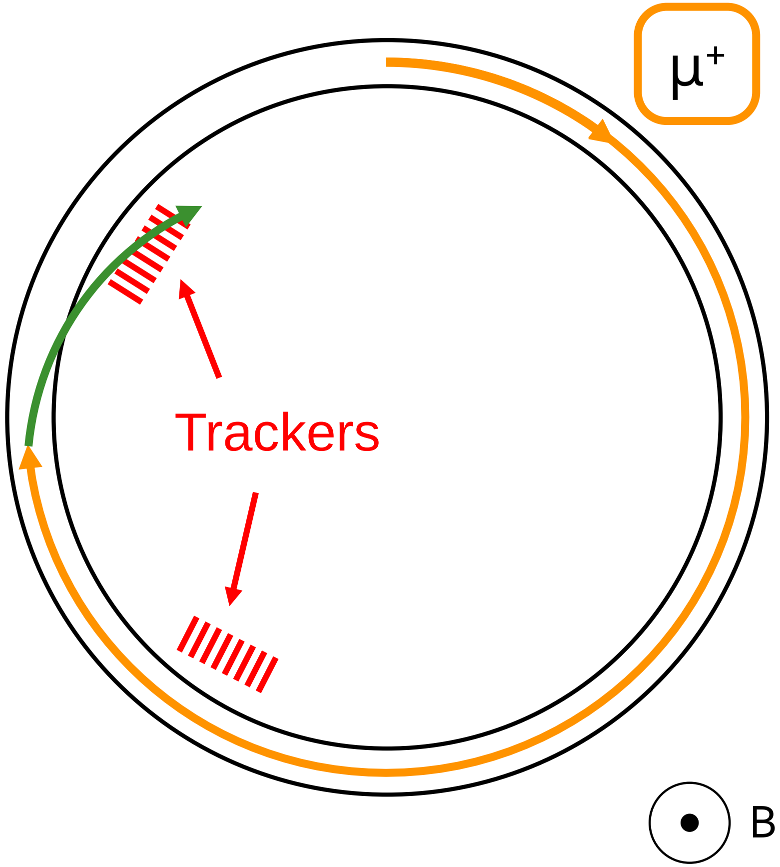


Each calorimeter: array of 9x6 PbF_2 crystals (2.5 x 2.5 cm^2 x 14 cm, $15X_0$), readout by SiPMs
Laser calibration system: gain corrected to ~10^{-4}

Trackers

Two trackers allow to see muon beam dynamics in real time by reconstruction of muon decay vertex



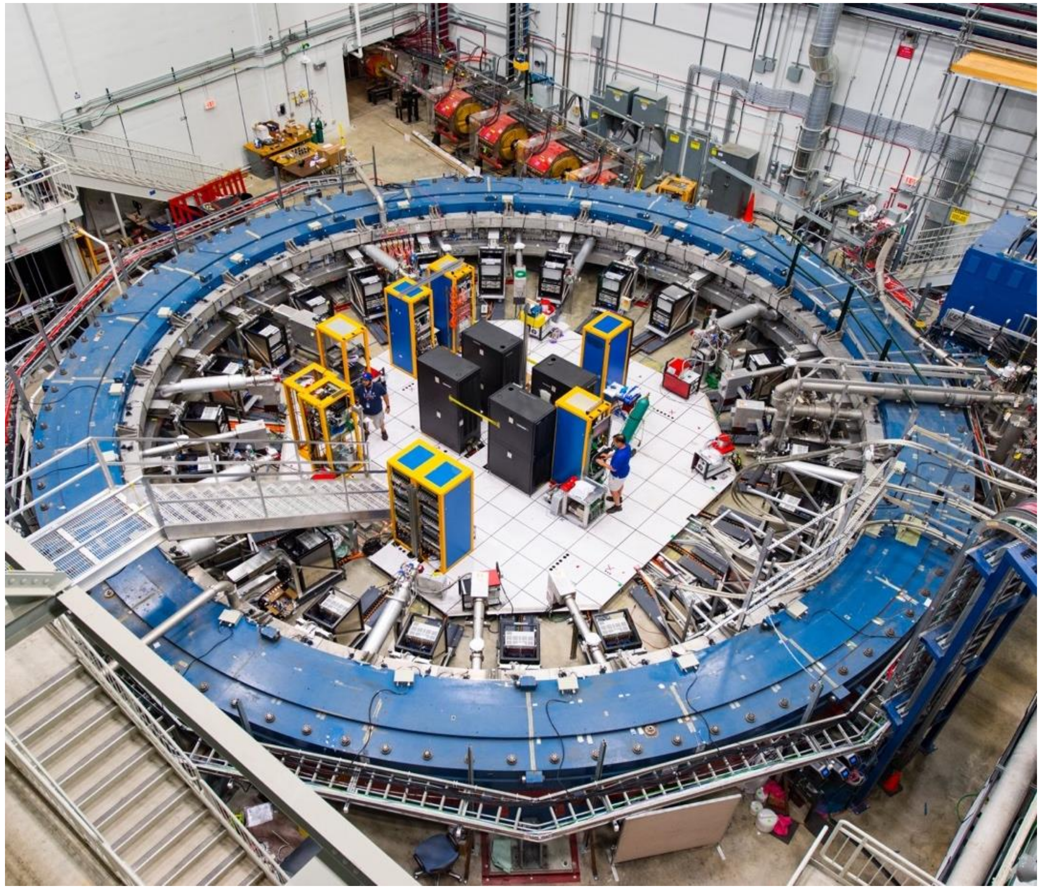


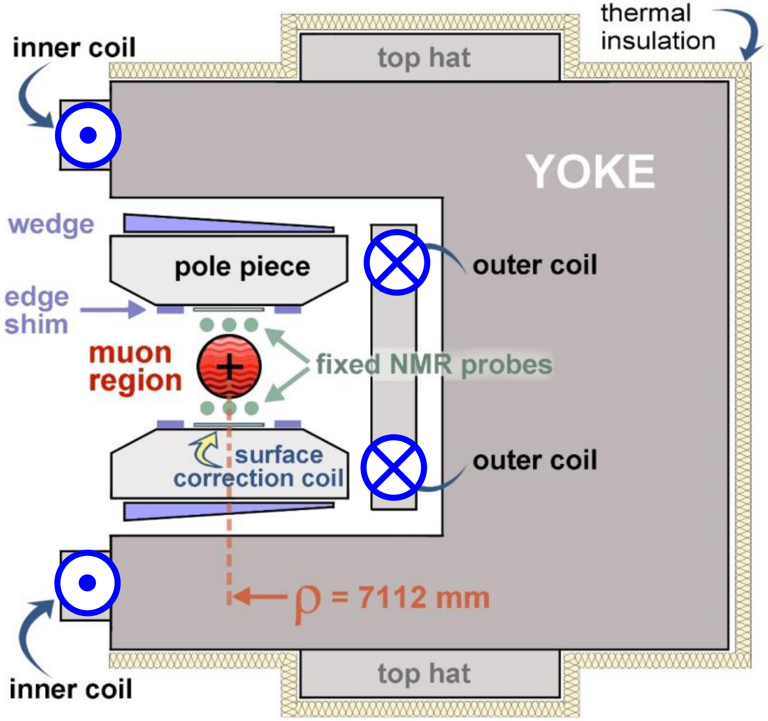
Inside the ring

Video editing: Simon Corrodi

The ring magnet

The storage ring is a 14 m diameter, 1.45 T C-shaped magnet



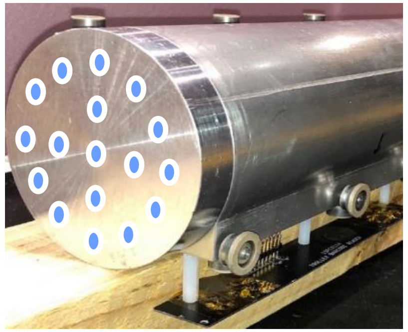


~10000 shimming knobs:

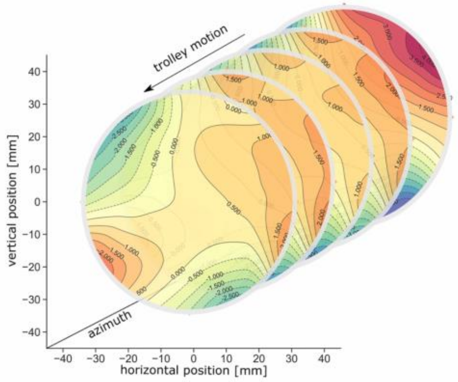
- 48 top / bottom hats to tune dipole
- 800 wedge shims to tune dipole
- 9000 iron foils to fine tune field
- 200 tunable coils for higher multipoles

Monitoring B field

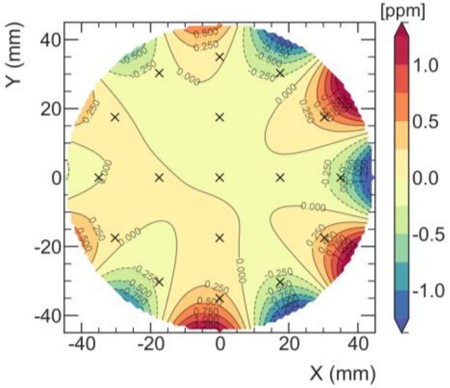
- In-vacuum NMR trolley **maps field every ~3 days**



17 petroleum jelly NMR probes

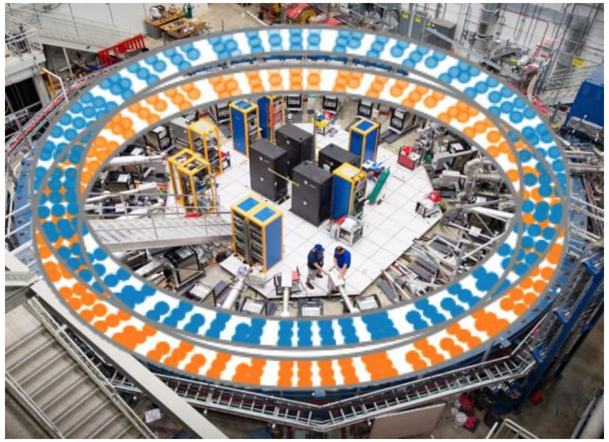


2D field maps (~8000 points)

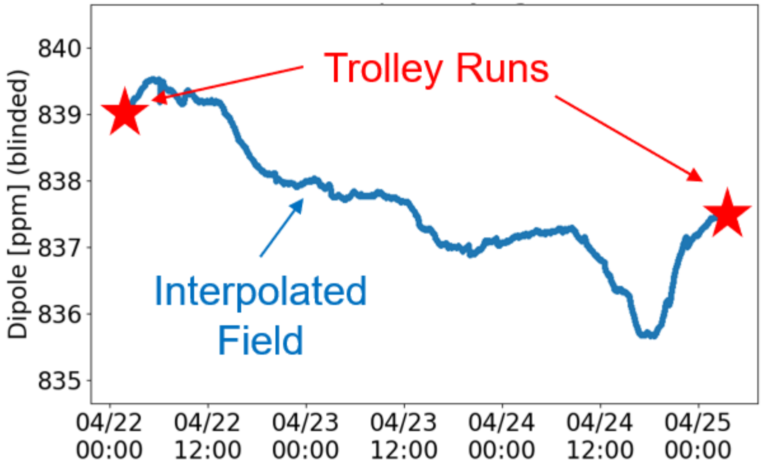


Azimuthally-Averaged **Variation < 1 ppm**

- **378 fixed probes** monitor field during muon storage at 72 locations



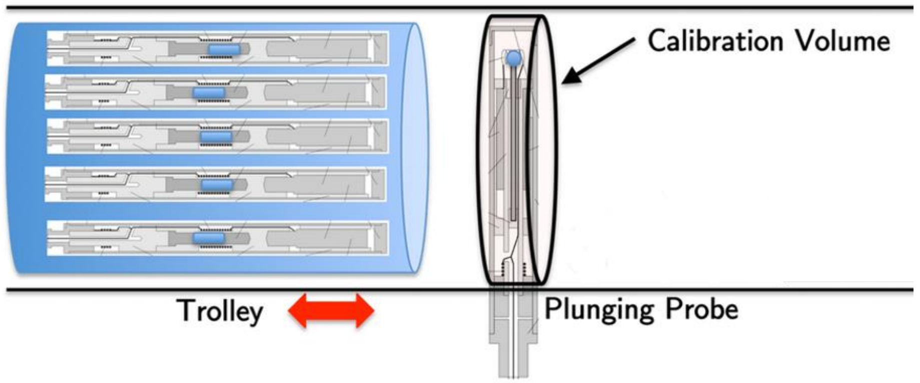
Fixed probes above/below muon storage region

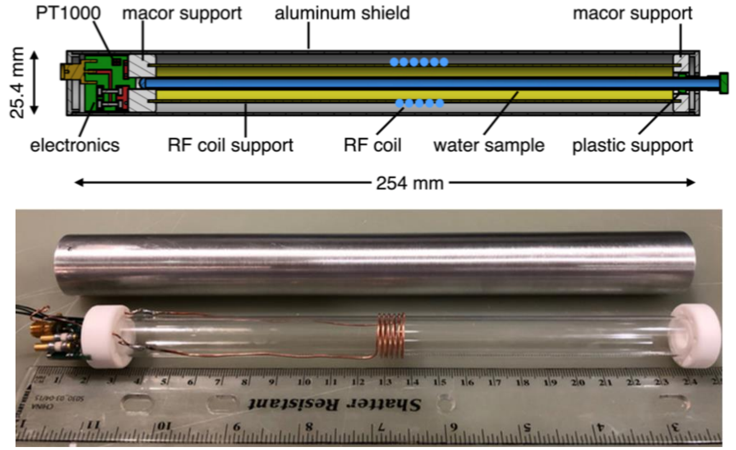


Field map is convoluted with muon spatial distribution to get an average field

Absolute calibration

- Cross-calibrate using a cylindrical **plunging H_2O probe** which repeatedly **changes places with trolley (petroleum jelly probes)**





- This probe is **checked against a spherical probe** using an MRI magnet at ANL
- Both also cross-checked against a **^{3}He probe** (different systematics)

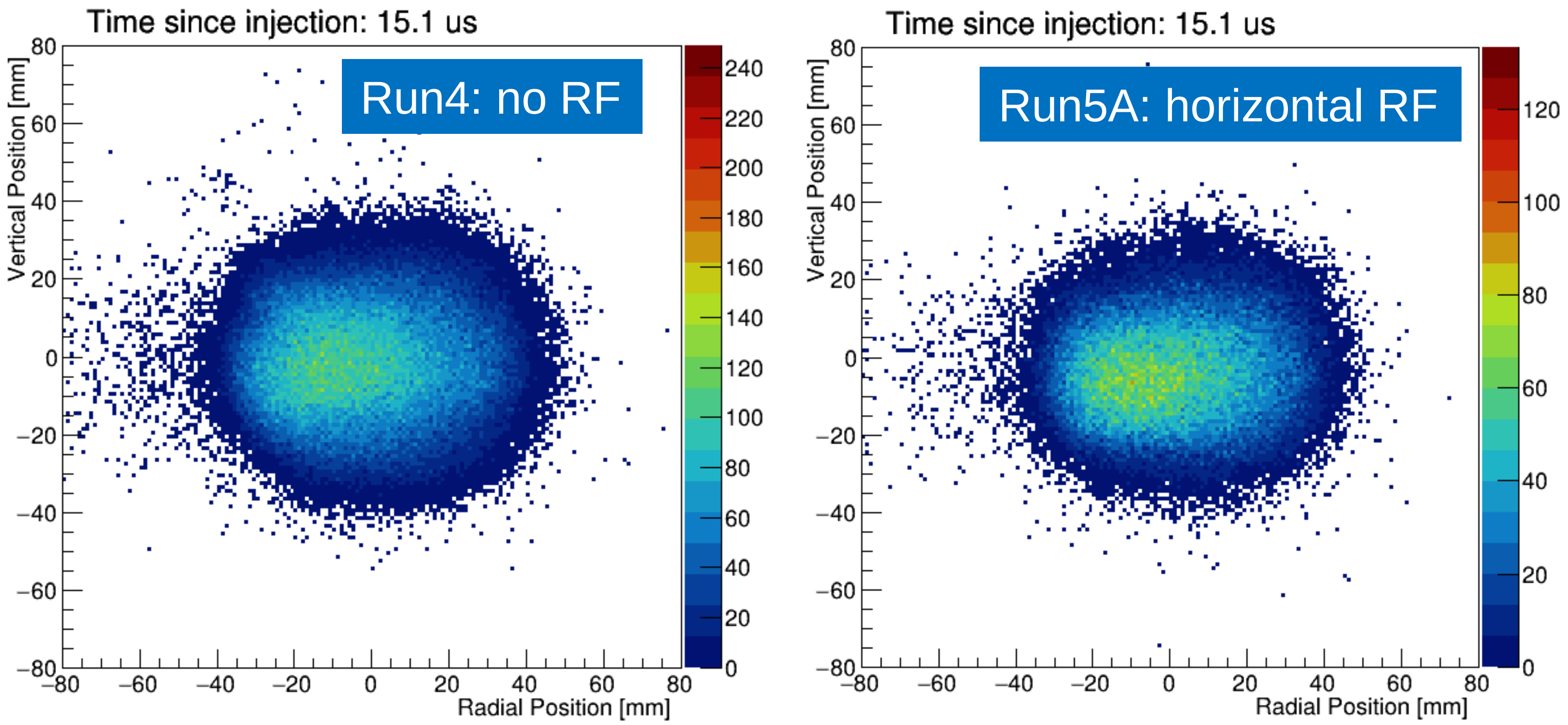
$$\Delta B/B \approx 5 \cdot 10^{-8}$$

H_2O Probe

^{3}He Probe

Muon Distribution from Trackers

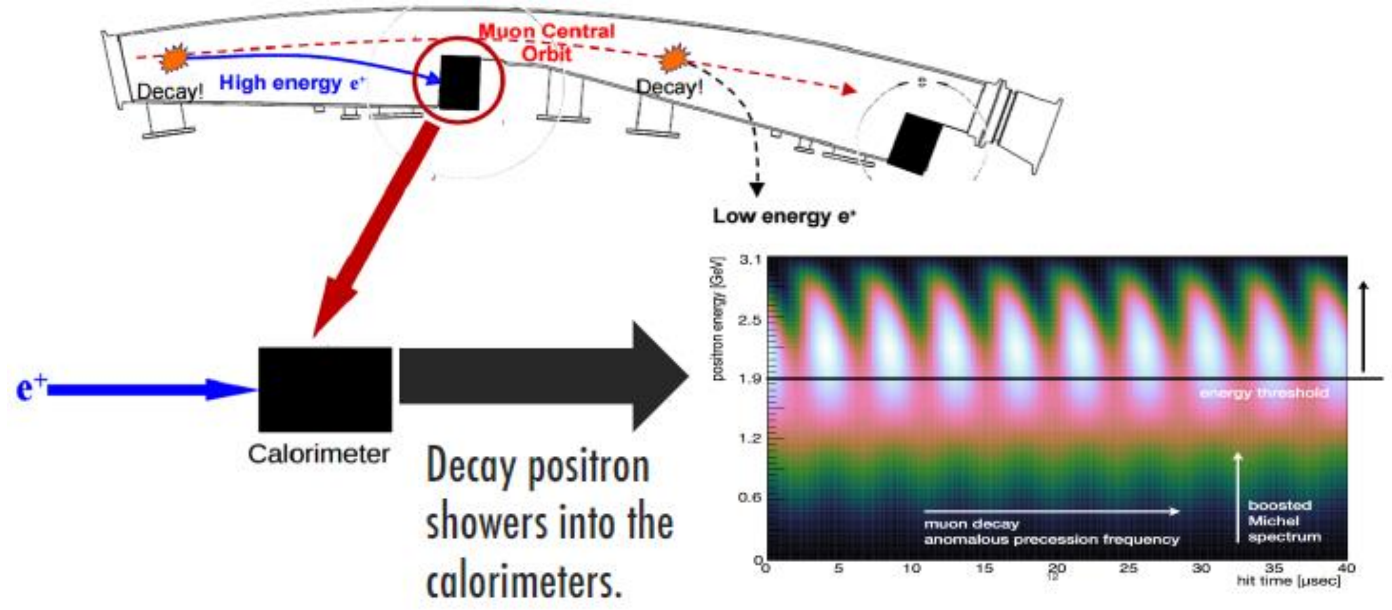
- Measure **beam oscillations** directly



Cross-checked using calorimeters and MiniSciFi – dedicated system to measure beam profile

Measuring ω_a

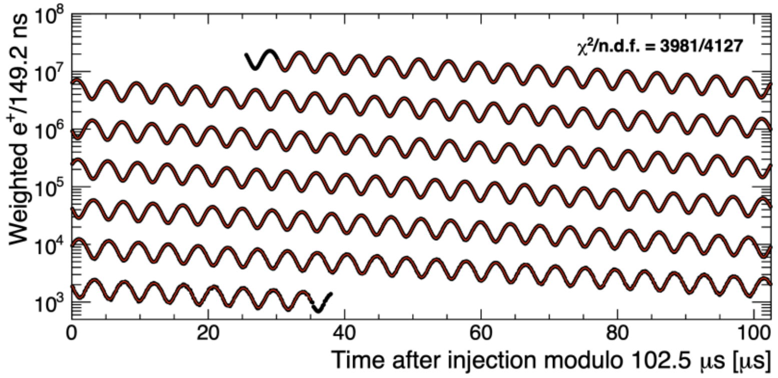
The energy distribution of positrons depends on spin direction, thus number of high energy positrons is modulated by precession frequency



Counting rate of high energy positrons

"Wiggle plot"

$$dN(e^+ > 1\ \text{GeV})/dt$$

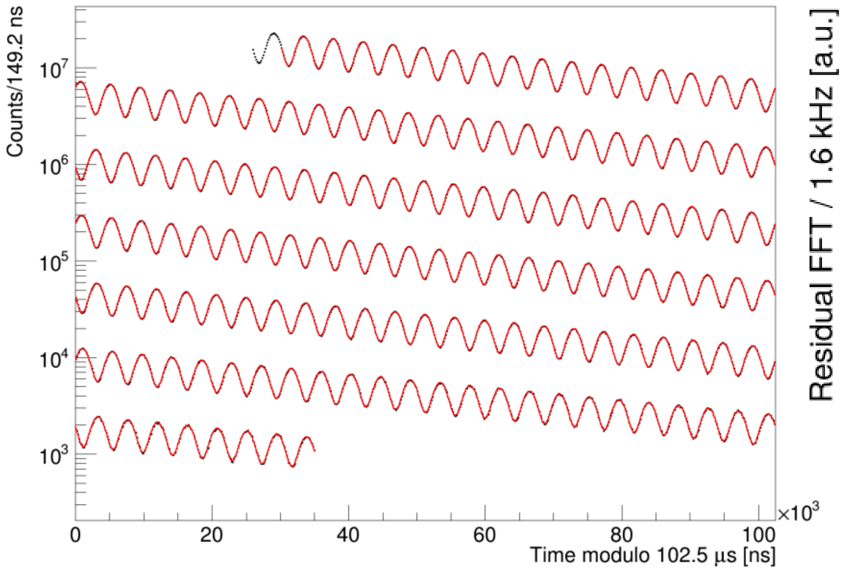


5-par fit

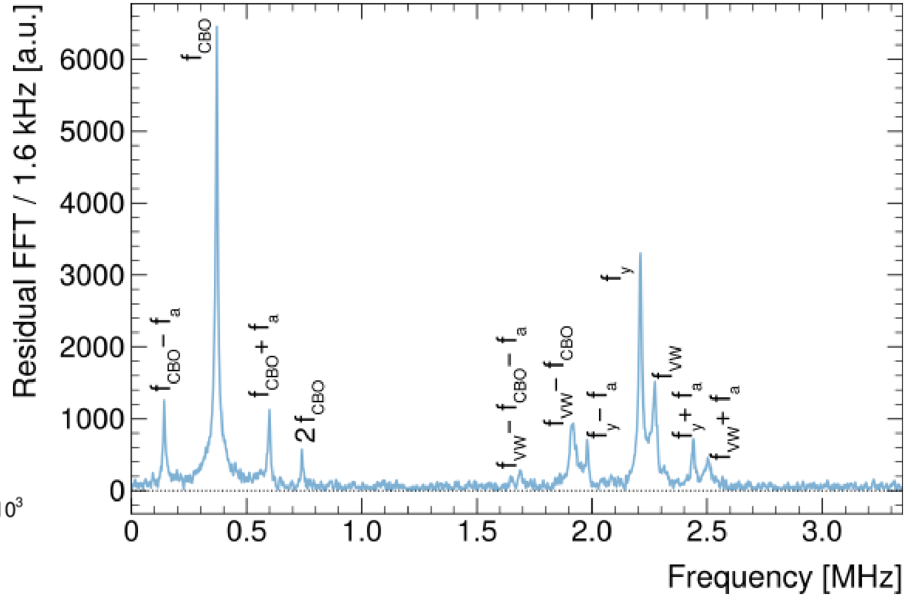
Simple model: exponential decay and precession

$$N(t) = \boldsymbol{N_0} e^{(-t/\boldsymbol{\tau})}[1 + \boldsymbol{A}\cos(\boldsymbol{\omega_a} t - \boldsymbol{\phi})]$$

χ^2/ndf = 51530/4150



Fourier transform of residuals

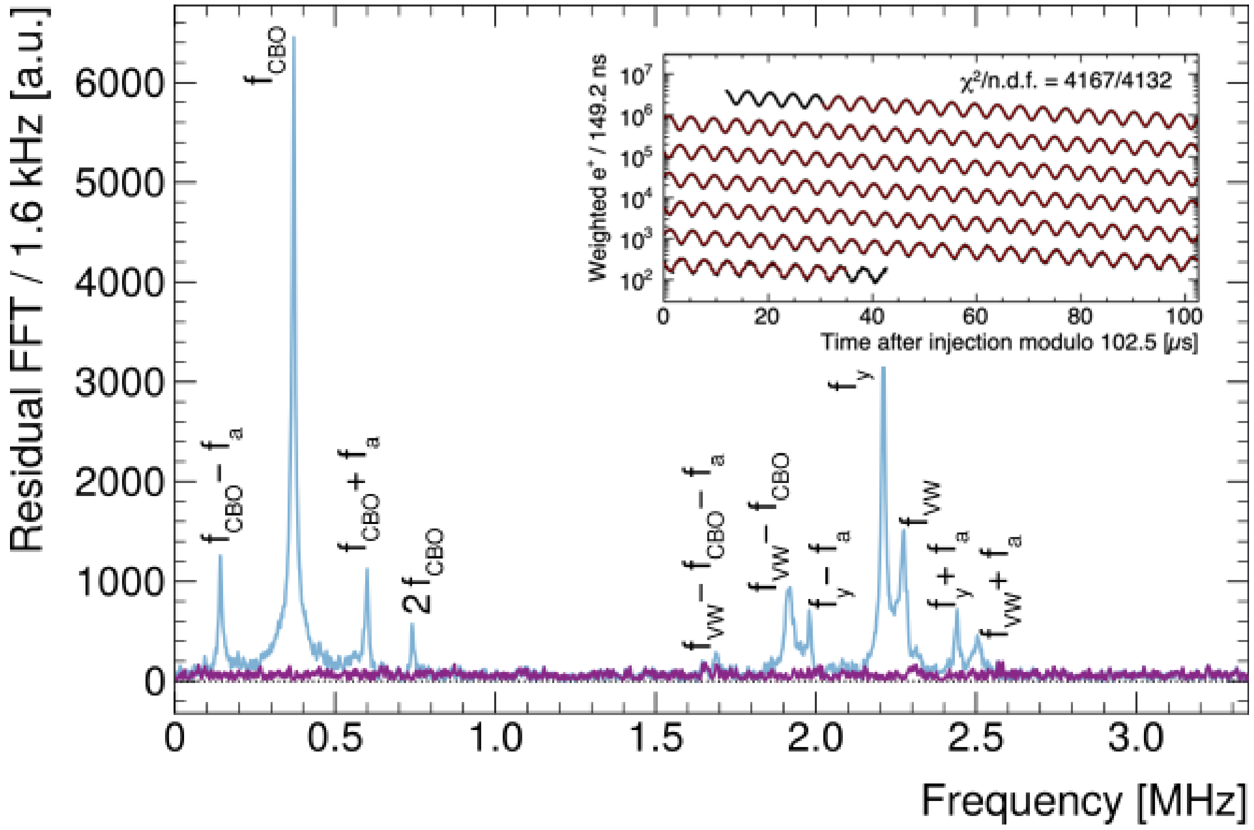


Realistic model must account for **detector effects**, **beam oscillations** that couple to acceptance, and **lost muons** that disrupt pure exponential

Complete fit

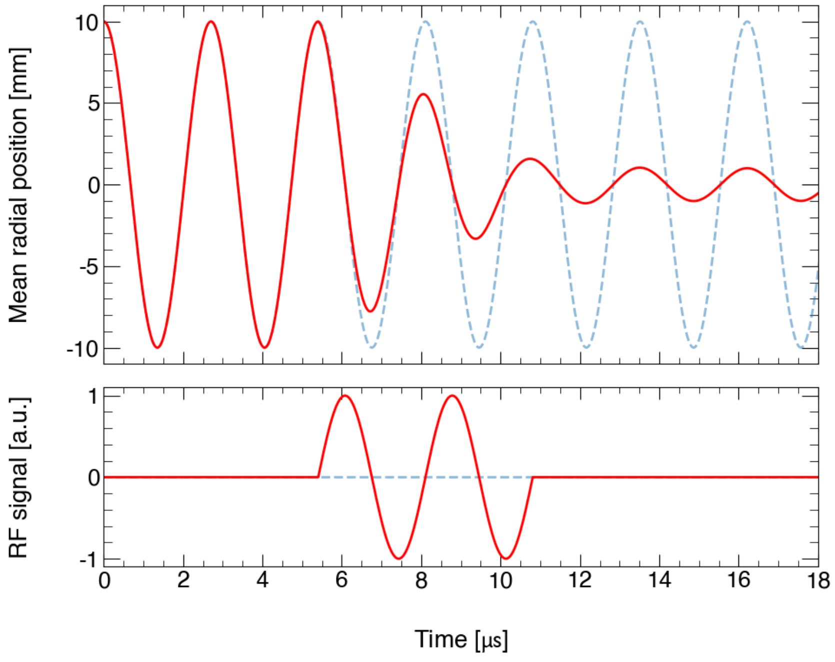
Realistic model allows to reach good fit quality. Many options:
5 analysis groups, 8 methods, 3 energy reconstructions, ...
These effect are important! ω_a shifts by 1-2 ppm.

Fourier transform of residuals



RF system

In Runs 5/6 RF modulation was applied to quadrupole plates in order to dump amplitude of coherent beam oscillations



- Added RF acts like a forced harmonic oscillator (for 6 μs)
- Tuned to the CBO frequency, it reduces oscillation of the **mean** of the particle ensemble (reduces the coherence)
- CBO effect (if not accounted for!):
  - —800 ppb without RF
  - —80 ppb with RF

Obtaining a_μ

We need to make corrections for several small effects:

$$\frac{\omega_a}{\omega_p} = \frac{\omega_a^m}{\omega_p^m} \frac{1 + C_e + C_p + C_{pa} + C_{dd} + C_{ml}}{1 + B_k + B_q}$$

E-field & vertical motion ($C_e + C_p$)

Phase change with time ($C_{pa} + C_{dd} + C_{ml}$)

Measured Values (ω_a^m, ω_p^m)

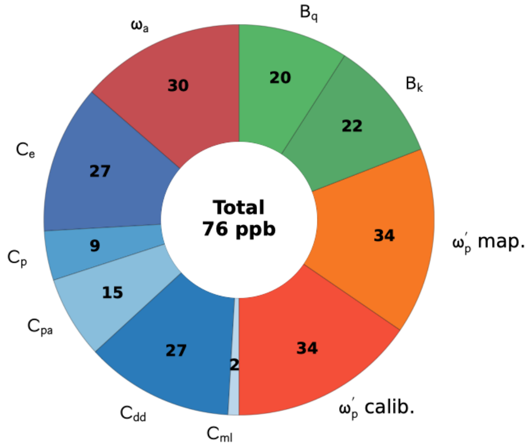
Transient Magnetic Fields due to Quads and Kickers ($B_k + B_q$)

Total correction is **572 ppb**, dominated by **E-field & Pitch**…

Systematics for Run 4/5/6

Run-4/5/6

Quantity		Correction (ppb)	Uncertainty (ppb)
ω_a^m (statistical)		$\cdots$	114
ω_a^m (systematic)		$\cdots$	30
C_e	Electric Field	347	27
C_p	Pitch	175	9
C_{pa}	Phase Acceptance	-33	15
C_{dd}	Differential Decay	26	27
C_{ml}	Muon Loss	0	2
$\langle \omega_p' \times M \rangle$ (mapping, tracking)		$\cdots$	34
$\langle \omega_p' \times M \rangle$ (calibration)		$\cdots$	34
B_k	Transient Kicker	-37	22
B_q	Transient ESQ	-21	20
μ_p'/μ_B		$\cdots$	4
m_μ/m_e		$\cdots$	22
Total systematic for $\mathcal{R}_\mu'$		$\cdots$	76
Total for a_μ		572	139



- TDR goal: 100 ppb ✓
- “evenly” distributed
  - No dominant source
  - Further improving would require to reduce in many categories

Comparison of runs

$\frac{\omega_a}{\tilde{\omega}'_p}$	Stat. Uncertainty (ppb)	Syst. Uncertainty (ppb)	Total Uncertainty (ppb)
Run-1	434	159*	462
Run-2/3	201	78*	216
Run-4/5/6	114	76	137
Run-1-6	98	78	**125**

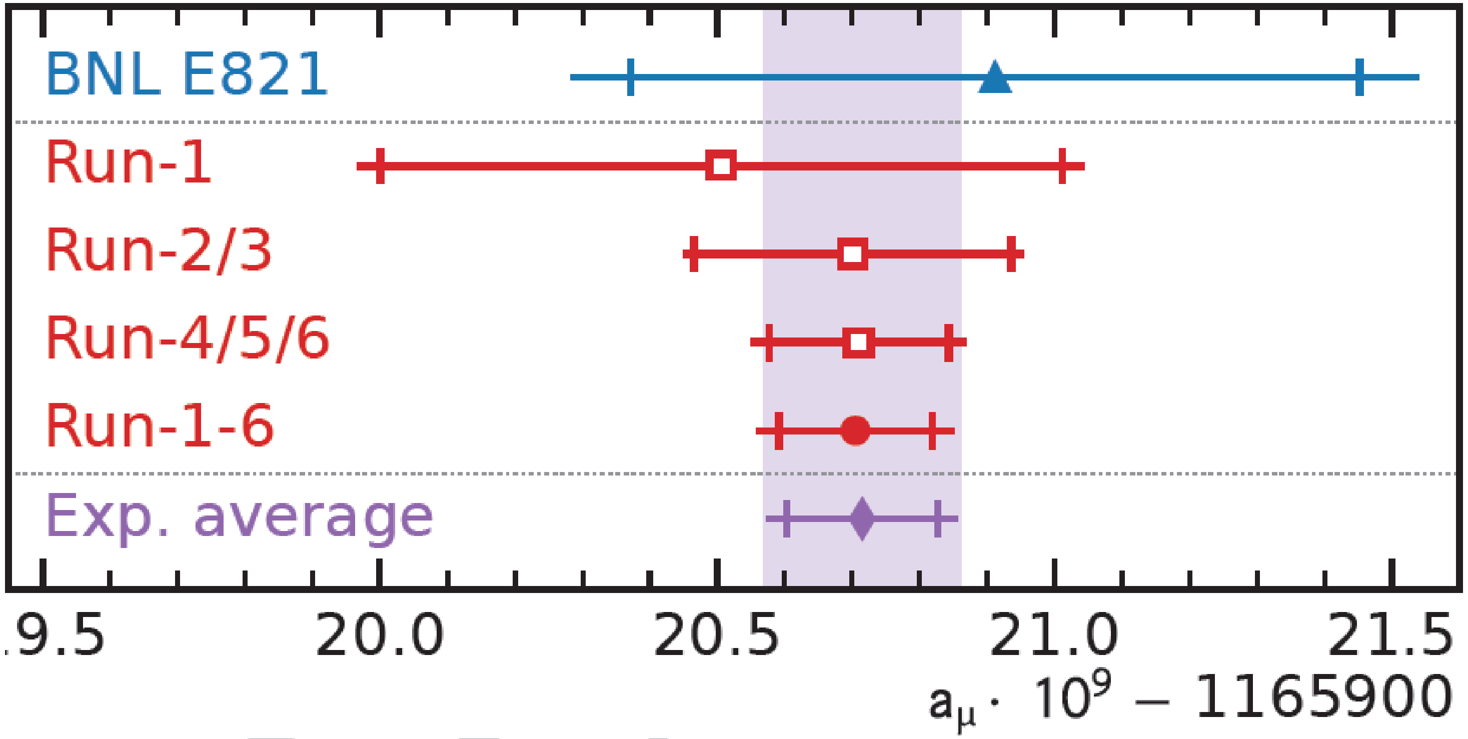
TDR goal 100 ppb ✓

TDR goal: 100 ppb ✓

TDR goal: 140 ppb ✓

Unblinding Meeting on 20 May 2025

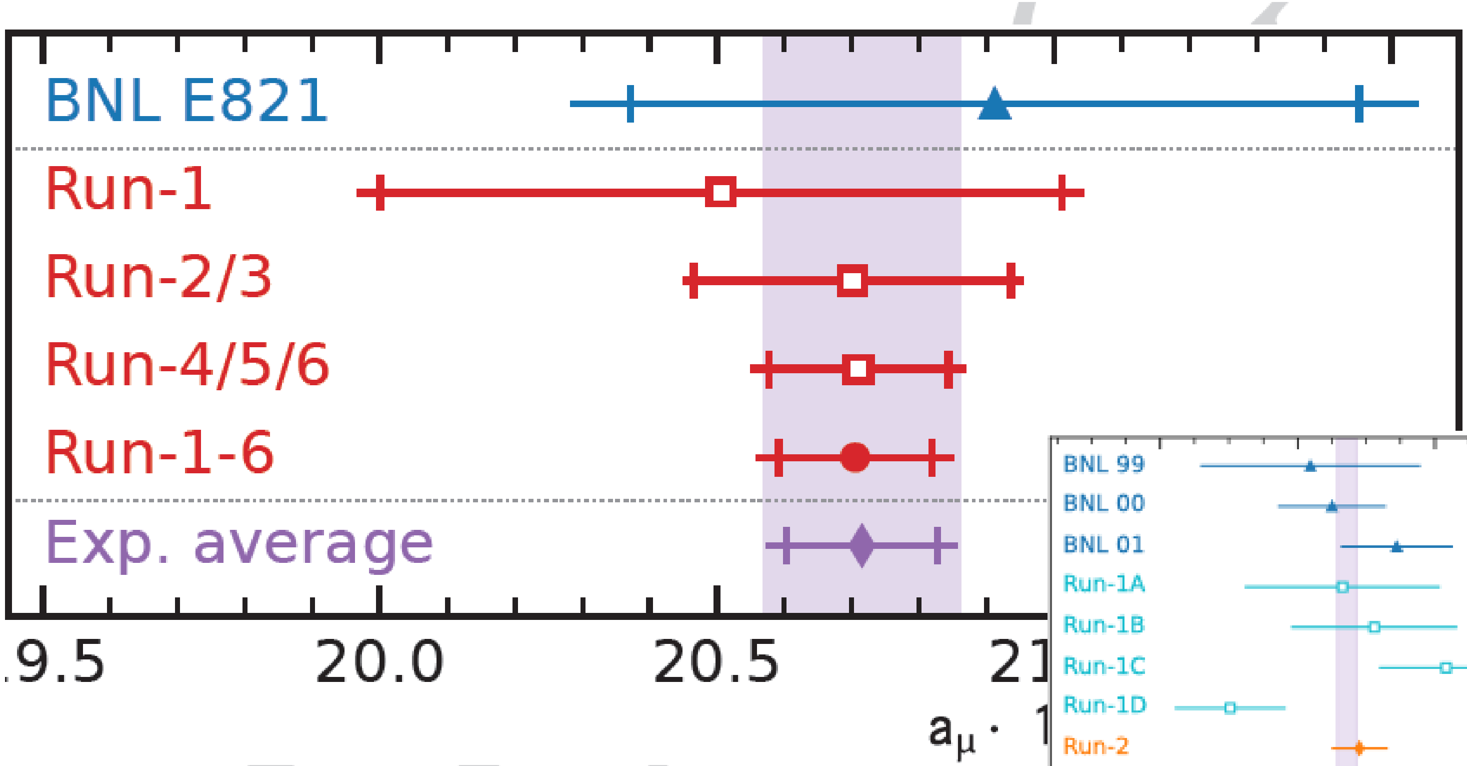
Muon G-2 final result

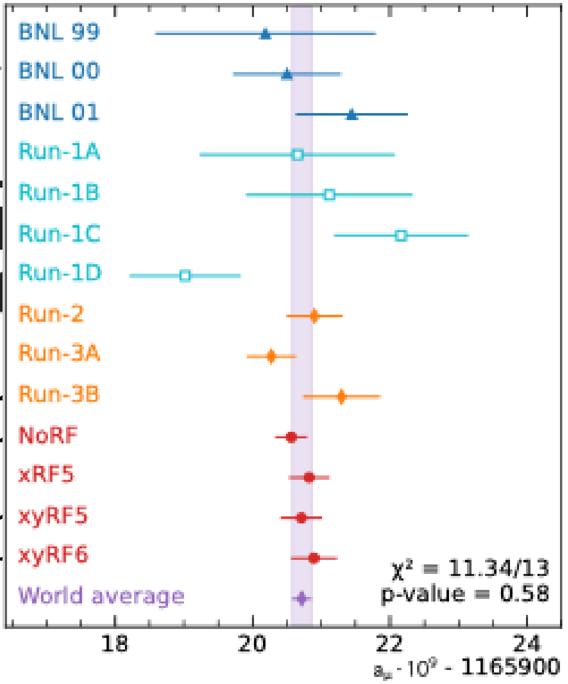


$$a_\mu(4/5/6) = 1.001\,165\,920\,710\,(162)$$ 139 ppb

$$a_\mu(exp) = 1.001\,165\,920\,715\,(145)$$ 124 ppb

Muon G-2 final result

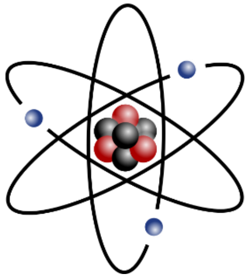




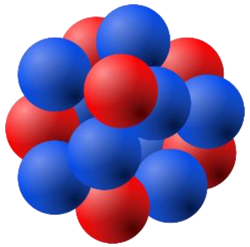
$$a_\mu(4/5/6) = 1.001\,165\,920\,7$$

$$a_\mu(exp) = 1.001\,165\,920\,7$$

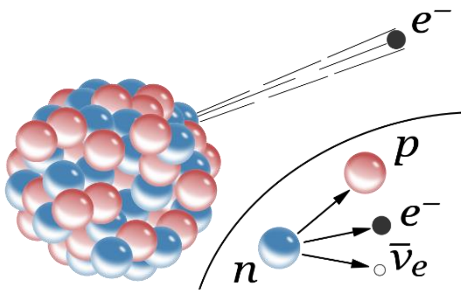
SM prediction for a_μ



Electromagnetic interactions



Strong interactions



Weak interactions

$0.001\ 165\ 847\ 19\ (0.1)$

???

$0.000\ 000\ 069\ 37\ (43)$

$0.000\ 000\ 001\ 54\ (1)$

$$a_\mu = 0.001\ 165\ 918\ 10\ (43)$$ WP2020

The uncertainty is dominated by contribution of strong interactions

Dispersive approach: $a_\mu(Had; LO) = \int \sigma_{e^+e^- \to hadrons}(s)\ K(s)\ ds$

Contribution of exclusive hadronic cross sections to a_μ

In exclusive approach, we calculate a_μ integral for each final state and sum them:

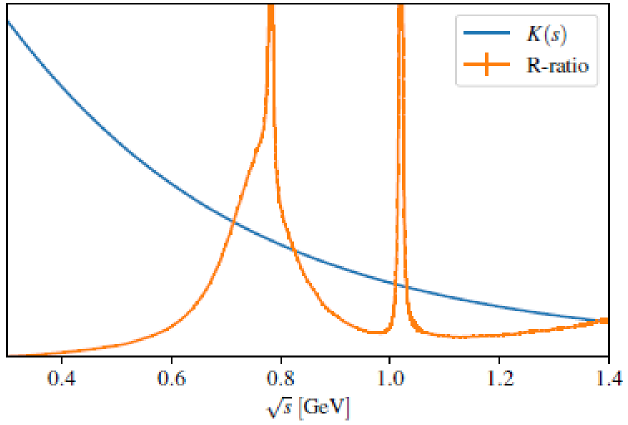
$$a_\mu^{had}(LO) = \sum_{X=\pi^0\gamma,\pi^+\pi^-,\dots} a_\mu^X(LO) = \sum_X \frac{1}{4\pi^3}\int \sigma^0(e^+e^- \to X)K_\mu(s)ds$$

Channel	$a_\mu^{\text{had,LO}}$ $[10^{-10}]$
$\pi^0\gamma$	$4.41 \pm 0.06 \pm 0.04 \pm 0.07$
$\eta\gamma$	$0.65 \pm 0.02 \pm 0.01 \pm 0.01$
$\pi^+\pi^-$	$507.85 \pm 0.83 \pm 3.23 \pm 0.55$
$\pi^+\pi^-\pi^0$	$46.21 \pm 0.40 \pm 1.10 \pm 0.86$
$2\pi^+2\pi^-$	$13.68 \pm 0.03 \pm 0.27 \pm 0.14$
$\pi^+\pi^-2\pi^0$	$18.03 \pm 0.06 \pm 0.48 \pm 0.26$
$2\pi^+2\pi^-\pi^0$ (η excl.)	$0.69 \pm 0.04 \pm 0.06 \pm 0.03$
$\pi^+\pi^-3\pi^0$ (η excl.)	$0.49 \pm 0.03 \pm 0.09 \pm 0.00$
$3\pi^+3\pi^-$	$0.11 \pm 0.00 \pm 0.01 \pm 0.00$
$2\pi^+2\pi^-2\pi^0$ (η excl.)	$0.71 \pm 0.06 \pm 0.07 \pm 0.14$
$\pi^+\pi^-4\pi^0$ (η excl., isospin)	$0.08 \pm 0.01 \pm 0.08 \pm 0.00$
$\eta\pi^+\pi^-$	$1.19 \pm 0.02 \pm 0.04 \pm 0.02$
$\eta\omega$	$0.35 \pm 0.01 \pm 0.02 \pm 0.01$
$\eta\pi^+\pi^-\pi^0$(non-ω,ϕ)	$0.34 \pm 0.03 \pm 0.03 \pm 0.04$
$\eta 2\pi^+2\pi^-$	$0.02 \pm 0.01 \pm 0.00 \pm 0.00$
$\omega\eta\pi^0$	$0.06 \pm 0.01 \pm 0.01 \pm 0.00$
$\omega\pi^0$ ($\omega \to \pi^0\gamma$)	$0.94 \pm 0.01 \pm 0.03 \pm 0.00$
$\omega 2\pi$ ($\omega \to \pi^0\gamma$)	$0.07 \pm 0.00 \pm 0.00 \pm 0.00$
ω (non-$3\pi, \pi\gamma, \eta\gamma$)	$0.04 \pm 0.00 \pm 0.00 \pm 0.00$
K^+K^-	$23.08 \pm 0.20 \pm 0.33 \pm 0.21$
K_SK_L	$12.82 \pm 0.06 \pm 0.18 \pm 0.15$

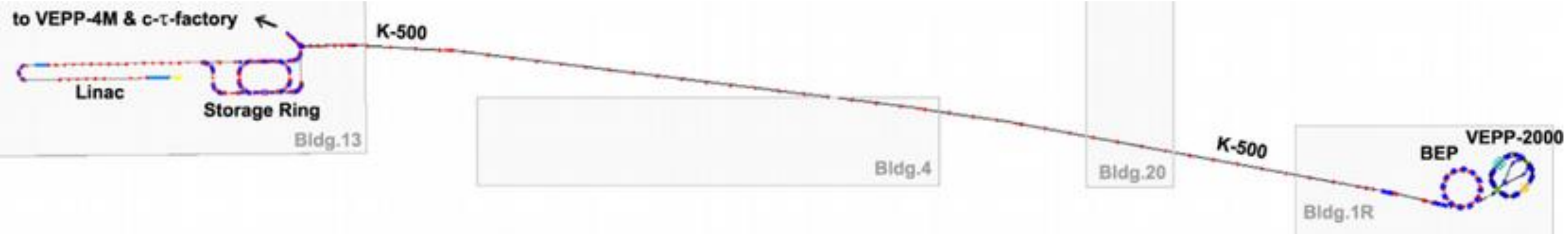
From DHMZ'19

The larger the contribution, the better relative precision is required

$e^+e^- \to \pi^+\pi^-$ is by far the most challenging and has got the most attention (74% of total hadronic contribution!)

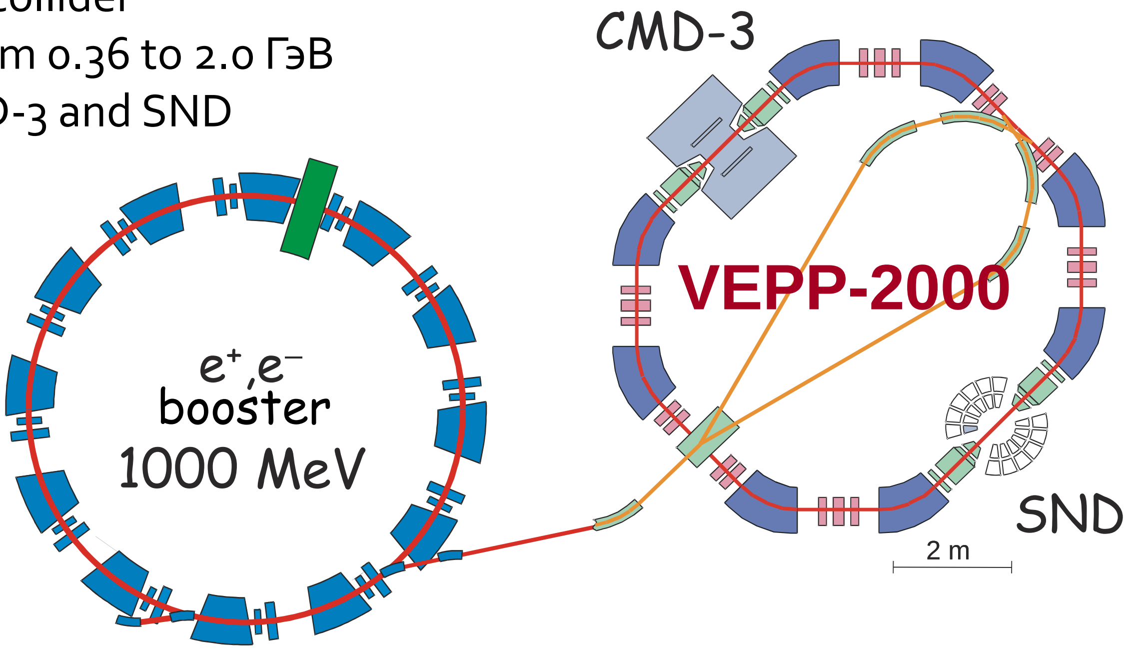


VEPP-2000 collider



Electron-positron collider
Covers c.m. energy range from 0.36 to 2.0 ГэВ
Two experiments – CMD-3 and SND

Design parameters @ 1 GeV	
Circumference	24.388 m
Beam energy	150 ÷ 1000 MeV
N of bunches	1×1
N of particles	1×10^{11}
Betatron tunes	4.14 / 2.14
Beta*	8.5 cm
BB parameter	0.1
Luminosity	1×10^{32} cm^{-2}s^{-1}

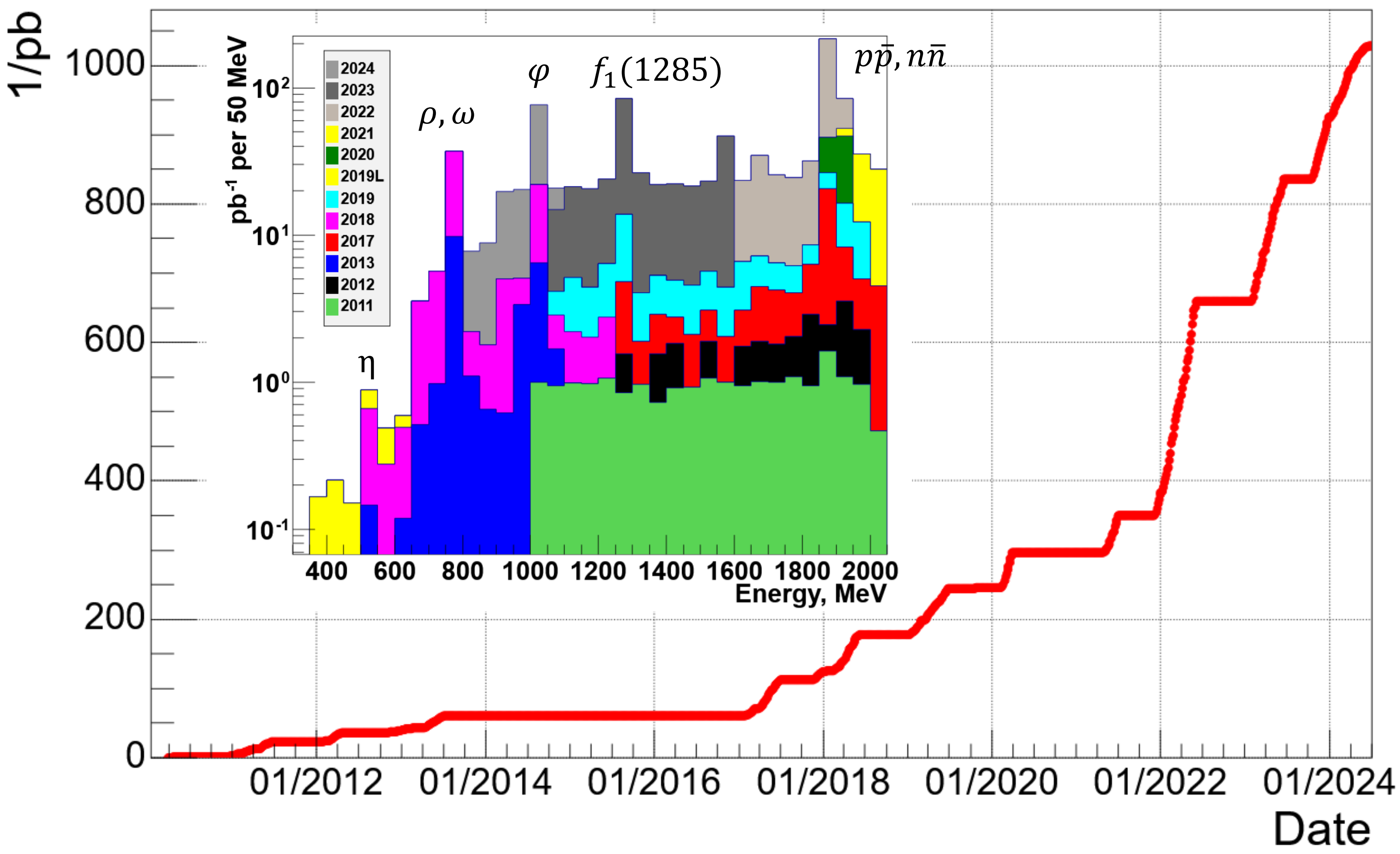


"Round beam" optics

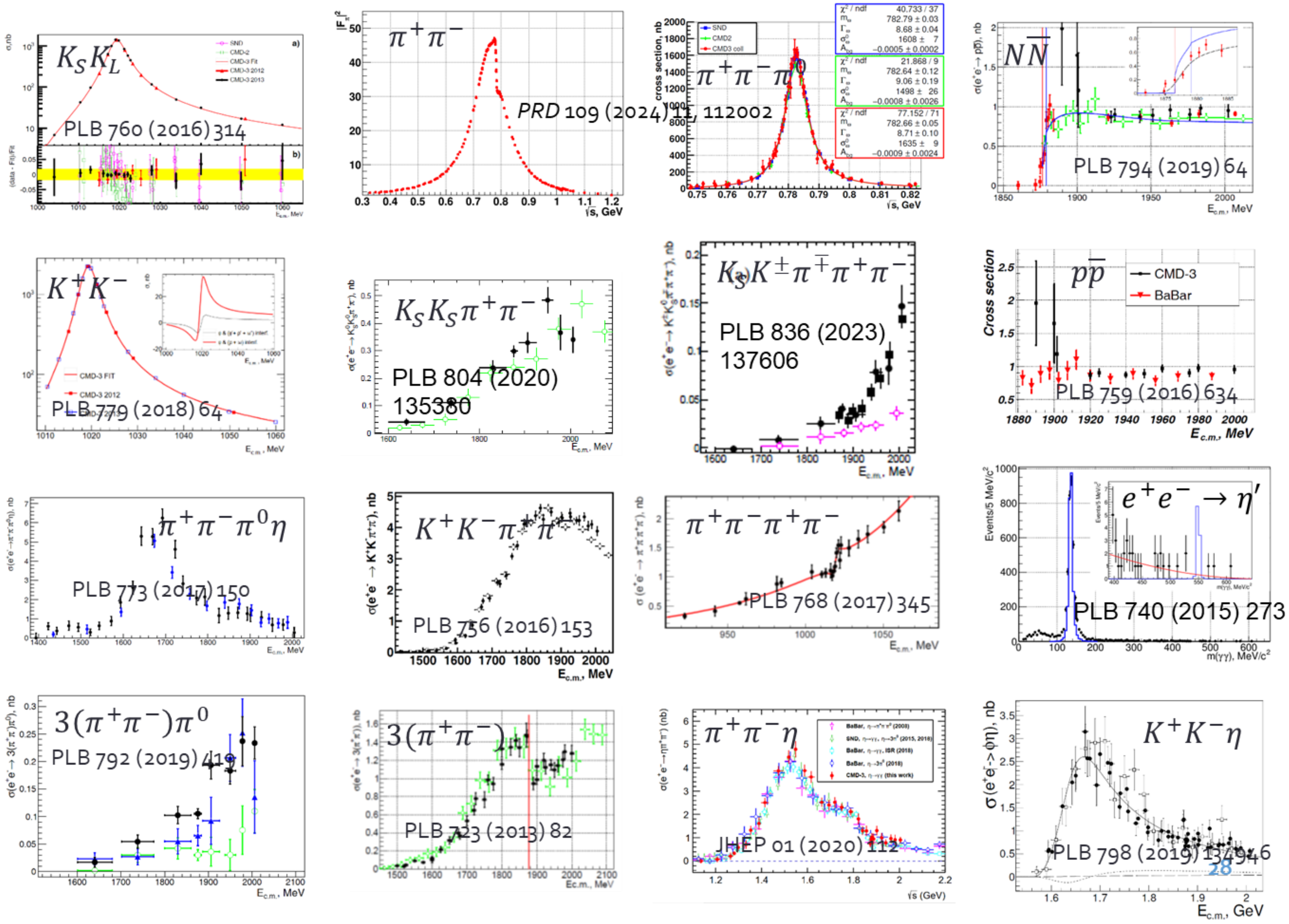
Energy monitoring by Compton backscattering ($\sigma_{\sqrt{s}} \approx 0.1$ MeV)

Collected data

Overpassed 1 1/fb per detector

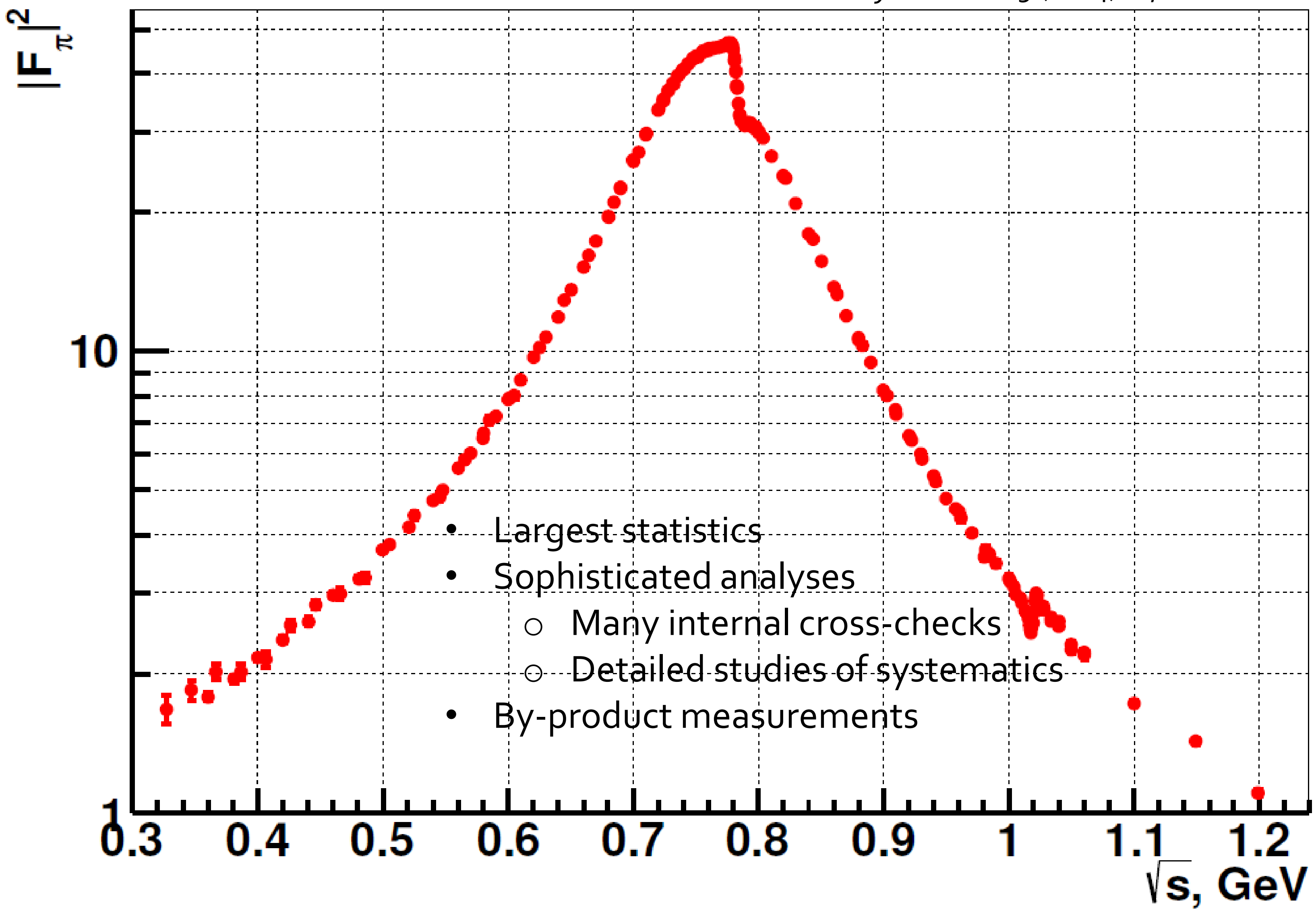


Study of exclusive channels of $e^+e^- \rightarrow hadrons$ at CMD-3



Phys.Rev.Lett. 132 (2024) 23, 231903
Phys.Rev.D 109 (2024) 11, 112002

CMD-3 measurement of $e^+e^- \to \pi^+\pi^-$

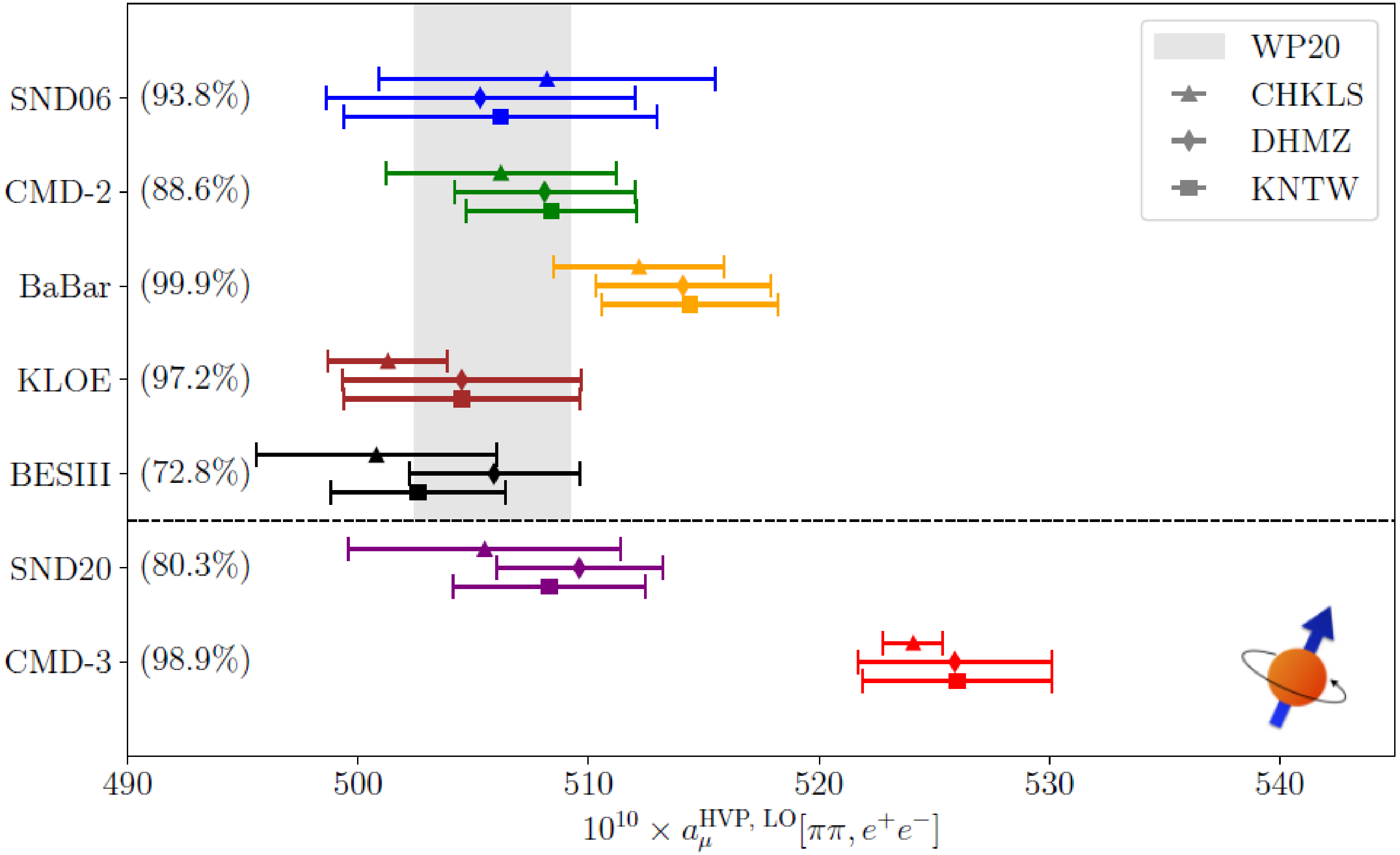


- Largest statistics
- Sophisticated analyses
  - Many internal cross-checks
  - Detailed studies of systematics
- By-product measurements

Larger than previous measurements -> lead to increase of $a_\mu(SM)$

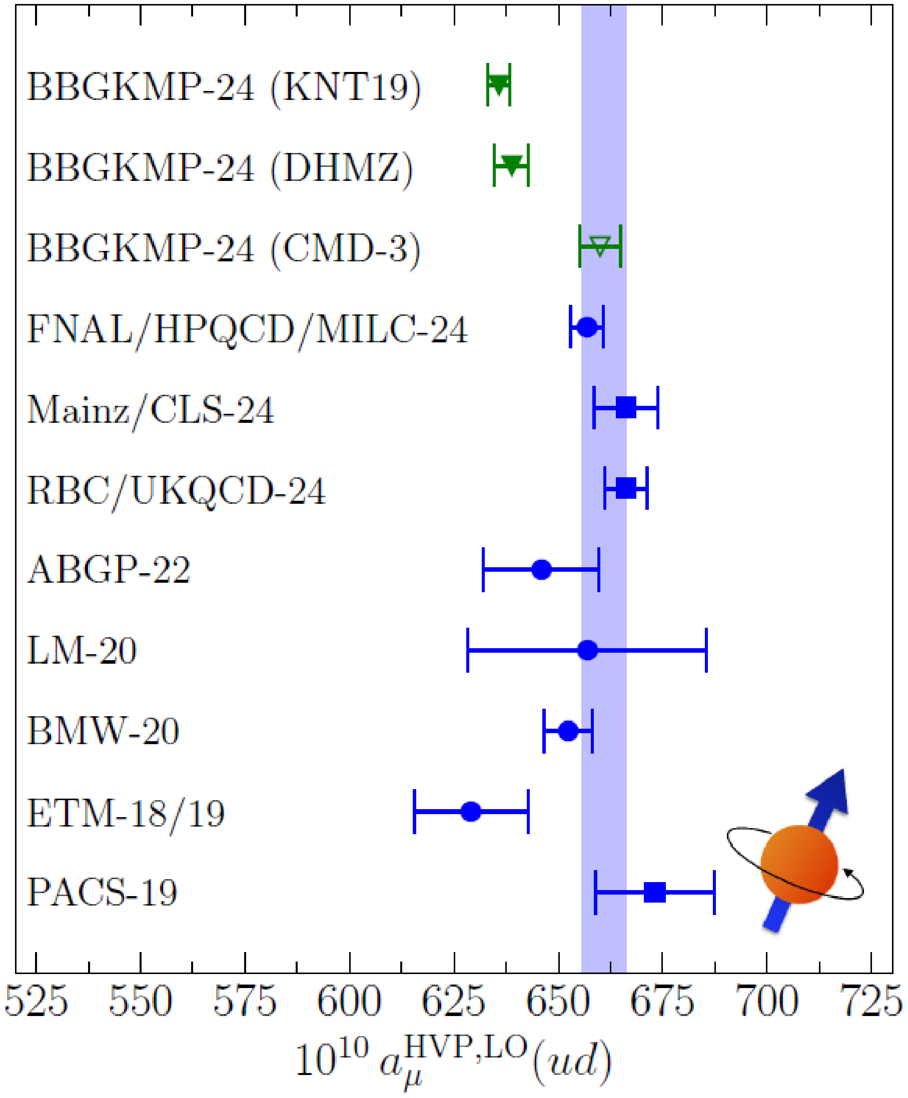
Hadronic contribution from 2π

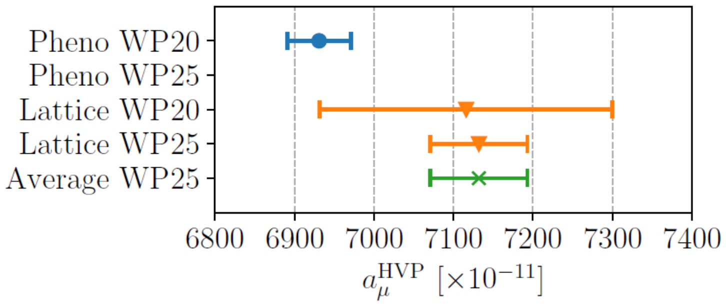
White paper 2025



White paper 2025

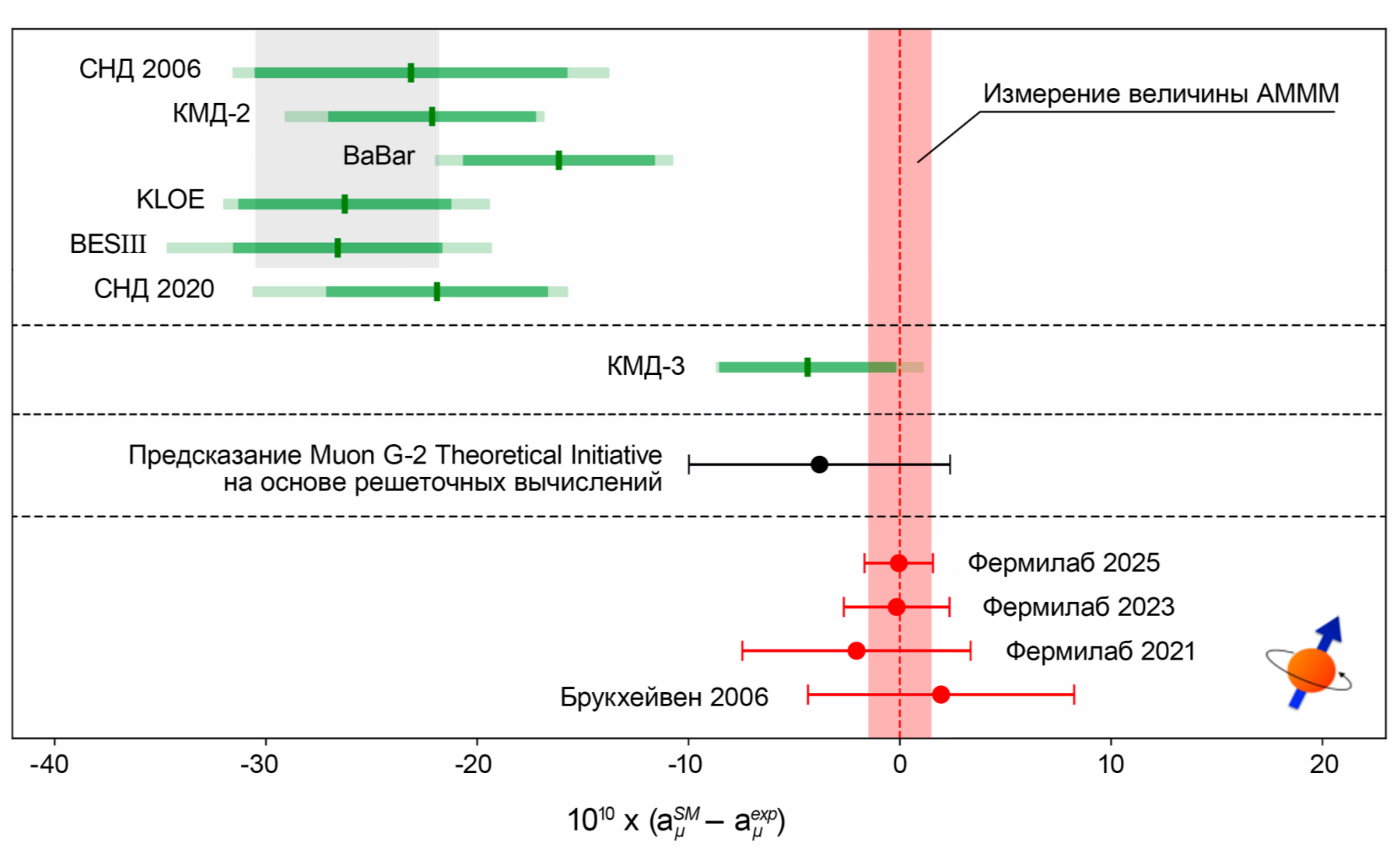
Lattice calculations





Experiment vs SM prediction

A view from 2025



Prospects for SM prediction

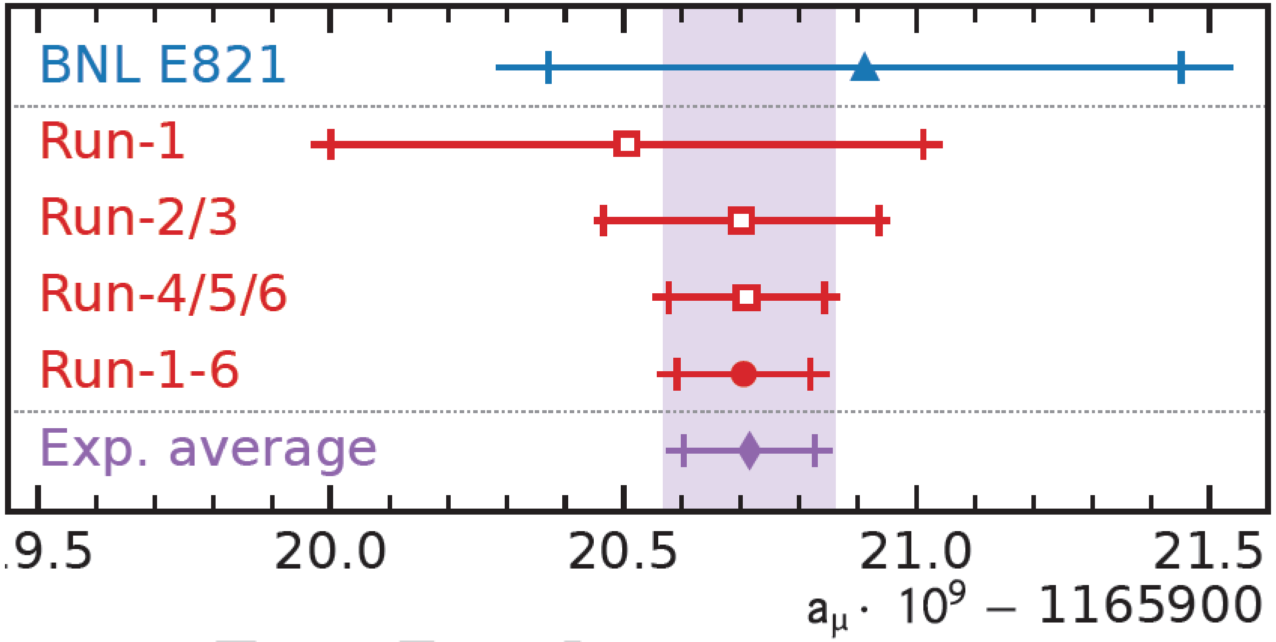
At the moment the comparison between experiment and theory is limited by the accuracy of the SM prediction

1. There are a lot of efforts to understand discrepancies in existing $\pi^+\pi^-$ data
   1. Expect result from SND @VEPP-2000 soon
   2. BABAR performs reanalysis of data using new independent approach
   3. KLOE: new analysis of 7x larger data set
   4. BES-III performs analysis of new data
   5. BELLE-II plans to measure hadronic cross section
   6. CMD-3 and SND continue to collect data
2. There is dedicated experiment, Muone, being prepared at CERN to measure hadronic contribution via $e\mu$ scattering
3. There is fast progress in lattice calculations – expect better precision
4. CMD-3 and SND teams are preparing new experiments at VEPP-2000 aimed at x3-4 fold improvements in precision of $\pi^+\pi^-$ and other hadronic cross sections

There are good chances to improve precision of SM prediction in coming years

New measurement of a_μ at J-PARC (next talk)

Summary



$$a_\mu(exp) \quad = 1.001\,165\,920\,715\,(145)$$

- **Most precise determination of a_μ for many years to come**
- Benchmark for any New Physics Models
  - Limited by the knowledge of the Standard model prediction