TWENTY-SECOND LOMONOSOV

CONFERENCE August, 21-27, 2025

ON ELEMENTARY PARTICLE PHYSICS

MOSCOW STATE UNIVERSITY

Near-threshold structures with heavy quarks

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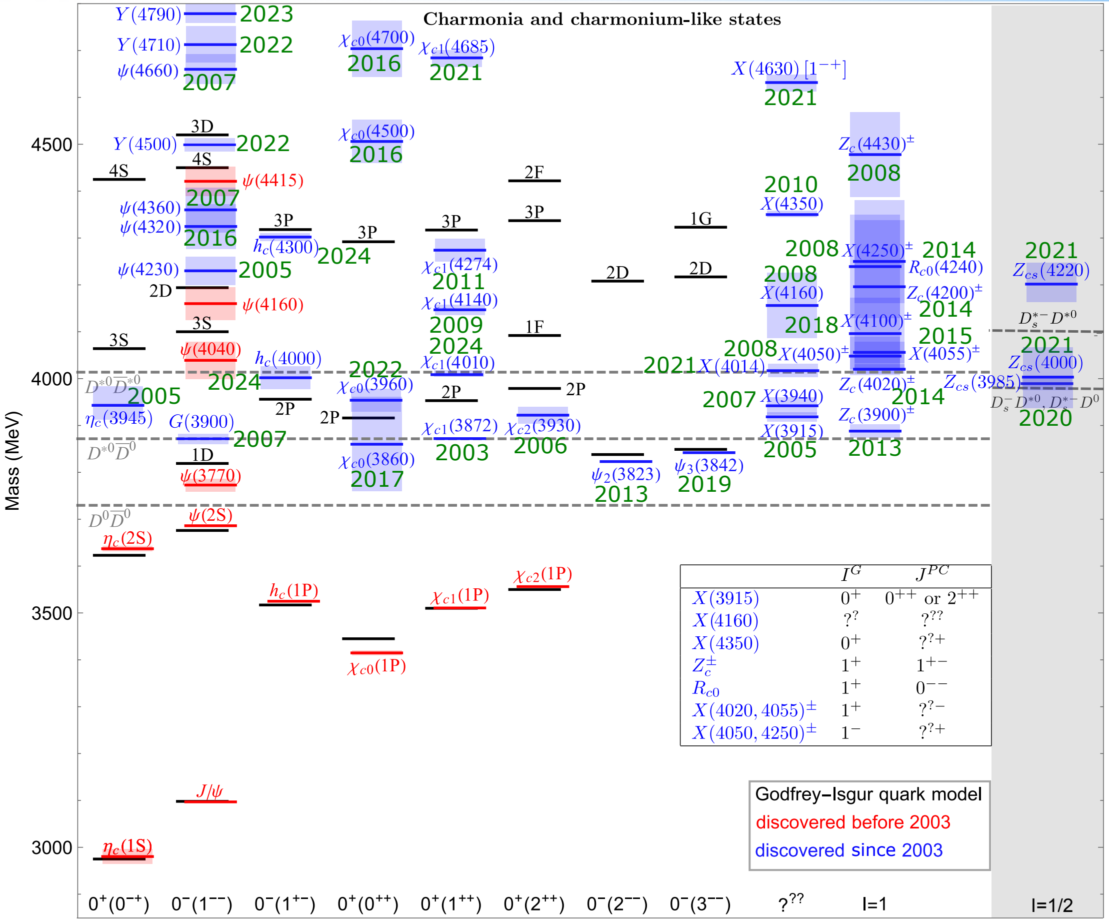
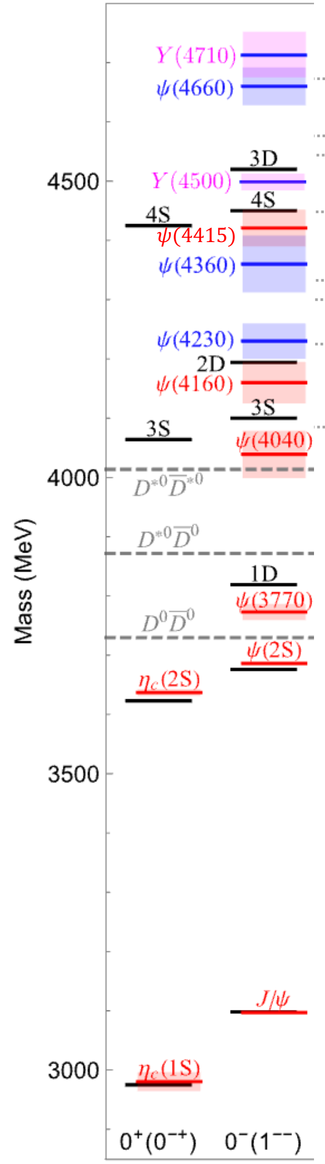
August 21, 2015



Many hidden-charm thresholds above 4 GeV



$\psi' f_0(980)$

$D_{s1}\bar{D}_s^*, \Lambda_c^+\bar{\Lambda}_c^-$

$\chi_{c0,1,2}\phi$

$D_{s0}^*\bar{D}_s^*, D_{s1}\bar{D}_s$

$D_2(2460)\bar{D}^*$

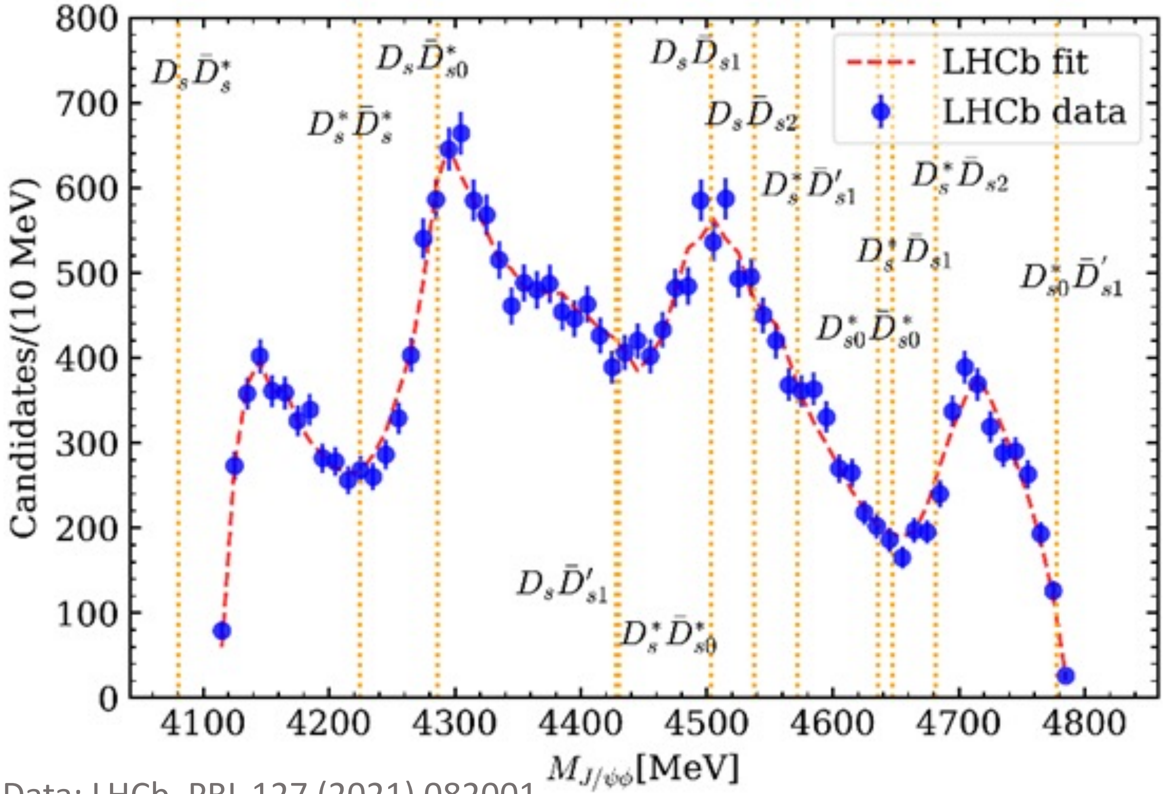
$D_1(2420)\bar{D}^*$

$D_1(2420)\bar{D}$

$\chi_{c0,1,2}\omega$

$J/\psi f_0(980)$

2-body S-wave thresholds



Data: LHCb, PRL 127 (2021) 082001

Plot: X.-K. Dong, FKG, B.-S. Zou, Progr. Phys. 41 (2021) 65 [arXiv:2101.01021]

Thresholds play an essential role

- Complicates extraction of resonance properties!
  Measurements on various final states are important
- Hadronic molecules?

Molecular line shapes at LO

- Scattering length approx.: $p \cot\delta = -\frac{1}{a} + \cdots$
- Poles: bound or virtual state ($\kappa = 1/|a|$)
  - Bound and virtual state can hardly be distinguished above threshold ($E > 0$)

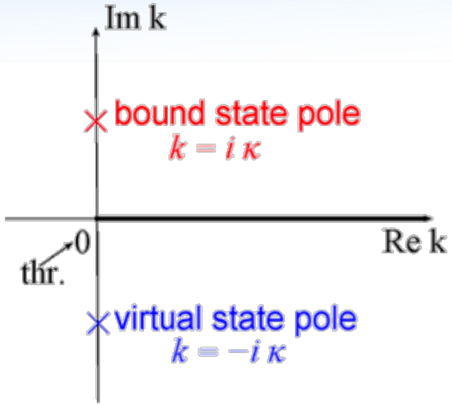
$$|T_{\rm NR}(E)|^2 \propto \left|\frac{1}{\pm\kappa + i\sqrt{2\mu E}}\right|^2 = \frac{1}{\kappa^2 + 2\mu E}$$

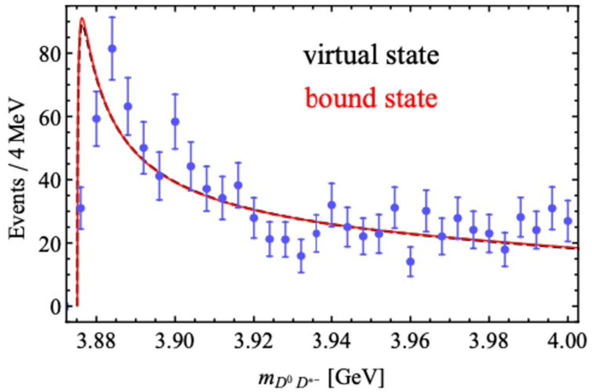
  - Different below threshold ($E < 0$)
    - bound state: peaked below threshold

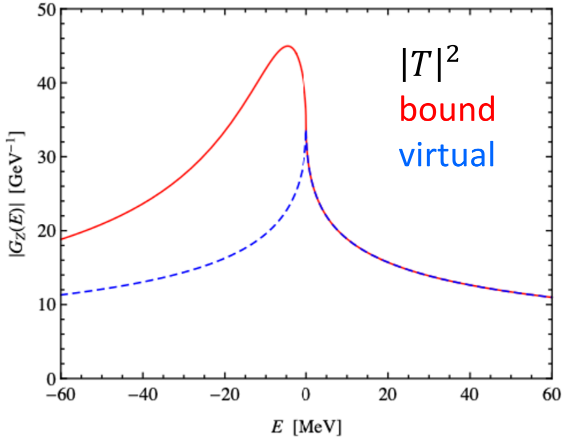
$$|T_{\rm NR}(E)|^2 \propto \frac{1}{(\kappa - \sqrt{-2\mu E})^2}$$

    - virtual state: sharp cusp at threshold

$$|T_{\rm NR}(E)|^2 \propto \frac{1}{(\kappa + \sqrt{-2\mu E})^2}$$

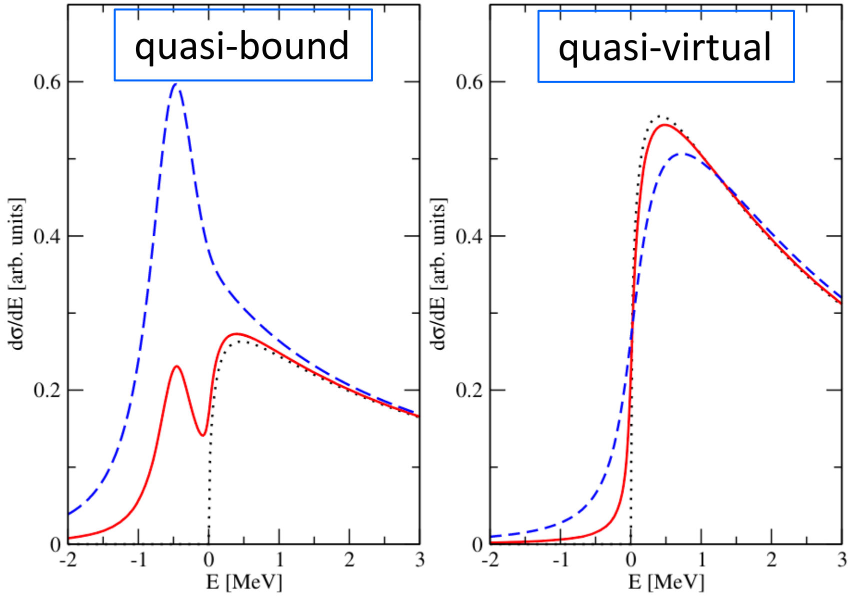






FKG, et al., RMP 90 (2018) 015004;
N. Brambilla et al., Phys.Rept. 873 (2020) 1

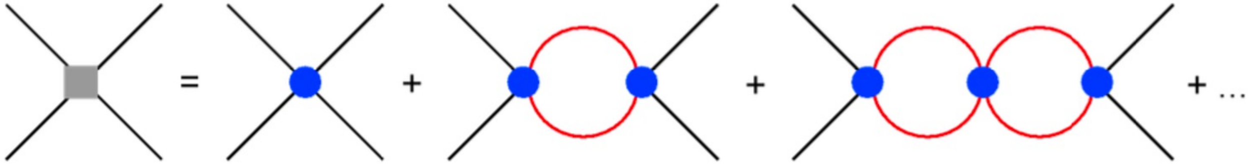
line shapes w/ phase space; one unstable constituent:



$\Gamma = 0$	0.1 MeV	1 MeV
dotted	dashed	solid

NREFT at LO for coupled channels

- Full threshold structure needs to be measured in a lower channel (ch-1) $\Rightarrow$ coupled channels
- Consider a two-channel system, construct a "nonrelativistic" effective field theory (NREFT)
  - Energy region around the higher threshold (ch-2), Σ_2
  - Expansion in powers of $E = \sqrt{s} - \Sigma_2$
  - Momentum in the lower channel can also be expanded



$$T(E) = 8\pi\Sigma_2 \begin{pmatrix} -\frac{1}{a_{11}} + ik_1 & \frac{1}{a_{12}} \\ \frac{1}{a_{12}} & -\frac{1}{a_{22}} - \sqrt{-2\mu_2 E - i\epsilon} \end{pmatrix}^{-1} = -\frac{8\pi\Sigma_2}{\det} \begin{pmatrix} \frac{1}{a_{22}} + \sqrt{-2\mu_2 E - i\epsilon} & \frac{1}{a_{12}} \\ \frac{1}{a_{12}} & \frac{1}{a_{11}} - ik_1 \end{pmatrix}$$

$$\det = \left(\frac{1}{a_{11}} - ik_1\right)\left(\frac{1}{a_{22}} + \sqrt{-2\mu_2 E - i\epsilon}\right) - \frac{1}{a_{12}^2}$$

- a_{22}: single-ch. scattering length of ch-2
- a_{11}: single-ch. interaction strength of ch-1 at Σ_2

Effective scattering length with open-channel effects becomes complex, $\mathrm{Im}\frac{1}{a_{22,\mathrm{eff}}} \leq 0$

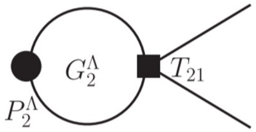
$$T_{22}(E) = -\frac{8\pi}{\Sigma_2}\left[\frac{1}{a_{22,\mathrm{eff}}} - i\sqrt{2\mu_2 E} + \mathcal{O}(E)\right]^{-1} \qquad \frac{1}{a_{22,\mathrm{eff}}} = \frac{1}{a_{22}} - \frac{a_{11}}{a_{12}^2(1+a_{11}^2k_1^2)} - i\frac{a_{11}^2k_1}{a_{12}^2(1+a_{11}^2k_1^2)}.$$

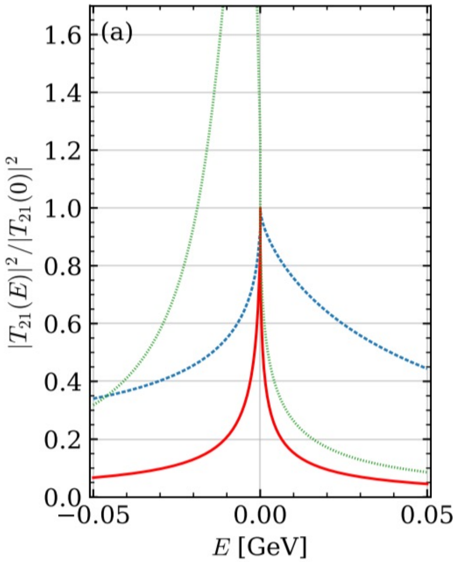
Distinct line shapes of the same pole

X.-K. Dong, FKG, B.-S. Zou, PRL 126 (2021) 152001

Line shapes of the same pole depend on the production mechanism. Consider production of particles in ch-1

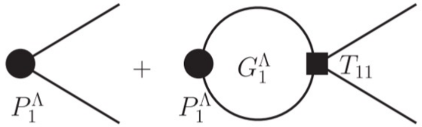
- Dominated by ch-2
  - Maximal at threshold for positive $\mathrm{Re}(a_{22,\mathrm{eff}})$ (attraction), FWHM $\propto 1/\mu$
    - more pronounced for heavier hadrons and stronger interactions
  - Peaking at pole for negative $\mathrm{Re}(a_{22,\mathrm{eff}})$

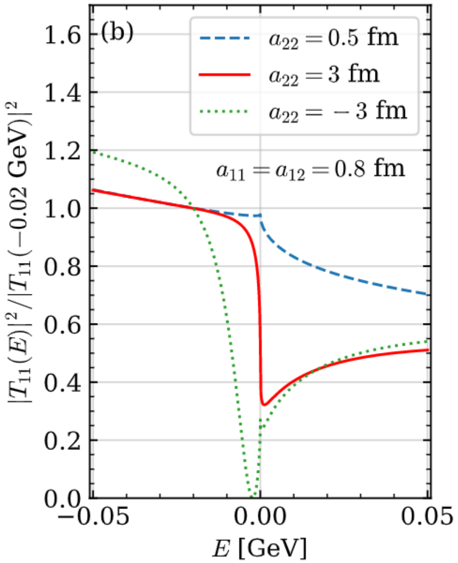




$$T_{21} \propto \frac{1}{a_{22,\mathrm{eff}}^{-1} - i\sqrt{2\mu_2 E}}$$

- Dominated by ch-1
  - One pole and one zero
  - Universality for large scattering length: for large $|a_{22}|$, there must be a dip around threshold (zero close to threshold)





$$T_{11} \propto \frac{a_{22}^{-1} - i\sqrt{2\mu_2 E}}{a_{22,\mathrm{eff}}^{-1} - i\sqrt{2\mu_2 E}}$$

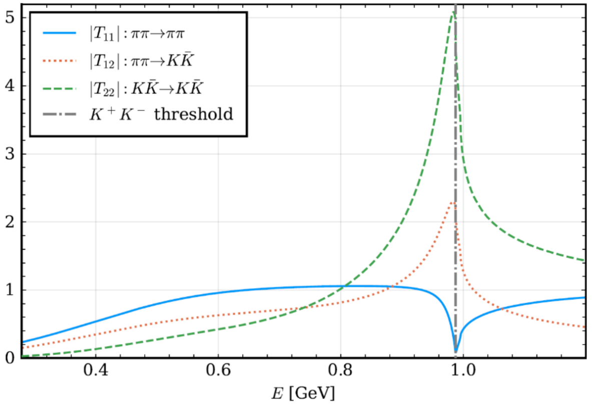
Process-dependent line shapes known since long; see, e.g., J. Taylor, *Scattering Theory*

✓ Classification of near-threshold structures and pole trajectories for two heavy channels: Z.-H. Zhang, FKG, PLB 863 (2025) 139387

Distinct line shapes of the same pole

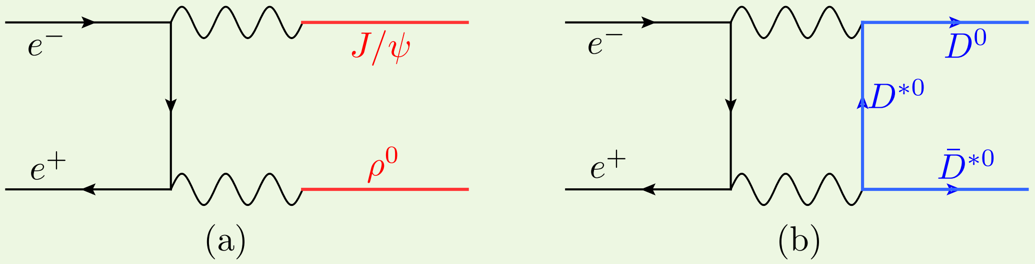
- Example-1: $f_0(980)$
  - T-matrix for $\pi\pi$ and $K\bar{K}$ coupled channels

    with the T-matrix from L.-Y. Dai, M. R. Pennington, PRD 90 (2014) 036004



- Example-2: direct production of $X(3872)$ in e^+e^-

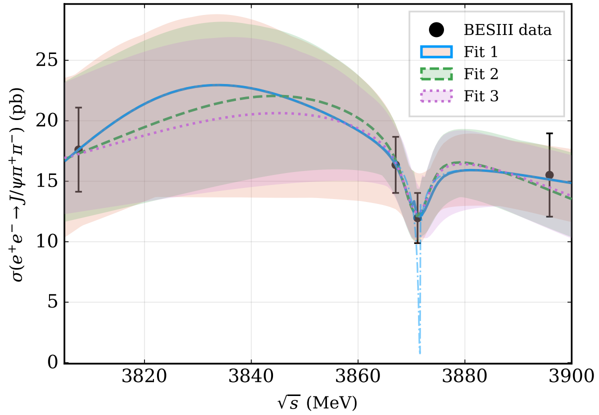
Baru, FKG, Hanhart, Nefediev, PRD 109 (2024) L111501



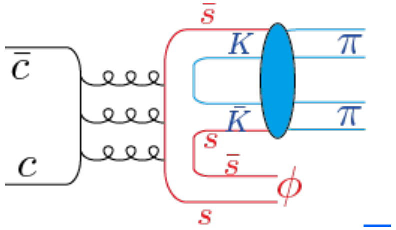
- Driving channel:

  J/ψ +light vector

- Prediction: dip around $D^*\bar{D}^*$ threshold

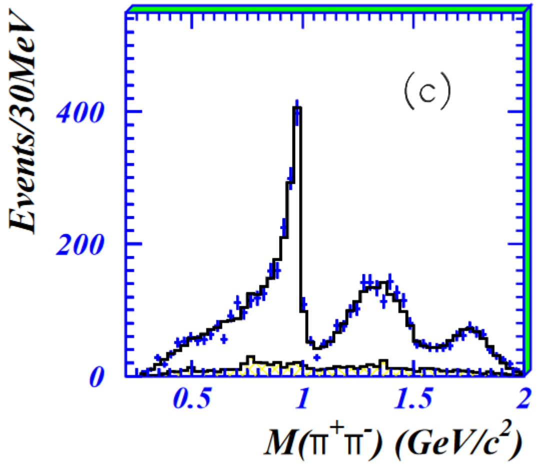


- $J/\psi \to \phi\pi^+\pi^-$



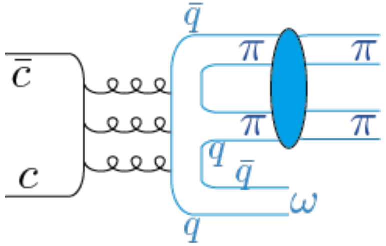
Driving channel: $K\bar{K}$

$J/\psi \to \phi K\bar{K} \to \phi\pi^+\pi^-$



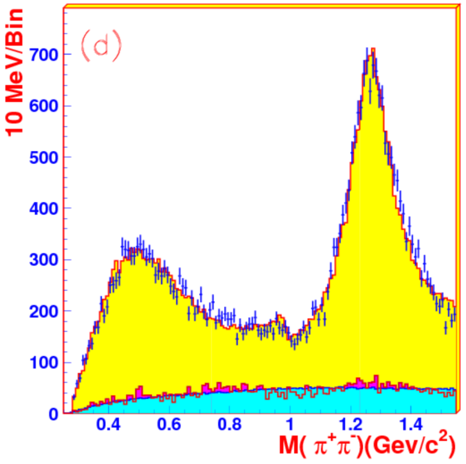
BES, PLB 607 (2005) 243

- $J/\psi \to \omega\pi^+\pi^-$



Driving channel: $\pi\pi$

$J/\psi \to \omega\pi\pi \to \omega\pi^+\pi^-$



BES, PLB 598 (2004) 149

Binding mechanism

- One-boson exchange — Vector + scalar exchanges: M. Voloshin, L. Okun, JETP Lett. 23 (1976) 333

One-pion exchange

N.A. Törnqvist, ZPC 61 (1994) 525; …

➢ systems like $D\bar{D}, \Sigma_c\bar{D}$ unbound

Composite	J^{PC}	Deuson
$D\bar{D}^*$	0^{-+}	$\eta_c(\approx 3870)$
$D\bar{D}^*$	1^{++}	$\chi_{c1}(\approx 3870)$
$D^*\bar{D}^*$	0^{++}	$\chi_{c0}(\approx 4015)$
$D^*\bar{D}^*$	0^{-+}	$\eta_c(\approx 4015)$
$D^*\bar{D}^*$	1^{+-}	$h_{c0}(\approx 4015)$
$D^*\bar{D}^*$	2^{++}	$\chi_{c2}(\approx 4015)$
$B\bar{B}^*$	0^{-+}	$\eta_b(\approx 10545)$
$B\bar{B}^*$	1^{++}	$\chi_{b1}(\approx 10562)$
$B^*\bar{B}^*$	0^{++}	$\chi_{b0}(\approx 10582)$
$B^*\bar{B}^*$	0^{++}	$\eta_b(\approx 10590)$
$B^*\bar{B}^*$	1^{+-}	$h_b(\approx 10608)$
$B^*\bar{B}^*$	2^{++}	$\chi_{b2}(\approx 10602)$

One-vector exchange S. Krewald, R. Lemmer, F. Sassen, PRD 69 (2004) 016003; …

➢ $D\bar{D}$ bound state predicted

C.-Y. Wong, PRC 69 (2004) 055202; Y.-J. Zhang et al., PRD 74 (2006) 014013; D. Gamermann et al., PRD 76 (2007) 074016; J. Nieves et al., PRD 86 (2012) 056004; …

✧ Lattice QCD S. Prelovsek et al., JHEP06 (2021) 035
Conflict: not in D.J. Wilson et al., PRL 132 (2024) 241901 solution?

➢ Hidden-charm pentaquarks >4 GeV (including $\Sigma_c\bar{D}$) predicted

J.-J. Wu, R. Molina, E. Oset, B.-S. Zou, PRL 105 (2010) 232001; …

- Soft-gluon exchanges: equivalent to OZI breaking $\pi\pi, K\bar{K}$, …

X.-K. Dong et al., Sci. Bull. 66 (2021) 1577

☞ Survey of the molecular spectrum in a simple model
- ➢ light-vector-meson exchanges
- ➢ single channel
- ➢ neglecting mixing

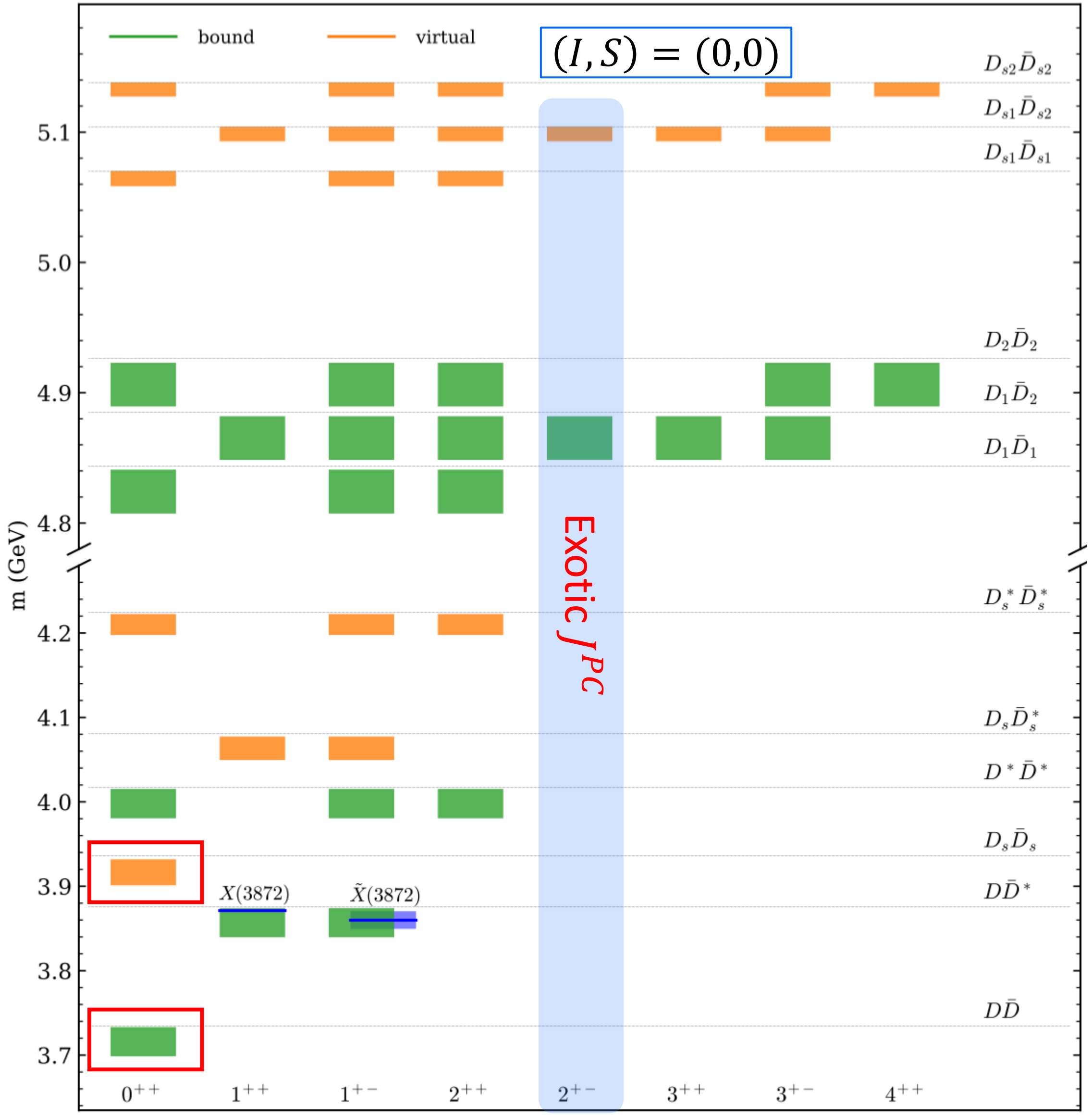
X.-K. Dong, FKG, B.-S. Zou, Progr. Phys. 41 (2021) 65; CTP 73 (2021) 015201

Extension of the survey including vector+scalar meson exchanges:
F.-Z. Peng, M. Sanchez-Sanchez, M.-J. Yan, M. Pavon Valderrama, PRD 105 (2022) 034028; M.-J. Yan, F.-Z. Peng, M. Pavon Valderrama, PRD 109 (2024) 014023

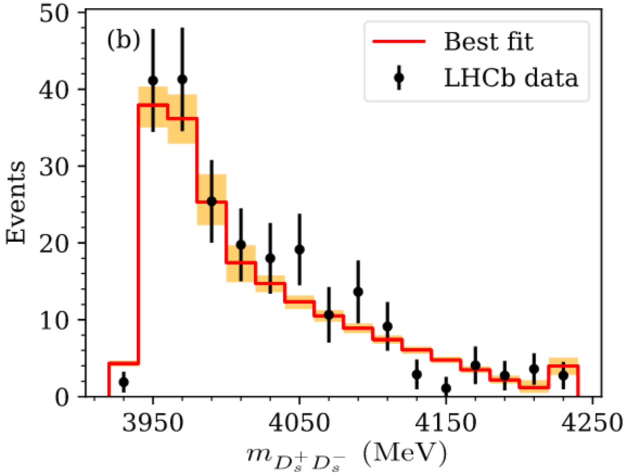
For a list of the literature on one-boson exchange models, see, e.g., Y.-R. Liu et al., PPNP 107 (2019) 237

Survey of hadronic molecules: hidden-charm mesons w/ $P = +$

X.-K. Dong, FKG, B.-S. Zou, Progr. Phys. 41 (2021) 65 [arXiv:2101.01021]



- ✓ > 200 hidden-charm hadronic molecules
- ✓ $X(3872)$ as a $\bar{D}D^*$ bound state
- ✓ $\tilde{X}(3872)$ COMPASS, PLB 783 (2018) 334
- ✓ $X(3960)$ in $B^+ \to D_s^+ D_s^- K^+$

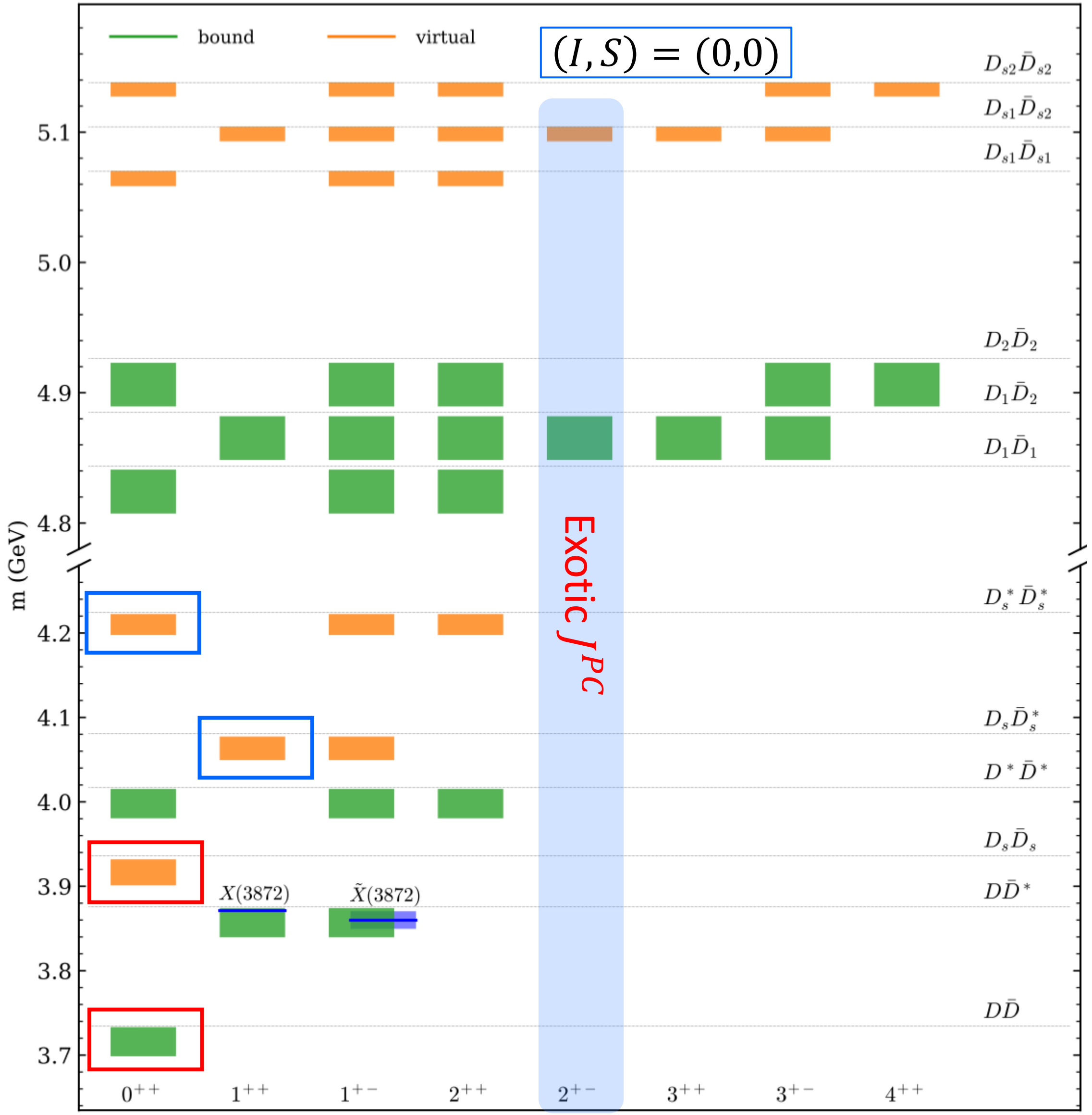


Data: LHCb, PRL 131 (2023) 071901

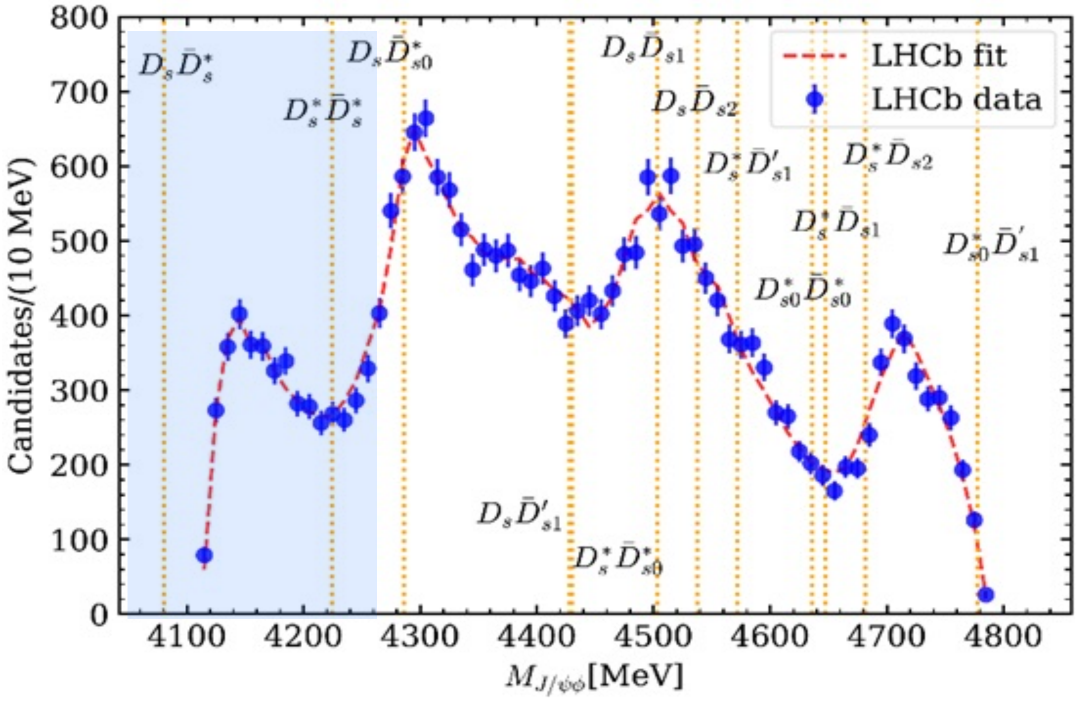
Fit in
T. Ji, X.-K. Dong, M. Albaladejo, M.-L. Du, FKG, J. Nieves, B.-S. Zou, Sci. Bull. 68 (2023) 2056

pole at $3936.5^{+0.4}_{-0.9} + i\,(16.1^{+4.2}_{-2.2})$ MeV

Survey of hadronic molecules: hidden-charm mesons w/ $P = +$



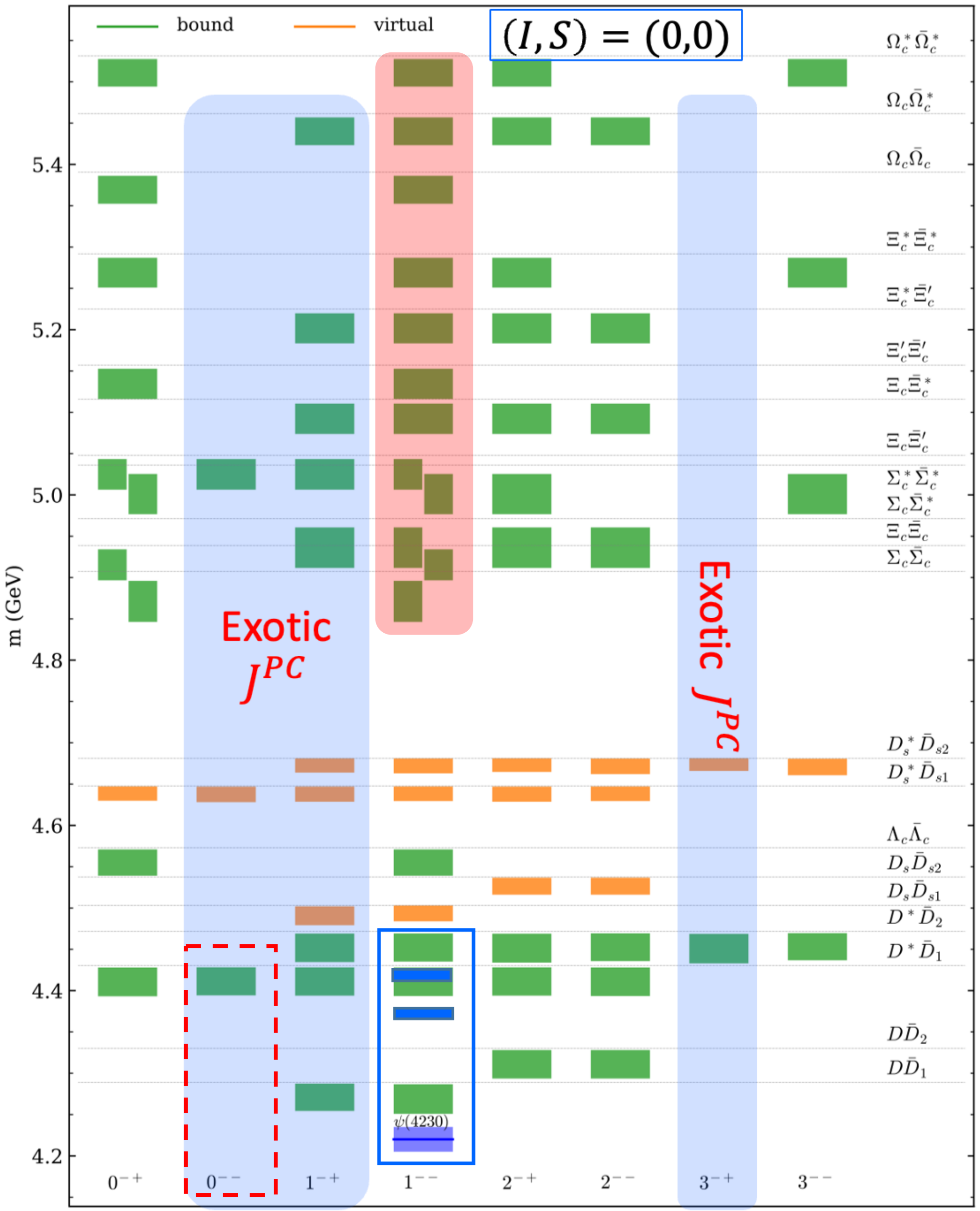
✓ $D_s\bar{D}_s^*$, $D_s^*\bar{D}_s^*$ virtual states?



Data: LHCb, PRL 127 (2021) 082001

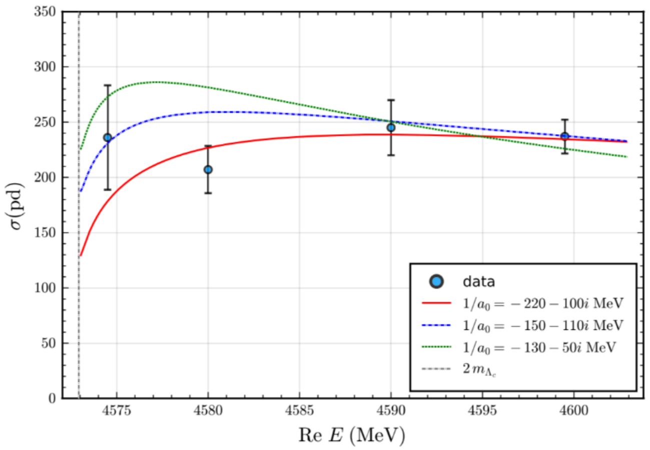
Virtual poles found from the fit in X. Luo, S.X. Nakamura, PRD 107 (2023) L011504

Hidden-charm mesons w/ $P = -$



- ✓ $Y(4260)/\psi(4230)$ as a $\bar{D}D_1$ bound state
- ✓ $\psi(4360), \psi(4415)$: $D^*\bar{D}_1, D^*\bar{D}_2$?
- ✓ Evidence for 1^{--} $\Lambda_c\bar{\Lambda}_c$ molecular state in BESIII data
  - Sommerfeld factor
  - near-threshold pole
  - different from $Y(4630)$

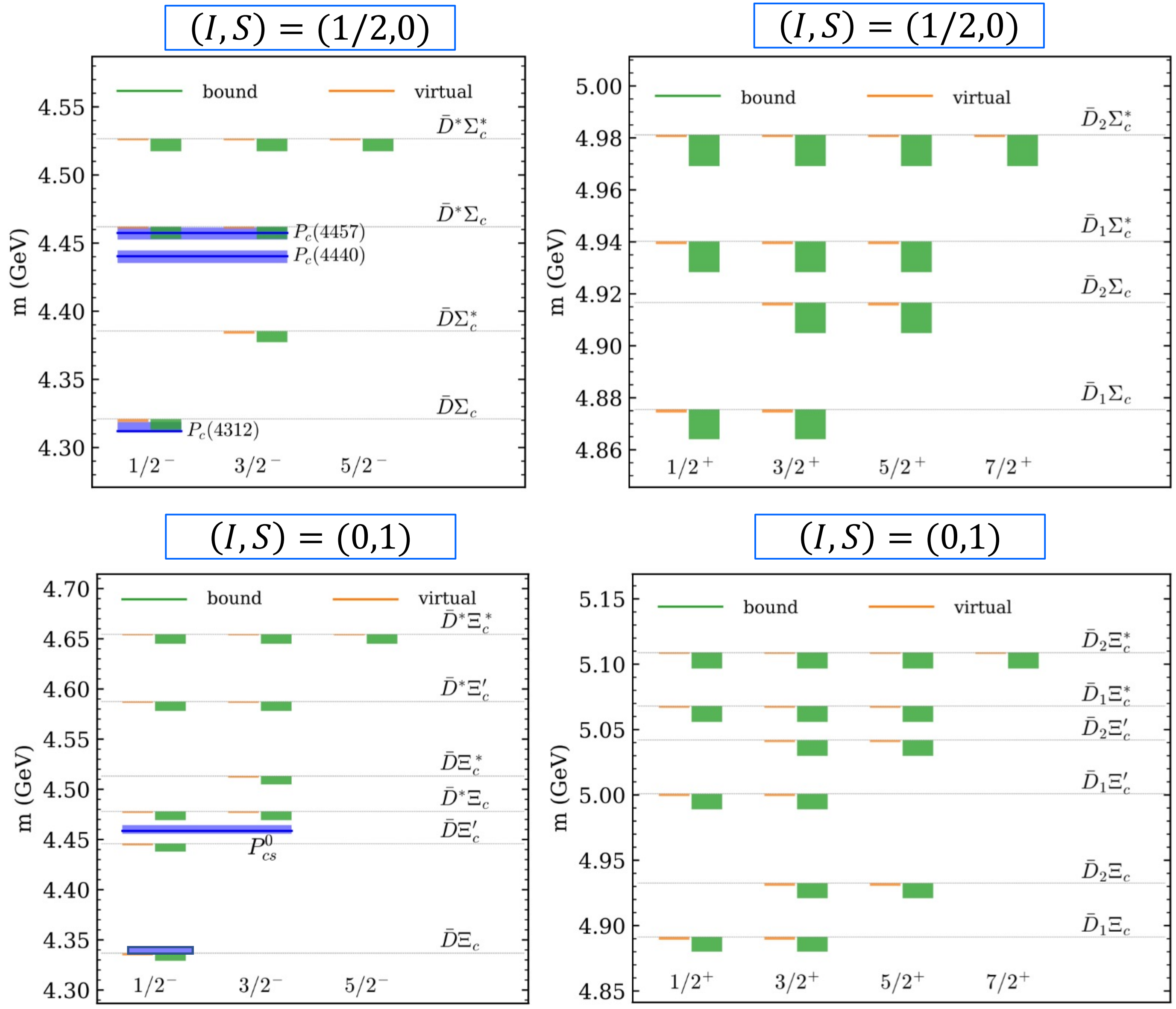
  Data from BESIII, PRL 120 (2018) 132001; see also Q.-F. Cao et al., PRD 100 (2019) 054040
  New BESIII data: PRL 131 (2024) 191901



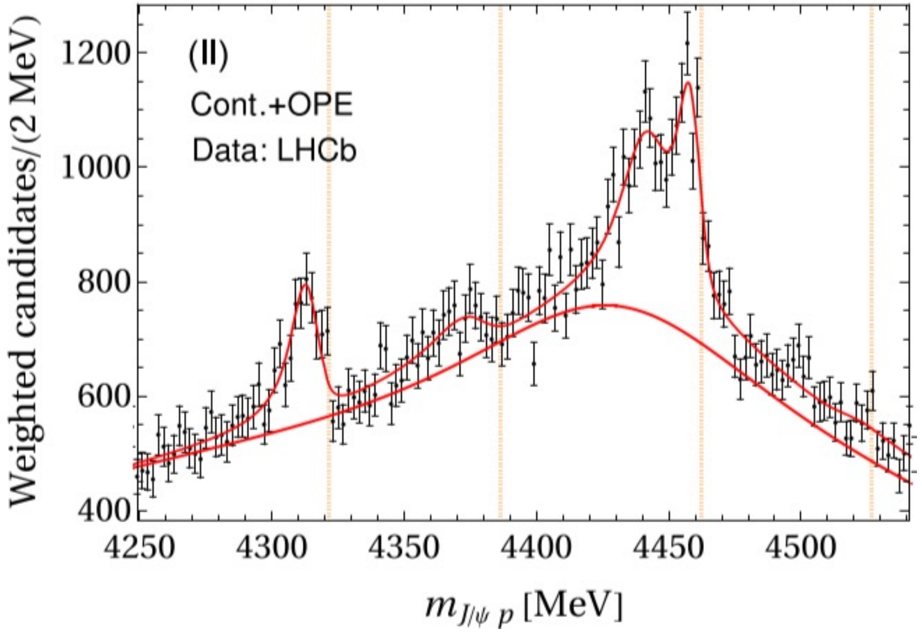
- ✓ Numerous states with exotic quantum numbers
  0^{--} $[\psi_0]$, $1^{-+}[\eta_{c1}]$, $3^{-+}[\eta_{c3}]$
  - ➢ 0^{--}: T. Ji, X.-K. Dong, FKG, B.-S. Zou, PRL 129 (2022) 102002

  e.g., $e^+e^- \to \gamma\eta_{c1,3}, \omega\eta_{c1,3}$; $\eta_{c1,3} \to D\bar{D}^*\pi, J/\psi\omega, \ldots$
- ✓ Many 1^{--} states in [4.8, 5.6] GeV: BEPCII-U, Belle II, LHCb, STCF/SCTF, ...

Hidden-charm pentaquarks



- ✓ P_c states as $\bar{D}^{(*)}\Sigma_c^{(*)}$ molecules
- ✓ The LHCb data can be well described in a pionful EFT

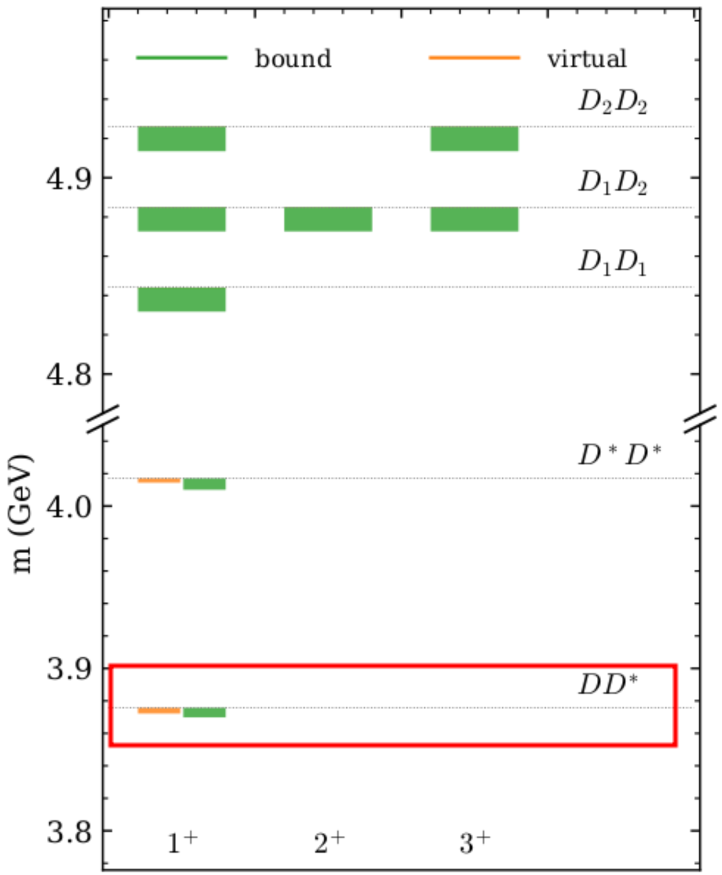


M.-L. Du et al., PRL 124 (2020) 072001; JHEP 08 (2021) 157

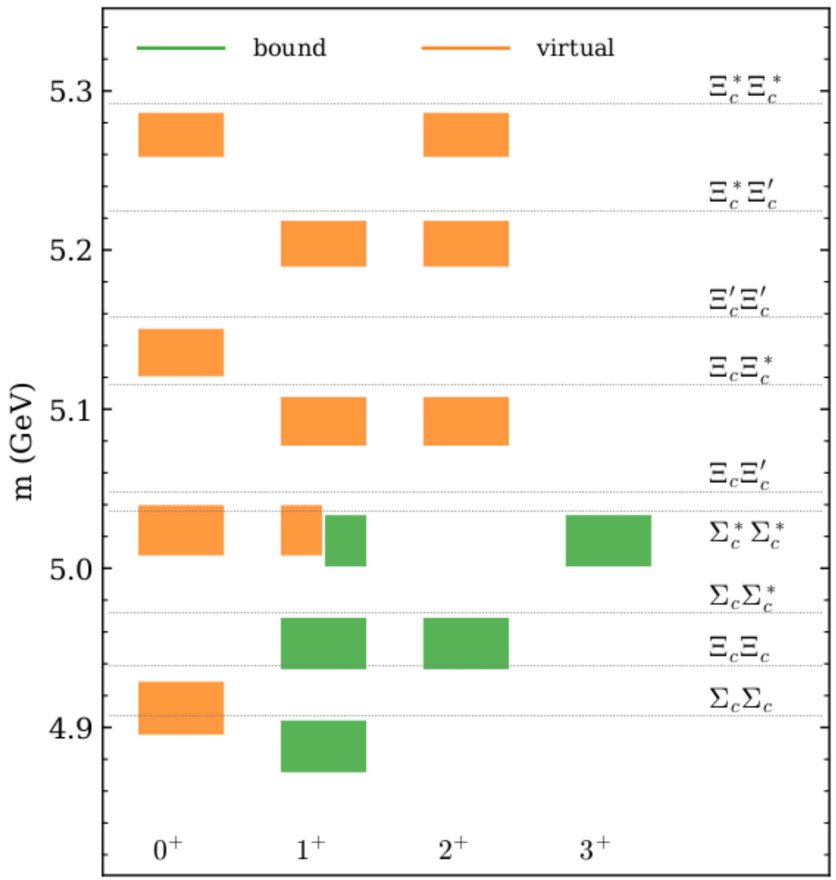
- ✓ $P_{cs}(4459)$: 2 $\bar{D}^*\Xi_c$ molecular states
- ✓ $P_{cs}(4338)$: $\bar{D}\Xi_c$ molecular state

Double-charm tetraquarks and dibaryons

$(I, S, B) = (0,0,0)$

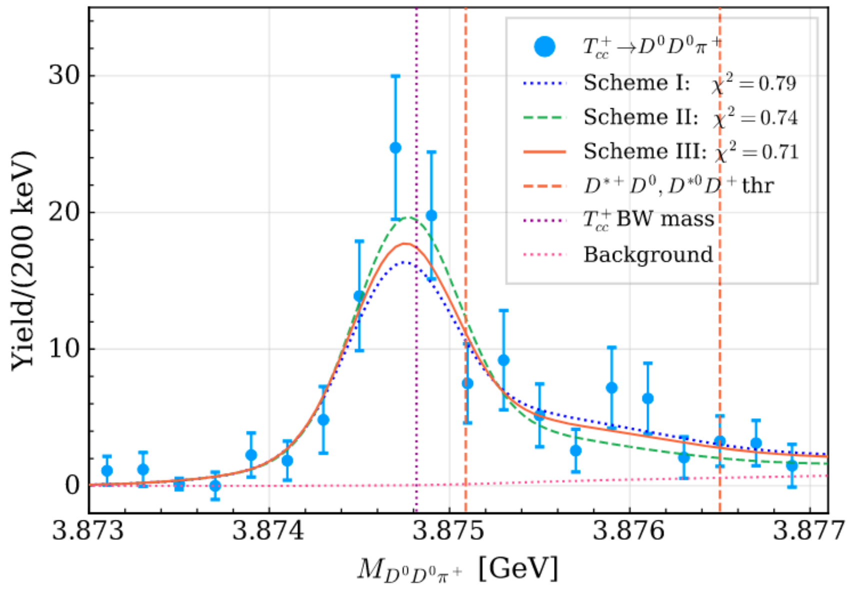


$(I, B) = (0,2)$



✓ $T_{cc}(3875)$ as D^*D molecule

✓ The LHCb data can be well described in a pionful EFT w/ 3-body effects



M.-L. Du et al., PRD 105 (2022) 014024

✓ isoscalar DD^* molecular state

✓ It has a spin partner 1^+ D^*D^* state

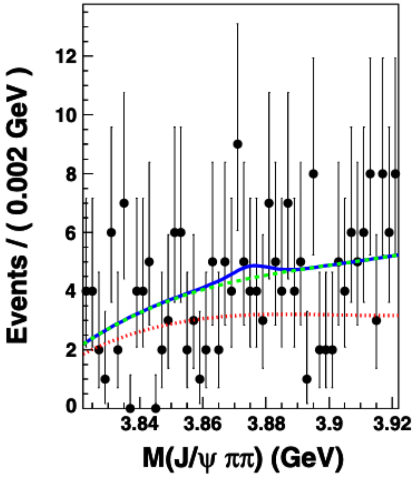
✓ Many (> 100) other similar double-charm molecular states

Closer look at $X(3872)$: Isospin-1 partner?

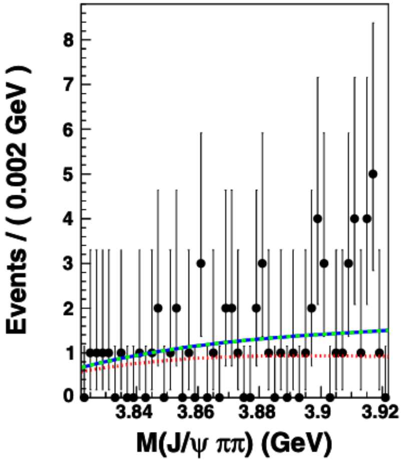
- Isospin-1 partner of X(3872) was predicted in the compact tetraquark model

L. Maiani, F. Piccinini, A.D. Polosa, V. Riquer, PRD 71 (2004) 014028

- No signal in the charged channel so far



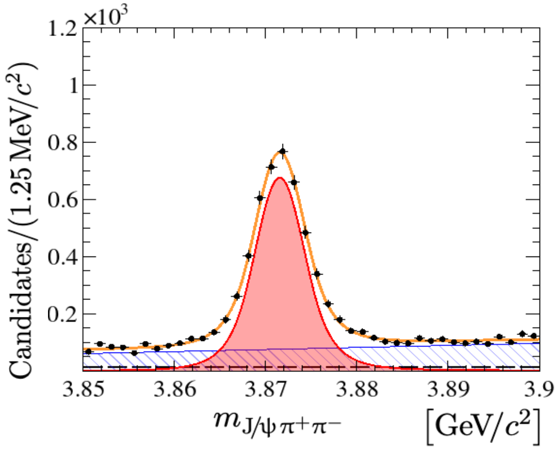
$\bar{B}^0 \to K^- \pi^+ \pi^0 J/\psi$



$B^+ \to K^0 \pi^+ \pi^0 J/\psi$

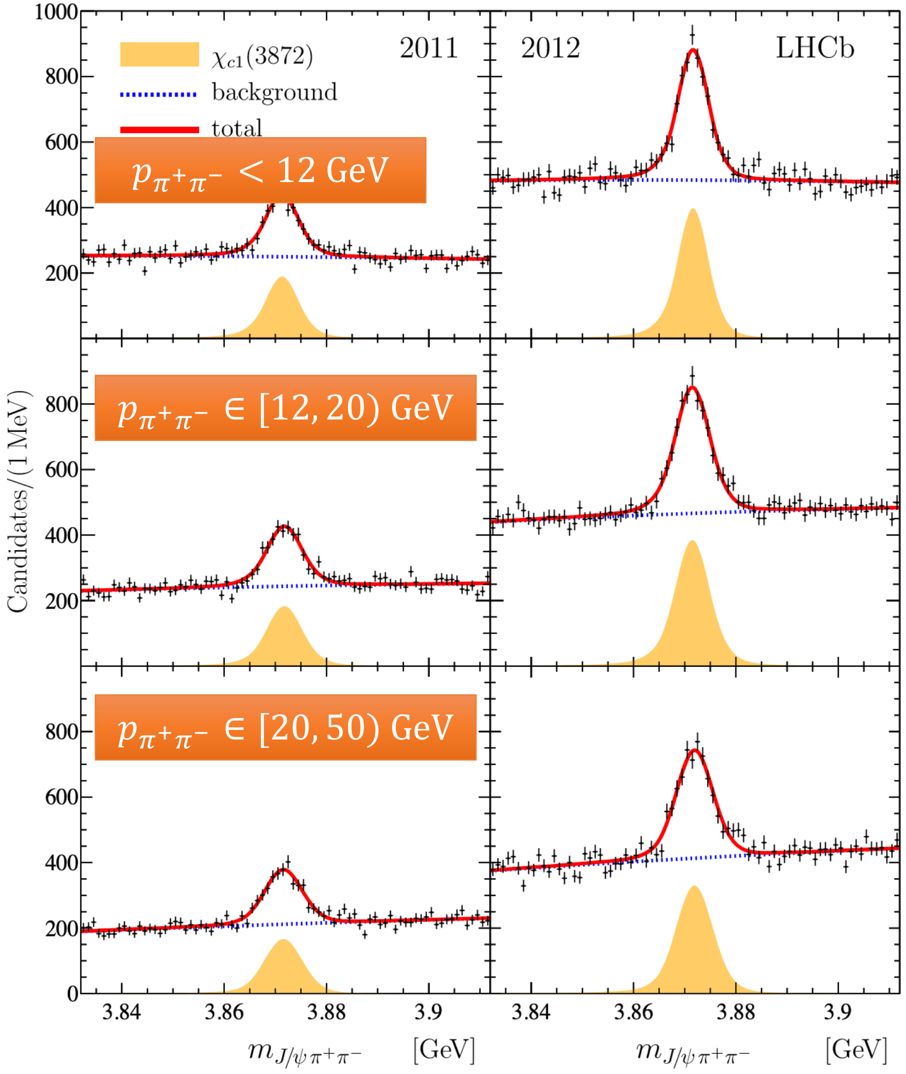
Belle, PRD 84 (2011) 052004

- No signal around the $D^+ D^{*-}$ threshold



$B^+ \to K^+ \pi^+ \pi^- J/\psi$

LHCb, JHEP 08 (2020) 123



$\pi^+ \pi^- J/\psi$ from b-hadrons

LHCb, PRD 102 (2020) 092005

$J^{PC} = 1^{++}$ sector

- Hadronic molecules: consider S-wave interactions between charm and anti-charm mesons
- For the $J^{PC} = 1^{++}$ sector, also two LECs at LO in nonrelativistic expansion:
  - $I = 0: C_{0X}$; $I = 1: C_{1X}$
- Two inputs from $X(3872)$ properties :
  - Mass

$$M_X = 3871.69^{+0.00+0.05}_{-0.04-0.13}\text{MeV}$$

LHCb, PRD 102 (2020) 092005

  - Isospin breaking in decays

$$M_{D^0} + M_{D^{*0}} = 3871.69(7)\ \text{MeV}$$

PDG 2024

$$R_X = \left|\frac{\mathcal{M}_{X(3872)\to J/\psi\rho^0}}{\mathcal{M}_{X(3872)\to J/\psi\omega}}\right| = 0.29 \pm 0.04$$

LHCb, PRD 108 (2023) L011103

- Neutral systems X and W_{c1}^0: coupled channels
  - ✓ $(D\bar{D}^*)_0 \equiv (D^0\bar{D}^{*0} - \bar{D}^0 D^{*0})/\sqrt{2}$
  - ✓ $(D\bar{D}^*)_\pm \equiv (D^+D^{*-} - D^-D^{*+})/\sqrt{2}$
- Charged systems $W_{c1}^\pm$: single channel

Nomenclature following M. Voloshin, PRD 83 (2011) 031502(R), for hidden-bottom heavy quark partners of Z_b predicted in A.E. Bondar et al., PRD 84 (2011) 054010

Extracted using BW for resonances;
Updated to 0.26 ± 0.03 using Omnes repr. for $\pi\pi$ P-wave $\mathcal{M}_{X\to J/\psi\pi\pi} = \mathcal{N}\varepsilon_{ijk}\varepsilon_\psi^i\varepsilon_X^j q_\pi^k P(s)\Omega(s)\left[1+\kappa_X G_\omega(s)\right]$
J. Dias et al., PRD 111 (2025) 014031

Omnes — $\rho\omega$ mixing

Prediction of an isospin vector partner of $X(3872)$

Z.-H. Zhang, T. Ji, X.-K. Dong, FKG, C.Hanhart, U.-G. Meißner, A. Rusetsky, JHEP 08 (2025) 130

- There must be near-threshold isovector W_{c1} states
  - Virtual state pole in the stable D^* limit
    - W_{c1}^{+} in $D^+\bar{D}^{*0}$ single-channel scattering amplitude: pole on the 2nd Riemann sheet (RS), 8^{+8}_{-5} MeV below D^0D^{*-} threshold

$$W_{c1}^{\pm}: \quad 3866.9^{+4.6}_{-7.7} - i(0.07 \pm 0.01)\ \text{MeV}$$

    - W_{c1}^{0} in $(D\bar{D}^*)_0 - (D\bar{D}^*)_\pm$ scattering amplitudes: pole on the 4th RS (RS_{+-}), $1.3^{+0.8}_{-0.0}$ MeV above D^+D^{*-} threshold

$$W_{c1}^{0}: \quad 3881.2^{+0.8}_{-0.0} + i1.6^{+0.7}_{-0.9}\ \text{MeV}$$

  - Must appear as threshold cusps!!!
  - Compact tetraquarks (Maiani et al. (2005)) cannot be virtual states as they do not feel the thresholds

- Virtual state W_{c1} was confirmed in lattice QCD calculation with $M_\pi = 280$ MeV

M. Sadl et al., PRD 111 (2025) 054513

J^{PC}	Interpolators	$1/a_0$ [fm^{-1}]	r_0 [fm]	$\chi^2/N_{\rm dof}$	$\Delta m_{\rm V}$ [MeV]
1^{+-}	All	$0.46^{+1.16}_{-0.45}$	$0.96^{+0.43}_{-0.73}$	0.13	$-3.0^{+3.0}_{-31.1}$
	$\eta_c\rho$ excl	$0.54^{+1.07}_{-0.44}$	$2.23^{+0.95}_{-1.08}$	0.24	$-2.8^{+2.6}_{-17.1}$
1^{++}	All	$0.62^{+1.30}_{-0.51}$	$1.78^{+0.25}_{-2.44}$	0.18	$-3.8^{+3.6\text{a}}$
	$J/\psi\rho, \eta_c a_0$ excl	$0.96^{+1.42}_{-0.91}$	$2.19^{+0.36}_{-1.00}$	0.15	$-6.7^{+6.7}_{-19.5}$

[a]Uncertainty is so large that it is unbounded from below.

sign convention different from ours

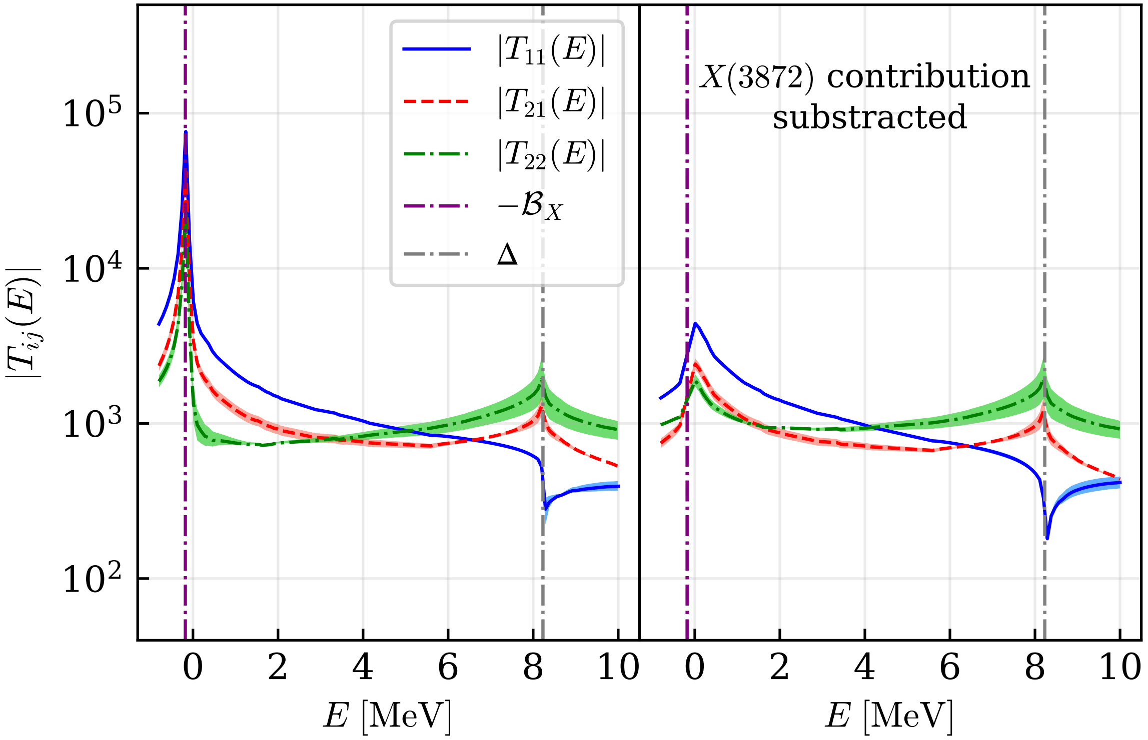
- Also in one-boson exchange model

X.-X. Chen, Z.-M. Ding, J. He, PRD 111 (2025) 114008

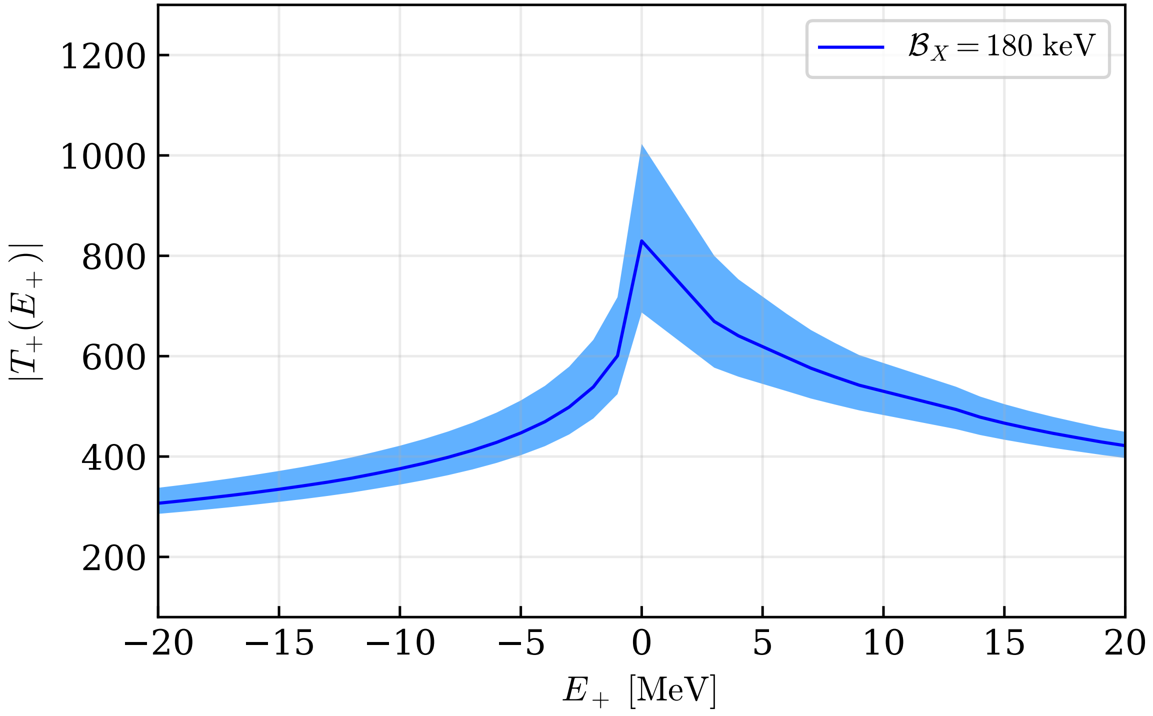
Prediction of an isospin vector partner of $X(3872)$

- There must be an isospin vector partner W_{c1}
  - Virtual state pole in the stable D^* limit $\Rightarrow$ explains why it has not observed so far!

➢ W_{c1}^0 in $D^0\bar{D}^{*0} - D^+D^{*-}$ scattering amplitudes



➢ W_{c1}^+ in $D^+\bar{D}^{*0}$ scattering amplitude



➢ to be searched for in high-statistic $J/\psi\pi^+\pi^0$ data

➢ Compact tetraquark (Maiani et al. (2005)) cannot be virtual state!

Combined analysis of BESIII and LHCb data for $X(3872)$

Teng Ji, X.-K. Dong, FKG, C. Hanhart, U.-G. Meißner, arXiv:2502.04458

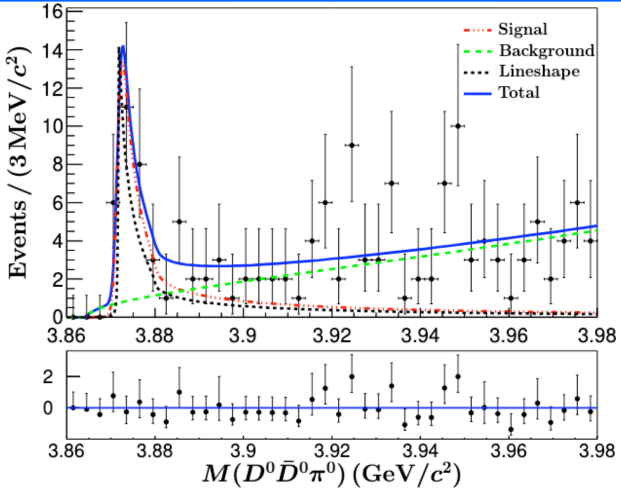
- $X(3872)$ line shapes ⇒ $X(3872)$ + possible $W_{c1}(3880)^0$
- $\pi^+\pi^-$ invariant mass distribution ⇒ isospin breaking, information on $I = 1$

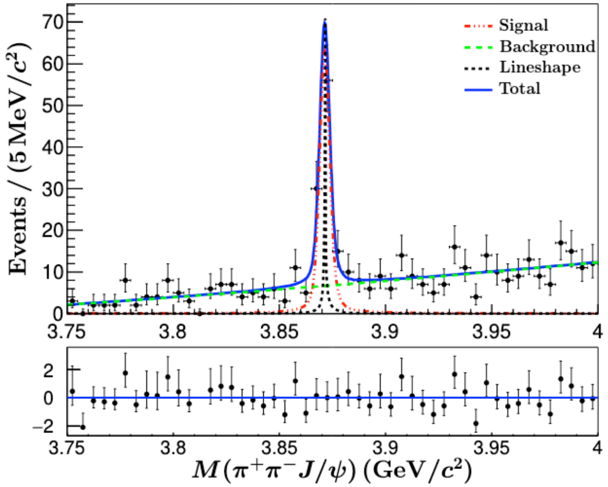
BESIII:

$e^+e^- \to \gamma[D^0\bar{D}^0\pi^0]$

$e^+e^- \to \gamma[J/\psi\pi^+\pi^-]$

BESIII, PRL 132 (2024) 151903

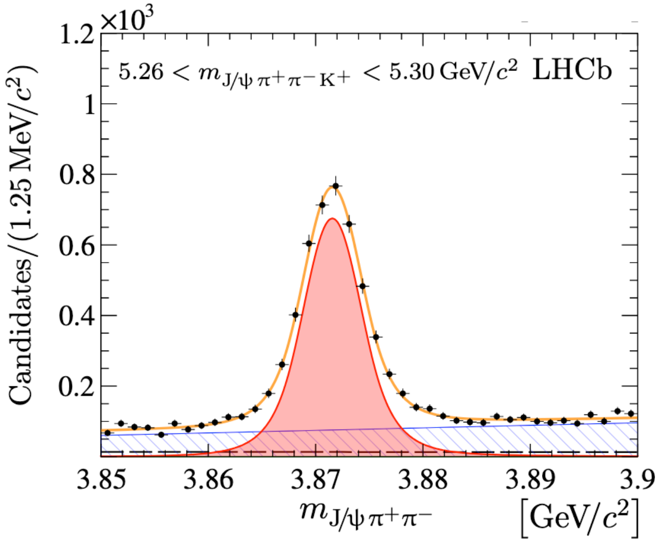


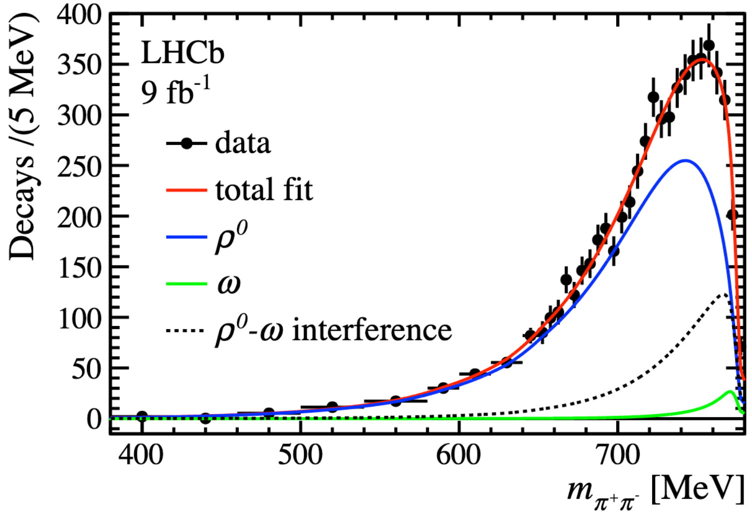


LHCb:

$B^+ \to K^+[J/\psi\pi^+\pi^-]$

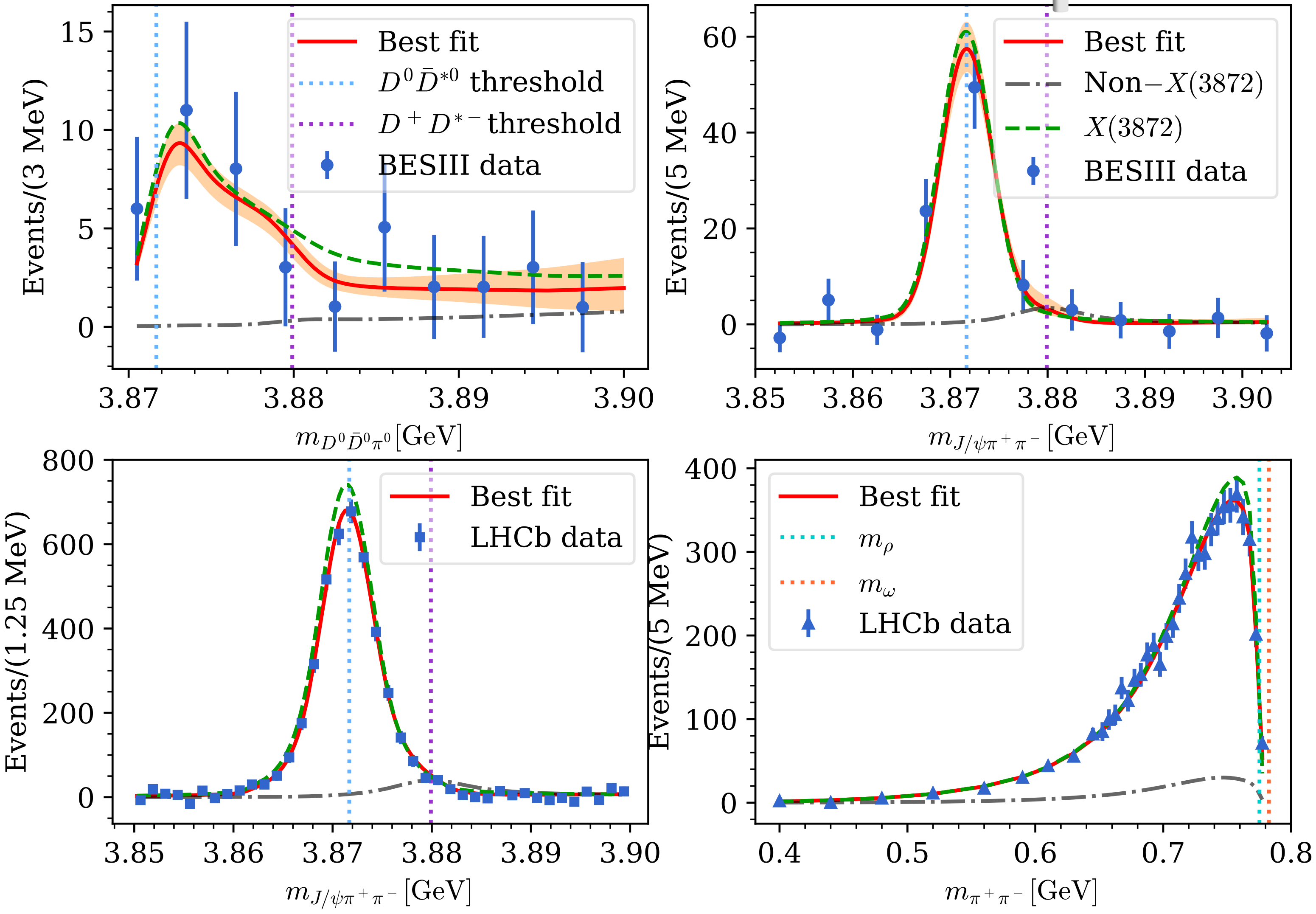
LHCb, JHEP 08 (2020) 123;
PRD 108 (2023) L011103





Combined analysis of BESIII and LHCb data for $X(3872)$

Results updated



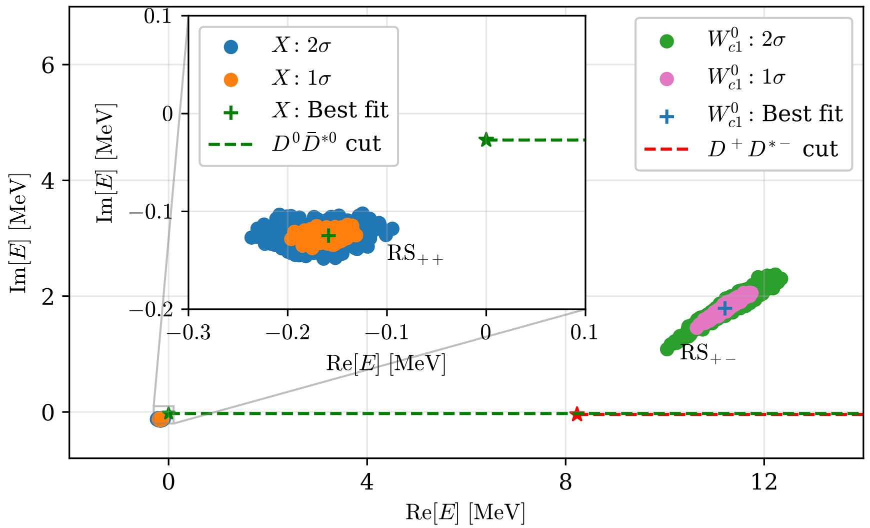
Best fit: $\chi^2/\mathrm{dof} = 57/(96-10) = 0.66$

- Poles
  - $X(3872)$ as a bound state below $D^0\bar{D}^{*0}$ threshold (2.8σ)

$$E_X = \left(-160^{+43+38}_{-47-57} - 125^{+18+15}_{-25-28}i\right)\ \mathrm{keV}$$

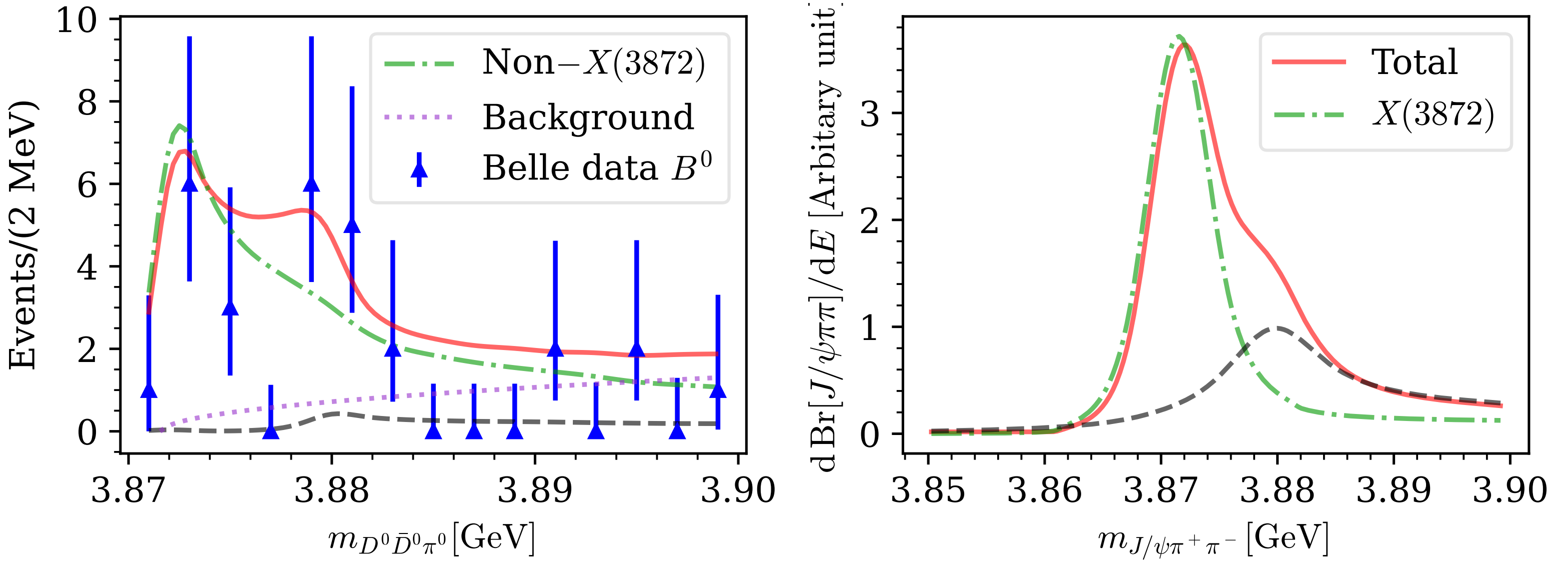
  - $W_{c1}(3880)^0$ pole on RS_{+-}, relative to the D^+D^{*-} threshold:

$$E_W = \left(3.1 \pm 0.7 + 1.3^{+1.9}_{-0.6}i\right)\ \mathrm{MeV}$$



Implications of the existence of $W_{c1}(3880)$

- $W_{c1}(3880)^0$ signal should be stronger in $B^0 \to K^0[D^0\bar{D}^0\pi^0, J/\psi\pi^+\pi^-]$ decays, to be checked @ LHCb, Belle II



- Cusp at $D^+\bar{D}^{*0}$ threshold in $J/\psi\pi^\pm\pi^0$

Summary

- General rules for (near-)threshold structures
  - S-wave attraction, more prominent for heavier particles and stronger attraction
  - Pole behavior: distinct line shapes depending on reaction mechanism
  - Universality: a dip (for large $|a_{22}|$) at the higher channel threshold in T_{11}
- Large number of hadronic molecular candidates
  - Prediction of an isovector partner of $X(3872)$: $W_{c1}^{\pm,0}$ as virtual states

:

Thank you for your attention!

Other properties of X(3872)

Teng Ji, X.-K. Dong, FKG, C. Hanhart, U.-G. Meißner, arXiv:2502.04458

- Width (twice of the imaginary part of the pole): 250^{+36+30}_{-50-56} keV
  - Branching fractions computed using the method in L.A. Heuser, G. Chanturia, FKG, C. Hanhart, M. Hoferichter, B. Kubis, EPJC 84 (2024) 599

Mode	$D^0\bar{D}^0\pi^0$	$D^0\bar{D}^0\gamma$	$J/\psi\pi^+\pi^-$	$J/\psi\pi^+\pi^-\pi^0$	others
BR(%)	41^{+3}_{-4}	22 ± 2	5^{+2}_{-1}	16^{+4}_{-3}	16 ± 2

- Isospin breaking ratio $R_X \equiv \left|\frac{g_{XJ/\psi\rho}}{g_{XJ/\psi\omega}}\right| = 0.26(2)$
- Compositeness using a formula including range corrections Y. Li, FKG, J.-Y. Pang, J.-J. Wu, PRD 105 (2022) L071502

$$X = 1 - \exp\left(\frac{1}{\pi}\int_0^\infty dE\,\frac{\mathrm{Re}\,\delta(E)}{E - \mathrm{Re}\,E_X}\right) = 0.97(2)$$

Results updated