# Quantum Cosmology in 3-Geometry Superspace

**M.L. Fil'chenkov, Yu.P. Laptev**

*RUDN University, Moscow, Russia*

*E-mail: fmichael@mail.ru*

# Outline

- **Introduction**
- **Wheeler-DeWitt's Equation**
- **WKB approximation in quantum geometrodynamics**
- **Time emergence in quantum cosmology**
- **Conclusion**
- **References**

# Introduction

The four-dimensional space-time is split into time and a three-dimensional space of instantaneous configurations, forming a space-time of 3-geometries being considered in quantum cosmology. On the other hand, its classical limit describes the Universe's time. Therein lies the time problem in quantum cosmology, which was considered by philosophers, mainly on the level of interpretations [1].

Consider the time problem in the framework of quantum geometrodynamics.

# Wheeler-DeWitt's Equation

In quantum cosmology, the Universe's wave function is described in space of 3-geometries, i.e. $\frac{\partial\psi(\ {}^3G)}{\partial t} = 0$ is assumed. Hence we obtain Wheeler-DeWitt's equation $\widehat{H}\psi = 0$. Since each 3-geometry describes a spatial configuration at a certain instant, time is present in 3-geometry superspace implicitly. The superspace of 3-geometries is imaginable in the form of a set of photos made at different instants. The combination of 3-geometries describes an implicit dependence of the superspace on time as well as it occurs during a film demonstration. The 3-geometries are space-like cross-sections of the 4-dimensional space-time, whose realization probability is determined by the absolute value squared $\left|\psi\left( {}^3G\right)\right|^2$ of the wave function.

For a homogeneous isotropic Universe the 1-st Fiedmann equation

$$\frac{1}{2}\left(\frac{da}{d\eta}\right)^2 - \frac{4\pi G\varepsilon a^2}{3c^2} = -\frac{kc^2}{2}, \tag{1}$$

is describable in the form of Hamiltonian connection

$$H = \frac{p_a^2}{2} + \frac{ka^2}{2} - \frac{4\pi G\varepsilon a^4}{3c^4} = 0\,, \tag{2}$$

where $\varepsilon$ is the energy density, $a$ the scale factor, $k = 0, \pm 1$ the model parameter, $p_a = \frac{da}{d\eta}$ the generalized momentum, $\eta$ the conformal time defined by the relation $cdt = ad\eta$.

Hence it follows that the Lagrangian

$$L = \frac{p_a^2}{2} - \frac{ka^2}{2} + \frac{4\pi G\varepsilon a^4}{3c^4} \tag{3}$$

and the generalized momentum

$$p_a = \sqrt{\frac{8\pi G\varepsilon a^4}{3c^4} - ka^2}. \tag{4}$$

Replacing, in the Hamiltonian connection, the quantity $p_a$ by the operator $\hat{p}_a = \frac{l_{pl}^2}{i}\frac{d}{da}$, we obtain Wheeler-DeWitt's equation in the minisuperspace of the scale factors [3]

$$\frac{d^2\psi}{da^2} - V(a)\psi = 0, \tag{5}$$

where

$$V(a) = \frac{1}{l_{pl}^4}\left(ka^2 - \frac{8\pi G\varepsilon a^4}{3c^4}\right). \quad (6)$$

From the relation $Ld\eta = p_a da$ we find the dependence of the synchronous time on the scale factor

$$t = \frac{1}{cl_{pl}^2}\int\frac{ada}{\sqrt{-V}}. \quad (7)$$

# WKB approximation of quantum geometrodynamics

Consider a WKB approximation of quantum geometrodynamics. The WKB wave function has the form $\psi \sim e^{\frac{iS}{\hbar}}$, where the action reads

$$S = \hbar \int \sqrt{-V}\, da. \tag{8}$$

Find a relation between $t(a)$ and $S(a)$ in the form

$$t = \frac{\hbar}{c l_{pl}^2} \int \frac{a da}{\frac{dS}{da}}\,. \tag{9}$$

Since time is determined by the WKB wave function phase, the classical world proves to be programmed on the quantum level [2].

For the multicomponent medium with

$$\varepsilon(a) = \varepsilon_0 \sum_n B_n \left(\frac{r_0}{a}\right)^n, \tag{10}$$

where $n = 3(1 + w)$, $\sum_n B_n = 1$, $\frac{1}{r_0^2} = \frac{8\pi G \varepsilon_0}{3c^4}$, $r_0$ is de Sitter's horizon.

In the case of a barotropic equation of state we have $p = w\varepsilon$. Consider the dependences of the scale factor on time and the corresponding ones for the WKB wave function phase on the scale factor for one-component media with $k = 0$.

The scale factor

$$a(t) = r_0 \left(\frac{nct}{2r_0}\right)^{\frac{2}{n}} \text{ for } n \neq 0, \tag{11}$$

$$a(t) = r_0 e^{\frac{ct}{r_0}} \text{ for } n = 0. \tag{12}$$

The wave function phase

$$\frac{S}{\hbar} = \frac{r_0^{\frac{n}{2}-1} a^{3-\frac{n}{2}}}{\left(3-\frac{n}{2}\right) l_{pl}^2} \text{ for } n \neq 6, \tag{13}$$

$$\frac{S}{\hbar} = \left(\frac{r_0}{l_{pl}}\right)^2 ln \frac{a}{r_0} \text{ for } n = 6. \tag{14}$$

Consider the above-mentioned dependences for de Sitter's vacuum, i.e. for $w = -1,\ n = 0$ :

$$a(t) = r_0 \left(\frac{nct}{2r_0}\right)^{\frac{2}{n}} , \frac{S}{\hbar} = \frac{a^3}{3 r_0 l_{pl}^2} . \tag{15}$$

De Sitter's vacuum responsible for the first inflation is unstable, since the sound velocity $v_s = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s}$ is imaginary in this case.

At $t \sim 10^{-33}\ s$ from the singularity there occurs the Big Bang being accompanied by creation of ultrarelativistic particles and radiation with the equation of state $w = \frac{1}{3},\ n = 1.$ In this case we obtain:

$$a(t) = r_0\sqrt{\frac{2ct}{r_0}}\ , \frac{S}{\hbar} = \frac{r_0 a}{l_{pl}^2}\ . \tag{16}$$

# Time emergence in quantum cosmology

Reduce Wheeler-DeWitt's equation in minisuperspace of 3-geometries, allowing for the relation

$$V(a) = \frac{2m_{pl}}{\hbar^2}[U(a) - E], \tag{17}$$

to an equation of Schrödinger's steady-state equation type

$$\frac{d^2\psi}{da^2} - \frac{2m_{pl}}{\hbar^2}[U(a) - E]\psi = 0, \tag{18}$$

where

$$E = \frac{m_{pl}c^2}{2}\left(\frac{r_0}{l_{pl}}\right)^2 B_4 \, . \tag{19}$$

This equation describes the Universe, behaving as an ultrarelativistic planckeon in the field formed by types of matter with $w \neq \frac{1}{3}$, to which corresponds the potential energy $U(a)$. The Universe's birth from de Sitter's vacuum, as a result of a quantum fluctuation, is interpreted as a tunnelling of the planckeon from pre-de Sitter's stage through a potential barrier [3] .

The tunnelling probability is given by Gamow's formula

$$D = exp\left(-\left|\frac{2}{\hbar}\int_{a_1}^{a_2}\sqrt{E-U}\,da\right|\right), \quad (20)$$

where $U(a_1) = U(a_2) = E$ .

Under the barrier, i.e. for $E < U,$ time is imaginary due to the imaginariness of the WKB action on which it depends. After tunnelling through the barrier, i.e. in the classical domain, there arise a real time, the dependence of the scale factor on which describes Friedmann's Universe evolution. Thus, although time is not present in quantum cosmology explicitly, it emerges in classical cosmology.

# Conclusion

In quantum geometrodynamics, the Universe's wave function is written in the minisuperspace of scale factors, implicitly dependent on time, whereas the WKB wave function phase depends on the scale factor, describing the Universe's evolution. A relation between the dependence of the scale factor on time and the dependence of the wave function phase on the scale factor has been found for one-component media, which predetermines the classical world on the quantum level. This proves to be possible, since each 3-geometry describes the spatial configuration at a certain instant, which means that the superspace of scale factors contains time implicitly. The Universe's birth makes explicit this dependence.

# References

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3. E.P. Tryon. "Nature", v. 246, p. 396(1973).