21 LOMONOSOV CONFERENCE ON ELEMENTARY PARTICLE PHYSICS

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EINSTEIN-MAXWELL TETRAD GRAND UNIFICATION

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TETRADS IN GEOMETRODYNAMICS.

$$f^{\mu\nu}{}_{;\nu} = 0$$

$$*f^{\mu\nu}{}_{;\nu} = 0$$

$$R_{\mu\nu} = f_{\mu\lambda}\, f_{\nu}{}^{\lambda} + *f_{\mu\lambda}\, *f_{\nu}{}^{\lambda}$$

$$f_{\mu\nu} = \left(G^{1/2} / c^2 \right) F_{\mu\nu}$$ geometrized electromagnetic field.

$$*f_{\mu\nu} = \frac{1}{2}\, \epsilon_{\mu\nu\sigma\zeta}\, f^{\sigma\zeta}$$ dual tensor of $f_{\mu\nu}$.

WE MAKE THE DEFINITION

$$\xi_{\eta\nu} = \cos\alpha \; f_{\eta\nu} - {}^{*}f_{\eta\nu} \sin\alpha$$

THE FIELD HAS UNDERGONE A DUALITY ROTATION BY AN ANGLE $-\alpha$.

$$\xi_{\eta\nu} = e^{-{}^{*}\alpha} f_{\eta\nu} \quad \text{(NOTATION)}$$

ASSUME THAT THE INVARIANTS

$f_{\eta\nu} f^{\eta\nu}$ AND $f_{\eta\nu} {}^{*}f^{\eta\nu}$ DO NOT VANISH.

- NON NULL FIELD -

CHOOSE THE ANGLE α SO THAT

α : LOCAL SCALAR
THE COMPLEXION

$$\xi_{\eta\nu} {}^{*}\xi^{\eta\nu} = 0 \quad \text{(MISNER AND WHEELER)}$$

$$\Rightarrow \quad \tan(2\alpha) = -(f_{\eta\nu} {}^{*}f^{\eta\nu}) / (f_{\sigma\delta} f^{\sigma\delta})$$

STRESS - ENERGY TENSOR

$$T_{\eta\nu} = f_{\eta\lambda}\, f_{\nu}{}^{\lambda} + {}^{*}f_{\eta\lambda}\, {}^{*}f_{\nu}{}^{\lambda}$$

$$f_{\eta\nu} = \xi_{\eta\nu} \cos\alpha + {}^{*}\xi_{\eta\nu} \sin\alpha$$ DUALITY ROTATION

$$\Rightarrow T_{\eta\nu} = \xi_{\eta\lambda}\, \xi_{\nu}{}^{\lambda} + {}^{*}\xi_{\eta\lambda}\, {}^{*}\xi_{\nu}{}^{\lambda}$$

$\xi_{\eta\nu}$ EXTREMAL FIELD

WE WOULD LIKE TO FIND A TETRAD THAT DIAGONALIZES $T_{\eta\nu}$ COVARIANTLY.

WE CONSIDER THE NON-NULL ELECTROMAGNETIC FIELDS /

$$Q = \xi_{\eta\nu}\, \xi^{\eta\nu} = -\sqrt{T_{\eta\nu} T^{\eta\nu}} \neq 0.$$

THEN, WE FIND FOUR VECTORS THAT DIAGONALIZE $T_{\eta\nu}$ IN GEOMETRODYNAMICS :

$$V_{(1)}^{\alpha} = \xi^{\alpha\lambda}\, \xi_{\rho\lambda}\, X^{\rho}$$

$$V_{(2)}^{\alpha} = \sqrt{\frac{-Q}{2}}\, \xi^{\alpha\lambda}\, X_{\lambda}$$

$$V_{(3)}^{\alpha} = \sqrt{\frac{-Q}{2}}\, {}^{*}\xi^{\alpha\lambda}\, Y_{\lambda}$$

$$V_{(4)}^{\alpha} = {}^{*}\xi^{\alpha\lambda}\, {}^{*}\xi_{\rho\lambda}\, Y^{\rho}$$

WE ARE FREE TO CHOOSE THE TWO VECTOR FIELDS X^{ρ} AND Y^{ρ} AS LONG AS THE FOUR VECTOR FIELDS ARE NOT TRIVIAL

X^{ρ} AND Y^{ρ} : GAUGE VECTORS.

IN A 4-DIM SPACE TIME TWO ANTISYMMMETRIC FIELDS SATISFY THE RELATION:

$$A_{\eta\alpha} B^{\nu\alpha} - {*B}_{\eta\alpha} {*A}^{\nu\alpha} = \frac{1}{2} \delta_\eta^\nu \left(A_{\alpha\beta} B^{\alpha\beta}\right)$$

USING THIS RELATION AND THE EXTREMAL FIELD CONDITION

$\xi_{\eta\nu} {*\xi}^{\eta\nu} = 0$ WE FIND:

$$\boxed{\xi_{\alpha\eta} {*\xi}^{\eta\nu} = 0} \quad (1)$$

USING THE SAME RELATION:

$$\boxed{\xi_{\eta\alpha} \xi^{\nu\alpha} - {*\xi}_{\eta\alpha} {*\xi}^{\nu\alpha} = \frac{1}{2} \delta_\eta^\nu \, Q} \quad (2)$$

$$Q = \xi_{\eta\nu} \xi^{\eta\nu} \neq 0$$

THEN, USING 1 AND 2 WE FIND

$$V_{(1)}^{\alpha} \, T_{\alpha}{}^{\beta} = \frac{a}{2} V_{(1)}^{\beta}$$

$$V_{(2)}^{\alpha} \, T_{\alpha}{}^{\beta} = \frac{a}{2} V_{(2)}^{\beta}$$

$$V_{(3)}^{\alpha} \, T_{\alpha}{}^{\beta} = -\frac{a}{2} V_{(3)}^{\beta}$$

$$V_{(4)}^{\alpha} \, T_{\alpha}{}^{\beta} = -\frac{a}{2} V_{(4)}^{\beta}$$

ELECTROMAGNETIC POTENTIALS IN GEOMETRODYNAMICS.

OUR GOAL: SIMPLIFY AS MUCH AS WE CAN THE EXPRESSION OF THE ELECTROMAGNETIC FIELD THROUGH THE USE OF AN ORTHONORMAL TETRAD

→ SO THAT ITS GEOMETRICAL PROPERTIES CAN BE UNDERSTOOD IN AN EASIER WAY.

IN GEOMETRODYNAMICS:

$$f^{\eta\nu}{}_{;\nu} = 0$$

$$*f^{\eta\nu}{}_{;\nu} = 0$$

$\Rightarrow$ EXISTENCE OF TWO POTENTIAL VECTORS:

$$f_{\eta\nu} = A_{\nu;\eta} - A_{\eta;\nu}$$

$$*f_{\eta\nu} = *A_{\nu;\eta} - *A_{\eta;\nu}$$

A_η AND $*A_\eta$ (NOTATION) NOT INDEPENDENT FROM EACH OTHER.

WE CAN MAKE THE CHOICE

$$X^{\rho} = A^{\rho}$$

$$Y^{\rho} = *A^{\rho}$$

X^{ρ} AND Y^{ρ} : WE ARE FREE TO CHOOSE THE "GAUGE VECTORS"

THEN, THE FOUR VECTORS ARE :

$$V_{(1)}^{\alpha} = \xi^{\alpha\lambda} \, \xi_{\rho\lambda} \, A^{\rho}$$

$$V_{(2)}^{\alpha} = \sqrt{\frac{-Q}{2}} \; \xi^{\alpha\lambda} \, A_{\lambda}$$

$$V_{(3)}^{\alpha} = \sqrt{\frac{-Q}{2}} \; *\xi^{\alpha\lambda} \, *A_{\lambda}$$

$$V_{(4)}^{\alpha} = *\xi^{\alpha\lambda} \, *\xi_{\rho\lambda} \, *A^{\rho}$$

WE ASSUME FOR SIMPLICITY

$$-V_{(1)}^{\alpha} \, V_{(1)\alpha} = V_{(2)}^{\alpha} \, V_{(2)\alpha} > 0 \; // \; V_{(3)}^{\alpha} \, V_{(3)\alpha} = V_{(4)}^{\alpha} \, V_{(4)\alpha} > 0$$

GAUGE GEOMETRY

ONCE WE MAKE THE CHOICE

$$X^\rho = A^\rho \qquad Y^\rho = {}^*A^\rho$$

WHAT HAPPENS TO THE TETRAD VECTORS WHEN WE MAKE THE TRANSFORMATIONS:

NEW CHOICE OF GAUGE VECTORS:

$$A_\alpha \longrightarrow A_\alpha + \Lambda_{,\alpha}$$

$$^*A_\alpha \longrightarrow {}^*A_\alpha + {}^*\Lambda_{,\alpha}$$

NOTATION

$$\Lambda_{,\alpha} = \Lambda_\alpha \qquad {}^*\Lambda_{,\alpha} = {}^*\Lambda_\alpha$$

Λ AND ${}^*\Lambda$ ARE SCALARS.

SCHOUTEN DEFINED WHAT HE CALLED A TWO-BLADED STRUCTURE IN A SPACETIME.

BLADE ONE : $\left(V^{\alpha}_{(1)}, V^{\alpha}_{(2)}\right)$

BLADE TWO : $\left(V^{\alpha}_{(3)}, V^{\alpha}_{(4)}\right)$.

GAUGE TRANSFORMATIONS ON BLADE ONE

$$\begin{cases} \tilde{V}^{\alpha}_{(1)} = V^{\alpha}_{(1)} + \boxed{\mathcal{J}^{\alpha\lambda}\mathcal{J}_{\rho\lambda}\Lambda^{\rho}} \\ \tilde{V}^{\alpha}_{(2)} = V^{\alpha}_{(2)} + \boxed{\sqrt{\frac{-d}{2}}\,\mathcal{J}^{\alpha\lambda}\Lambda_{\lambda}} \end{cases}$$

$$\mathcal{J}^{\alpha\lambda}\mathcal{J}_{\rho\lambda}\Lambda^{\rho}V_{(3)\alpha} = \mathcal{J}^{\alpha\lambda}\mathcal{J}_{\rho\lambda}\Lambda^{\rho}V_{(4)\alpha} = 0$$

$$\sqrt{\frac{-d}{2}}\,\mathcal{J}^{\alpha\lambda}\Lambda_{\lambda}V_{(3)\alpha} = \sqrt{\frac{-d}{2}}\,\mathcal{J}^{\alpha\lambda}\Lambda_{\lambda}V_{(4)\alpha} = 0.$$

WE WRITE THEN,

$$\begin{cases} \tilde{V}_{(1)}^{\alpha} = V_{(1)}^{\alpha} + C\, V_{(1)}^{\alpha} + D\, V_{(2)}^{\alpha} \\ \tilde{V}_{(2)}^{\alpha} = V_{(2)}^{\alpha} + E\, V_{(1)}^{\alpha} + F\, V_{(2)}^{\alpha}. \end{cases}$$

USING RELATIONS 1 AND 2

WE FIND: $E = D$

$F = C$

$$\begin{cases} C = \left(-\frac{\alpha}{2}\right) V_{(1)\sigma} \Lambda^{\sigma} / \left(V_{(2)\beta} V_{(2)}^{\beta}\right) \\ D = \left(-\frac{\alpha}{2}\right) V_{(2)\sigma} \Lambda^{\sigma} / \left(V_{(1)\beta} V_{(1)}^{\beta}\right). \end{cases}$$

$$\begin{cases} \tilde{V}_{(1)}^{\alpha} \tilde{V}_{(1)\alpha} = \left[(1+C)^2 - D^2\right] V_{(1)}^{\alpha} V_{(1)\alpha} \\ \tilde{V}_{(2)}^{\alpha} \tilde{V}_{(2)\alpha} = \underbrace{\left[(1+C)^2 - D^2\right]}_{\text{SCALAR CAUSALIJY FACTOR}} V_{(2)}^{\alpha} V_{(2)\alpha} \end{cases}$$

$V_{(1)}^{\alpha} V_{(1)\alpha} = -V_{(2)}^{\alpha} V_{(2)\alpha}.$

CASES: $$\left[(1+c)^2 - D^2\right] > 0$$

① $1+c>0$

$$\frac{\tilde{V}_{(1)}^{\alpha}}{\sqrt{-\tilde{V}_{(1)}^{\beta}\tilde{V}_{(1)\beta}}} = \frac{(1+c)}{\sqrt{(1+c)^2-D^2}}\frac{V_{(1)}^{\alpha}}{\sqrt{-V_{(1)}^{\beta}V_{(1)\beta}}} + \frac{D}{\sqrt{(1+c)^2-D^2}}\frac{V_{(2)}^{\alpha}}{\sqrt{V_{(2)}^{\beta}V_{(2)\beta}}}$$

$$\frac{\tilde{V}_{(2)}^{\alpha}}{\sqrt{\tilde{V}_{(2)}^{\beta}\tilde{V}_{(2)\beta}}} = \frac{D}{\sqrt{(1+c)^2-D^2}}\frac{V_{(1)}^{\alpha}}{\sqrt{-V_{(1)}^{\beta}V_{(1)\beta}}} + \frac{(1+c)}{\sqrt{(1+c)^2-D^2}}\frac{V_{(2)}^{\alpha}}{\sqrt{V_{(2)}^{\beta}V_{(2)\beta}}}$$

AN ELECTROMAGNETIC GAUGE TRANSFORMATION GENERATES A BOOST TRANSFORMATION ON THE NORMALIZED $\left(\frac{V_{(1)}^{\alpha}}{\sqrt{-V_{(1)}^{\beta}V_{(1)\beta}}}, \frac{V_{(2)}^{\alpha}}{\sqrt{V_{(2)}^{\beta}V_{(2)\beta}}}\right)$.

② $1+c<0$

$$\frac{\tilde{V}^{\alpha}_{(1)}}{\sqrt{-\tilde{V}^{\beta}_{(1)}\tilde{V}_{(1)\beta}}} = \frac{[-(1+c)]}{\sqrt{(1+c)^2-D^2}}\frac{\left(-V^{\alpha}_{(1)}\right)}{\sqrt{-V^{\beta}_{(1)}V_{(1)\beta}}} + \frac{[-D]}{\sqrt{(1+c)^2-D^2}}\frac{\left(-V^{\alpha}_{(2)}\right)}{\sqrt{V^{\beta}_{(2)}V_{(2)\beta}}}$$

$$\frac{\tilde{V}^{\alpha}_{(2)}}{\sqrt{\tilde{V}^{\beta}_{(2)}\tilde{V}_{(2)\beta}}} = \frac{[-D]}{\sqrt{(1+c)^2-D^2}}\frac{\left(-V^{\alpha}_{(1)}\right)}{\sqrt{-V^{\beta}_{(1)}V_{(1)\beta}}} + \frac{[-(1+c)]}{\sqrt{(1+c)^2-D^2}}\frac{\left(-V^{\alpha}_{(2)}\right)}{\sqrt{V^{\beta}_{(2)}V_{(2)\beta}}}$$

AN ELECTROMAGNETIC GAUGE TRANSFORMATION GENERATES THE COMPOSITION OF AN INVERSION AND A BOOST OF THE NORMALIZED

$$\left(\frac{V^{\alpha}_{(1)}}{\sqrt{-V^{\beta}_{(1)}V_{(1)\beta}}}\,,\;\frac{V^{\alpha}_{(2)}}{\sqrt{V^{\beta}_{(2)}V_{(2)\beta}}}\right).$$

SPECIAL
IMPROPER
TRANSFORMATIONS
ON BLADE 1.

$$\left[(1+c)^2 - D^2\right] < 0$$

$$\tilde{V}_{(1)}^{\alpha}\,\tilde{V}_{(1)\alpha} = \left[-(1+c)^2 + D^2\right]\left(-V_{(1)}^{\alpha}\,V_{(1)\alpha}\right)$$

$$\left(-\tilde{V}_{(2)}^{\alpha}\,\tilde{V}_{(2)\alpha}\right) = \left[-(1+c)^2 + D^2\right]\left(V_{(2)}^{\alpha}\,V_{(2)\alpha}\right).$$

3 $\qquad 1+c > 0$

$$\frac{\tilde{V}_{(1)}^{\alpha}}{\sqrt{\tilde{V}_{(1)}^{\beta}\,\tilde{V}_{(1)\beta}}} = \frac{(1+c)}{\sqrt{-(1+c)^2+D^2}}\,\frac{V_{(1)}^{\alpha}}{\sqrt{-V_{(1)}^{\beta}\,V_{(1)\beta}}} + \frac{D}{\sqrt{-(1+c)^2+D^2}}\,\frac{V_{(2)}^{\alpha}}{\sqrt{V_{(2)}^{\beta}\,V_{(2)\beta}}}$$

$$\frac{\tilde{V}_{(2)}^{\alpha}}{\sqrt{-V_{(2)}^{\beta}\,V_{(2)\beta}}} = \frac{D}{\sqrt{-(1+c)^2+D^2}}\,\frac{V_{(1)}^{\alpha}}{\sqrt{-V_{(1)}^{\beta}\,V_{(1)\beta}}} + \frac{(1+c)}{\sqrt{-(1+c)^2+D^2}}\,\frac{V_{(2)}^{\alpha}}{\sqrt{V_{(2)}^{\beta}\,V_{(2)\beta}}}.$$

BOOST $\begin{pmatrix} \frac{D}{\sqrt{-(1+c)^2+D^2}} & \frac{(1+c)}{\sqrt{-(1+c)^2+D^2}} \\ \frac{(1+c)}{\sqrt{-(1+c)^2+D^2}} & \frac{D}{\sqrt{-(1+c)^2+D^2}} \end{pmatrix}$ COMPOSED WITH

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ NOT A LORENTZ TRANSF.

REFLECTION

(4) $\quad 1+C<0$

$$\frac{\tilde{V}_{(1)}^{\alpha}}{\sqrt{\tilde{V}_{(1)}^{\beta}\,\tilde{V}_{(1)\beta}}} = \frac{[-(1+C)]}{\sqrt{-(1+C)^2+D^2}}\,\frac{(-V_{(1)}^{\alpha})}{\sqrt{-V_{(1)}^{\beta}\,V_{(1)\beta}}} + \frac{[-D]}{\sqrt{-(1+C)^2+D^2}}\,\frac{(-V_{(2)}^{\alpha})}{\sqrt{V_{(2)}^{\beta}\,V_{(2)\beta}}}$$

$$\frac{\tilde{V}_{(2)}^{\alpha}}{\sqrt{-\tilde{V}_{(2)}^{\beta}\,\tilde{V}_{(2)\beta}}} = \frac{[-D]}{\sqrt{-(1+C)^2+D^2}}\,\frac{(-V_{(1)}^{\alpha})}{\sqrt{-V_{(1)}^{\beta}\,V_{(1)\beta}}} + \frac{[-(1+C)]}{\sqrt{-(1+C)^2+D^2}}\,\frac{(-V_{(2)}^{\alpha})}{\sqrt{V_{(2)}^{\beta}\,V_{(2)\beta}}}$$

$D = 1+C \quad$ OR $\quad D = -(1+C)$

MAP INTO THE INTERSECTION WITH THE LOCAL LIGHT CONE 4 SOLUTIONS.

BOOST $\begin{pmatrix} \dfrac{[-D]}{\sqrt{-(1+C)^2+D^2}} & \dfrac{[-(1+C)]}{\sqrt{-(1+C)^2+D^2}} \\ \dfrac{[-(1+C)]}{\sqrt{-(1+C)^2+D^2}} & \dfrac{[-D]}{\sqrt{-(1+C)^2+D^2}} \end{pmatrix}$ COMPOSED WITH $-\mathbb{1}_{2\times 2}$ AND COMPOSED WITH $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ NOT A LORENTZ TRANSFORMATION REFLECTION

GAUGE TRANSFORMATIONS ON BLADE TWO

$$\begin{cases} \tilde{V}^{\alpha}_{(3)} = V^{\alpha}_{(3)} + \sqrt{\frac{-\mathcal{Q}}{2}} \; {}^*f^{\alpha\lambda}\, {}^*\Lambda_{\lambda} \\ \tilde{V}^{\alpha}_{(4)} = V^{\alpha}_{(4)} + {}^*f^{\alpha\lambda}\, {}^*f_{\rho\lambda}\, {}^*\Lambda^{\rho} \end{cases}$$

$$\begin{cases} \sqrt{\frac{-\mathcal{Q}}{2}} \; {}^*f^{\alpha\lambda}\, {}^*\Lambda_{\lambda} \, V_{(1)\alpha} = \sqrt{\frac{-\mathcal{Q}}{2}} \; {}^*f^{\alpha\lambda}\, {}^*\Lambda_{\lambda} \, V_{(2)\alpha} = 0 \\ {}^*f^{\alpha\lambda}\, {}^*f_{\rho\lambda}\, {}^*\Lambda^{\rho} \, V_{(1)\alpha} = {}^*f^{\alpha\lambda}\, {}^*f_{\rho\lambda}\, {}^*\Lambda^{\rho} \, V_{(2)\alpha} = 0 \end{cases}$$

$$\begin{cases} \tilde{V}^{\alpha}_{(3)} = V^{\alpha}_{(3)} + K\, V^{\alpha}_{(3)} + L\, V^{\alpha}_{(4)} \\ \tilde{V}^{\alpha}_{(4)} = V^{\alpha}_{(4)} + M\, V^{\alpha}_{(3)} + N\, V^{\alpha}_{(4)} \end{cases}$$

USING RELATIONS 1 AND 2 $\Rightarrow$

$$K = N \qquad L = -M$$

$$M = \left(-\frac{\mathcal{Q}}{2}\right) V_{(3)\sigma}\, {}^*\Lambda^{\sigma} / \left(V_{(4)\beta}\, V^{\beta}_{(4)}\right)$$

$$N = \left(-\frac{\mathcal{Q}}{2}\right) V_{(4)\sigma}\, {}^*\Lambda^{\sigma} / \left(V_{(3)\beta}\, V^{\beta}_{(3)}\right).$$

$$\begin{cases} \tilde{V}_{(3)}^{\beta} \tilde{V}_{(3)\beta} = \left[(1+N)^2 + M^2\right] V_{(3)}^{\beta} V_{(3)\beta} \\ \tilde{V}_{(4)}^{\beta} \tilde{V}_{(4)\beta} = \left[(1+N)^2 + M^2\right] V_{(4)}^{\beta} V_{(4)\beta} \end{cases}$$

$$\frac{\tilde{V}_{(3)}^{\alpha}}{\sqrt{\tilde{V}_{(3)}^{\beta} \tilde{V}_{(3)\beta}}} = \frac{(1+N)}{\sqrt{(1+N)^2 + M^2}} \frac{V_{(3)}^{\alpha}}{\sqrt{V_{(3)}^{\beta} V_{(3)\beta}}} - \frac{M}{\sqrt{(1+N)^2 + M^2}} \frac{V_{(4)}^{\alpha}}{\sqrt{V_{(4)}^{\beta} V_{(4)\beta}}}$$

$$\frac{\tilde{V}_{(4)}^{\alpha}}{\sqrt{\tilde{V}_{(4)}^{\beta} \tilde{V}_{(4)\beta}}} = \frac{M}{\sqrt{(1+N)^2 + M^2}} \frac{V_{(3)}^{\alpha}}{\sqrt{V_{(3)}^{\beta} V_{(3)\beta}}} + \frac{(1+N)}{\sqrt{(1+N)^2 + M^2}} \frac{V_{(4)}^{\alpha}}{\sqrt{V_{(4)}^{\beta} V_{(4)\beta}}}$$

FOR $(1+N)^2 + M^2 > 0$, THE $*A^{\alpha} \longrightarrow *A^{\alpha} + *\Lambda^{\alpha}$ GAUGE TRANSFORMATION, GENERATES A ROTATION TRANSFORMATION OF THE NORMALIZED VECTORS:

$$\left(\frac{V_{(3)}^{\alpha}}{\sqrt{V_{(3)}^{\beta} V_{(3)\beta}}} , \frac{V_{(4)}^{\alpha}}{\sqrt{V_{(4)}^{\beta} V_{(4)\beta}}} \right).$$

GROUP ISOMORPHISM

LB1 (LORENTZ BLADE ONE)

THE GROUP OF SO(1,1) BOOST TETRAD TRANSFORMATIONS ON BLADE ONE

PLUS $-\mathbb{1}_{2\times 2}$ PLUS $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ NOT A LORENTZ TRANSF.

PLUS 4 LIGHT CONE SOLUTIONS $\Lambda^0{}_0 = \Lambda^1{}_1 = 0$ $\Lambda^0{}_1 = \Lambda^1{}_0 = 1$

LB2 (LORENTZ BLADE TWO)

THE GROUP OF LORENTZ TETRAD TRANSFORMATIONS (ROTATIONS) SO(2) ON BLADE TWO.

LB1 proper ISOMORPHIC TO LB2

LB1 $4 \longrightarrow 1$ LB2

ISOMORPHISM ON BLADE ONE

THEOREM: THE MAPPING

BETWEEN THE LOCAL GAUGE GROUP
AND THE GROUP LB1
HAS A KERNEL $\{\Lambda_\eta / \Lambda_\eta \in \text{plane } 2\}$

KERNEL OF MEASURE ZERO

MINUS THE KERNEL → ISOMORPHISM

ISOMORPHISM ON BLADE TWO

THEOREM: THE MAPPING

BETWEEN THE LOCAL GAUGE GROUP

AND THE GROUP LB2 HAS A KERNEL

$\{\Lambda_\eta / \Lambda_\eta \in \text{plane } 1\}$ KERNEL OF MEASURE ZERO

THEOREM: THE MAPPING BETWEEN THE LOCAL GAUGE GROUP AND THE GROUP

$LB1 \otimes LB2$ IS AN ISOMORPHISM PIECEWISE

EXAMPLE:

REISSNER-NORDSTRÖM

$$ds^2 = -\left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right)dt^2 + \left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right)^{-1} dr^2$$
$$+ r^2\left(d\theta^2 + \sin^2\theta \, d\phi^2\right)$$

$$U^t = -\frac{(|q|/q)}{\sqrt{1 - \frac{2m}{r} + \frac{q^2}{r^2}}}$$

$$V^r = \sqrt{1 - \frac{2m}{r} + \frac{q^2}{r^2}}$$

$$Z^\theta = -\frac{|\cos\theta|}{r\cos\theta}$$

$$W^\phi = -\frac{|q|\,|\cos\theta|}{q\, r \sin\theta \cos\theta}$$

ENERGY MOMENTUM CURRENTS

$$T^{\alpha}{}_{\beta}\,\xi^{\beta}_{j} \qquad j = 1 \dots 4$$

$$\xi_1 = (1, 0, 0, 0) \qquad \xi_2 = (0, \sin\phi, \cos\phi \cot\theta, 0)$$

$$\xi_3 = (0, \cos\phi, -\sin\phi \cot\theta, 0) \qquad \xi_4 = (0, 0, 0, 1)$$

ξ^{β}_{j} : KILLING VECTORS

$$J(\xi_1) = -\frac{q^2}{r^4}\,\xi_1 \qquad J(\xi_a) = \frac{q^2}{r^4}\,\xi_a \qquad a = 2, 3, 4$$

$J(\xi_1)$ BELONGS TO PLANE ONE SPANNED BY (U^{α}, V^{α})

$J(\xi_a)$ BELONGS TO PLANE TWO SPANNED BY (Z^{α}, W^{α}).

GENERAL CURRENTS FROM EINSTEIN-MAXWEL EQS.

$$\xi^{\eta\nu}{}_{;\nu} = - * \xi^{\eta\nu} \alpha_\nu$$ ON PLANE TWO

$$* \xi^{\eta\nu}{}_{;\nu} = \xi^{\eta\nu} \alpha_\nu$$ ON PLANE ONE

$$\alpha_\nu = \alpha_{,\nu} \neq 0$$ NON-TRIVIAL

THESE CURRENTS ARE CONSERVED

$$\xi^{\eta\nu}{}_{;\nu\eta} = -(* \xi^{\eta\nu} \alpha_\nu)_{;\eta} = 0$$

$$* \xi^{\eta\nu}{}_{;\nu\eta} = (\xi^{\eta\nu} \alpha_\nu)_{;\eta} = 0$$

ORTHONORMAL TETRAD

$$U^{\alpha} = \xi^{\alpha\lambda}\,\xi_{\rho\lambda}\,A^{\rho} \Big/ \left(\sqrt{\frac{-Q}{2}}\;\sqrt{A_{\eta}\,\xi^{\eta\sigma}\,\xi_{\nu\sigma}\,A^{\nu}}\right)$$

$$V^{\alpha} = \xi^{\alpha\lambda}\,A_{\lambda} \Big/ \left(\sqrt{A_{\eta}\,\xi^{\eta\sigma}\,\xi_{\nu\sigma}\,A^{\nu}}\right)$$

$$Z^{\alpha} = {*\xi}^{\alpha\lambda}\,{*A}_{\lambda} \Big/ \left(\sqrt{{*A}_{\eta}\,{*\xi}^{\eta\sigma}\,{*\xi}_{\nu\sigma}\,{*A}^{\nu}}\right)$$

$$W^{\alpha} = {*\xi}^{\alpha\lambda}\,{*\xi}_{\rho\lambda}\,{*A}^{\rho} \Big/ \left(\sqrt{\frac{-Q}{2}}\;\sqrt{{*A}_{\eta}\,{*\xi}^{\eta\sigma}\,{*\xi}_{\nu\sigma}\,{*A}^{\nu}}\right).$$

$$-U^{\alpha}\,U_{\alpha} = V^{\alpha}\,V_{\alpha} = Z^{\alpha}\,Z_{\alpha} = W^{\alpha}\,W_{\alpha} = 1.$$

$$g_{\alpha\beta} = -U_\alpha U_\beta + V_\alpha V_\beta + Z_\alpha Z_\beta + W_\alpha W_\beta$$

$$T_{\alpha\beta} = \left(\frac{Q}{2}\right)\left[-U_\alpha U_\beta + V_\alpha V_\beta - Z_\alpha Z_\beta - W_\alpha W_\beta\right]$$

$$f_{\alpha\beta} = -2\sqrt{\frac{-Q}{2}}\cos(\alpha)\ U_{[\alpha} V_{\beta]} +$$

$$+ 2\sqrt{\frac{-Q}{2}}\sin(\alpha)\ Z_{[\alpha} W_{\beta]} .$$

$i^2 = -1$.

NULL TETRAD

$$K_\alpha = \frac{1}{\sqrt{2}}\left(U_\alpha + V_\alpha\right) \qquad L_\alpha = \frac{1}{\sqrt{2}}\left(U_\alpha - V_\alpha\right)$$

$$T_\alpha = \frac{1}{\sqrt{2}}\left(Z_\alpha + i W_\alpha\right) \qquad \bar{T}_\alpha = \frac{1}{\sqrt{2}}\left(Z_\alpha - i W_\alpha\right)$$

BIVECTORS

$$U_{\alpha\beta} = \bar{T}_\alpha L_\beta - \bar{T}_\beta L_\alpha$$

$$V_{\alpha\beta} = K_\alpha T_\beta - K_\beta T_\alpha$$

$$W_{\alpha\beta} = T_\alpha \bar{T}_\beta - T_\beta \bar{T}_\alpha - K_\alpha L_\beta + K_\beta L_\alpha$$

SELF-DUAL ELECTROMAGNETIC BIVECTOR

$$\Phi_{\alpha\beta} = f_{\alpha\beta} + i \, {*f_{\alpha\beta}} = \underbrace{\boxed{-\sqrt{\frac{-Q}{2}}\, e^{-i\alpha}}}_{=\phi_1} W_{\alpha\beta}$$

VACUUM EINSTEIN - MAXWELL EQUATIONS

(NEWMAN - PENROSE TETRAD FORMALISM)

$$D\phi_1 = 2\rho\phi_1$$

$$\delta\phi_1 = -2\tau\phi_1$$

$$\bar{\delta}\phi_1 = -2\pi\phi_1$$

$$\Delta\phi_1 = -2\eta\phi_1 .$$

WE CONSIDER THE TETRAD VECTORS

$$V^{\alpha}_{(1)} = \zeta^{\alpha\lambda} \zeta_{\rho\lambda} X^{\rho}$$

$$V^{\alpha}_{(2)} = \zeta^{\alpha\lambda} X_{\lambda}$$

$$V^{\alpha}_{(3)} = {}^{*}\zeta^{\alpha\lambda} Y_{\lambda}$$

$$V^{\alpha}_{(4)} = {}^{*}\zeta^{\alpha\lambda} \, {}^{*}\zeta_{\rho\lambda} Y^{\rho}$$

IF WE CHOOSE $X^{\rho} = Y^{\rho} = A^{\rho}$

$$V^{\alpha}_{(1)} - V^{\alpha}_{(2)} = g^{\alpha\lambda} g_{\rho\lambda} A^{\rho} - {}^{*}g^{\alpha\lambda}\, {}^{*}g_{\rho\lambda} A^{\rho} =$$

$$= \frac{a}{2} A^{\alpha}$$

USING IDENTITY 2

$$V^{\alpha}_{(1)} V_{(1)\alpha} = \left(-\frac{a}{2}\right)\left(A_{\eta}\, g^{\eta\sigma} g_{\nu\sigma} A^{\nu}\right) =$$

$$= - V^{\alpha}_{(2)} V_{(2)\alpha} \left(\frac{a}{2}\right)$$

WE CALL $C = \dfrac{\left(V^{\alpha}_{(1)} V_{(1)\alpha}\right)}{\left(V^{\beta}_{(2)} V_{(2)\beta}\right)}$

$$A^{\alpha} = -\left(V^{\alpha}_{(1)} - V^{\alpha}_{(2)}\right) / C$$

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$$\tan(2\alpha) = - f_{\mu\nu} \, {}^{*}f^{\mu\nu} \Big/ f_{\lambda\rho} f^{\lambda\rho}$$

$$\xi_{\alpha\beta} = -2\sqrt{\frac{-Q}{2}} \; U_{[\alpha} V_{\beta]}$$

$$^{*}\xi_{\alpha\beta} = 2\sqrt{\frac{-Q}{2}} \; Z_{[\alpha} W_{\beta]}$$

$$f_{\alpha\beta} = -2\sqrt{\frac{-Q}{2}} \cos\alpha \; U_{[\alpha} V_{\beta]}$$

$$+ 2\sqrt{\frac{-Q}{2}} \sin\alpha \; Z_{[\alpha} W_{\beta]}$$

COULOMB CASE $(-+++)$

$$f_{tr} = \frac{e}{r^2} \qquad A_t = \frac{e}{r} \quad A_r = 0$$

$$V^t_{(1)} = \xi^{tr}\, \xi_{tr}\, A^t = |\xi_{tr}|^2 A_t$$

$$V^r_{(1)} = \xi^{rt}\, \xi_{rt}\, A^r = 0$$

$$V^t_{(2)} = |\xi_{tr}|\, \xi^{tr} A_r = 0$$

$$V^r_{(2)} = |\xi_{tr}|\, \xi^{rt} A_t = |\xi_{tr}|\, \xi_{tr} A_t$$

$$Q = -2\, |\xi_{tr}|^2$$

$$V^{\alpha}_{(1)} V_{(1)\alpha} = V^t_{(1)} V_{(1)t} + V^r_{(1)} V_{(1)r} = -|\xi_{tr}|^4 (A_t)^2 + $$

$$+ |\xi_{tr}|^4 (A_r)^2 =$$

$$= -|\xi_{tr}|^4 (A_t)^2$$

$$V^{\alpha}_{(2)} V_{(2)\alpha} = V^{t}_{(2)} V_{(2)t} + V^{r}_{(2)} V_{(2)r} =$$

$$= -|\zeta_{tr}|^4 (A_r)^2 + |\zeta_{tr}|^4 (A_t)^2 =$$

$$= |\zeta_{tr}|^4 (A_t)^2$$

$$V^{\alpha}_{(1)} V_{(1)\alpha} = -V^{\alpha}_{(2)} V_{(2)\alpha} \neq 0$$

THEN, WE TRANSFORM THE GAUGE

$$f_{tr} = \frac{e}{r^2} = \zeta_{tr} \quad A_t = \frac{e}{r} \quad A_r^{new} = -\frac{e}{r}$$

GAUGE TRANSFORMATION: $\Lambda_t = 0 \quad \Lambda_r = -\frac{e}{r}$

$\Lambda_{,\eta} = \Lambda_{\eta}$ NOTATION

$$V^{\alpha}_{(1)} V_{(1)\alpha} = |\zeta_{tr}|^4 \left(-(A_t)^2 + (A_r^{new})^2\right)$$

$$V^{\alpha}_{(2)} V_{(2)\alpha} = |\zeta_{tr}|^4 \left((A_t)^2 - (A_r^{new})^2\right)$$

$$\tilde{V}_{(1)}^{\alpha}\ \tilde{V}_{(1)\alpha} = \left[(1+C)^2 - D^2\right] V_{(1)}^{\alpha}\ V_{(1)\alpha}$$

$$\tilde{V}_{(2)}^{\alpha}\ \tilde{V}_{(2)\alpha} = \left[(1+C)^2 - D^2\right] V_{(2)}^{\alpha}\ V_{(2)\alpha}$$

$$C = \left(-\frac{a}{2}\right) \frac{V_{(1)\sigma}\, \Lambda^{\sigma}}{V_{(2)\beta}\, V_{(2)}^{\beta}}$$

$$D = \left(-\frac{a}{2}\right) \frac{V_{(2)\sigma}\, \Lambda^{\sigma}}{V_{(1)\beta}\, V_{(1)}^{\beta}}$$

$$\left[(1+C)^2 - D^2\right] = (1-0)^2 - \left(\frac{\Lambda r}{A t}\right)^2 = 0$$

REISS-NORDSTRÖM CASE

$$g_{tt} = -\left(1 - \frac{2M}{r} + \frac{e^2}{r^2}\right)$$

$$g_{rr} = \left(1 - \frac{2M}{r} + \frac{e^2}{r^2}\right)^{-1}$$

$$g_{\theta\theta} = r^2$$

$$g_{\phi\phi} = r^2 \sin^2\theta$$

$$V^t_{(1)} = \zeta^{tr} \zeta_{tr} A^t = -g^{tt} |\zeta_{tr}|^2 A_t$$

$$V^r_{(1)} = \zeta^{rt} \zeta_{rt} A^r = -g^{rr} |\zeta_{tr}|^2 A_r$$

$$V^t_{(2)} = |\zeta_{tr}| \zeta^{tr} A_r = -|\zeta_{tr}| \zeta_{tr} A_r$$

$$V^r_{(2)} = |\zeta_{tr}| \zeta^{rt} A_t = |\zeta_{tr}| \zeta_{tr} A_t$$

$$Q = -2 |\zeta_{tr}|^2$$

GAUGE TRANSFORMATION Λ AND NULL CONDITION $\Rightarrow$

$$\Lambda_t - g_{tt}\Lambda_r = -A_t$$

$$\Lambda_H = \exp\left(-i\int_a^r \left(\frac{\omega}{g_{tt}} + \kappa\right) dr\right) \, .$$

$$\cdot \exp\left[i\,(\kappa r - \omega t)\right]$$

a, κ, ω CONSTANTS.

$$\Lambda_{INH} = -\frac{e}{r\left(1 - \frac{2M}{r} + \frac{e^2}{r^2}\right)} = \frac{A_t}{g_{tt}}$$

$$D = \pm (1 + c)$$

$D = -(1 + c)$ $\quad \Lambda_r = g_{rr} A_t$ INH

FOR SU(2) LOCAL GAUGE TRS.
AN ANALOGOUS PROCEDURE.

EXAMPLE : RARE DECAYS

$$b \rightarrow s\gamma$$

LET US CONCENTRATE ON
THE DIAGRAM WITH LOOP $c\bar{c}$
THERE WILL BE A CURRENT
ASSOCIATED TO $c \rightarrow \bar{c} + \gamma$

$$J^{\alpha}_{[c\bar{c}\gamma]}$$

quarks have electric charge $\Rightarrow$

$$E^{\rho}_{[c\bar{c}\gamma]\alpha}$$

WE CAN ASSOCIATE
TO THE VERTEX

$$X^{\rho}_{[c\bar{c}\gamma]} = Y^{\rho}_{[c\bar{c}\gamma]} = J^{\alpha}_{[c\bar{c}\gamma]} E^{\rho}_{[c\bar{c}\gamma]\alpha}$$

INITIAL GUGE FOR SOME SU(2) S

IN GENERAL

$$X^{\sigma} = Y^{\sigma} = \mathrm{Tr}\left[\Sigma^{\alpha\beta} E^{\rho}_{\alpha} E^{\lambda}_{\beta} * J^{\sigma}_{\rho} * J_{\lambda\gamma} A^{\gamma} \right]$$

A^{γ} GAUGED BY SU(2)

FOR SOME SU(2) S, INITIALLY WE KNOW $X^{\sigma} = Y^{\sigma}$

THEN WE PROCEED WITH A LOCAL SU(2) GAUGE TRANSFORMATION AND MAKE THE VECTORS IN PLANE ONE, NULL.

DRELL - YAN PROCESS

LET US FOCUS ON THE DIAGRAM INCLUDING A VIRTUAL PHOTON WITH EMISSION OF A PAIR LEPTON - ANTILEPTON $\gamma^* \rightarrow e^+ + e^-$

THERE WILL BE A CURRENT

$$J^{\alpha}_{[\gamma e^+ e^-]}$$

e^+ AND e^- HAVE ELECTRIC CHARGE $\Rightarrow$

$$E^{\rho}_{[\gamma e^+ e^-]\alpha}$$

TO THE VERTEX $\gamma^* \rightarrow e^+ + e^-$

WE ASSOCIATE INITIAL GAUGE VECTORS

$$X^{\rho}_{[\gamma e^+ e^-]} = Y^{\rho}_{[\gamma e^+ e^-]} = J^{\alpha}_{[\gamma e^+ e^-]} \; E^{\rho}_{[\gamma e^+ e^-]\alpha}$$

UNDER A LOCAL $S \in SU(2)$ THE PLANE ONE VECTORS BECOME NULL.

SUMMARY

- NEW ORTHONORMAL TETRAD FOR NON-NULL ELECTROMAGNETIC FIELDS IN CURVED SPACETIMES.

- ISOMORPHISMS BETWEEN THE LOCAL GAUGE GROUP AND LOCAL LB1 AND LB2 GROUPS.
(GEOMETRIZATION OF GAUGE THEORIES).

- MAXIMUM SIMPLIFICATION OF RELEVANT TENSORS AND FIELD EQUATIONS.

- NEW TETRADS ENCODE GRAVITATIONAL AND GAUGE INFORMATION.

- EXPLICIT RELATIONSHIP BETWEEN "GAUGE" AND "GRAVITY".

- WE ARE INTRODUCING AND EXPLICIT "LINK" BETWEEN THE "INTERNAL" AND THE "SPACETIME", SO FAR DETACHED FROM EACH OTHER.

- FOR OTHER THEORIES
OTHER FIELD EQUATIONS
NON-ABELIAN

→ NEW CHOICES FOR THE "GAUGE VECTORS" ALLOW TO PROVE THEOREMS IN THE NON-ABELIN CASE.

SPACELIKE AND TIMELIKE VECTOR TRANSFORM INTO NULL VECTORS

A. Garat, Timelike and Spacelike vector transform into null vectors through local gauge transfs.

- EXTENSION OR GENERALIZATION TO NON-ABELIAN THEORIES

$SU(2) \times U(1)$

IS IT POSSIBLE TO BUILD THE NEW TETRADS WITH SKELETON - GAUGE VECTOR STRUCTURE IN HIGHER DIMENSIONAL SPACETIMES ?

IS IT POSSIBLE IN HIGHER DIMENSIONAL SPACETIMES TO PROVE NEW RESULTS IN GROUP THEORY FOR THE ASSOCIATED SYMMETRIES ?

AHARONOV - BOHM KIND OF EXPERIMENTS → FULL INVERSION

FULL SPACETIME INVERSION GENERATED BY EM ABEL GAUGE TRANSF. QUANTUM STUDIES: M&F 8 337-349 (2021)

- EXTENSION OR GENERALIZATION TO NON-ABELIAN THEORIES

$SU(3) \times SU(2) \times U(1)$.

Int. J. Geom. Meth. Mod. Phys. 15(3) (2018)

- FUTURE RESEARCH (FOLLOWING RESEARCH ONGOING)

1. CAN WE ASSOCIATE CLASSICAL SPACETIMES TO MICROPARTICLES?

2. TETRAD FIELDS CAN BE ASSOCIATED TO MICROPARTICLES?

3. ARE PARTICLE MULTIPLETS ASSOCIATED TO SYMMETRY PLANES IN 4-DIM LORENTZIAN SPACETIMES?

Tetrads in SU(N) Yang-Mills geometrodynamics

Int. J. Mod. Phys. A 34(29) (2019)

NULL FIELD

$$f_{\eta\nu} = K_\eta V_\nu - K_\nu V_\eta$$

$$V_\eta V^\eta = 1 \qquad K_\nu K^\nu = 0$$

$$K_\eta V^\eta = 0$$

$$R_{\eta\nu} = 2 K_\eta K_\nu$$

HOW TO GAUGE THE TETRAD WHEN THE MAXWELL EQUATIONS HAVE SOURCES J^{α}

$$f^{\mu\nu}{}_{;\nu} = J^{\alpha}$$

$$*f^{\mu\nu}{}_{;\nu} = 0$$

JUST ONE POTENTIAL $A^{\alpha} = X^{\alpha}$

FOR THE COULOMB CASE IN MINKOWSKY SPACETIME

$(-+++)$ $\quad (t, \rho, \theta, z)$ $\quad (-1, 1, \rho^2, 1)$

$$\sqrt{-g} = \rho$$

$$K_t{}^{\mu} = (1,0,0,0) \quad K_{\rho}{}^{\mu} = (0,1,0,0)$$

$$K_{\theta}{}^{\mu} = (0,0,\tfrac{1}{\rho},0) \quad K_z{}^{\mu} = (0,0,0,1)$$

WE CAN CHOOSE FOR SOME GEOMETRIES:

$$Y^{\alpha} = K_t^{\alpha}$$

OR EVEN $Y^{\alpha} = K_t^{\alpha} + A^{\alpha}$

FOR EXAMPLE FOR THE SOLENOID CASE WHERE THE GEOMETRY IS CYLINDRICAL AND

$$f_{\theta z} = B_{\rho} \qquad f_{\rho\theta} = B_z$$

$$\xi_{\rho\theta} = f_{\rho\theta} \qquad {}^*\xi_{tz} = -f_{\theta\rho} \qquad {}^*\xi_{t\rho} = f_{\theta z}$$

$$V_{(3)}^{\rho} = {}^*\xi^{\rho t} Y_t = - {}^*\xi_{t\rho} Y^t$$

$$V_{(3)}^{z} = {}^*\xi^{z t} Y_t = - {}^*\xi_{tz} Y^t$$

$$V_{(4)}^{t} = {}^*\xi^{t\rho} \, {}^*\xi_{t\rho} Y^t + {}^*\xi^{tz} \, {}^*\xi_{tz} Y^t$$