Quantum electrodynamics with empty fermion vacuum. Possibilities of experimental verification.

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Introduction

In standard QED, we use the Dirac equation with a bispinor wave function to describe fermion states. The Dirac equation has solutions with positive and negative energies. The physical vacuum of the Dirac equation is described in the language of completely occupied states with the negative energies (Dirac sea). The holes in the Dirac sea are interpreted as availability of antiparticles. In the Stükelberg-Feynman theory, positrons are electrons with the negative energies moving in the reverse direction in space-time. The QED fermion vacuum is non-empty, virtual creation and annihilation of particles and antiparticles is theoretically assumed therein.

Introduction

The motion of fermions in quantum theory can be described by equations with spinor wave functions. Earlier, we have examined the two possibilities: the Foldy-Wouthuysen (FW) representation and the representation with Klein-Gordon-type equation for fermions (KG). For these representations, the $(QED)_{FW}$ and $(QED)_{KG}$ formalisms were developed and some physical effects were calculated.

In the lowest order of the perturbation theory, the cross-sections of the Column scattering of an electron, electron scattering on a proton, the Compton effect and annihilation of an electron-positron pair were calculated. The electron’s self-energy, self-energy of photon, anomalous magnetic moment of electron, the Lamb shift of atomic energy levels were calculated. The final results completely coincide with the appropriate results in the standard QED with the Dirac equation.

Introduction

As against the standard QED, the following is new in $(QED)_{FW}$ and $(QED)_{KG}$:

- ✓ When calculating physical effects, it is suffice to use solutions with positive energies of fermions. It refers both to real and virtual intermediate fermion states.
- ✓ We use two separate equations for electrons and positrons. These equations differ from each other by the sign of the electric charge and the signs in front of summands with masses of an electron and a positron.
- ✓ In analogy with the vacuum of the Schrödinger equation, the fermion vacuum is empty. In this case, existence of the sea of solutions with negative energies (Dirac sea), processes of the virtual creation and annihilation of electron-positron pairs, the concept of vacuum polarization become excessive. In the future, this conclusion can be verified experimentally either in case of the successful development of exawatt-power optical lasers or in the experiments with collisions of heavy ions with the total $Z > 170 \div 175$.

Introduction

The aim of our effort:

We present a necessity of correction of the standard QED with the Dirac equation and with the bispinor wave function in view of $(QED)_{FW}$ and $(QED)_{KG}$ versions. When calculating physical effects in the updated QED theory, we will use only solutions of the Dirac free equation with positive energies. It refers both to real and virtual intermediate fermion states. We will use two separate equations for electrons and positrons. These equations differ from each other by the sign of the electric charge and the signs in front of summands with masses of an electron and a positron.

On QED formalism

In the standard QED, the Dirac equation with the bispinor wave function is used. The Dirac equation for an electron with mass *m* and electrical charge *e<0*, interacting with an electromagnetic field $A^{\mu}(\mathbf{r},t)$, can be written as:

$$p^0\psi_D = H_D\psi_D = \left(\boldsymbol{\alpha}\left(\mathbf{p}-e\mathbf{A}\right)+\beta m+eA^0\right)\psi_D,$$

where H_D is the Dirac Hamiltonian; $p^0 = i\left(\partial/\partial t\right),\ \mathbf{p} = -i\vec{\nabla},$

$A^0(\mathbf{r},t),\ A^l(\mathbf{r},t)$ are electromagnetic potentials;

$\alpha^{\mu} = \begin{cases} 1 \\ \alpha^k \end{cases},\ \alpha^k, \beta$ are four-dimensional Dirac matrixes, $k,l=1,2,3.$

The bispinor $\psi_D(\mathbf{r},t) = \begin{pmatrix} \varphi(\mathbf{r},t)\,U_S \\ \chi(\mathbf{r},t)\,U_S \end{pmatrix}.$

Here, U_S are normalized Pauli spinors $\left(\text{for } S_z = \frac{1}{2}\ \ U_S = \begin{pmatrix} 1 \\ 0 \end{pmatrix},\ \text{for } S_z = -\frac{1}{2}\ \ U_S = \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right).$

On QED formalism

In the free case (without interaction), Dirac equation has the following normalized solutions with positive and negative energies

$$(\psi_D)_0^{(+)}(\mathbf{r},t) = \frac{1}{(2\pi)^{3/2}}\sqrt{\frac{E+m}{2E}}\begin{pmatrix} U_S \\ \dfrac{\boldsymbol{\sigma}\mathbf{p}}{E+m}U_S \end{pmatrix} e^{-iEt+i\mathbf{pr}}, \quad (\psi_D)_0^{(-)}(\mathbf{r},t) = \frac{1}{(2\pi)^{3/2}}\sqrt{\frac{E+m}{2E}}\begin{pmatrix} \dfrac{\boldsymbol{\sigma}\mathbf{p}}{E+m}U_S \\ U_S \end{pmatrix} e^{iEt-i\mathbf{pr}}.$$

Here, $E=\sqrt{m^2+\mathbf{p}^2}$, σ^k are two-dimensional Pauli matrixes.

Solutions were obtained by using matrixes α^k, β in the Dirac-Pauli representation. The similar solutions can be obtained with Dirac matrixes in the spinor representation, widely used in the Standard model. QED with spinor equations for fermions and with the spinor representation of Dirac matrixes is represented in paper [1] for $(\text{QED})_{\text{FW}}$ and in paper [2] for $(\text{QED})_{\text{KG}}$. The final results in papers [1] and [2] coincide with the results of the standard QED and with the results in paper [3] obtained by using matrixes α^k, β in the Dirac-Pauli representation.

[1] V. P. Neznamov, Part. Nucl. 43, 36 (2012), arxiv: 1107.0693 (physics. gen-ph), [2] L. S. Hostler, J. Math. Phys. 26, 1348 (1985).
[3] V. P. Neznamov, Part. Nucl. 37, 86 (2006), arxiv: hep-th/0411050; V. P. Neznamov and V. E. Shemarulin, Int. J. Mod. Phys. A 36, 2150086 (2021).

QED in the Foldy-Wouthuysen representation

In the FW-representation, the Dirac equation for an electron, interacting with an electromagnetic field $A^{\mu}(\mathbf{r},t)$, can be obtained in the form of series in the electromagnetic coupling constant by applying a sequence of unitary transformations $U_{FW}=\left(1+e\delta_1+e^2\delta_2+e^3\delta_3+...\right)U_0.$

Here $U_{FW}^{+}=U_{FW}^{-1},\ \psi_{FW}=U_{FW}\psi_D.$

As the result, we obtain the equation of

$$p^0\psi_{FW}=H_{FW}\psi_{FW}=\left(\beta E+eK_1^{FW}\left(+m,A^{\mu}\right)+e^2K_2^{FW}\left(+m,A^{\mu},A^{\nu}\right)+e^3K_3^{FW}\left(+m,A^{\mu},A^{\nu},A^{\gamma}+...\right)\right)\psi_{FW}.$$

Here, designation +*m* points to the use of the positive sign in front of βm in the Dirac equation. In the equation there are not summands with the negative sign in front of a mass *m*. It is follows from the structure of expressions $K_1^{FW},\ K_2^{FW}$ ... In the free case $p^0\left(\psi_{FW}\right)_0=\beta E\left(\psi_{FW}\right)_0.$

For the positive energy of $p^0=E$

$$\left(\psi_{FW}\right)_0^{(+)}(\mathbf{r},t)=U_{FW}^0\left(\psi_D\right)_0^{(+)}=\frac{1}{(2\pi)^{3/2}}\begin{pmatrix}U_S\\0\end{pmatrix}e^{-iEt+i\mathbf{pr}},$$

for the negative energy of $p^0=-E,$

$$\left(\psi_{FW}\right)_0^{(-)}(\mathbf{r},t)=U_{FW}^0\left(\psi_D\right)_0^{(-)}=\frac{1}{(2\pi)^{3/2}}\begin{pmatrix}0\\U_S\end{pmatrix}e^{iEt-i\mathbf{pr}}.$$

QED in the Foldy-Wouthuysen representation

In the FW-representation, Dirac equation has a noncovariant form and a Hamiltonian H_{FW} is nonlocal. In this case, in the quantum field theory, it is difficult to use standard methods of secondary quantization. However, we can use the S-matrix approach and the Feynman method of the propagation function. In this method, QED processes are described by integral equations.

Equation for four-dimensional *x*, *y* can be written as

$$\psi_{FW}(x) = (\psi_{FW})_0^{(\pm)}(x) + \int d^4y\, S_{FW}(x-y) K^{FW}(y) \psi_{FW}(y),$$

where $K^{FW}(y) = \sum_{n=1}^{\infty} e^n K_n^{FW}(y)$ is the interaction Hamiltonian; $S_{FW}(x-y)$ is the Feynman propagator in the Foldy-Wouthuysen (FW) representation

$$S_{FW}(x-y) = \frac{1}{(2\pi)^4} \int d^4y\, e^{-ip(x-y)} \frac{p^0 + \beta E}{p^2 - m^2 + i\varepsilon}.$$

The elements of the *S*-matrix can be written as

$$S_{fi} = \delta_{fi} - i\varepsilon_f \int d^4y \left[(\bar{\psi}_{FW})_0^{(\pm)}(y) \right]_f K^{FW}(y) \left[\psi_{FW}(y) \right]_i .$$

Here, the dash above the function ψ_{FW} means the Hermitian conjugation, $\varepsilon_f = \pm 1$.

QED in the Foldy-Wouthuysen representation

Let us note a number of essential points:

- Hamiltonians H_{FW} and $K^{FW}(y)$ are diagonal relative to mixing of upper and lower components of a bispinor ψ_{FW}. Each of the Dirac equations includes two independent equations with spinor wave functions $\sim U_s$. One of the equations describes states with positive energies, the second equation – the states with negative energies. *S*-matrix elements can be calculated taking into account only the states with positive energies. In this case, the states with negative energies are not used in the calculations of the QED physical processes. They are needed just for the mathematical completeness in expansions of operators and wave functions.
- The essential peculiarity of the theory in the case when four-momenta of external fermion lines lie on a mass surface $\left(\left(p^0\right)^2 - \mathbf{p}^2 = m^2\right)$ is the compensation for the contribution of the diagrams with fermion propagators and the contribution of the corresponding summands in propagator-free diagrams determined by formula (12) [1].

[1] V. P. Neznamov, Part. Nucl. 37, 86 (2006), arxiv: hep-th/0411050.

QED in the Foldy-Wouthuysen representation

- In the standard QED with the Dirac equation, positrons are electrons with negative energies moving in the reverse direction in space-time. In the Foldy-Wouthuysen representation, the situations changes. If in the equation for the *S*-matrix elements, on the left, we use $\left[\left(\bar{\psi}_{FW}\right)_0^{(+)}\right]_f$, and, on the right, we use $-\left[\left(\psi_{FW}\right)_0^{(-)}\right]_i$, then, due to the structure of bispinors and due to evenness of an interaction Hamiltonian $K^{FW}(y)$ in all the orders of the perturbation theory we will obtain zero values of the corresponding elements of the *S*-matrix.

$$\left\langle \frac{1}{(2\pi)^{3/2}} e^{iE_f t - i\mathbf{p_f r}} \begin{pmatrix} \bar{U}_S & 0 \end{pmatrix} \middle| M \middle| \begin{pmatrix} 0 \\ U_S \end{pmatrix} \frac{1}{(2\pi)^{3/2}} e^{iE_i t - i\mathbf{p_i r}} \right\rangle = 0!!!$$

even operator by definition

We will obtain the similar result if in the indicated equation, on the left, we use $\left[\left(\bar{\psi}_{FW}\right)_0^{(-)}\right]_f$, and, on the right, we use $-\left[\left(\psi_{FW}\right)_0^{(+)}\right]_i$.

So, the positrons in FW-representation cannot be described by electron states with negative energies. The positrons in the FW-representation should be described by the states with positive energies of the special equation for positrons.

QED in the Foldy-Wouthuysen representation

- To describe physical processes in $(QED)_{FW}$ with participation of real antiparticles, it was found out that in the initial Dirac equations, particle and antiparticle masses should have opposite signs. It is connected with our nonuse in theory the states with negative energies.

QED in the Foldy-Wouthuysen representation

- The Dirac equation for positrons has the form of

$$p^0\psi_D^C = \left(\boldsymbol{\alpha}\left(\mathbf{p} + e\mathbf{A}\right) - \beta m - eA^0\right)\psi_D^C.$$

Here $\psi_D^C = \beta\Sigma_2\psi_D^*,\ \Sigma_2 = \begin{pmatrix} \sigma_2 & 0 \\ 0 & \sigma_2 \end{pmatrix},\ \psi_D^*$ is a complex conjugate bispinor.

This equation differs from Dirac equation by signs of charge and summand with βm.

In the FW-representation the equation has the form of

$$p^0\psi_{FW}^C = \left(\beta E - eK_1^{FW}\left(-m, A^\mu\right) + e^2K_2^{FW}\left(-m, A^\mu, A^\nu\right) - e^3K_3^{FW}\left(-m, A^\mu, A^\nu, A^\gamma\right) + \ldots\right)\psi_{FW}^C.$$

Here there are not the summands with the positive sign before the mass *m*.

In paper [1] without the use of states with negative energies of fermions, the $(\text{QED})_{FW}$ formalism is developed and some physical effects were calculated. The final results of the calculations coincide with the results in the standard QED.

[1] V. P. Neznamov, Part. Nucl. 37, 86 (2006), arxiv: hep-th/0411050.

$(QED)_{KG}$ with the Klein-Gordon-type equation for fermions

Self-conjugate equations for electrons and positrons with spinor wave functions were obtained in papers [1], [2]. These equations have the form of

$$\left[\left(p^0 - eA^0\right)^2 - m^2 - \left(p^0 - eA^0 + m\right)^{1/2} \boldsymbol{\sigma}\left(\mathbf{p} - e\mathbf{A}\right)\frac{1}{p^0 - eA^0 + m}\boldsymbol{\sigma}\left(\mathbf{p} - e\mathbf{A}\right)\left(p^0 - eA^0 + m\right)^{1/2}\right]\Phi = 0,$$

$$\left[\left(p^0 + eA^0\right)^2 - m^2 - \left(p^0 + eA^0 - m\right)^{1/2} \boldsymbol{\sigma}\left(\mathbf{p} + e\mathbf{A}\right)\frac{1}{p^0 + eA^0 - m}\boldsymbol{\sigma}\left(\mathbf{p} + e\mathbf{A}\right)\left(p^0 + eA^0 - m\right)^{1/2}\right]\Phi^C = 0.$$

In equations we can perform expansion as a series in a charge *e*

$$\left[p_0^2 - \mathbf{p}^2 - m^2 \mp eV_1\left(\pm m, A^\mu\right) - e^2 V_2\left(\pm m, A^\mu, A^\nu\right) \mp e^3 V_3\left(\pm m, A^\mu, A^\nu, A^\gamma - \ldots\right)\right]\Phi\left(\pm m, \mathbf{r}, t\right) = 0.$$

Here, the upper signs in front of the charge and mass correspond to equation for an electron, the lower signs correspond to equation for a positron, $\Phi\left(+m,\mathbf{r},t\right) = \Phi$, $\Phi\left(-m,\mathbf{r},t\right) = \Phi^C$, $\Phi^C = \sigma_2 \Phi^*$.

$\Phi = g_\varphi \varphi\left(\mathbf{r},t\right) U_S$, where $g_\varphi = \left(p^0 - eA^0 + m\right)^{-1/2}$.

The algorithm for determining an interaction operator $V = eV_1 + e^2 V_2 + e^3 V_3 + \ldots$ is provided in paper [2].

[1] V. P. Neznamov, I. I. Safronov, J. Exp. Theor. Phys. 128, 672 (2019), arxiv: 1907.03579 (physics. gen-ph, hep-th).

[2] V. P. Neznamov, Int. J. Mod. Phys. A, 2150173 (2021), arxiv: 2110.03530 (physics. gen-ph).

$(QED)_{KG}$ with the Klein-Gordon-type equation for fermions

In the free case, the self-conjugated equations for electrons and positrons become the Klein-Gordon equations with spinor wave functions

$$\left(p_0^2-\mathbf{p}^2-m^2\right)\Phi_0\left(\mathbf{r},t\right)=0,\ \left(p_0^2-\mathbf{p}^2-m^2\right)\Phi_0^C\left(\mathbf{r},t\right)=0.$$

The orthonormal solutions of these equations have the form of

$$\Phi_0^{(\pm)}\left(\mathbf{r},t\right)=\frac{1}{\left(2\pi\right)^{3/2}}\frac{1}{\sqrt{2E}}e^{\mp iEt\pm i\mathbf{pr}}U_S,\ \left(\Phi_0^C\right)^{(\pm)}\left(\mathbf{r},t\right)=\frac{1}{\left(2\pi\right)^{3/2}}\frac{1}{\sqrt{2E}}e^{\mp iEt+i\mathbf{pr}}\sigma_2 U_S.$$

In the paper [1], the $(QED)_{KG}$ formalism was developed and some physical effects were calculated. As well as in the FW-representation, the final computational results coincide with the results in the standard QED.

In $(QED)_{KG}$, real and virtual states with negative energies of self-conjugated equations for electrons and positrons are not used either. In the equations, the masses of particles and antiparticles have opposite signs.

[1] V. P. Neznamov, Int. J. Mod. Phys. A, 2150173 (2021), arxiv: 2110.03530 (physics. gen-ph).

Standard QED without the Dirac sea and with opposite signs before particle and antiparticle masses

In standard QED, we use the Dirac equation with a bispinor wave function.

In this section, we answer the question: is it possible to develop the QED formalism with the Dirac equation with the opposite signs of charges and masses for particles and antiparticles and with the use of only positive energies for real and virtual fermion states?

As is known, the unitary transformations of the Hamiltonians and wave functions in the quantum theory preserve all physical characteristics of considered objects of researches. But at transition from the Dirac representation to the Foldy-Wouthuysen representation, we encounter with other physical picture.

Standard QED without the Dirac sea and with opposite signs before particle and antiparticle masses

In the Dirac representation in the quantum theory there is a connection between the solutions with positive and negative energies of fermions. In the Foldy-Wouthuysen representation, there is not such kind of connection.

Obviously, in the Dirac representation there is an unnecessary nonphysical information that used in the formalism of the standard QED. This information is connected with the negative energies of fermions. For recovery of parity with the Foldy-Wouthuysen representation, we must refuse to use the solutions with the negative energies of fermions in the standard QED. The solutions with the negative energies are necessary only for mathematical completeness in expansions of operators and wave functions.

Standard QED without the Dirac sea and with opposite signs before particle and antiparticle masses

So, in the updated QED with the Dirac equation and bispinor function, we will use two individual equations for electrons and positrons. These equations differ from each other by the sign of the electric charge and the signs in front of summands with masses of an electron and a positron. The second change of the Feynman rules is the use of two individual retarded propagators for virtual electrons and positrons. In calculating with the retarded Green functions, it should take into account only positively frequency poles.

Standard QED without the Dirac sea and with opposite signs before particle and antiparticle masses

As a result in updated QED, when calculating physical effects, it is suffice to use solutions with positive energies of fermions. It refers both to real and virtual intermediate fermion states.

In updated QED, the fermion vacuum is empty. In a theory, the processes of virtual creation and annihilation of particles and antiparticles are absent .

The results of calculated electrodynamics phenomena in updated QED coincides with the results in standard (QED) and also in $(QED)_{FW}$ and $(QED)_{KG}$ versions.

Possibilities of experimental verifications of fermion vacuum in QED

In standard QED, the fermion vacuum is non-empty. The processes of creation and annihilation of virtual electron-positron pairs are present therein. In intense electromagnetic fields, the vacuum creation of real electron-positron pairs is possible. The well-known example thereof is the Schwinger effect: vacuum creation of real pairs in the strong uniform electrical field.

In QED variants without the use of fermion states with negative energies, the fermion vacuum is empty. The possibility of vacuum creation of real and virtual pairs is absent therein.

The direct answer to the question about the content of the fermion vacuum is experimental confirmation of existence (or absence) of the Schwinger effect. The intensity of the critical Schwinger field is $\sim 5\cdot 10^{29}$ W/cm^2.

Possibilities of experimental verifications of fermion vacuum in QED

To achieve such an intensity, two possibilities are available:

- ❑ Development of exawatt-power optical lasers (see, for instance, the XCELS-project). At the XCELS facility, the achievement of $\sim 10^{24} \div 10^{25}$ W/cm^2 intensity of a laser field is anticipated. The calculations were shown that at the use of a single focused laser pulse, the necessary critical field for production of electron-positron pairs decreases up to $I = 10^{28}$ W/cm^2. At collisions of two and more focused laser pulses, the threshold critical field decreases up to $I = 10^{25} \div 10^{26}$ W/cm^2.

Possibilities of experimental verifications of fermion vacuum in QED

- Experiments with collisions of heavy ions with the total $Z>170 \div 175$. Such experiments were carried out at the GSI facility (Darmstadt, Germany) and Argonne national laboratory (USA) in 1970-ies–1980-ies. However, they did not lead to the unambiguous conclusion about the possibility of vacuum creation of pairs in super-critical fields. New experiments devoted to this subject can be carried out in the FAIR (Darmstadt, Germany), HIAF (China), NICA (Dubna, Russia) acceleration centers currently under construction.

Conclusions

In the report, three QED versions with opposite signs of particle and antiparticle masses in Dirac equations and with empty fermion vacuum without the sea of negative energies are considered. The first version is $(QED)_{FW}$ in the Foldy-Wouthuysen representation. The second version is $(QED)_{KG}$ with equations for Klein-Gordon-type fermions. The third version is the updated standard QED.

In all the versions, in calculations of QED physical effects, in real and virtual intermediate states, only the states with positive energies are used.

The QED versions were tested by calculations of physical processes.

In the lowest order of the perturbation theory, cross-sections of the Coulomb electron scattering, scattering of an electron on a proton, the Compton-effect, annihilation of electron-positron pairs were calculated. The self-energy of electron, self-energy of photon, anomalous magnetic moment of electron, the Lamb shift of energy levels were calculated. The final results completely coincide with the results of the standard QED with the Dirac sea.

Conclusions

In the QED versions under discussion, the new conception of fermion vacuum leads to new physical consequences.

1. In these QED versions, there is no “Zitterbewegung” of fermion coordinates. This fact associated with absence of virtual interaction between the states of fermions with positive and negative energies was already mentioned in the first Foldy-Wouthuysen’s paper.
2. For the same reasons, there is no Klein paradox in these QED versions.
3. In the QED versions under discussion, there are no processes of vacuum creation of particle-antiparticle pairs in intense electromagnetic field. In particular, there is no Schwinger effect, i.e., vacuum creation of pairs in a strong uniform electrical field.

This conclusion can be experimentally verified in the future. The intensity of the critical field necessary for vacuum creation of pair, can be achieved both at the facilities with exawatt-power optic lasers and in the experiments in collision of heavy ions with total $Z \geq 170 \div 175$ in the FAIR, HIAF, NICA acceleration centers under construction.

Thank you for your attention

Description of some quantum electrodynamics effects in updated QED

Below, we will use the formulas with designations like in Standard model (see, for example, [1])

$$\hat{p} = \gamma_\mu p^\mu, \ \gamma_\mu = \gamma_0 \alpha_\mu, \ \gamma_0 = \beta.$$

If $\left(\psi_D\right)_0^{(+)}(x) = \sqrt{\frac{m}{E(2\pi)^3}} u^e(p,s,m) e^{-ipx}$, then the Dirac free equation has the form of

$$\left(\hat{p} - m\right) u^e(p,s,m) = 0.$$

By analogy, if $\left(\psi_D^C\right)_0^{(+)} = \sqrt{\frac{m}{E(2\pi)^3}} u^p(p,s,-m) e^{-ipx}$, then the Dirac free equation for positrons has the form of

$$\left(\hat{p} + m\right) u^p(p,s,-m) = 0.$$

[1] J. D. Bjorken, S. D. Drell, The Relativistic Quantum Theory, McGraw-Hill Book Company (1964).

Description of some quantum electrodynamics effects in updated QED

The retarded propagators for electrons and positrons are equal to, correspondingly

$$\frac{i}{\hat{p} \pm m + i\varepsilon}.$$

A condition of completeness for $u^e(p,s,m)$ and $u^p(p,s,-m)$ are equal to

$$\sum_{\pm s} u^e_\beta(p,s,m)\bar{u}^e_\lambda(p,s,m) = \left(\frac{\hat{p}+m}{2m}\right)_{\beta\lambda},$$

$$\sum_{\pm s} u^p_\beta(p,s,-m)\bar{u}^p_\lambda(p,s,-m) = \left(\frac{\hat{p}-m}{2m}\right)_{\beta\lambda}.$$

Here and further, the dash above the function means the Hermitian conjugation with the next multiplication by a matrix γ^0.

Description of some quantum electrodynamics effects in updated QED

- ☐ Compton scattering of an photon on the electrons (positrons).

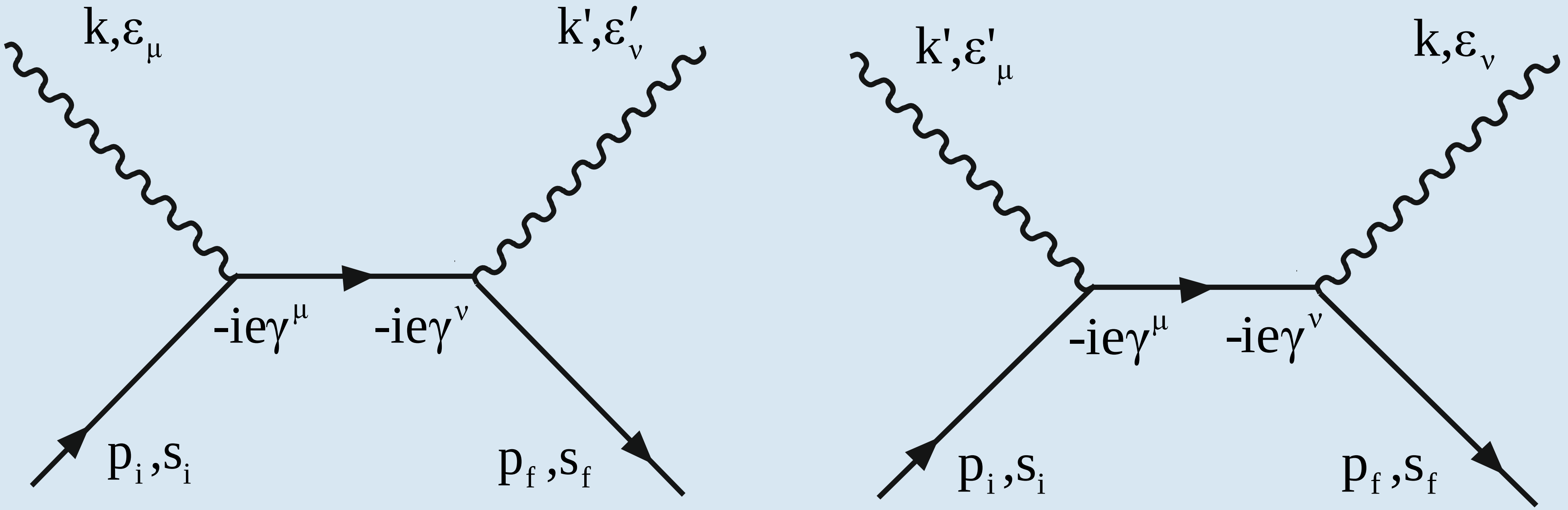

Fig. 1. Feynman diagrams of the second order of the perturbation theory.

In Fig. 1 p_i, p_f, s_i, s_f are either four-momentum and spins of electrons or four-momentum and spins of positrons. $k, \varepsilon, k', \varepsilon'$ are momenta and polarizations of absorbed and emitted photons.

Description of some quantum electrodynamics effects in updated QED

A matrix element of S-matrix is equal to

$$S_{fi}^{compt} = \frac{e^2}{V^2}\sqrt{\frac{m^2}{E_f E_i}}\frac{1}{\sqrt{2k2k'}}(2\pi)^4\delta^4\left(p_f + k' - p_i - k\right)\overline{u}\left(p_f, s_f, \pm m\right)\times$$

$$\times\left[\left(-i\hat{\varepsilon}'\right)\frac{i}{\hat{p}_i + \hat{k}_+ \mp m}\left(-i\hat{\varepsilon}\right) + \left(-i\hat{\varepsilon}\right)\frac{i}{\hat{p}_i - \hat{k}_+ \mp m}\left(-i\hat{\varepsilon}'\right)\right]u\left(p_i, s_i, \pm m\right).$$

The cross-section for nonpolarized electron or positron is equal to (see, for example, [1])

$$\frac{d\sigma}{d\Omega} = \frac{1}{2}\sum_{\pm s_i, s_f}\frac{d\sigma}{d\Omega} = \frac{a^2}{2}\left(\frac{k'}{k}\right)^2 \mathrm{Sp}\,\frac{\hat{p}_f \pm m}{2m}\left(\frac{\hat{\varepsilon}'\hat{\varepsilon}k}{2kp_i} + \frac{\hat{\varepsilon}\hat{\varepsilon}'k'}{2k'p_i}\right)\frac{\hat{p}_i \pm m}{2m}\left(\frac{k\hat{\varepsilon}\hat{\varepsilon}'}{2kp_i} + \frac{k'\hat{\varepsilon}'\hat{\varepsilon}}{2k'p_i}\right).$$

Here, the upper sign in front of a mass *m* should be used in the calculating of scattering cross-section of a photon on an electron, the lower sign - in the calculating of scattering cross-section of a photon on an positron. Both cross-sections are equal and coincide with the cross-section calculated in the standard QED.

[1] J. D. Bjorken, S. D. Drell, The Relativistic Quantum Theory, McGraw-Hill Book Company (1964).

Description of some quantum electrodynamics effects in updated QED

- Annihilation of electron-positron pair

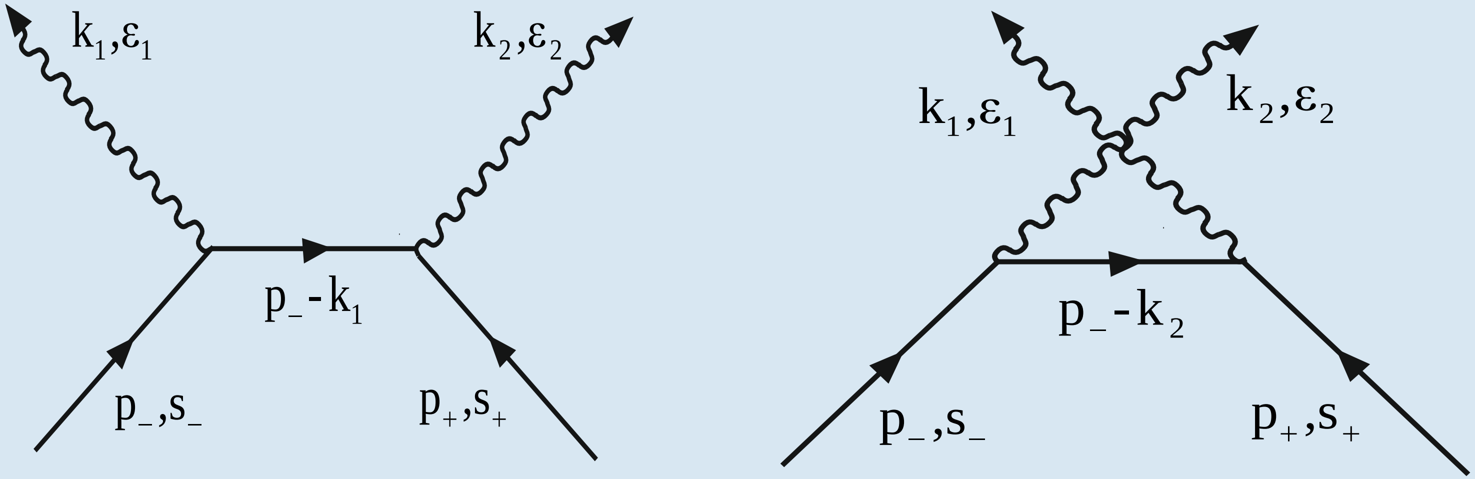

Fig. 1. Feynman diagrams of the second order of perturbation theory.

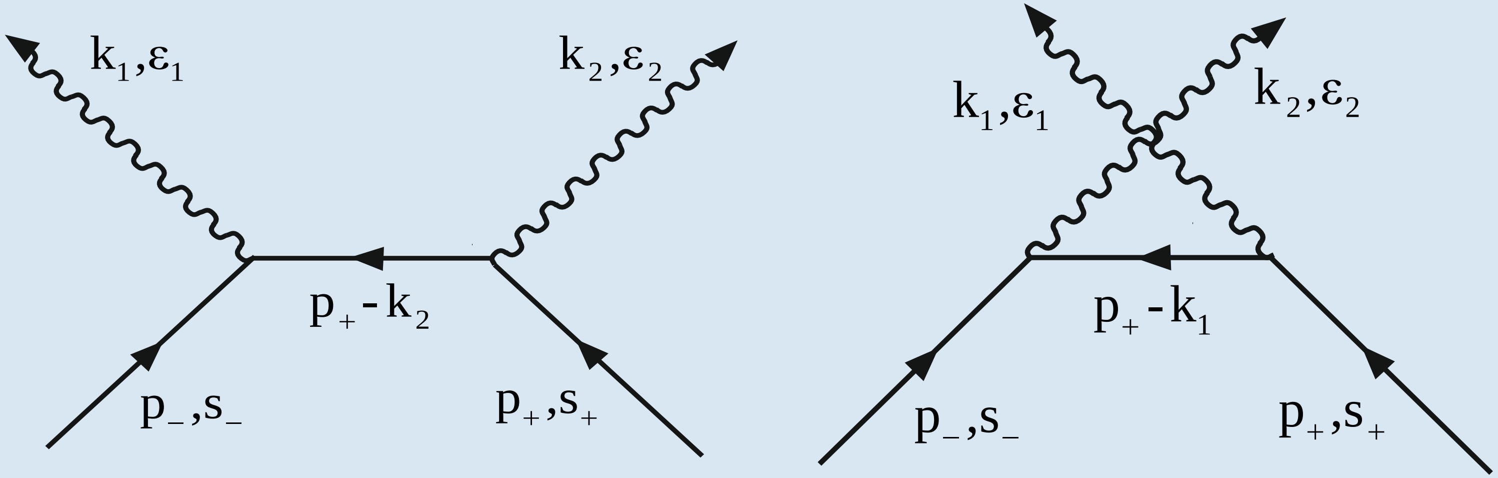

Fig. 2. Feynman diagrams of the second order of perturbation theory.

Description of some quantum electrodynamics effects in updated QED

In Fig.1, we use the electron propagators $i/\left(\hat{p}_- - \hat{k}_1 - m + i\varepsilon\right)$ and $i/\left(\hat{p}_- - \hat{k}_2 - m + i\varepsilon\right)$, in Fig.2 we use the positron propagators $i/\left(\hat{p}_+ - \hat{k}_2 + m + i\varepsilon\right)$ and $i/\,\hat{p}_+ - \hat{k}_1 + m + i\varepsilon.$

The diagrams of Figs. 1 and 2 are equivalent to each other since they coincide at $p_+ \leftrightarrow p_-$, $s_+ \leftrightarrow s_-$, $+m \leftrightarrow -m.$ When calculating the annihilation cross-section, one can be used either the diagrams of Fig. 1, or the diagrams of Fig.2.

By analogy with the standard QED (see, for example, [1]), differential annihilation cross-section corresponding, for example, to the diagrams of Figs. 1 is equal to

$$d\sigma = \frac{e^4}{(2\pi)^2}\int \frac{m}{E_+\beta_+}\frac{1}{4}\mathrm{Sp}\,\frac{\hat{p}_+ - m}{2m}\left(\frac{\hat{\varepsilon}_2\hat{k}_1\hat{\varepsilon}_1}{2p_-k_1} + \frac{\hat{\varepsilon}_1\hat{k}_2\hat{\varepsilon}_2}{2p_-k_2}\right)\frac{\hat{p}_- + m}{2m}\left(\frac{\hat{\varepsilon}_1\hat{k}_1\hat{\varepsilon}_2}{2p_-k_1} + \frac{\hat{\varepsilon}_2\hat{k}_2\hat{\varepsilon}_1}{2p_-k_2}\right)\frac{d^3k_1}{2k_1}\frac{d^3k_2}{2k_2}\delta^4\left(k_1 + k_2 - p_- - p_+\right).$$

This expression is cited for the case of nonpolarized positron collided with nonpolirized electron at rest in laboratory system. It fully coincides with corresponding expression in the standard QED.

[1] J. D. Bjorken, S. D. Drell, The Relativistic Quantum Theory, McGraw-Hill Book Company (1964).

Description of some quantum electrodynamics effects in updated QED

- Self-energies of electron and positron

In Fig. 3, p is momentum of an electron (positron). In calculations one can used either retarded electron propagator or retarded positron propagator.

The self-energy operator in the second order of perturbation theory is equal to

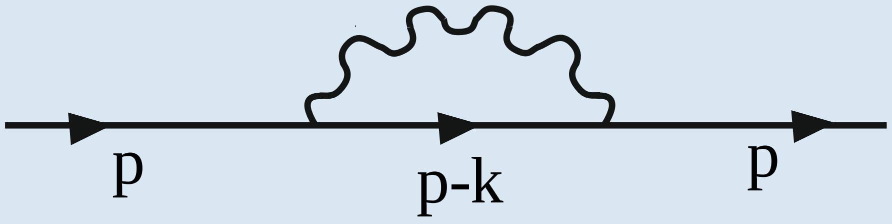

Fig. 3. Feynman diagram in the second order of perturbation theory.

$$M(p) = -\frac{8\pi i}{(2\pi)^4} e^2 \int \frac{\pm 2m - \hat{p} + \hat{k}}{(p-k)^2 - m^2} \frac{d^4 k}{k^2}.$$

Here, the upper sign in front of the mass correspond to the self-energy of electron, and the lower sign correspond to the self-energy of positron. As a result, after calculations, both expressions coincide with each other and with the expressions for self-energies of electron and positron in the standard QED.

Description of some quantum electrodynamics effects in updated QED

- Self-energy function of photon

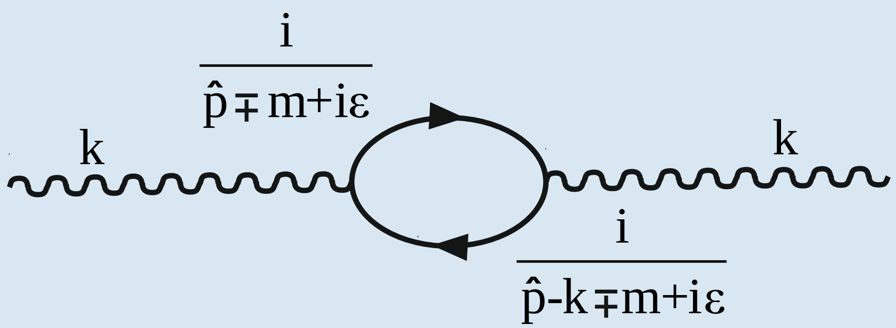

Fig. 4. Feynman diagrams in the second order of perturbation theory

In the second order of perturbation theory, the photon propagator can be written as

$$-iD_{\alpha\beta}=\frac{(-i)g_{\alpha\beta}}{k^2}+\frac{(-i)g_{\alpha\mu}}{k^2}i\Pi_{\mu\nu}\frac{(-i)g_{\nu\beta}}{k^2}.$$

Tensor $\Pi_{\mu\nu}(k)$ is equal to

$$\Pi_{\mu\nu}(k)=-(-ie)^2\int\frac{d^4p}{(2\pi)^4}\mathrm{Sp}\left(\gamma^\mu\frac{i}{\hat{p}-m+i\varepsilon}\gamma^\nu\frac{i}{\hat{p}-\hat{k}-m+i\varepsilon}+\gamma^\mu\frac{i}{\hat{p}+m+i\varepsilon}\gamma^\nu\frac{i}{\hat{p}-\hat{k}+m+i\varepsilon}\right).$$

Tensor $\Pi_{\mu\nu}(k)$ coincides with the appropriate tensor in the standard QED in the final expressions.