Influence of relativistic rotation on QCD properties

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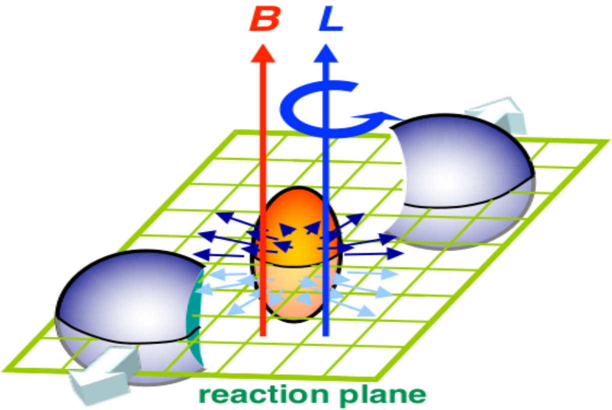
Outline:

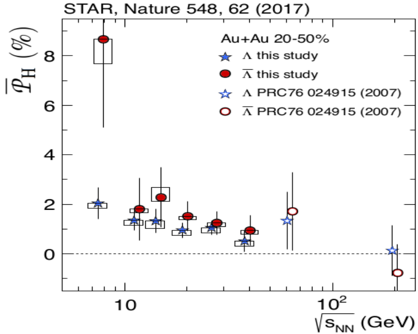
- Introduction
- Critical temperatures
- Moment of inertia of QGP
- Inhomogeneous phase transitions in QGP
- Conclusion

Rotation of QGP in heavy ion collisions



- QGP is created with non-zero angular momentum in non-central collisions

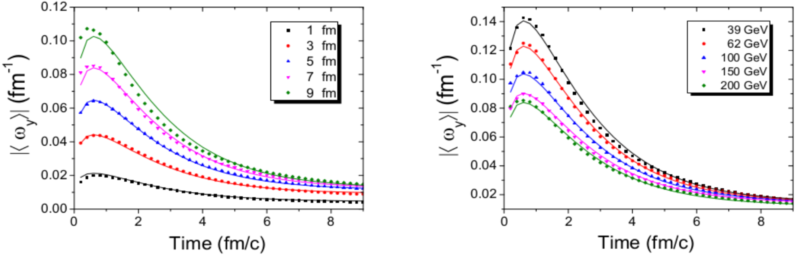
Rotation of QGP in heavy ion collisions



Angular velocity from STAR (Nature 548, 62 (2017))

- $\Omega = (P_\Lambda + P_{\bar\Lambda})\frac{k_B T}{\hbar}$ (Phys. Rev. C 95, 054902 (2017))
- $\Omega \sim 10$ MeV ($v \sim c$ at distances 10-20 fm)
- Relativistic rotation of QGP

Rotation of QGP in heavy ion collisions



Hydrodynamic simulations (Phys.Rev.C 94, 044910 (2016))

- Au-Au: *left* $\sqrt{s} = 200$ GeV, *right* $b = 7$ fm,
- $\Omega \sim (4 - 28)$ MeV
- Relativistic rotation of QGP

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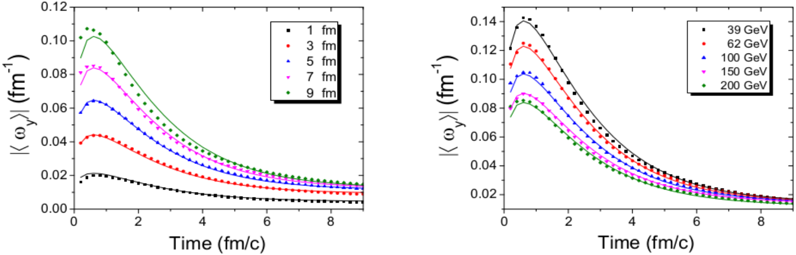

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How relativistic rotation influences QCD?

Study of rotating QGP

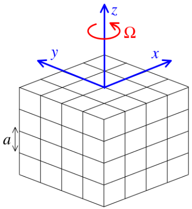
- Our aim: study rotating QCD within lattice simulations
- Rotating QCD at thermodynamic equilibrium
  - At the equilibrium the system rotates with some Ω
  - The study is conducted in **the reference frame which rotates with QCD matter**
  - QCD in external gravitational field
- **Boundary conditions are very important!**

Details of the simulations

- Gluodynamics is studied at thermodynamic equilibrium in external gravitational field
- The metric tensor

$$g_{\mu\nu} = \begin{pmatrix} 1 - r^2\Omega^2 & \Omega y & -\Omega x & 0 \\ \Omega y & -1 & 0 & 0 \\ -\Omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- Geometry of the system: $N_t \times N_z \times N_x \times N_y = N_t \times N_z \times N_s^2$



Details of the simulations

- Partition function ($\hat{H}$ is conserved)

$$Z = \mathrm{Tr}\,\exp\left[-\beta \hat{H}\right] = \int DA\ \exp\left[-S_G\right]$$

- Euclidean action

$$S_G = -\frac{1}{2g_{YM}^2}\int d^4x\,\sqrt{g_E}\,g_E^{\mu\nu}\,g_E^{\alpha\beta}\,F_{\mu\alpha}^{(a)}F_{\nu\beta(a)}$$

$$S_G = \frac{1}{2g_{YM}^2}\int d^4x \mathrm{Tr}\Big[(1-r^2\Omega^2)F_{xy}^a F_{xy}^a + (1-y^2\Omega^2)F_{xz}^a F_{xz}^a +$$

$$+(1-x^2\Omega^2)F_{yz}^a F_{yz}^a + +F_{x\tau}^a F_{x\tau}^a + F_{y\tau}^a F_{y\tau}^a + F_{z\tau}^a F_{z\tau}^a -$$

$$-2iy\Omega(F_{xy}^a F_{y\tau}^a + F_{xz}^a F_{z\tau}^a) + 2ix\Omega(F_{yx}^a F_{x\tau}^a + F_{yz}^a F_{z\tau}^a) - 2xy\Omega^2 F_{xz}F_{zy}\Big]$$

Details of the simulations

- *Ehrenfest–Tolman effect:* **In gravitational field the temperature is not constant in space at thermal equilibrium**

$$T(r)\sqrt{g_{00}} = const = 1/\beta$$

$$T(r)\sqrt{1 - r^2\Omega^2} = 1/\beta$$

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- We use the designation $T = T(r = 0) = 1/\beta$

Details of the simulations

Boundary conditions

- **Periodic b.c.:**
  - $U_{x,\mu} = U_{x+N_i,\mu}$
  - Not appropriate for the field of velocities of rotating body
- **Dirichlet b.c.:**
  - $U_{x,\mu}\big|_{x\in\Gamma} = 1, \quad A_\mu\big|_{x\in\Gamma} = 0$
  - Violate Z_3 symmetry
- **Neumann b.c.:**
  - Outside the volume $U_P = 1, \quad F_{\mu\nu} = 0$
- *The dependence on boundary conditions is the property of all approaches*
- *One can expect that boundary conditions influence our results considerably, but their influence is restricted due to the screening*

Details of the simulations

Sign problem

$$S_G = \frac{1}{2g_{YM}^2} \int d^4x \mathrm{Tr}\big[(1 - r^2\Omega^2) F^a_{xy} F^a_{xy} + (1 - y^2\Omega^2) F^a_{xz} F^a_{xz} +$$

$$+(1 - x^2\Omega^2) F^a_{yz} F^a_{yz} + +F^a_{x\tau} F^a_{x\tau} + F^a_{y\tau} F^a_{y\tau} + F^a_{z\tau} F^a_{z\tau} -$$

$$-2iy\Omega(F^a_{xy} F^a_{y\tau} + F^a_{xz} F^a_{z\tau}) + 2ix\Omega(F^a_{yx} F^a_{x\tau} + F^a_{yz} F^a_{z\tau}) - 2xy\Omega^2 F_{xz} F_{zy}\big]$$

- The Euclidean action has imaginary part (**sign problem**)
- Simulations are carried out at imaginary angular velocities $\Omega \to i\Omega_I$
- The results are analytically continued to real angular velocities
- This approach works up to sufficiently large Ω

Details of the simulations: critical temperatures

Confinement/deconfinement phase transition

- Polyakov line

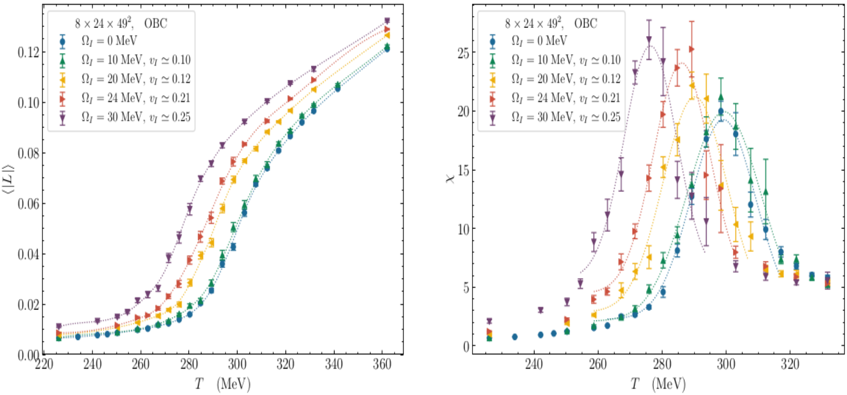
$$L = \left\langle \mathrm{Tr}\mathcal{T} \exp\left[ig \int_{[0,\beta]} A_4\, dx^4\right]\right\rangle$$

- Susceptibility of the Polyakov line

$$\chi = N_s^2 N_z \left(\langle |L|^2 \rangle - \langle |L| \rangle^2\right)$$

- T_c is determined from Gaussian fit of the $\chi(T)$

Results of the calculation (Neumann b.c.)



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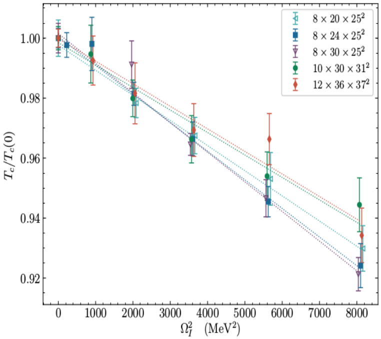
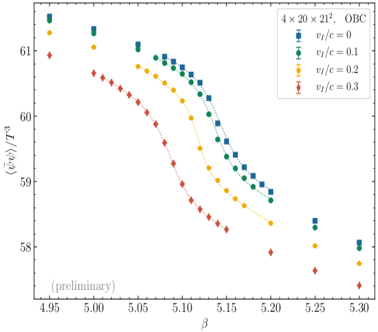


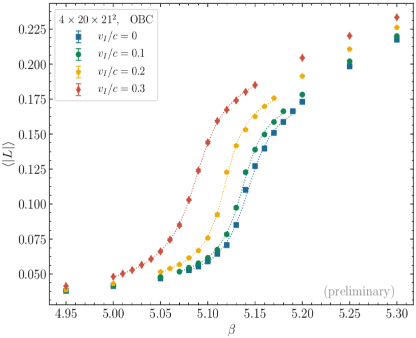
- The results can be well described by the formula ($C_2 > 0$)

$$\frac{T_c(\Omega_I)}{T_c(0)} = 1 - C_2\Omega_I^2 \Rightarrow \frac{T_c(\Omega)}{T_c(0)} = 1 + C_2\Omega^2$$

- **The critical temperature rises with angular velocity**
- The results weakly depend on lattice spacing and the volume in z-direction

Simulation with fermions





- Lattice simulation with Wilson fermions: $m_\pi \simeq 700\,\mathrm{MeV}$
- Critical couplings of both transitions coincide
- Critical temperatures are increased (at least for $m_\pi \simeq 700\,\mathrm{MeV}$)

EoS of rotating gluodynamics

- Free energy of rotating QGP

$$F(T,R,\Omega) = F_0(T,R) - \frac{1}{2} I(T,R)\Omega^2, \quad I(T,\Omega) = -\frac{1}{\Omega}\left(\frac{\partial F}{\partial \Omega}\right)_T$$

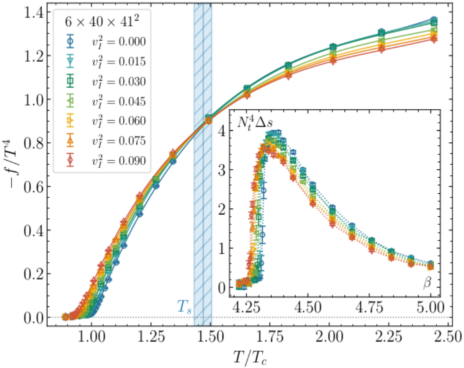
- $I = I_{fluct} + I_{cond}$
  $I_{fluct} = \ll J_3^2 \gg_T$, $I_{cond} = \frac{1}{3}\int d^3x r^2 \ll H^2 \gg_T$
- Moment of inertia is related to the scale anomaly in QCD

$$\ll E^2 \gg_T + \ll H^2 \gg_T \sim -T^{\mu}_{\mu}$$

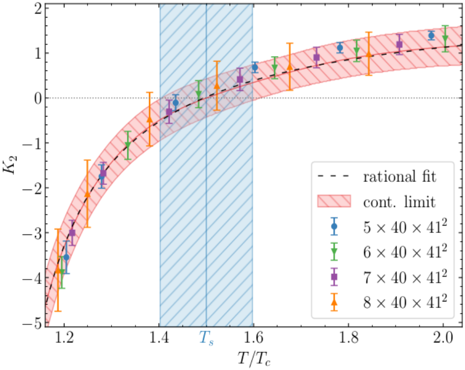
- Instead of $I(T,R)$ we calculate $K_2 = -\frac{I(T,R)}{F_0(T,R)R^2}$

Calculation of free energy on the lattice

- $F = -T \log Z$ impossible to calculate on the lattice
- $\frac{\partial F}{\partial \beta} \sim \langle \Delta s(\beta) \rangle = s(\beta)_T - s(\beta)_{T=0}, \quad \beta = \frac{6}{g^2}$
- $\frac{F(T)}{T^4} \sim \int_{\beta_0}^{\beta_1} d\beta' \langle \Delta s(\beta') \rangle$

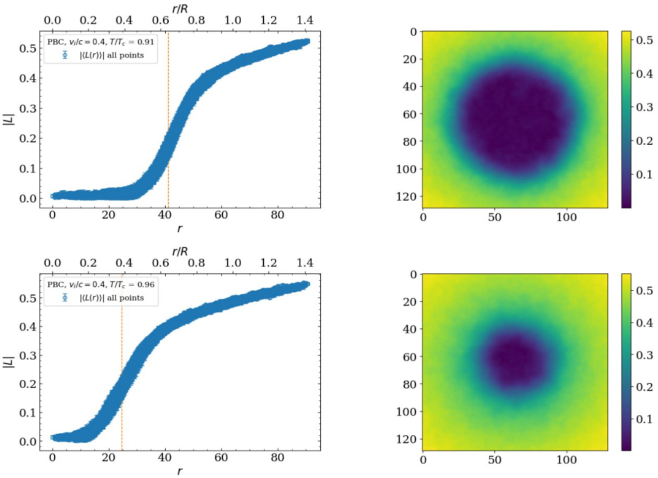


Moment of inertia of gluon plasma



- $I(T, R) = -F_0(T, R) K_2 R^2$
- $I < 0$ for $T < 1.5T_c$ and $I > 0$ for $T > 1.5T_c$
- Negative moment of inertia indicates a thermodynamic instability of rigid rotation
- The region of $I < 0$ is related to magnetic condensate and the scale anomaly
- We believe that the same is true for QCD

Inhomogeneous phase transitions in QGP



- Polyakov loop strongly depends on distance to the rotation axis
- Confinement in the center and deconfinement close to boundary

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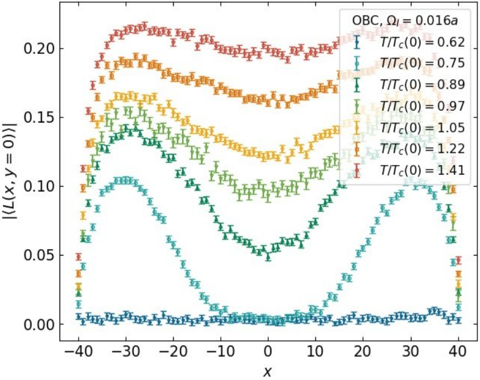

- Strong inhomogeneity of Polyakov loop close to $\sim T_c$
- Weak dependence on distance at large temperatures

Conclusion

- Lattice study of rotating gluodynamics and QCD have been carried out
- Critical temperatures rise with rotation
- We calculated the moment of inertia of GP. It is negative at temperatures $T < 1.5T_c$ and positive at larger temperatures
- Negative moment of inertia indicates a thermodynamic instability of rigid rotation
- We observed inhomogeneous phase transitions in GP
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THANK YOU!