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Polarization through the elastic scattering in magnetic field

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Outline

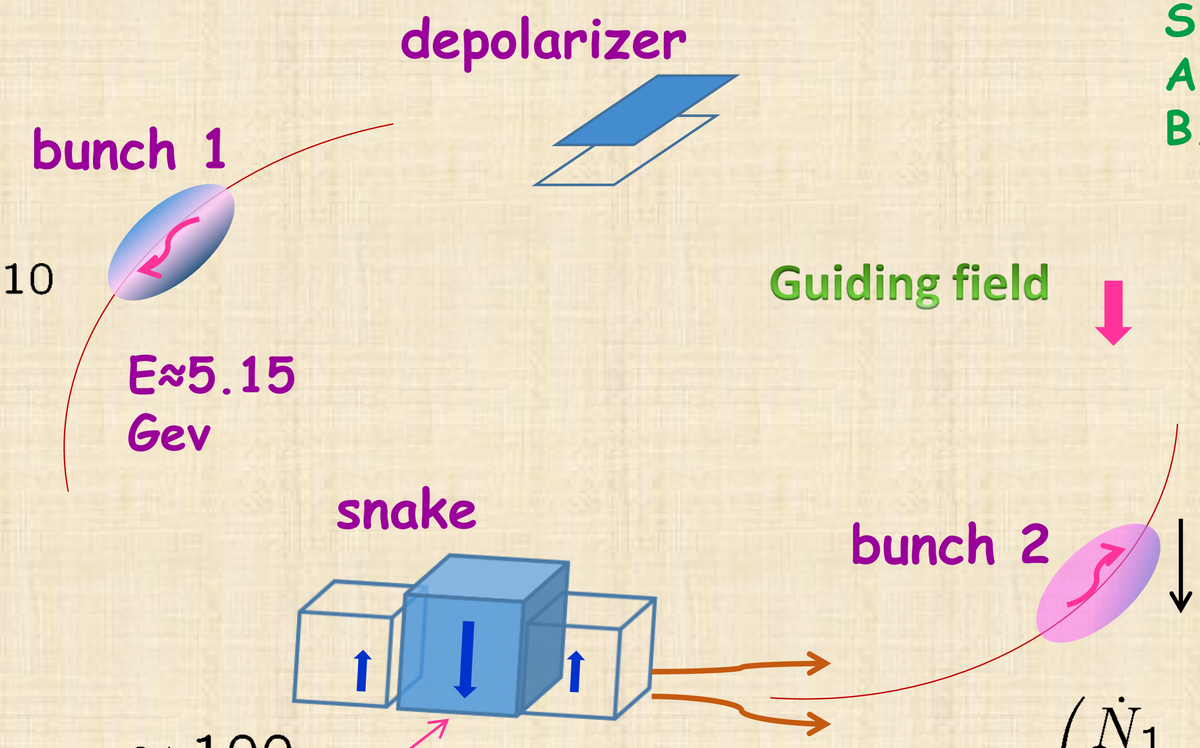
1. Experimental background
2. Conceptual scheme of separation: effusional polarization
3. Spin-orbital interaction
4. Effects of spin-spin interaction
5. EP vs RP: comparison
6. Remark: no residual depolarization in EP

1 Experimental background: Novosibirsk VEPP-4 experiment (1984)

A. E. Bondar, B. A. L. Saldin Nucl. Instr. Meth. 195 577 (1982)

S.A. Belomestnykh, A. E. Bondar, M. N. B. Yegorychev et al. Nucl. Instr. Meth. 227 173 (1984)

$N \sim 10^{10}$



$\zeta \simeq 0.7$

$\tau_{pulse} \simeq 100 ns$

Central gap magnetic field ~ 10÷20 kGs

$$\left(\frac{\dot{N}_1}{\dot{N}_2} - 1\right) \times 10^4$$

Nuclear Instruments and Methods in Physics Research 227 (1984) 173–181

S.A. Belomestnykh, A. E. Bondar, M. N. Yegorychev et al.

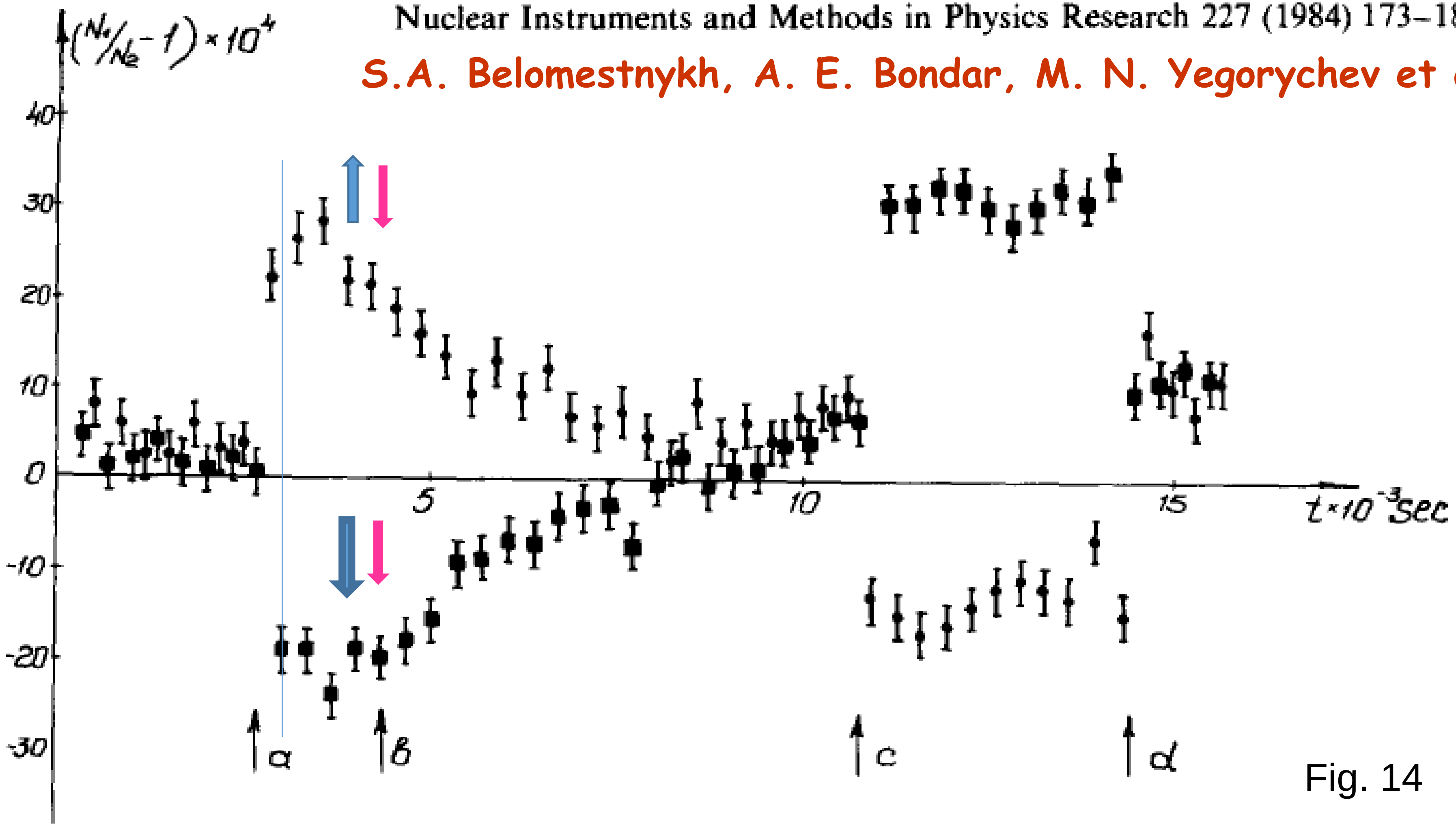


Fig. 14

■ - the sign of the field in the central gap [of the ‘snake’] coincides with the sign of the storage ring guiding field; ● - the sign of the field in the ‘snake’ is opposite to the storage ring guiding field. Within the time interval (a,b) the first bunch is depolarized. In the time interval (b,c) the depolarizer is switched off. Within the time interval (c,d) the second bunch is depolarized. Starting at the moment d both bunches are depolarized simultaneously.

2. Conceptual scheme of separation

$$\exp(i\Delta W) = \exp\left(\frac{i}{2}\int\int JD_cJ\right), \quad |e^{(i\Delta W)}|^2 = e^{-2\Im\Delta W}$$

$$J_\alpha(x) = j_\alpha^{orb}(x) + j_\alpha^{s}(x)$$

LS: *SQS'05, JINR, Proc. Int. Conf., B. M. Zupnik ed., 344 (2005); Int.J.Mod.Phys. 37 #15 (2022)*

$$\Delta W = -(\Delta m_{or} + \Delta m_{so} + \Delta m_{ss})T$$

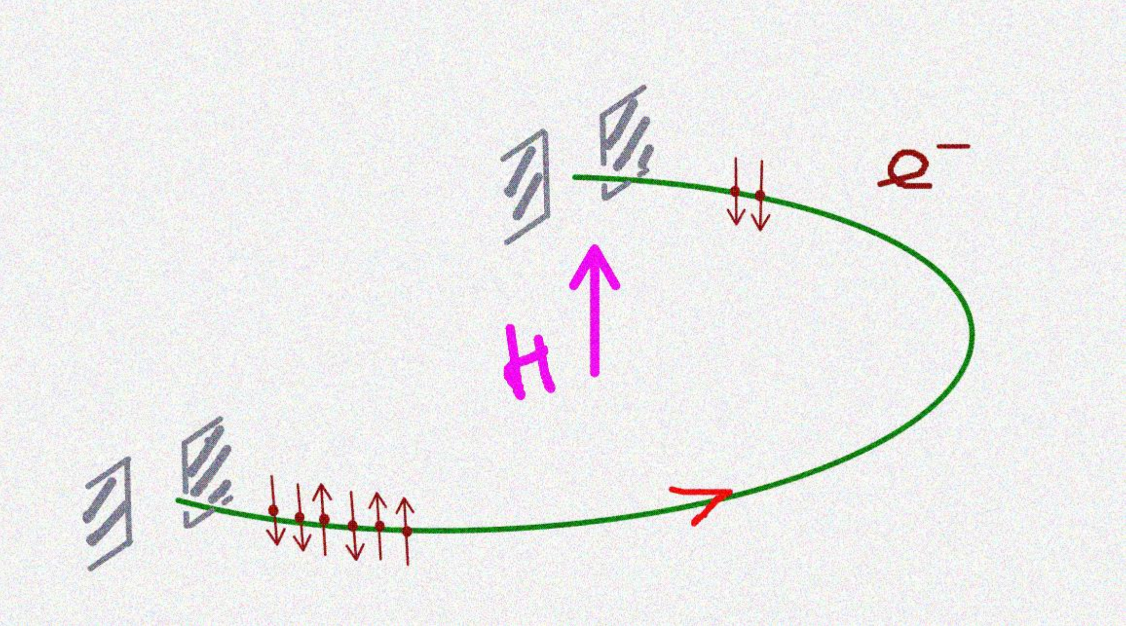
$$\lambda = -2\Im\Delta m$$

$$\gamma \simeq 10^4 \div 10^5\,,\ \Delta n \simeq 10^{11} \div 10^{12} \ll n \sim 10^{17}$$ ← **Landau quantum number**

Radiation reaction Effects:

$$\frac{\Delta n}{n} \sim 10^{-5} \sim \chi = \frac{1}{m^3}\sqrt{(eF_{\mu\nu}p_\nu)^2}$$

If no gain is imparted to the bunch, the number of particles moving along the unperturbed trajectory is to be gradually diminished to help confining the particles with 'rightly' orientated spins over the designed orbit



The probability distribution of spins within the elastically scattered electrons

$$dN(\vec{\zeta},0) = N_0 \frac{d\Omega}{4\pi} \,.$$

$$dN(\vec{\zeta},T) = N_0 \frac{d\Omega}{4\pi} \, e^{-\lambda(\vec{\zeta})T} \,,$$

$$\frac{dN(\vec{\zeta},T)}{N(T)} \equiv w(\vec{\zeta},T) d\Omega$$

$$N(T) = \int \frac{d\Omega}{4\pi} e^{-\lambda(\vec{\zeta})T} N_0 \,,$$

$$\langle \zeta_3 \rangle = \int \zeta_3 w(\vec{\zeta},T) d\Omega \,.$$

Isotropic factors like

$$N_0 \,, \exp\left(-\lambda_{or} T\right) , \exp\left(-k\vec{\zeta}^2 T\right)$$

do not enter the distribution $w(\vec{\zeta},T)$

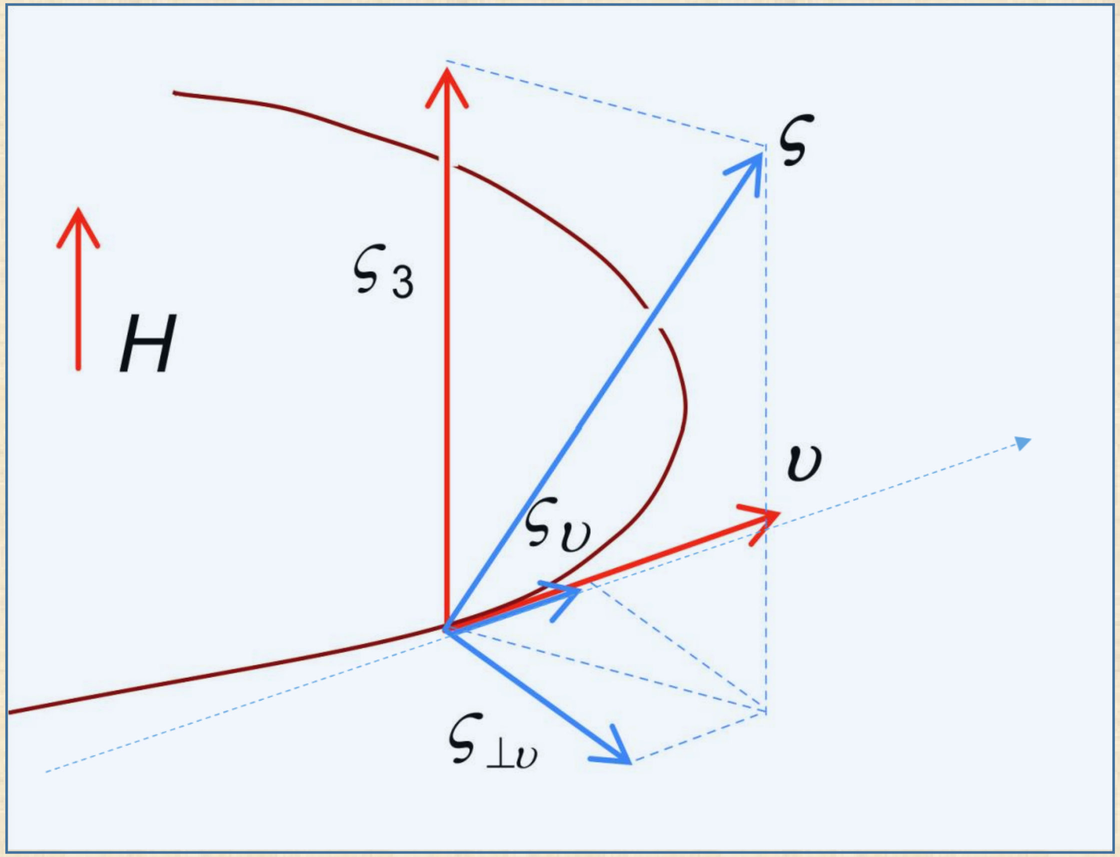
3. Spin-orbital interaction

$$\lambda(\vec{\zeta}) = -2\Im\Delta m^{CED} = \lambda_{or} + \lambda_{so} + \lambda_{ss}$$

$$\sim \chi \qquad \sim \chi^2 \qquad \sim \chi^3$$

$$\lambda(\vec{\zeta}) = \lambda_{or} + a\zeta_3 + k\vec{\zeta}^2 - b\zeta_3^2 - c\zeta_v^2$$

$$\vec{\zeta}^2 = \zeta_{\perp v}^2 + \zeta_v^2 + \zeta_3^2$$



Langevin approximation

$$\langle\zeta_3\rangle_L = \int \zeta_3 w_L(\vec{\zeta}, T) d\Omega\,, \qquad w_L(\vec{\zeta}, T) = \frac{1}{Z_0} e^{-a\zeta_3 T}$$

$$Z_0 = 4\pi \sinh\theta/\theta\,,\ \theta = |\vec{\zeta} a_{BMT}| T\,, \qquad \langle\zeta_3\rangle_L = -|\vec{\zeta}| L(\theta) \equiv -|\vec{\zeta}| \left(\coth\theta - \frac{1}{\theta}\right).$$

$$\langle \zeta_3 \rangle_L = -|\vec{\zeta}| \left(1 - \frac{1}{\theta} \right) \, , \ \theta \gg 1 \, , (T \gg T_{so}/\gamma)$$

The (laboratory) running time: $$\boxed{T_{so} = 2\sqrt{3} a_B \gamma \chi^{-2} \simeq 0.1 \div 1 ms}$$

$$\langle \zeta_3 \rangle_L^{as} = -|\vec{\zeta}| \qquad N(T) \sim \exp\left(-\lambda T\right) N_0$$

Remark (LS, 2016):

$$\begin{tikzcd} & a_{BMT} \\ a \arrow[ur] \arrow[dr] & \\ & a_{Fr} \end{tikzcd}$$

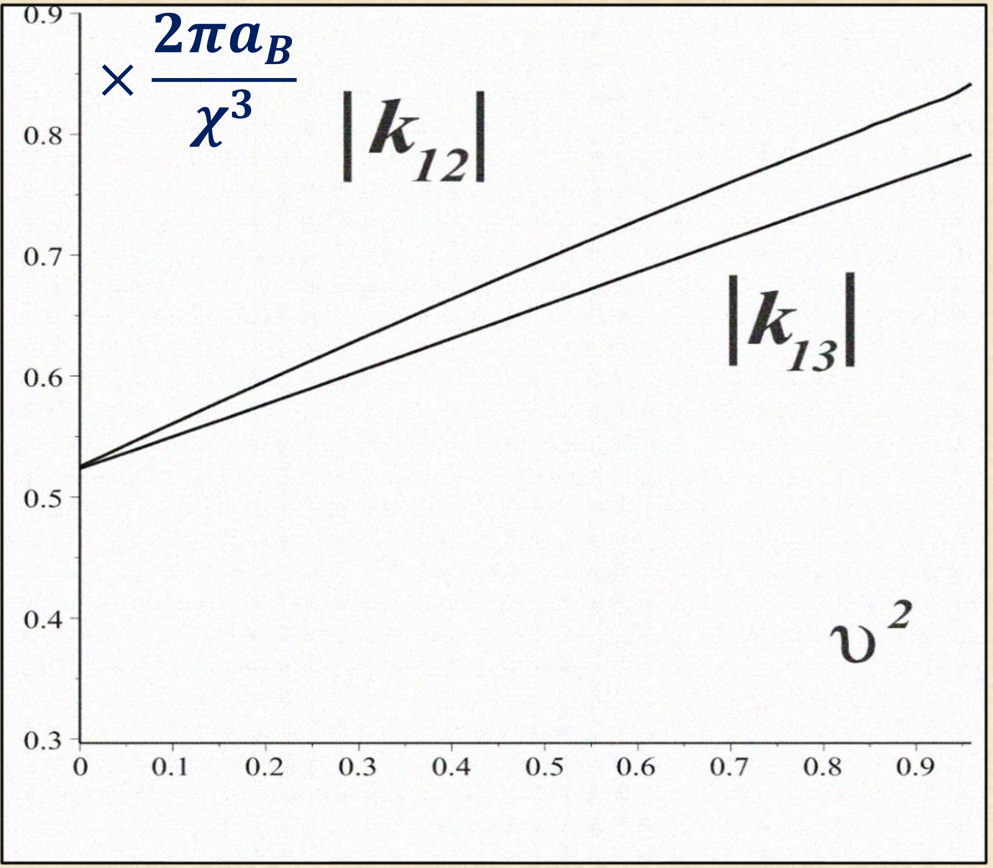
$$\frac{a_{Fr}}{a_{BMT}} = -\frac{3}{2} \ (\gamma \gg 1) \qquad a_{BMT} = \frac{\chi^2}{2\sqrt{3} a_B}$$

$$\theta_{Fr} = |\vec{\zeta} a_{Fr}| T \, , \qquad \langle \zeta_3 \rangle_L = |\vec{\zeta}| L(\theta_{Fr})$$

4. Effects of spin-spin interaction

$$\Delta W = \frac{1}{2}\int\int j^{(s)} D_c j^{(s)} = -\Delta m_{ss} T$$

$$\lambda_{ss} = -2\Im\Delta m_{ss} \sim \chi^3$$



$$\Im\Delta m_{ss} = k_0\vec{\zeta}^2 + k_{12}\zeta_{\perp v}^2 + k_{13}\zeta_v^2$$

$$= (k_0 + k_{12})\vec{\zeta}^2 - k_{12}\zeta_3^2 - (k_{12} - k_{13})\zeta_v^2$$

$$k_{12} - k_{13} = -\frac{\chi^3\gamma^{-5}}{4\pi a_B}\int_0^\infty dw \frac{v^2(w\cos w - \sin w)^2}{(w^2 - v^2\sin^2(w))^3}$$

Uniform motion

Symmetry: $$\lambda_{ss} = k\vec{\zeta}^2 - b\zeta_3^2 - c\zeta_v^2 \rightarrow k\zeta_\perp^2$$ **(k=b, c=0)**

a) neutron, F= const
b) electron in Wien filter $$\lambda_{so} = 0\,,\, \lambda_{ss} = \frac{\mu^2}{6\pi}|2\mu H_{RF}|^3\zeta_\perp^2$$

$$\lambda_{ss}|_{\zeta_\perp^2 \to 4} = W_{\uparrow\downarrow}^{QED}\gamma \quad (= \lambda^{QED}$$ **for neutron case)**

$$\lambda_{ss}|_{\zeta_\perp^2 \to 4} = \lambda^{QED}$$ **"1/4 - rule"** → full ***Q***↔***C*** coincidence

$W_{\uparrow\downarrow}^{QED}$ **I.Ternov, V. Bagrov, A. Khapaev (1965)**

Classical derivation (of spin-flip probability rate & radiation power) in:

V. Lyuboshitz (1966)
V. Bordovitsyn, V. Gushchina (1994)
A. Lobanov, O. Pavlova (2000)

Quadratic spin terms contribution (charged magneton)

$$Z = \int d\Omega \, e^{-aT\zeta_3} \, e^{bT\zeta_3^2 + cT\zeta_v^2} \, , \quad \langle \zeta_3 \rangle = -\frac{\partial}{T\partial a} \ln Z$$

$$\langle \zeta_3 \rangle = -|\vec{\zeta}| + \frac{a_B}{T} \left(\frac{c_1}{\chi^2} - \frac{c_2}{\chi} + c_3 + \dots \right) + \dots$$

$$\sim T^{-2}$$

$$c_1 = 2\sqrt{3}, \; c_2 = \frac{29\sqrt{3}}{8} |\vec{\zeta}|, \; c_3 = \frac{421\sqrt{3}}{64} |\vec{\zeta}|^2$$ **(BMT)**

$$c_1 = -4/\sqrt{3}, \; c_2 = ?, \; c_3 = ?$$ **(Frenkel)**

$$a_B \chi^{-2} \lesssim T \ll a_B \chi^{-3}$$

5. EP vs RP: comparison

Total radiation rate of electron:

$$\lambda^{QED} = \lambda^{(nf)} + \lambda^{(f)}, \quad \lambda^{(f)} = \gamma W_{\uparrow\downarrow}$$

J. Schwinger, W. Tsai (1972)

$$\lambda^{QED}(\zeta_3) = -2\Im \Delta m^{QED} \quad \zeta_3^2 = 1$$

V. Baier,
V. Katkov,
V. Strakhovenko (1970):

$$\vec{\zeta} = \langle \vec{\sigma}(t) \rangle$$

$$W_{\uparrow\downarrow} = \frac{1}{2T_{QED}} \Bigg(\underbrace{1 + \zeta_3 \frac{8\sqrt{3}}{15}}_{} - \frac{2}{9}\zeta_v^2 \Bigg)$$

A. Sokolov,
I.Ternov (1963)

$$\lambda(\vec{\zeta}) = \lambda_{or} + a\zeta_3 + k\vec{\zeta}^2 - b\zeta_3^2 - c\zeta_v^2 \qquad b = \frac{1}{4}\frac{\gamma}{T_{QED}}, \; c = \frac{1}{15}b$$

Do not contribute in EP

Charged magneton

$$\langle\zeta_3\rangle^{as} = \langle\zeta_3\rangle|_{T\gtrsim T_{QED}} = -\frac{W_{\uparrow\downarrow}(+1)-W_{\uparrow\downarrow}(-1)}{W_{\uparrow\downarrow}(+1)+W_{\uparrow\downarrow}(-1)}+$$

$$\mathcal{O}(e^{-T/T_{QED}}) = -\frac{8\sqrt{3}}{15} = -0.924$$

RP

$$\langle\zeta_3\rangle^{as} = \langle\zeta_3\rangle|_{T\gtrsim T_{so}} = -|\vec{\zeta}| + \mathcal{O}(T_{so}/T)$$

EP

$$\boxed{\langle\cdots\rangle_{\mathcal{R}P} \neq \langle\cdots\rangle_{\mathcal{E}P}}$$

$$T_{QED} = \frac{8\sqrt{3}}{15}\frac{a_B}{c}\gamma\chi^{-3} \simeq 1\,\text{hour}$$

lab. Sokolov-Ternov relaxation time

$$T_{so} = 2\sqrt{3}a_B\gamma\chi^{-2} \simeq 0.1 \div 1\text{ms}$$

Do not contain $\hbar$

Neutral magneton,
Circular motion, $\vec{H} \neq 0$

$$\lambda(\vec{\zeta}) = \lambda_{ss} = k\vec{\zeta}^2 - b\zeta_3^2 - c\zeta_v^2$$

$$\left.\frac{\langle\zeta_v^2\rangle}{\langle\zeta_3^2\rangle}\right|_{T>T_{QED}} \simeq const > 0$$

Residual depolarization?

$$Z = \int d\Omega\, e^{bT\zeta_3^2 + cT\zeta_v^2} = 2\pi e^{bT|\vec{\zeta}|^2} \int_0^1 \frac{e^{-A\xi} I_0(\xi B)}{\sqrt{1-\xi}} d\xi$$

$$A = |\vec{\zeta}|^2 T\left(b - \frac{c}{2}\right), \qquad B = T\frac{c}{2}|\vec{\zeta}|^2 \qquad \langle\zeta_3\rangle = 0\,,$$

$$\langle\zeta_3^2\rangle = |\vec{\zeta}|^2 - \left(\frac{1}{2(b-c)} + \frac{1}{2b}\right)\frac{1}{T} + \cdots \quad \langle\zeta_v^2\rangle = \frac{1}{2(b-c)}\frac{1}{T} + \cdots$$

$$\langle\zeta_v^2\rangle / \langle\zeta_3^2\rangle \sim \mathcal{O}(1/T)$$

Conclusions

i. EP scheme could be used to separate charged particles according to orientation of their spins; it is determined by the total radiation rate and could be viewed as an alternative to RP process which is characteristic of storage ring model of spin relaxation. Both of schemes use the spin-dependent properties of SR ($\lambda(\vec{\zeta})$ in EP , and $W_{\uparrow\downarrow}(\vec{\zeta})$ in RP)

ii. **The straightforward application of EP mates with the loss of about $N_0(1-e^{-\lambda T})$ particles by the moment *T* (and this is the main drawback of the scheme).**

iii. The advantage seems to be the reduction in the temporal expenditures: the running time of EP process, T_{so} , is much less than the running time in RP process:

$$T_{so} \simeq 0.1 \div 1\, ms \qquad T_{QED} \sim 1\,\text{hour}$$

(T_{QED} is Sokolov – Ternov relaxation time)

iv. **The total radiation rate for neutral particle traversing through the constant background field has classical origin and coincides with QED spin-flip radiation rate thus giving no advantage in applications of EP to polarization experiments.**

v. The effect of residual depolarization does not take place within EP scheme giving for the asymptotic polarization: $\langle \zeta_3^2 \rangle_{as} \simeq |\vec{\zeta}|^2$

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