Energy Loss for Heavy Quarks in Strong Magnetic Field

Pavel Slepov

Based on papers
JHEP 07, 161 (2021) [arXiv:2011.07023],
Theoret. and Math. Phys., 206:3 (2021) [arXiv:2012.05758]
and arXiv:2305.06345
with I.Ya.Aref'eva, K.Rannu and A.Hajilou

Steklov Mathematical Institute of Russian Academy of Sciences

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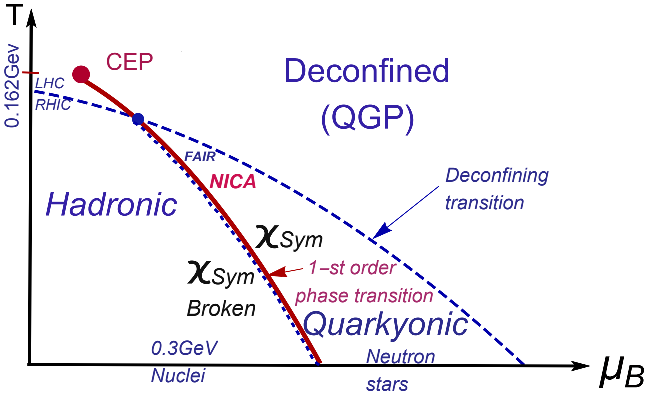
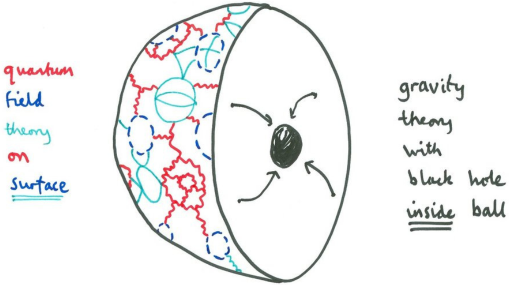
Expected QCD Phase Diagram



AdS/CFT correspondence

J.Maldacena, D.Witten, S.Gubser and others



AdS/CFT correspondence $\Longrightarrow$ $\frac{\eta}{s} = \frac{1}{4\pi}$

G.Policastro, D.Son, A.Starinets, PRL '01

Motivation

Purpose: Study of the QCD phase diagram in (μ, T) plane for the fully anisotropic background

Multiplicity $\mathcal{M} \propto s_{AdS}^{0.33}$ vs $\mathcal{M} \propto s_{LHC}^{0.155}$

$\mathcal{M} \propto s^{\frac{1}{\nu+2}}$, $\nu = 4.5$ I.Aref'eva, A.Golubtsova, JHEP '14

Strong magnetic field at the early stages of HIC: $eB \sim 0.3 GeV^2$

Action and metric. Twice Anisotropic Background

$$\mathcal{L} = R - \frac{f_0(\phi)}{4} F_0^2 - \frac{f_1(\phi)}{4} F_1^2 - \frac{f_3(\phi)}{4} F_3^2 - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$$

$$A_\mu^0 = A_t(z)\delta_\mu^0 \qquad F_1 = q_1\ dx^2 \wedge dx^3 \qquad F_3 = q_3\ dx^1 \wedge dx^2$$

$A_t(0) = \mu$ $\qquad g(0) = 1 \qquad$ *Dudal et al. (2019)*

$A_t(z_h) = 0 \qquad g(z_h) = 0 \qquad \phi(z_0) = 0 \ \rightarrow\ \sigma_{\text{string}}$

$$ds^2 = \frac{L^2}{z^2} b(z)\left[-g(z)\ dt^2 + dx_1^2 + \left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dx_2^2 + e^{c_B z^2}\left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dx_3^2 + \frac{dz^2}{g(z)}\right]$$

Aref'eva, Golubtsova (2014), Giataganas (2013) *Gürsoy, Järvinen et al. (2019)*

$b(z) = e^{2\mathcal{A}(z)} \ \rightarrow\$ quarks mass "Bottom-up approach"

$\mathcal{A}(z) = -\ cz^2/4 \ \rightarrow\$ heavy quarks background (b, t) *Andreev, Zakharov (2006)*

$\mathcal{A}(z) = -\ a\ln(bz^2+1) \ \rightarrow\$ light quarks background (d, u) *Li, Yang, Yuan (2017)*

Twice Anistropic solution for heavy quarks

$$g = e^{c_B z^2}\left\{1 - \frac{\Gamma\left(1+\frac{1}{\nu}\right) - \Gamma\left(1+\frac{1}{\nu}\ ;\ \frac{3}{4}(2c_B - c)z^2\right)}{\Gamma\left(1+\frac{1}{\nu}\right) - \Gamma\left(1+\frac{1}{\nu}\ ;\ \frac{3}{4}(2c_B - c)z_h^2\right)} - \right.$$

$$- \frac{\mu^2\,(2c_B - c)^{-\frac{1}{\nu}}}{4L^2\left(1 - e^{(c-2c_B)\frac{z_h^2}{4}}\right)^2}\left(\Gamma\left(1+\frac{1}{\nu}\right) - \Gamma\left(1+\frac{1}{\nu}\ ;\ \frac{3}{4}\,(2c_B - c)\,z^2\right)\right)\times$$

$$\left.\times\left[1 - \frac{\Gamma\left(1+\frac{1}{\nu}\right) - \Gamma\left(1+\frac{1}{\nu}\ ;\ \frac{3}{4}\,(2c_B - c)z^2\right)}{\Gamma\left(1+\frac{1}{\nu}\right) - \Gamma\left(1+\frac{1}{\nu}\ ;\ \frac{3}{4}\,(2c_B - c)z_h^2\right)}\ \frac{\Gamma\left(1+\frac{1}{\nu}\right) - \Gamma\left(1+\frac{1}{\nu}\ ;(2c_B - c)z_h^2\right)}{\Gamma\left(1+\frac{1}{\nu}\right) - \Gamma\left(1+\frac{1}{\nu}\ ;(2c_B - c)z^2\right)}\right]\right\},$$

$$A_t = \mu\,\frac{e^{\frac{1}{4}(c-2c_B)z^2} - e^{\frac{1}{4}(c-2c_B)z_h^2}}{1 - e^{\frac{1}{4}(c-2c_B)z_h^2}}, \qquad f_B = -\,2\left(\frac{z}{L}\right)^{-\frac{2}{\nu}} e^{-\frac{1}{2}cz^2}\,\frac{c_B z}{q_B^2}\,g\left(\frac{3cz}{2} + \frac{2}{\nu z} - c_B z - \frac{g'}{g}\right),$$

$$f_1(z) = 4\left(\frac{z}{L}\right)^{2-\frac{4}{\nu}} e^{-\frac{1}{2}(c-2c_B)z^2}\,\frac{\nu-1}{\nu z}\,g(z)\left(\frac{\nu+1}{\nu z} + \frac{3c-2c_B}{4}\,z - \frac{g'(z)}{2g(z)}\right),$$

$$\phi = \int_{z_0}^{z}\frac{1}{\nu\xi}\sqrt{4\nu - 4 + (4\nu c_B + 3(3c - 2c_B)\nu^2)\,\xi^2 + \left(\frac{3}{2}\,\nu^2 c^2 - 2c_B^2\right)\xi^4}\ d\xi \qquad z_0 \neq 0,$$

$$V(z) = -\frac{e^{\frac{1}{2}cz^2}}{4L^2\nu^2}\left\{\left[8(1+2\nu)(1+\nu) + 2(3+2\nu)(3c - 2c_B)\nu z^2 + (3c - 2c_B)^2\nu^2 z^4\right]g(z) - \right.$$

$$\left. - \left[2(4+5\nu) + 3(3c - 2c_B)\nu z^2\right] g'(z) + 2g''(z)\nu^2 z^2\right\}$$

I.Y.Aref'eva, K.A.Rannu, P.S. JHEP'21

Spatial Wilson loops. Parametrization

We use the representation of the rotation matrix $M(\phi, \theta, \psi)$ in 3-dimensional space:

$$x^i = \sum_{j=1,2,3} a_{ij}(\phi, \theta, \psi)\, \zeta^j, \qquad i = 1, 2, 3,$$

in terms of the Euler angles ϕ, θ, ψ:

$$M(\phi, \theta, \psi) = \begin{pmatrix} a_{11}(\phi, \theta, \psi) & a_{12}(\phi, \theta, \psi) & a_{13}(\phi, \theta, \psi) \\ a_{21}(\phi, \theta, \psi) & a_{22}(\phi, \theta, \psi) & a_{23}(\phi, \theta, \psi) \\ a_{31}(\phi, \theta, \psi) & a_{32}(\phi, \theta, \psi) & a_{33}(\phi, \theta, \psi) \end{pmatrix}$$

$$\begin{aligned}
a_{11}(\phi, \theta, \psi) &= \cos\phi\cos\psi - \cos\theta\sin\phi\sin\psi, \\
a_{12}(\phi, \theta, \psi) &= -\cos\psi\sin\phi - \cos\phi\cos\theta\sin\psi, \\
a_{13}(\phi, \theta, \psi) &= \sin\theta\sin\psi, \\
a_{21}(\phi, \theta, \psi) &= \cos\theta\cos\psi\sin\phi + \cos\phi\sin\psi, \\
a_{22}(\phi, \theta, \psi) &= \cos\phi\cos\theta\cos\psi - \sin\phi\sin\psi, \\
a_{23}(\phi, \theta, \psi) &= -\cos\psi\sin\theta, \\
a_{31}(\phi, \theta, \psi) &= \sin\phi\sin\theta, \\
a_{32}(\phi, \theta, \psi) &= \cos\phi\sin\theta, \\
a_{33}(\phi, \theta, \psi) &= \cos\theta.
\end{aligned}$$

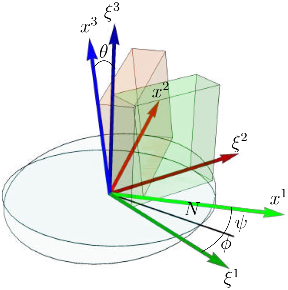
Here ϕ is the angle between ζ^1-axis and the node line (N), θ is the angle between ζ^3 and x^3-axes, ψ is the angle between the node line N and x^1-axis.

Spatial Wilson loops. Parametrization

To describe the nesting of the 2-dimensional world sheet in 5-dimensional space time we use

$$
\begin{aligned}
X^0(\xi) &= const, \\
X^i(\xi) &= \sum_{\alpha=1,2} a_{i\alpha}(\phi,\theta,\psi)\,\xi^\alpha, \qquad i=1,2,3, \\
X^4(\xi) &= z(\xi^1),
\end{aligned}
$$

where x^i are spatial coordinates and $a_{ij}(\phi,\theta,\psi)$ are entries of the rotation matrix.



Nambu-Goto action for SWL

$$\mathcal{S}_{SWL} = \int_{\mathcal{W}} \left(\frac{L^2 b_s}{z^2}\right) \sqrt{\left(\mathfrak{g}_1\mathfrak{g}_2 a_{33}^2 + \mathfrak{g}_1\mathfrak{g}_3 a_{23}^2 + \mathfrak{g}_2\mathfrak{g}_3 a_{13}^2 + \frac{z'^2}{g}\,\bar{g}_{22}\right)}\, d\xi^1 d\xi^2$$

$$\mathcal{V}_{SWL}(z) = \left(\frac{L^2 b_s}{z^2}\right) \sqrt{\mathfrak{g}_1\mathfrak{g}_2 a_{33}^2 + \mathfrak{g}_1\mathfrak{g}_3 a_{23}^2 + \mathfrak{g}_2\mathfrak{g}_3 a_{13}^2}$$

This result can be compared with the action and the effective potential for holographic entanglement entropy:

$$\mathcal{S}_{HEE} = \int_{\mathcal{P}} \left(\frac{L^2 b_s}{z^2}\right)^{3/2} \sqrt{\left(\mathfrak{g}_1\mathfrak{g}_2\mathfrak{g}_3 + \frac{z'^2}{g}\left(\bar{g}_{22}\bar{g}_{33} - \bar{g}_{23}^2\right)\right)}\, d\xi^1 d\xi^2 d\xi^3,$$

$$\mathcal{V}_{HEE}(z) = \left(\frac{L^2 b_s}{z^2}\right)^{3/2} \sqrt{\mathfrak{g}_1\mathfrak{g}_2\mathfrak{g}_3},$$

g, $\mathfrak{g}_1$, $\mathfrak{g}_2$, $\mathfrak{g}_3$ are functions of z and $\bar{g}_{22}$, $\bar{g}_{33}$, $\bar{g}_{23}$ are functions of z and the Euler angles.

I. Y. Aref'eva, A. Patrushev, P.S. JHEP'20

Born-Infeld type action

$$\mathcal{S} = \int_{-\ell/2}^{\ell/2} M(z(\xi))\sqrt{\mathcal{F}(z(\xi)) + (z'(\xi))^2}\, d\xi, \quad V(z(\xi)) = M(z(\xi))\sqrt{\mathcal{F}(z(\xi))}$$

We have two options to have $\ell \to \infty$ I. Aref'eva, EPJ Web Conf.'18

1) The existence of a stationary point of $\mathcal{V}(z)$: $\mathcal{V}'\Big|_{z_{DW}} = 0$.

$$\ell \underset{z\to z_*}{\sim} \frac{1}{\sqrt{F(z_{DW})}}\sqrt{\frac{\mathcal{V}(z_{DW})}{\mathcal{V}''(z_{DW})}}\log(z - z_*),$$

$$\mathcal{S} \underset{z\to z_*}{\sim} M(z_{DW})\sqrt{\frac{\mathcal{V}(z_{DW})}{\mathcal{V}''(z_{DW})}}\log(z - z_*).$$

$$\mathcal{S} \sim M(z_{DW}) \cdot \sqrt{F(z_{DW})} \cdot \ell,$$

$$\sigma_{DW} = M(z_{DW})\sqrt{F(z_{DW})}.$$

Born-Infeld type action

2) There is no stationary point of $\mathcal{V}(z)$ in the region $0 < z < z_h$, and we suppose it to be near horizon

$$F(z) = \mathfrak{F}(z_h)(z_h - z) + \mathcal{O}((z_h - z)^2),$$

if $M(z) \underset{z \to z_h}{\to} \infty$ as

$$M(z) \underset{z \sim z_h}{\sim} \frac{\mathcal{M}(z_h)}{\sqrt{z - z_h}},$$

$$\ell \underset{z \to z_h}{\sim} \frac{1}{\sqrt{\mathfrak{F}(z_h)}} \frac{1}{\sqrt{-\frac{2\mathcal{V}'(z_h)}{\mathcal{V}(z_h)}}} \log(z - z_h),$$

$$\mathcal{S} \underset{z \to z_*}{\sim} \mathcal{M}(z_h) \frac{1}{\sqrt{-\frac{2\mathcal{V}'(z_h)}{\mathcal{V}(z_h)}}} \log(z - z_h).$$

$$\sigma_h = \mathcal{M}(z_h) \sqrt{\mathfrak{F}(z_h)} = M(z_h) \sqrt{F(z_h)}.$$

Particular cases

1) $\phi = 0, \theta = 0, \psi = 0$; $a_{11} = a_{22} = a_{33} = 1$, $a_{12} = a_{21} = a_{31} = a_{31} = a_{32} = a_{23} = 0$:

$$\mathcal{S}_{xY_1} = \int_{\mathcal{P}} \left(\frac{L^2 b_s}{z^2}\right) \sqrt{\left(\mathfrak{g}_1\mathfrak{g}_2 + \frac{z'^2}{g}\mathfrak{g}_2\right)}\, d\xi^1 d\xi^2, \quad \mathcal{V}_{xY_1}(z) = \left(\frac{L^2 b_s}{z^2}\right) \sqrt{\mathfrak{g}_1\mathfrak{g}_2};$$

2) $\phi = \psi = 0$, $\theta = \pi/2$; $a_{11} = -a_{23} = a_{32} = 1$, $a_{12} = a_{13} = a_{21} = a_{22} = a_{31} = a_{33} = 0$:

$$\mathcal{S}_{xY_2} = \int_{\mathcal{P}} \left(\frac{L^2 b_s}{z^2}\right) \sqrt{\left(\mathfrak{g}_1\mathfrak{g}_3 + \frac{z'^2}{g}\mathfrak{g}_3\right)} d\xi^1 d\xi^2, \quad \mathcal{V}_{xY_2}(z) = \left(\frac{L^2 b_s}{z^2}\right) \sqrt{\mathfrak{g}_1\mathfrak{g}_3};$$

3) $\phi = \theta = -\psi = \pi/2$, $a_{22} = a_{31} = -a_{13} = 1, a_{11} = a_{12} = a_{21} = a_{23} = a_{32} = a_{33} = 0$:

$$\mathcal{S}_{y_1Y_2} = \int_{\mathcal{P}} \left(\frac{L^2 b_s}{z^2}\right) \sqrt{\left(\mathfrak{g}_2\mathfrak{g}_3 + \frac{z'^2}{g}\mathfrak{g}_2\right)} d\xi^1 d\xi^2, \quad \mathcal{V}_{y_1Y_2}(z) = \left(\frac{L^2 b_s}{z^2}\right) \sqrt{\mathfrak{g}_2\mathfrak{g}_3}.$$

DW equations

The equations for the DW for SWL in particular cases for different potentials:

$$\mathcal{DW}_{xY_1} = \mathcal{DW}_{Xy_1} \equiv \left. \frac{2b_s'(z)}{b_s(z)} + \frac{\mathfrak{g}_1'(z)}{\mathfrak{g}_1(z)} + \frac{\mathfrak{g}_2'(z)}{\mathfrak{g}_2(z)} - \frac{4}{z} \right|_{z=z_{DW}} = 0,$$

$$\mathcal{DW}_{xY_2} \equiv \left. \frac{2b_s'(z)}{b_s(z)} + \frac{\mathfrak{g}_1'(z)}{\mathfrak{g}_1(z)} + \frac{\mathfrak{g}_3'(z)}{\mathfrak{g}_3(z)} - \frac{4}{z} \right|_{z=z_{DW}} = 0,$$

$$\mathcal{DW}_{y_1Y_2} \equiv \left. \frac{2b_s'(z)}{b_s(z)} + \frac{\mathfrak{g}_2'(z)}{\mathfrak{g}_2(z)} + \frac{\mathfrak{g}_3'(z)}{\mathfrak{g}_3(z)} - \frac{4}{z} \right|_{z=z_{DW}} = 0.$$

String tension for SWLs and drag forces

For solution $\mathfrak{g}_1 = 1$, $\mathfrak{g}_2 = (z/L)^{2-2/\nu}$, $\mathfrak{g}_3 = (z/L)^{2-2/\nu} e^{c_B z^2}$:

$$\sigma_{xY_1} = \sigma_{Xy_1} = \left(\frac{L^2 b_s(z)}{z^2}\right) \sqrt{\mathfrak{g}_1 \mathfrak{g}_2} = \left(\frac{L^{1+1/\nu} b_s(z)}{z^{1+1/\nu}}\right),$$

$$\sigma_{xY_2} = \left(\frac{L^2 b_s(z)}{z^2}\right) \sqrt{\mathfrak{g}_1 \mathfrak{g}_3} = \left(\frac{L^{1+1/\nu} b_s(z)}{z^{1+1/\nu}}\right) e^{c_B z^2/2},$$

$$\sigma_{y_1 Y_2} = \left(\frac{L^2 b_s(z)}{z^2}\right) \sqrt{\mathfrak{g}_2 \mathfrak{g}_3} = \left(\frac{L^{2/\nu} b_s(z)}{z^{2/\nu}}\right) e^{c_B z^2/2},$$

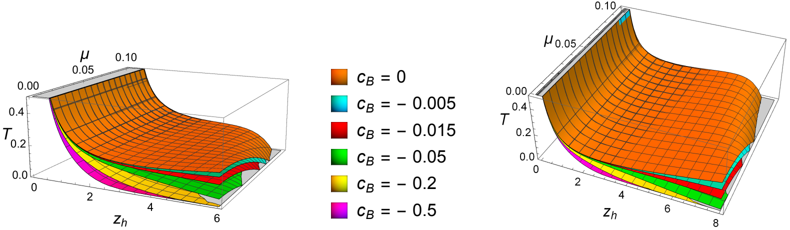
where $z = z_h$ or $z = z_{DW}$ (if the DW exists). The answers can be compared with drag forces I. Aref'eva Phys.Part.Nucl. 51 no.4, 489-496 (2020), O.Andreev, Mod. Phys. Lett. A 33, no.06 (2018), S. J. Sin and I. Zahed, Phys.Lett. B 648, 318 (2007).

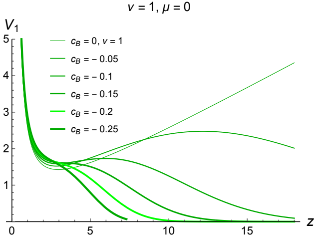
The drag forces for metric with $\mathfrak{g}_1 = 1$:

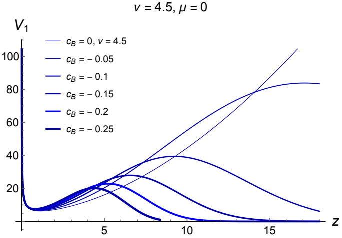
$$p_x = v_x \frac{b_s(z)}{z^2} \quad p_{y_1} = v_{y_1} \frac{b_s(z)}{z^2} \mathfrak{g}_2(z) \quad p_{y_2} = v_{y_2} \frac{b_s(z)}{z^2} \mathfrak{g}_3(z),$$

$$v_x = v\sqrt{\mathfrak{g}_2}, \quad v_{y_1} = v\,\frac{\sqrt{\mathfrak{g}_3}}{\mathfrak{g}_2}, \quad v_{y_2} = v\,\frac{\sqrt{\mathfrak{g}_2}}{\sqrt{\mathfrak{g}_3}} \quad \text{with some constant } v$$

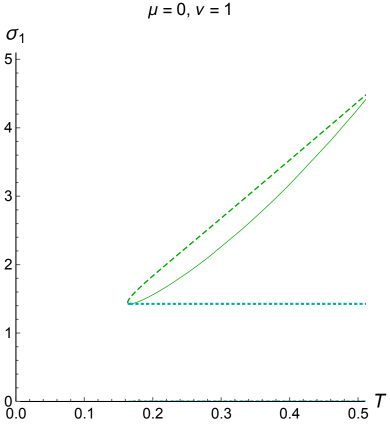
Temperature and effective potential $\mathcal{V}$

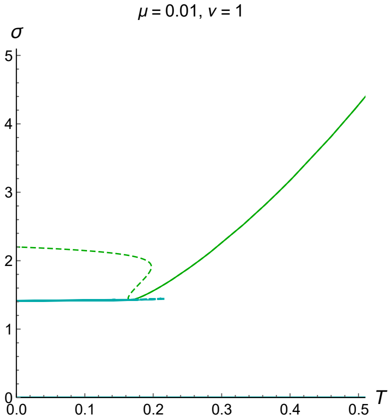




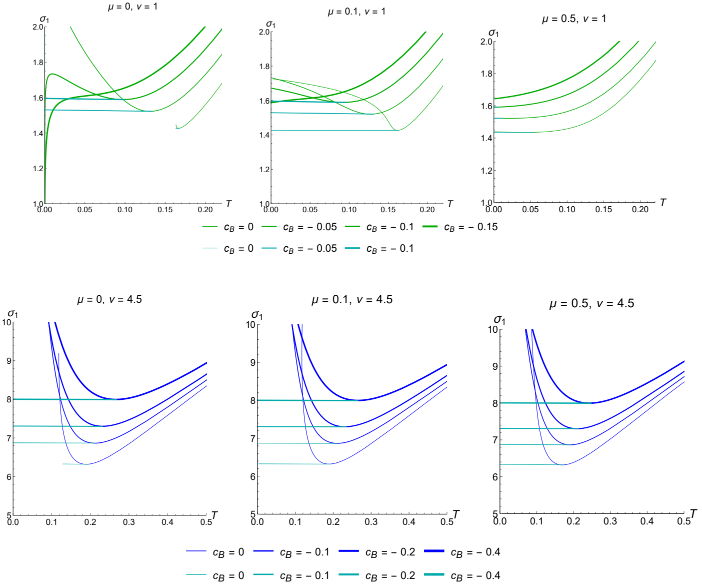


String tension for isotropic case and $c_B = 0$

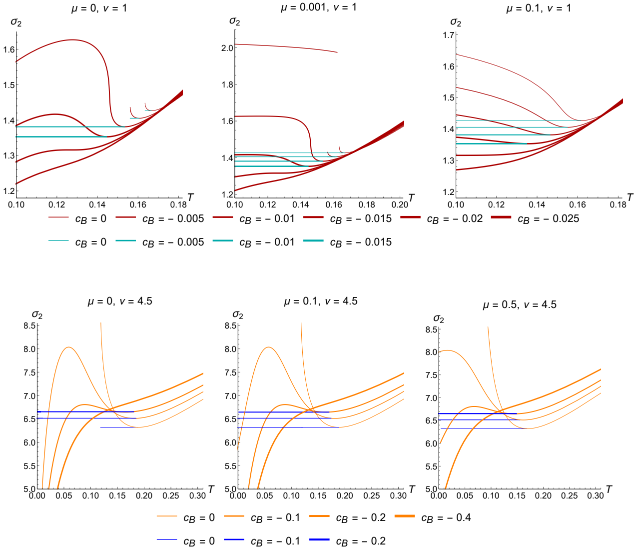




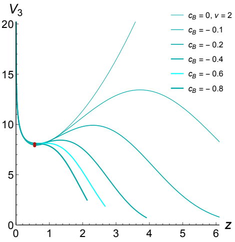
String tension σ_1 in isotropic and anisotropic cases ($\nu = 4.5$)



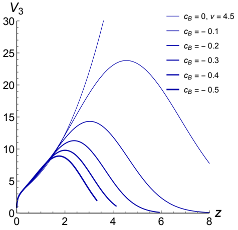
String tension σ_2 in isotropic and anisotropic cases ($\nu = 4.5$)



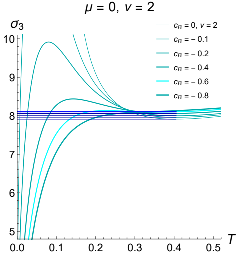
String tension σ_3 in isotropic and anisotropic cases ($\nu = 2$ and $\nu = 4.5$)



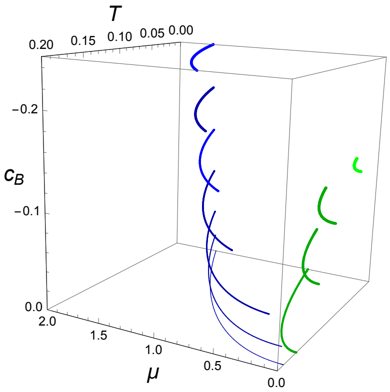
$\nu = 2$



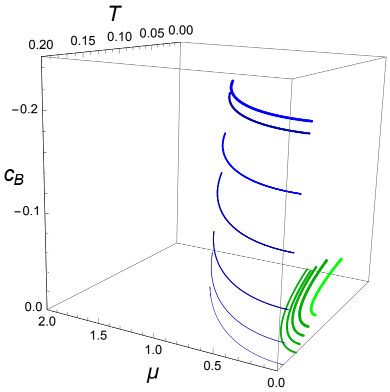
$\nu = 4.5$



Phase transitions of $\mathcal{V}_1$ and $\mathcal{V}_2$ for $\nu = 1, 4.5$



PT for $\mathcal{V}_1$

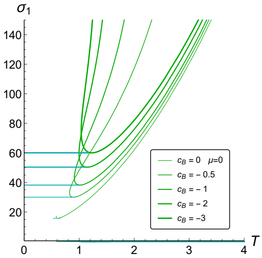


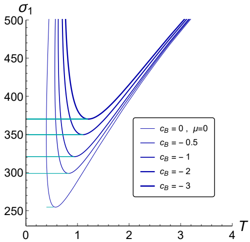
PT for $\mathcal{V}_2$

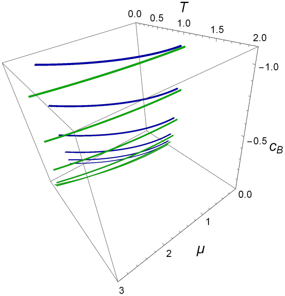
Phase transitions of $\mathcal{V}_1$ for $\nu = 1, 4.5$ in magnetic catalysis model with modified warp-factor

NEW Warp-factor: $\mathcal{A}(z) = -cz^2/4 - (p - c_B\, q_3)z^4$

Aref'eva, Hajilou, Rannu and P. S., arXiv:2305.06345







Conclusion

- The expressions for the string tension σ describing differently oriented SWL in the fully anisotropic background are obtained
- Under variation of thermodynamic parameters – temperature T, chemical potential μ and magnetic field – the string tensions undergo the phase transition.
- Temperature of this PT for magnetic catalysis model increases with increasing magnetic field in isotropic and anisotropic cases unlike the model with inverse magnetic catalysis

What's next?
Different orientations of SWL in MC model
Fully anisotropic hybrid model with external magnetic field

Thank you for your attention!