Higher Derivative Quantum Gravity in General Parametrization and General Gauge Conditions

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Theorem of Equivalence

S - matrix of renormalizable theory does not depend on non-linear, local reparametrisation of fields

$$\varphi^j \to \varphi^j + (\varphi^2)^j + (\varphi^3)^j + \cdots ,$$

Kallosh R.E., Tyutin I.V.,(1973), Yadernaya Fisika, v. 17, p. 190 (in Russian)
Y.M.P.Lam, (1973), Phys. Rev., D7, p. 2043
Tyutin I.V., (1982) Yadernaya Fisika, v. 35, p. 222 (in Russian)

Gravitational Theory

$$\underline{g}^{*\mu\nu} = \underline{g}^{\mu\nu}(-g)^p.$$

$$\underline{g}^*_{\mu\nu} = \underline{g}_{\mu\nu}.$$

Dynamical variables

Gravitational Theory

$$\underline{g}^{*\mu\nu} = \underline{g}^{\mu\nu}(-g)^p.$$

$$\underline{g}^*_{\mu\nu} = \underline{g}_{\mu\nu}.$$

Einstein Theory

$$\underline{g}^*_{\mu\nu} = \underline{g}_{\mu\nu}.$$

Goroff M.H. & Sagnotti (1986), ucl. Phys. B266, p 709

$$\underline{g}^{*\mu\nu} = \underline{g}^{\mu\nu}(-g)^{1/2}.$$

Capper D.M. et al (1973), Phys. Rev. D8;(1974)Phys. Rev. D9; (1974)Phys. Rev. D10

$$\underline{g}^{*\mu\nu} = \underline{g}^{\mu\nu} exp(-g)^{1/2}.$$

A.E.M. van de Ven, (1992), Nucl. Phys., B378, p. 309

Kallosh-DeWitt Theorem

Yang-Mills Theory
DeWitt B.S. (1965) "Dynamical Theory Groups and Fields"
Kallosh R.E. (1974), Nucl. Phys., B78,"Renormalization in non-abelian gauge theories"
Einstein gravity
Kallosh R.E. et al,(1978), Nucl. Phys., B137,"One-loop finitness of quantum gravity off mass shell"
Special case(Yang-Mills Theory)
McKeon G. et. al (1986), Nucl. Phys. B267

$$\mathcal{L}_{\rm gf} = \frac{1}{2\alpha}\left(n^{\mu}A^{a}_{\mu}\right)\hat{f}\left(n^{\nu}A_{a\nu}\right),$$

where $\hat{f}$ takes the next forms

$$\hat{f} = -1,$$

$$\hat{f} = \frac{\partial^2}{n^2},$$

$$\hat{f} = (n\partial)^2\left(n^2\right)^2.$$

Gauge and Parametrical Dependence of Quantum Einstein Gravity

Ichinose I.(1993), Nucl. Phys. B395
Lagrangian

$$\mathcal{L} = \sqrt{-g}R(g)$$

Dynamical variables

$$\underline{g}_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu}.$$

Gauge fixing

$$F_\mu = \nabla_\nu h^{*\nu}_{\ \mu} - l_1 \nabla_\mu h^* + \alpha J_\mu{}^{\alpha\beta} h_{\alpha\beta} + \delta K_\mu{}^{\lambda\alpha\beta} \nabla_\lambda h_{\alpha\beta}$$

where

$$J_\mu{}^{\alpha\beta} = J_\mu{}^{\alpha\beta}[\partial_\nu R, \partial_\nu R^{\alpha\beta}]$$

and

$$K_\mu{}^{\lambda\alpha\beta} = K_\mu{}^{\lambda\alpha\beta}[R, R_{\alpha\beta}, R_{\mu\nu\alpha\beta}]$$

One-loop divergences

$$\Delta\Gamma^{(1)}_{\infty} \sim R_{\mu\nu\alpha\beta} R^{\alpha\beta\lambda\delta} R_{\lambda\delta}{}^{\mu\nu} \quad (1)$$

Higher Derivative Quantum Gravity

$$\mathcal{L} = \sqrt{-g}\left(\alpha R + \beta R^2 + \gamma R_{\mu\nu}R^{\mu\nu} + \delta\nabla_\alpha\nabla^\alpha R + \Lambda\right),$$

where $\alpha, \beta, \gamma, \delta, \Lambda$ are unknown parameters
Dynamical variables are

$$g^*_{\mu\nu} = \varphi(\sqrt{-g})g_{\mu\nu}$$

So

$$\underline{g}_{\mu\nu} = g_{\mu\nu} + ah_{\mu\nu} + bg_{\mu\nu}h + ch_{\mu\nu}h + dh^\alpha_\mu h_{\alpha\nu} + eg_{\mu\nu}h^2 + fg_{\mu\nu}h^{\alpha\beta}h_{\alpha\beta} + \text{etc.}$$

One-Loop Divergences in Higher Derivative Quantum Gravity

$$\Delta\Gamma^{(1)}_{\infty} = -\frac{1}{32\pi(n-4)}\int \mathrm{d}^4x\sqrt{-g}\,\Big(\Lambda^2 C_\Lambda + C_{\Lambda R}\Lambda R + C_{(1R^2)}R_{\mu\nu}R^{\mu\nu} + \\ + C_{(2R^2)}R^2 + C_{(3R^2)}R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} + C_R R + C_5\Big)\,,$$

where

$$C_i = C_i(a, b, c, d....., l_k, n_j)$$

and

$$\mathcal{L}_{\text{gf}} = l_1 A_\nu^2 + l_2(\partial_\mu\phi)^2 + l_3 A_\mu\partial^\mu\phi + l_4(\Box\phi)^2 + l_5 F_{\mu\nu}^2 + l_6\left(A^\mu{}_{,\mu}\right)^2 + \\ + n_1 h^2 + n_2 h_{\mu\nu}h^{\mu\nu}$$

where we used the usual expressions $\phi = h^\alpha_\alpha$,

$$A_\mu = \partial^\nu h_{\mu\nu}, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

Gauge Dependence

Kazakov K.A., Kalmykov M.Yu., Pronin P.I., Stepanyantz K.V. (1998) Class. Quant. Grav. v.15, (1999)Phys. Rev.D59

$$\hat{\xi}\Gamma^{(1)}_{\infty} = a^{(1)} g_{\mu\nu} \frac{\delta S_{(0)}}{\delta h_{\mu\nu}}$$

$$a^{(1)} = a^{(1)}(a, b, c, d....., l_k, n_j)$$

To get gauge independent result is need to find

$$a^{(1)} = 0$$

Particular Cases

Kazakov K.A., Pronin P.I., (2000),Phys. Rev.D
Ilya L.Shapiro et al (2002), Phys. Rev.D
Ohta N., Percacci R. (2015-2019) Nucl. Phys.

Conformal Gravity

Conformal Parametrization

General Case

$$X^T W_1 X + V_1 X + s_1 = 0$$
$$X^T W_2 X + V_2 X + s_2 = 0$$
$$X^T W_3 X + V_3 X + s_3 = 0$$
$$X^T W_4 X + V_4 X + s_4 = 0,$$

General Case (2)

$$\begin{cases} a_1x^2 + b_1y^2 + c_1z^2 + d_1t^2 + m_1xy + n_1xz + k_1xt + l_1yz+ \\ +p_1yt + q_1zt + f_1x + g_1y + h_1z + r_1t + s_1 = 0, \\ a_2x^2 + b_2y^2 + c_2z^2 + d_2t^2 + m_2xy + n_2xz + k_2xt + l_2yz+ \\ +p_2yt + q_2zt + f_2x + g_2y + h_2z + r_2t + s_2 = 0, \\ a_3x^2 + b_3y^2 + c_3z^2 + d_3t^2 + m_3xy + n_3xz + k_3xt + l_3yz+ \\ +p_3yt + q_3zt + f_3x + g_3y + h_3z + r_3t + s_3 = 0, \\ a_4x^2 + b_4y^2 + c_4z^2 + d_4t^2 + m_4xy + n_4xz + k_4xt + l_4yz+ \\ +p_4yt + q_4zt + f_4x + g_4y + h_4z + r_4t + s_4 = 0, \\ a_5x^2 + b_5y^2 + c_5z^2 + d_5t^2 + m_5xy + n_5xz + k_5xt + l_5yz+ \\ +p_5yt + q_5zt + f_5x + g_5y + h_5z + r_5t + s_5 = 0. \end{cases}$$

Concluding Remarks

- There are 1024 solutions for gauge independent results
- Only 2 have been found
- Physical parametrization
- Refusion on quantum gravity
- Refusion "metrical"quantum theory of gravitation

Concluding Remarks II

Gauge affine-metric gravity

$$L = \frac{1}{16\pi G}\{R + Q_{\lambda\mu\nu}(a_1 Q^{\lambda\mu\nu} + a_2 Q^{\nu\mu\lambda} + a_3 g^{\lambda\nu} Q^{\alpha\mu}{}_{\alpha}) + \\ + R^{\alpha\beta\mu\nu}(b_1 R_{\alpha\beta\mu\nu} + b_2 R_{\mu\nu\alpha\beta} + b_3 R_{\alpha\mu\beta\nu} + \\ + b_4 R_{\alpha\mu} g_{\beta\nu} + b_5 R_{\mu\alpha} g_{\nu\beta} + b_6 g_{\mu\alpha} g_{\nu\beta} R)\}.$$

$g_{\beta\nu}$ - classical variable,

$\Gamma^{\lambda\mu\nu}$ - quantum variable

Thank You !