Lattice QCD with $N_c = 2$ at nonzero temperature and quark density

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28.08.2023

"21st Lomonosov Conference", Moscow State University

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Based on

- Phys.Rev.D 105 (2022) 11, 114505
- Phys.Rev.D 102 (2020) 114511
- JHEP 05 (2019) 171

OUTLINE

- Motivation
- QC_2D, lattice setup
- Confinement-deconfinement transition at low temperature
- Gluon propagators
- Conclusions

Motivation

- There are still problems with getting solid results in lattice QCD at nonzero μ_B (sign problem)
- Study of SU(2) QCD should help to check various methods and approaches to real QCD :

1) lattice methods (analytic continuation, Taylor expansion, etc.) can be checked

2) predictive power of other approaches (DSE, FRG, ChPT, effective actions,…) to nonperturbative QCD by comparison of their results for QCD-like theories with respective lattice results

3) SU(2) QCD reflects some properties of real QCD

Lattice studies of QC_2D

Dedicated workshop YITP workshop 'Probing the physics of high-density and low-temperature matter with ab initio calculations in 2-color QCD', 2020

Recent review **Viktor Braguta,** Phase Diagram of Dense Two-Color QCD at Low Temperatures, Symmetry 2023, 15, 1466.

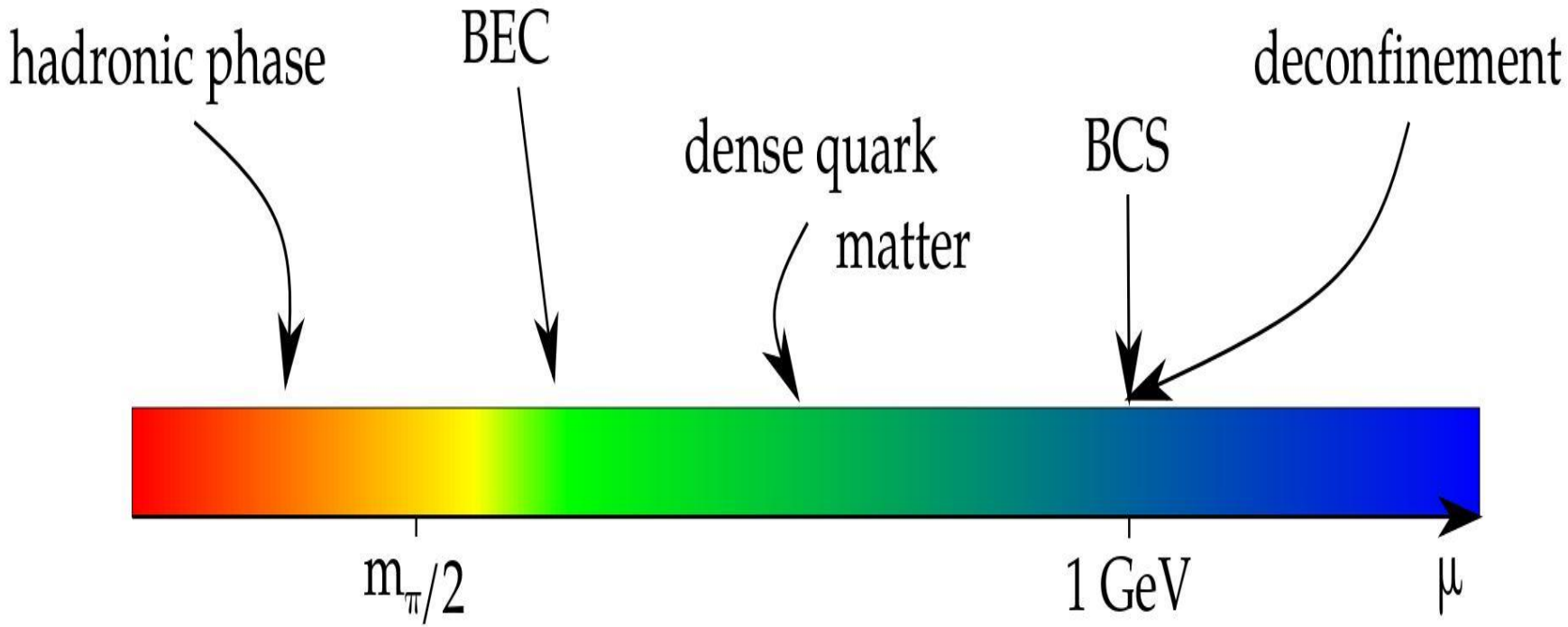
$N_f = 2$, staggered

- **Braguta, Ilgenfritz, Kotov, Molochkov, Nikolaev**, Study of the phase diagram of dense two-color QCD within lattice simulation, Phys. Rev. D 94 (2016)114510
- **Holicki, Wilhelm, Smith, Wellegehausen and von Smekal**, Two-colour QCD at finite density with two flavours of staggered quarks, PoS(LATTICE2016)052
- **Astrakhantsev, Bornyakov, Braguta, Ilgenfritz, Kotov,** Lattice study of static quark-antiquark interactions in dense quark matter, JHEP 05 (2019) 171

$N_f = 2$, Wilson

- **Boz, Giudice, Hands and Skullerud**, Dense Two-Color QCD Towards Continuum and Chiral Limits, Phys. Rev. D101 (2020) 074506
- **Iida, Itou, Lee,** Relative scale setting for two-color QCD with N_f=2 Wilson fermions, PTEP 2021 (2021)1, 013B05
- **Iida, Itou, Lee,** Two-colour QCD phases and the topology at low temperature and high density, JHEP 01 (2020) 181

Phase Diagram of QC_2D at T=0



At small μ this phase diagram is supported by CHPT, Lattice results, …

Lattice setup I

- SU(2) lattice QCD with $N_f = 2$ flavors of staggered Dirac operator
- Lattice size 40^4 and 32^4
- Lattice spacing $a = 0.048$ fm (fixed by $r_0 = 0.468$ fm)
- Pion mass $m_\pi = 680(40)$ MeV
- $L_1 = 1.92$ fm, $T_1 = 103$ MeV; $L_2 = 1.54$ fm, $T_2 = 128$ MeV
- $0 \leq \mu_q \lesssim 2000$ MeV ($0 \leq a\mu_q \leq 0.5$)

Lattice setup II

$$S_G = \frac{\beta}{2}\left[\, c_0 \sum_{pl} \mathrm{Re\,Tr}\,(1 - U_{pl}) + c_1 \sum_{pl} \mathrm{Re\,Tr}\,(1 - U_{rt})\,\right]$$

$$S_{stag} = \sum_x \bar{\psi}_x \left[\sum_\mu \frac{\eta_{x,\mu}}{2} \left(U_{x,\mu} e^{a\mu_q \delta_{\mu,0}} \psi_{x+\mu} - U^{\dagger}_{x-\mu,\mu}\, e^{-a\mu_q \delta_{\mu,0}} \psi_{x-\mu} \right) + am\, \psi_x \right] +$$

$$+ \sum_x \frac{1}{2} \lambda \left[\psi_x^T \sigma_2 \psi_x + \bar{\psi}_x \sigma_2 \bar{\psi}_x^T \right]$$

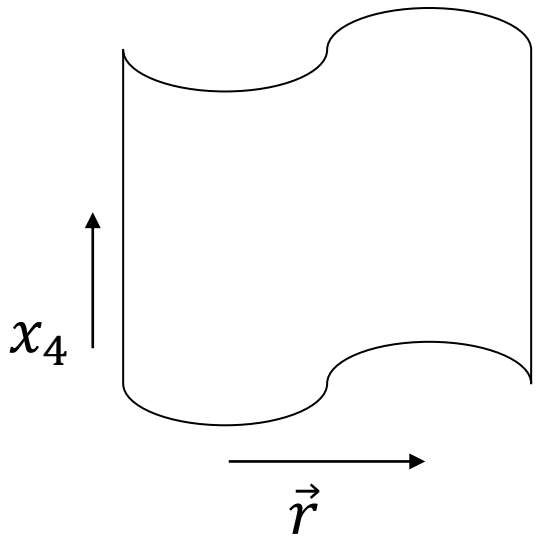
- $c_0 = 5/3, \quad c_1 = -1/12, \quad U_{x,\mu}$ are stout smeared variables
- $\eta_{x,\mu}$ - staggered sign function
- $\beta = 1.75, \quad am_q = 0.0075, \quad \lambda = 0.00075 = \frac{am_q}{10}$

Definitions

Wilson loop

$$W(C) = \frac{1}{N_c} Tr\{ P \exp(i \oint_C dx_\mu A_\mu(x)) \}$$

To compute $V_{\bar{q}q}(r)$ the contour C is



$$< W(r,t) >= C_0 e^{-E_0(r)t} + C_1 e^{-E_1(r)t} + \ldots$$

$$E_0(r) = V_{\bar{q}q}(r)$$

$$V_{\bar{q}q}(r) = -\lim_{t \to \infty} \frac{1}{t} \log < W(r,t) >$$

Spectral representation of WL

Confinement phase:

Ground state – hadron string for $r < r_{sb}$,

2 static-light mesons for $r > r_{sb}$

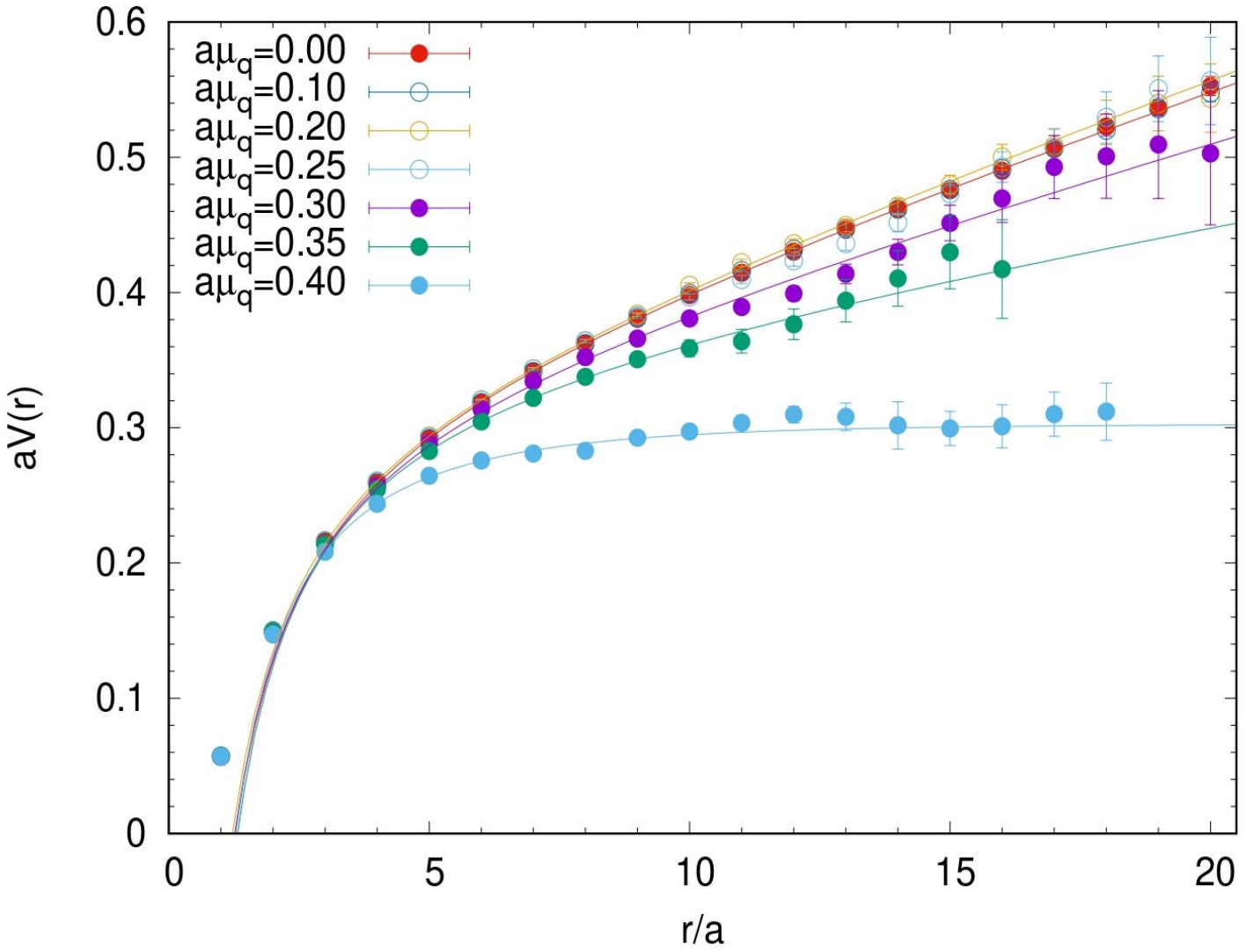
We use the fact that WL has very small overlap with s-l mesons state, C_{sl} <<1

For this reason one does not see string breaking, but clearly see hadron string state

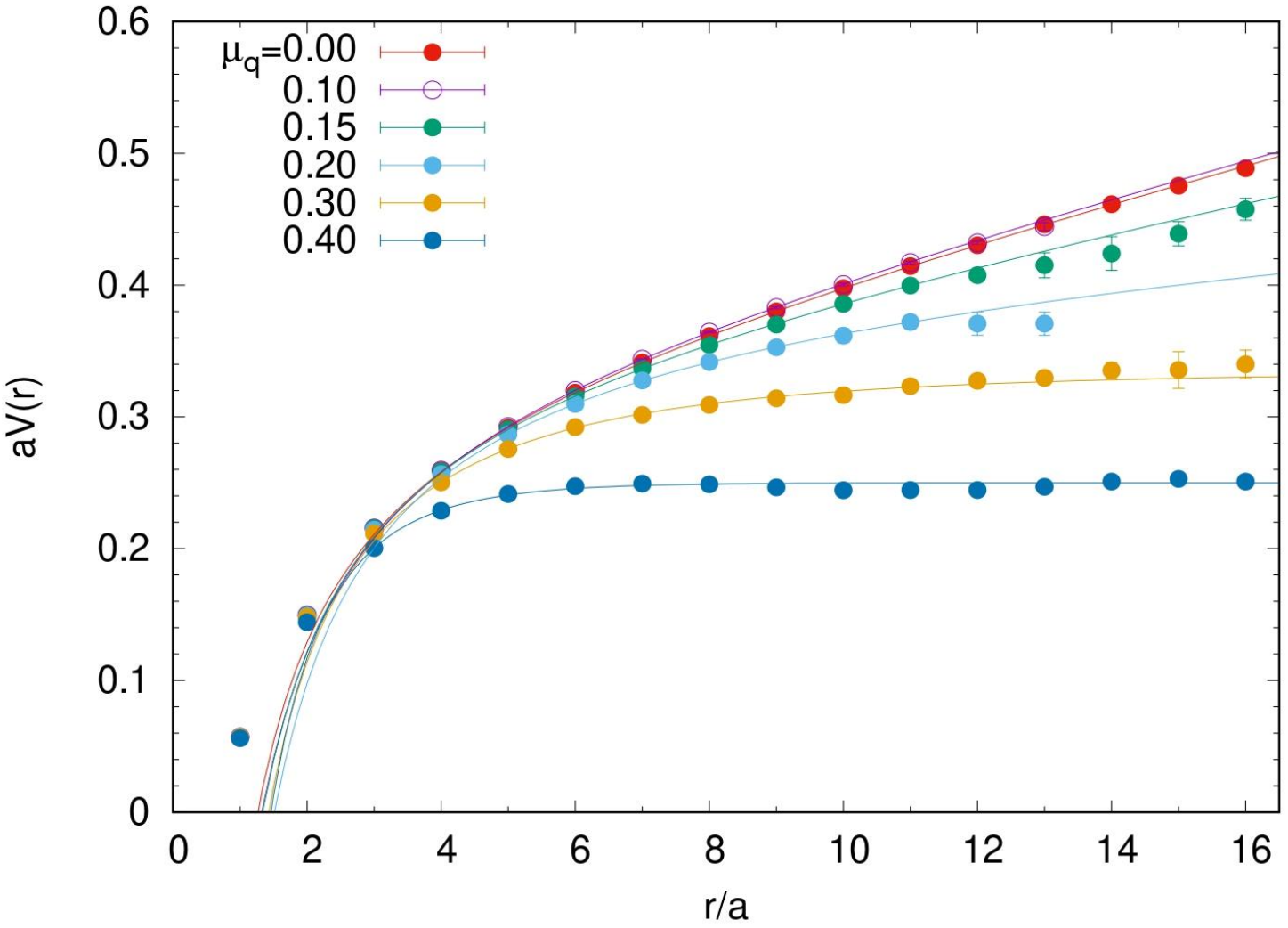
Deconfinement phase:

Ground state – color interaction is screened, Debye screening

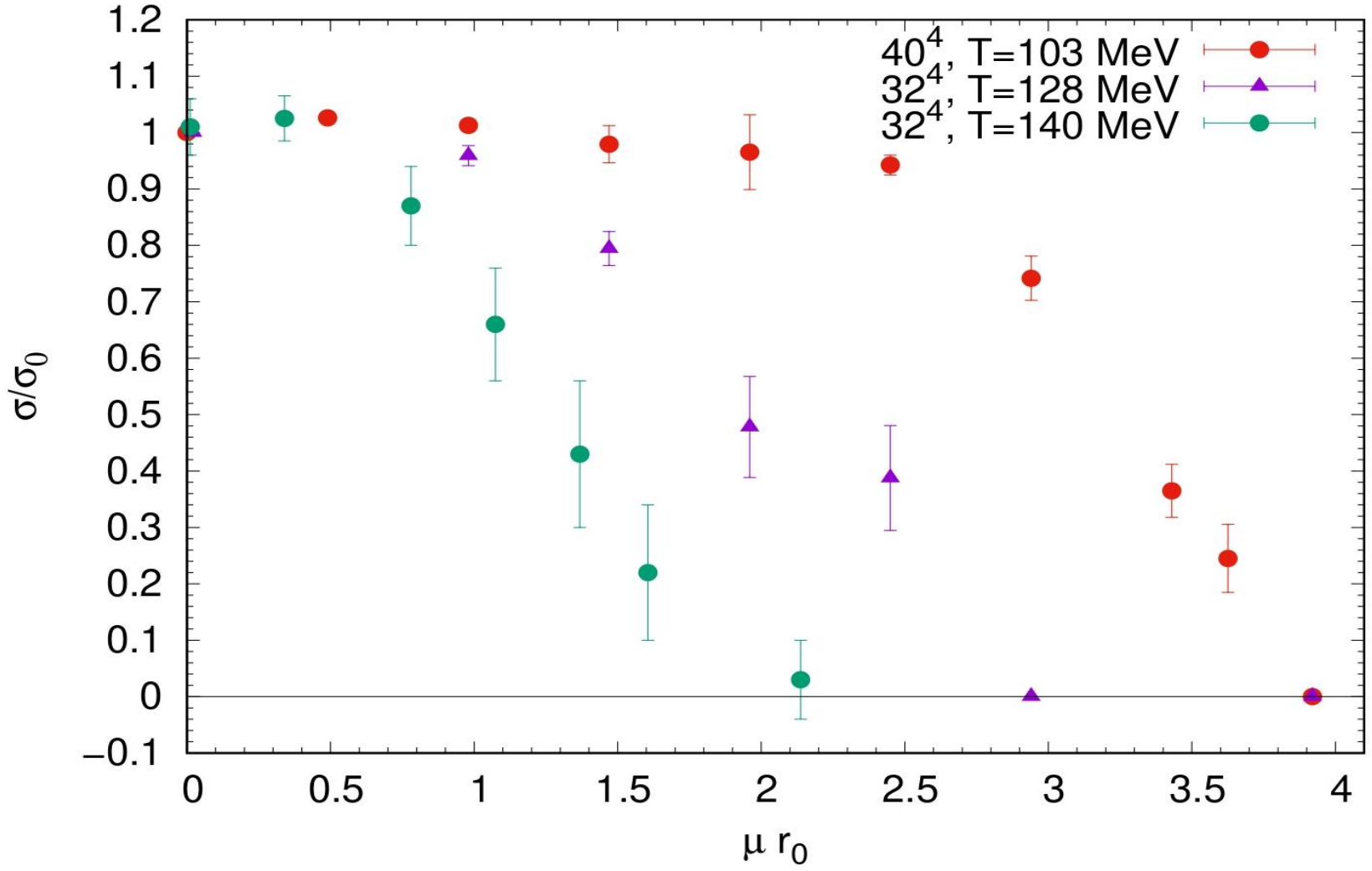
Static potentials for 40^4 lattice (T=103 MeV)



Static potentials for 32^4 lattice (T=128 MeV)

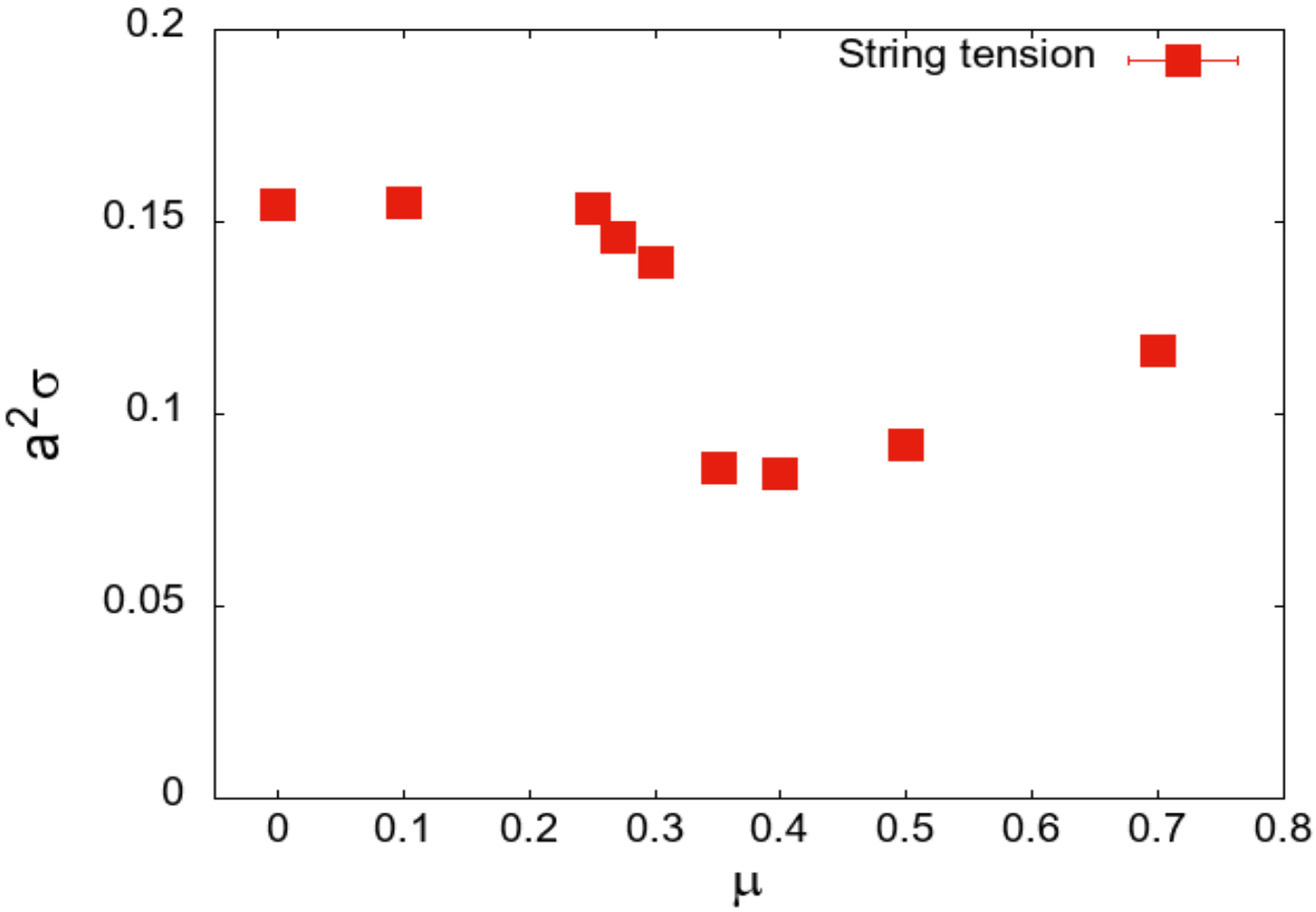


String tension vs. chemical potential

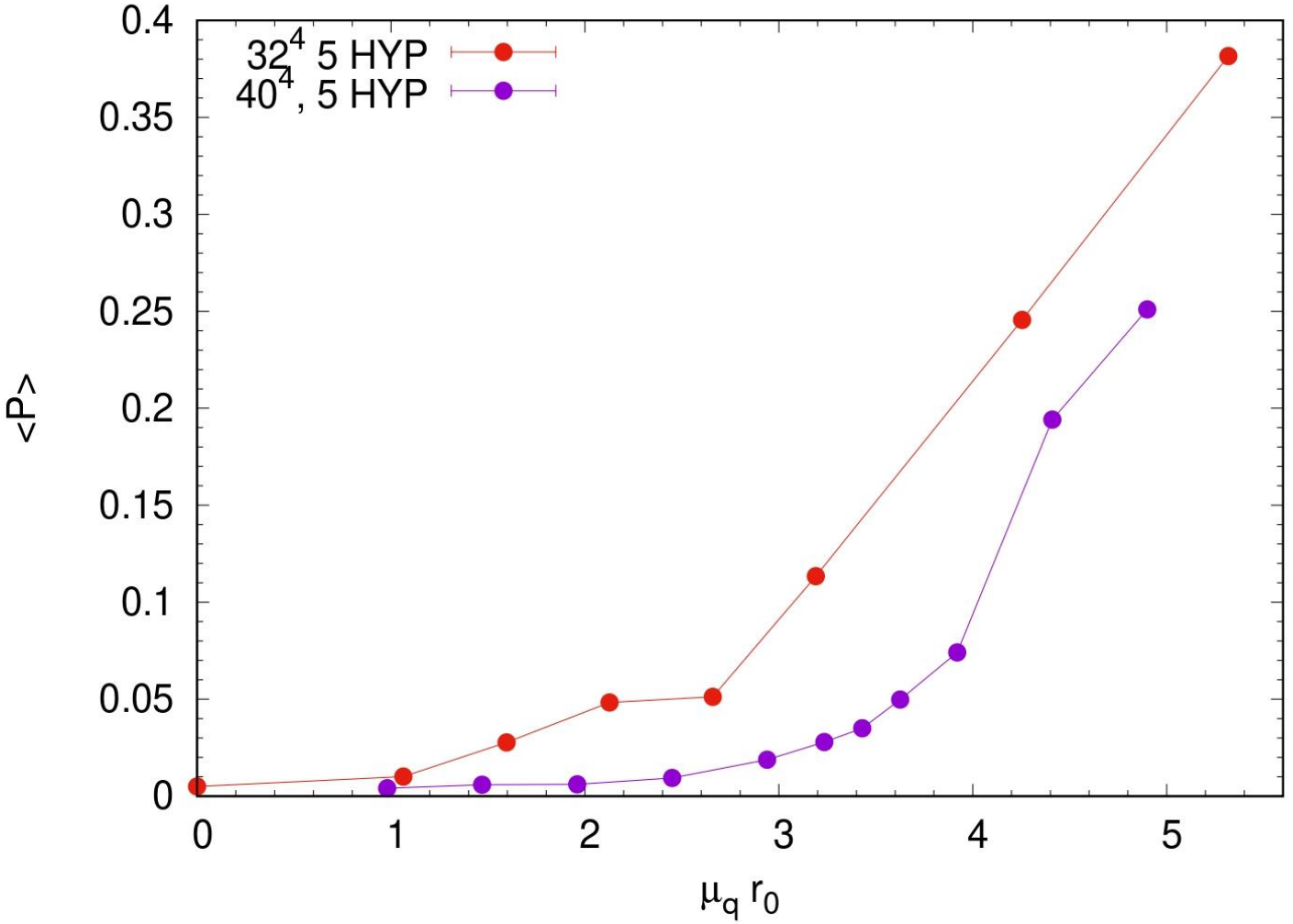


Recent results for Wilson fermion action

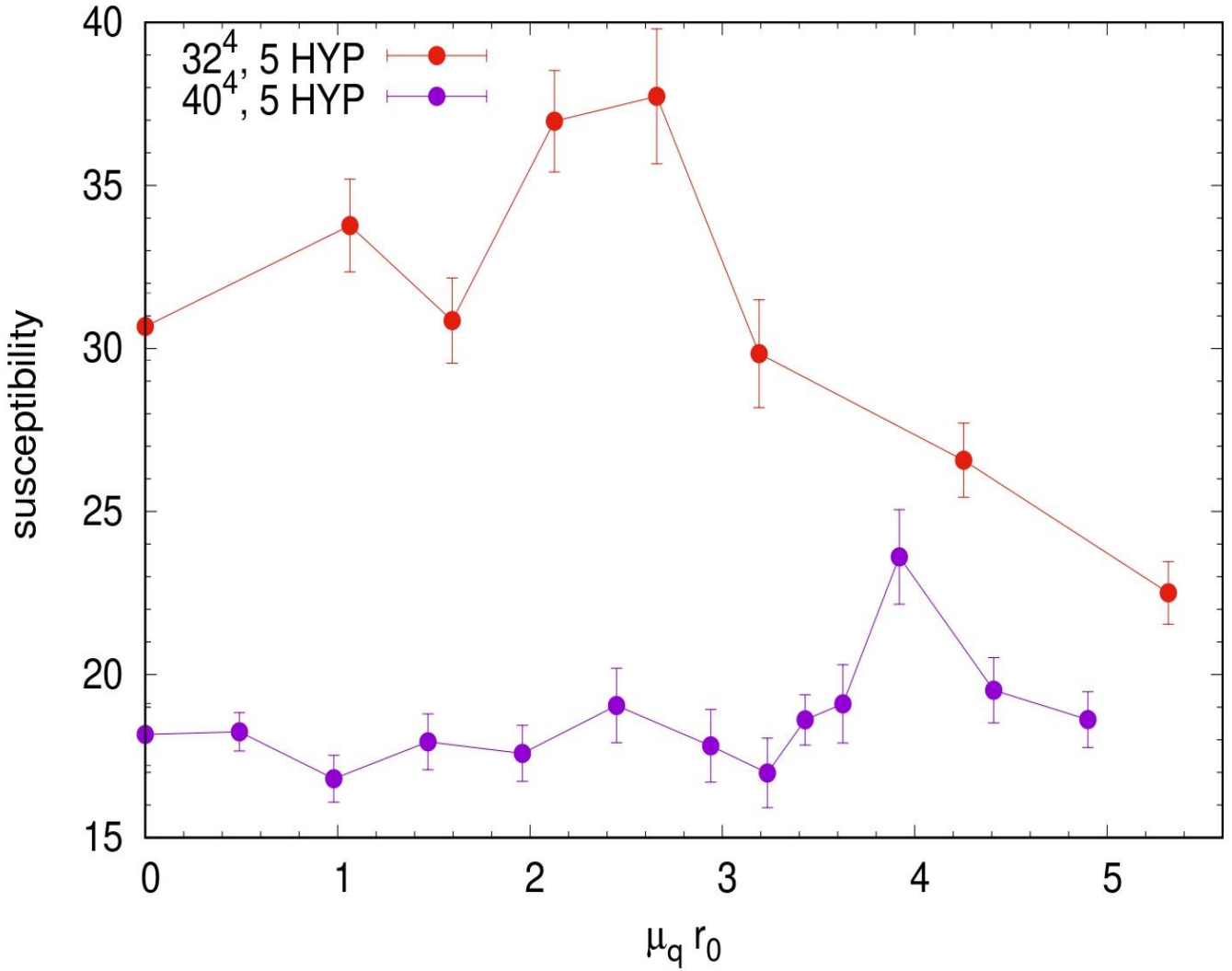
Ishiguro, Iida, Itou, Flux tube profiles in two-color QCD at low temperature and high density, PoS LATTICE2021 (2022) 063



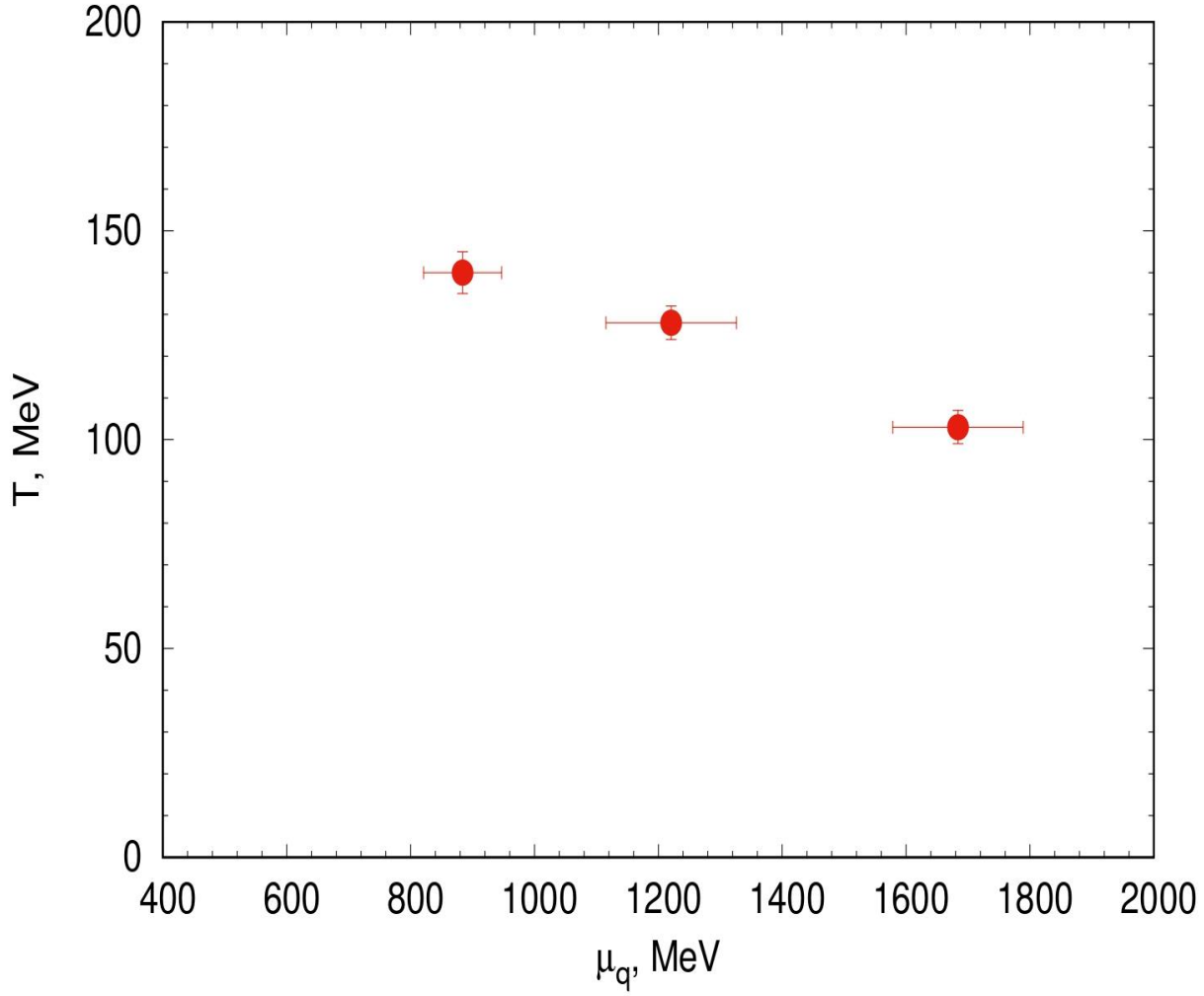
Polyakov loop for 40^4 and 32^4 lattices



Polyakov loop susceptibility for 40^4 and 32^4 lattices

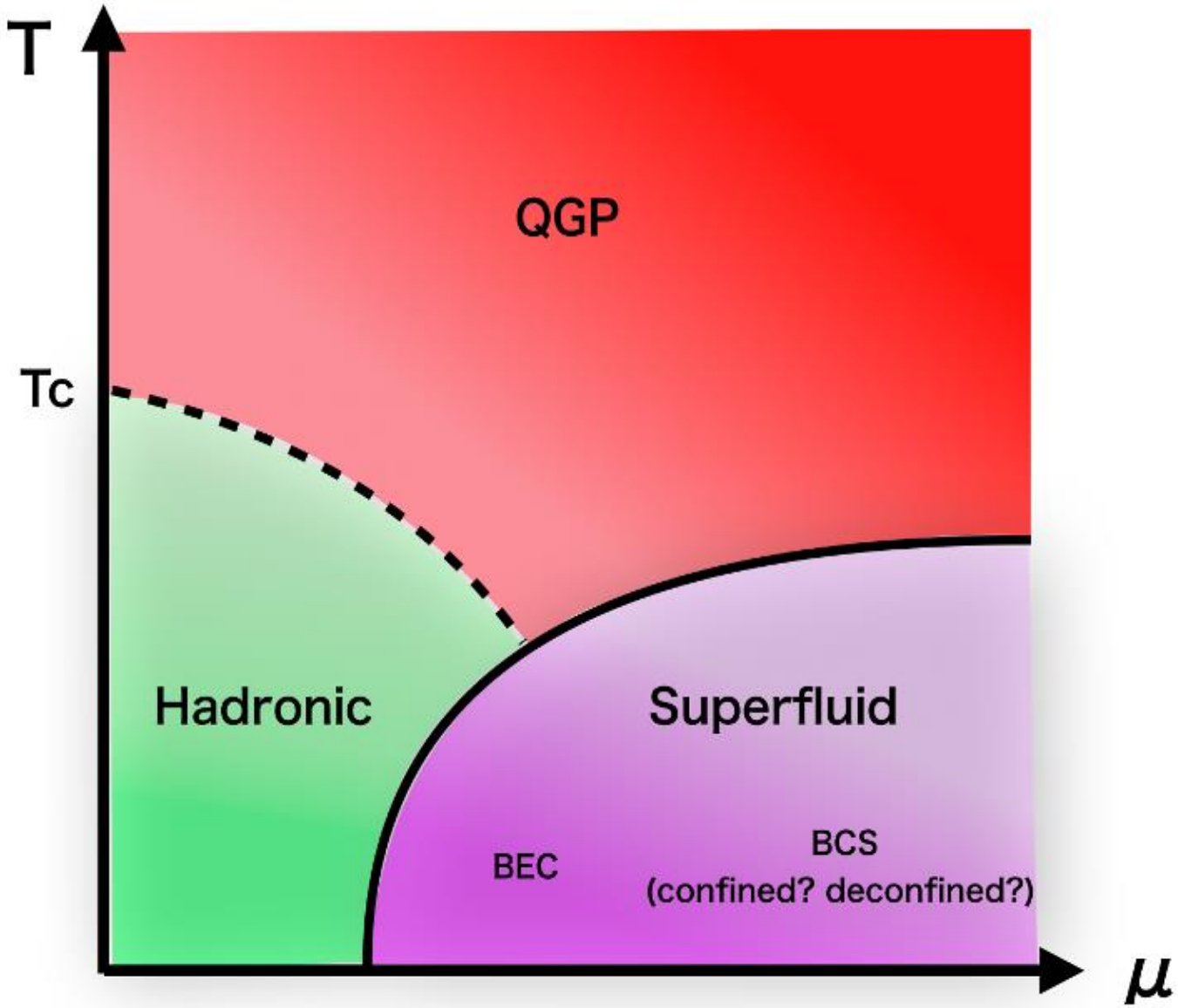


Transition line in the $\mu_q - T$ plane



Schematic phase diagram of two-colour QCD

borrowed from **Iida, Itou, Lee**, JHEP 01 (2020) 181



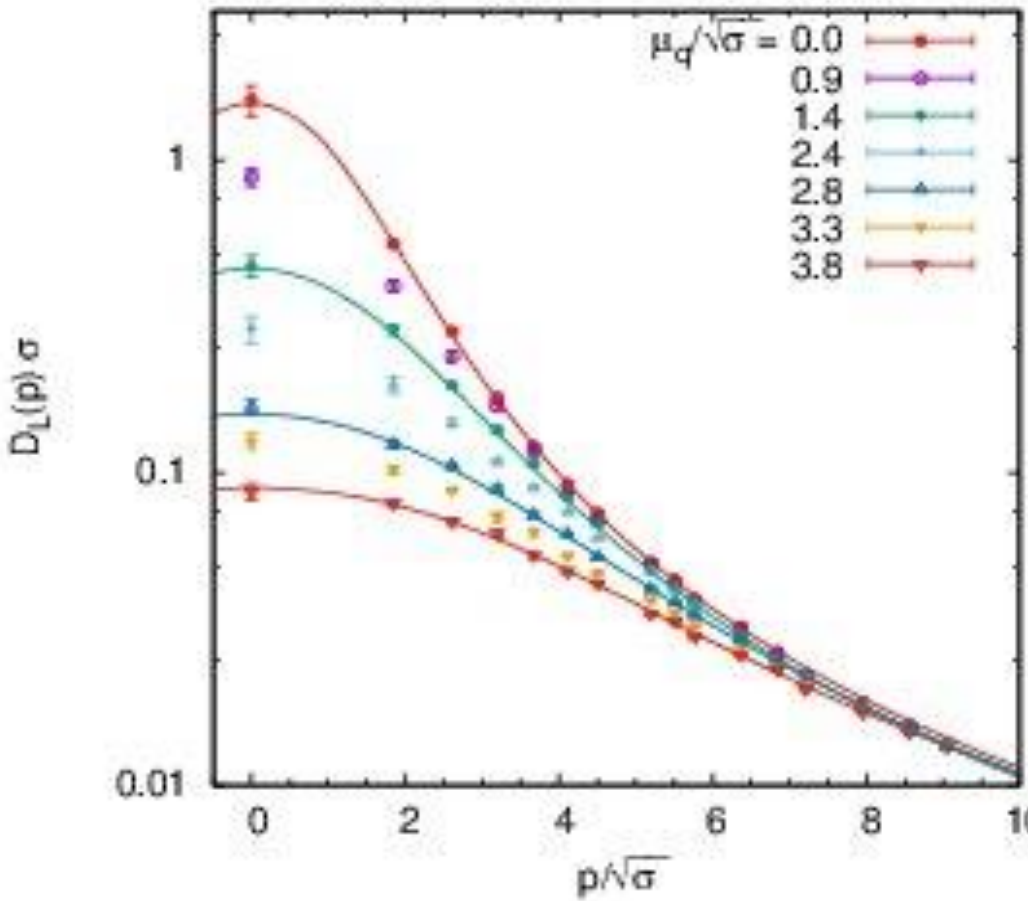
The gluon propagators in lattice QC_2D (Landau gauge)

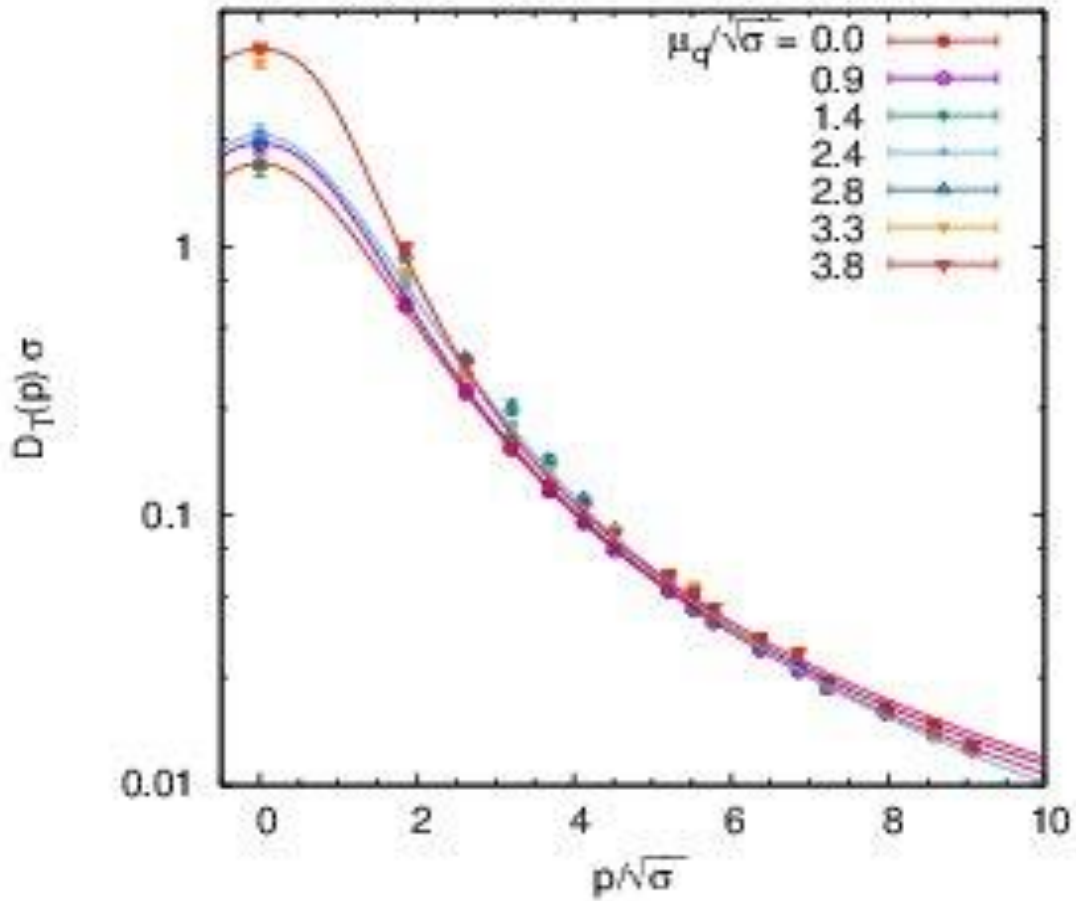
- $N_f = 2$, staggered quarks
- Lattice: 32^4
- $\beta = 1.8$, $a = 0.044$ fm, $L_s \approx 1.4$ fm
- $am_q = 0.0075$, $\lambda = 0.00075$, $m_\pi = 740(40)$ MeV

Ref.: **VB, Braguta, Nikolaev, R.N. Rogalyov,** Effects of Dense Quark Matter on Gluon Propagators in Lattice QC_2, Phys.Rev.D 102 (2020) 114511

Another study: **Boz, Hajizadeh, Maas, Skullerud**, Finite-density gauge correlation functions in QC2D, Phys.Rev.D 99 (2019) 7, 074514

$D_L(p)$, $D_T(p)$ propagators



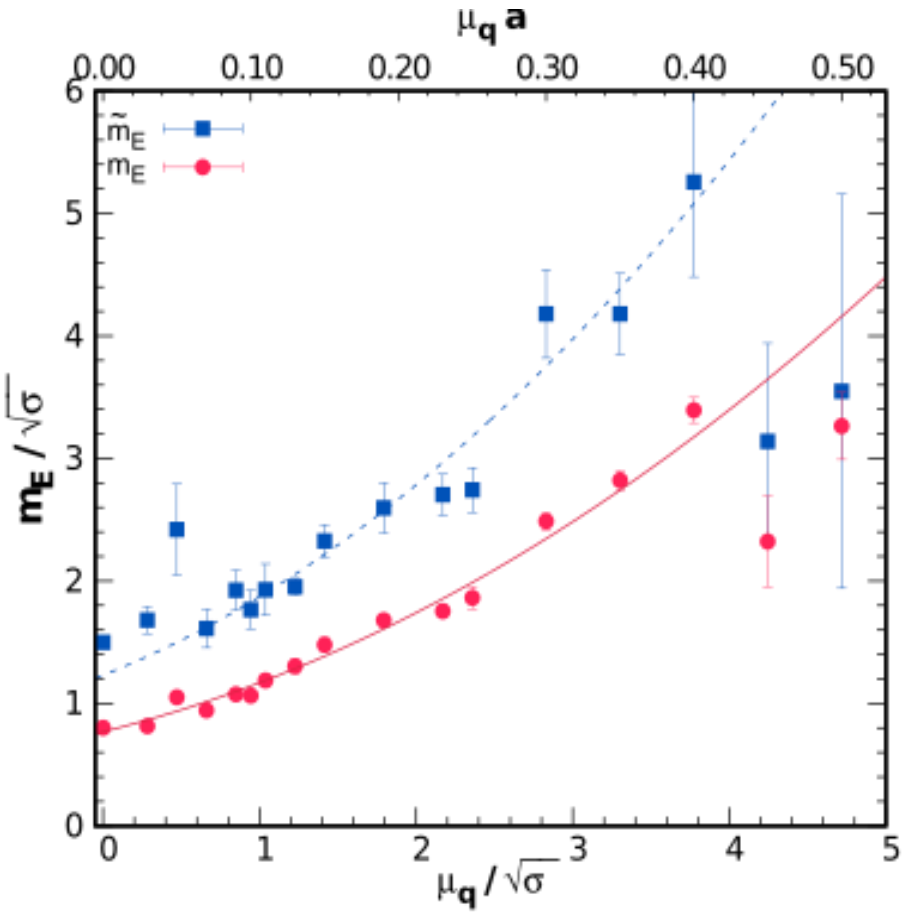


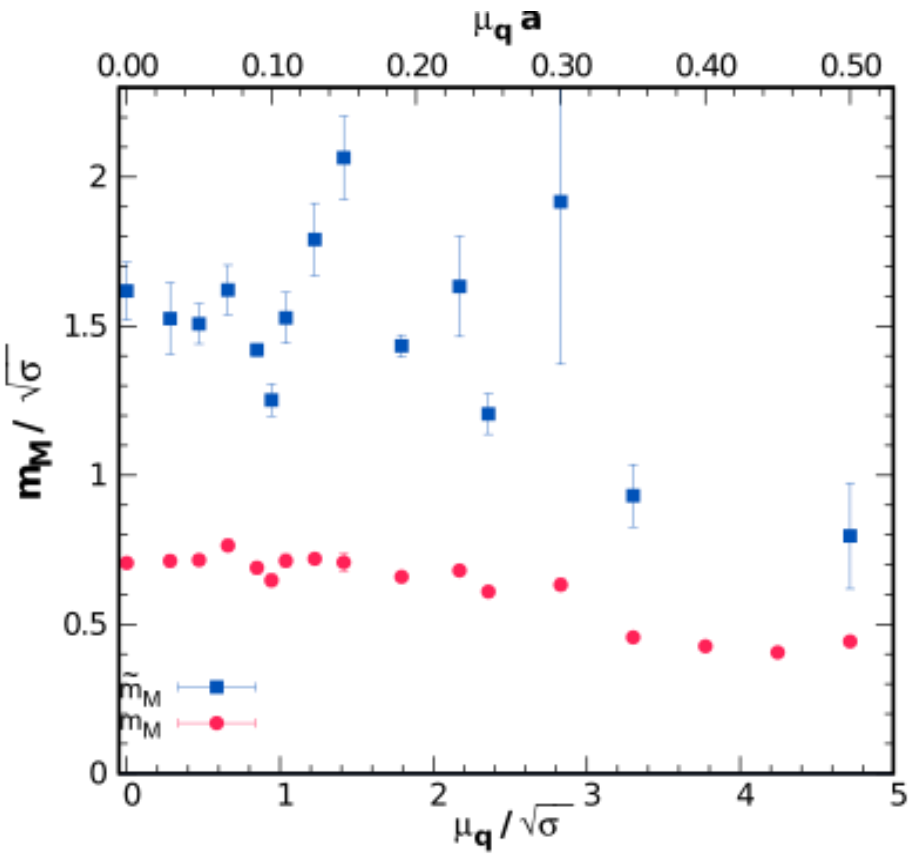
Definitions

- $m_E^2 = \frac{1}{D_L(0)} \qquad m_M^2 = \frac{1}{D_T(0)}$

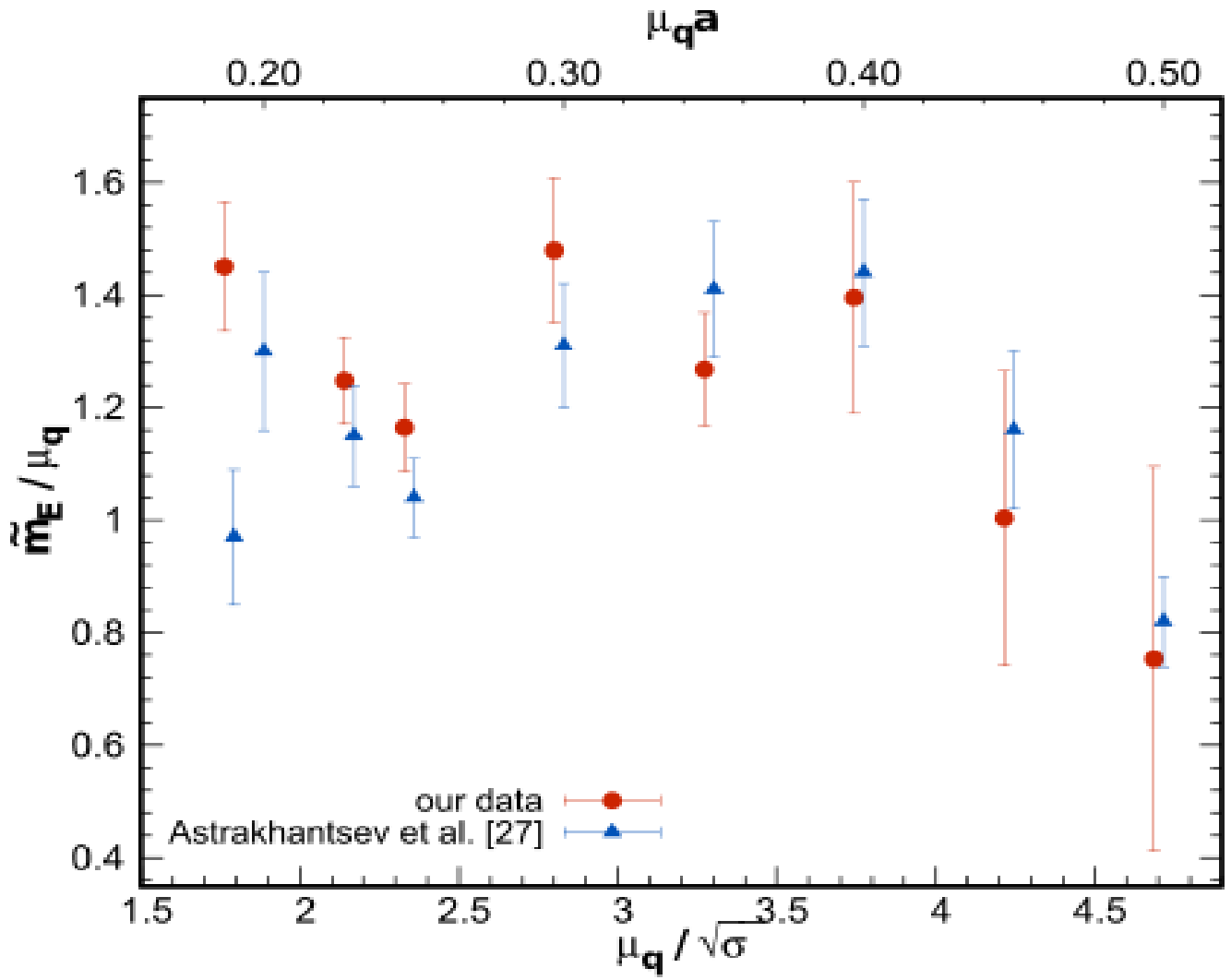
- $D_{L,T}^{-1}(p) = Z^{-1}(\tilde{m}_{E,M}^2 + p^2 + c_4 \cdot (p^2)^2)$

Drastic difference from results of **Boz et al**. for m_E





Comparison of $\tilde{m}_E$ and m_D



Conclusions and Outlook

- Simulations of lattice $\mathrm{QC_2D}$ with $N_f = 2$ staggered Dirac operator on large 40^4 and 32^4 lattices with small lattice spacing $a = 0.048$ fm

- String tension, Polyakov loop and its susceptibility were used to locate the deconfinement transition

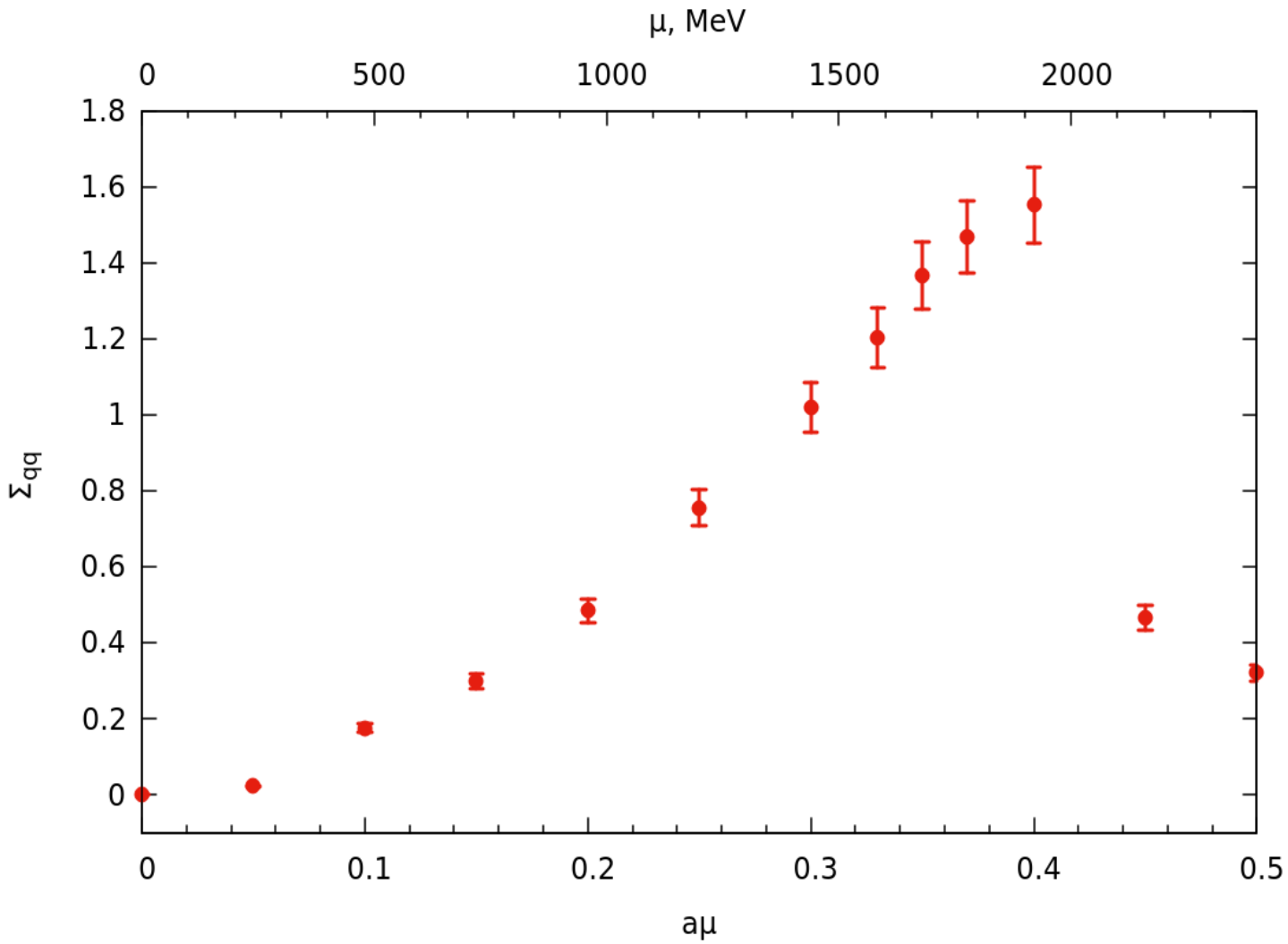
- The deconfinement transition line was determined in the ranges

$800 \lesssim \mu_q \lesssim 1700$ MeV, $100 \lesssim T \lesssim 140$ MeV

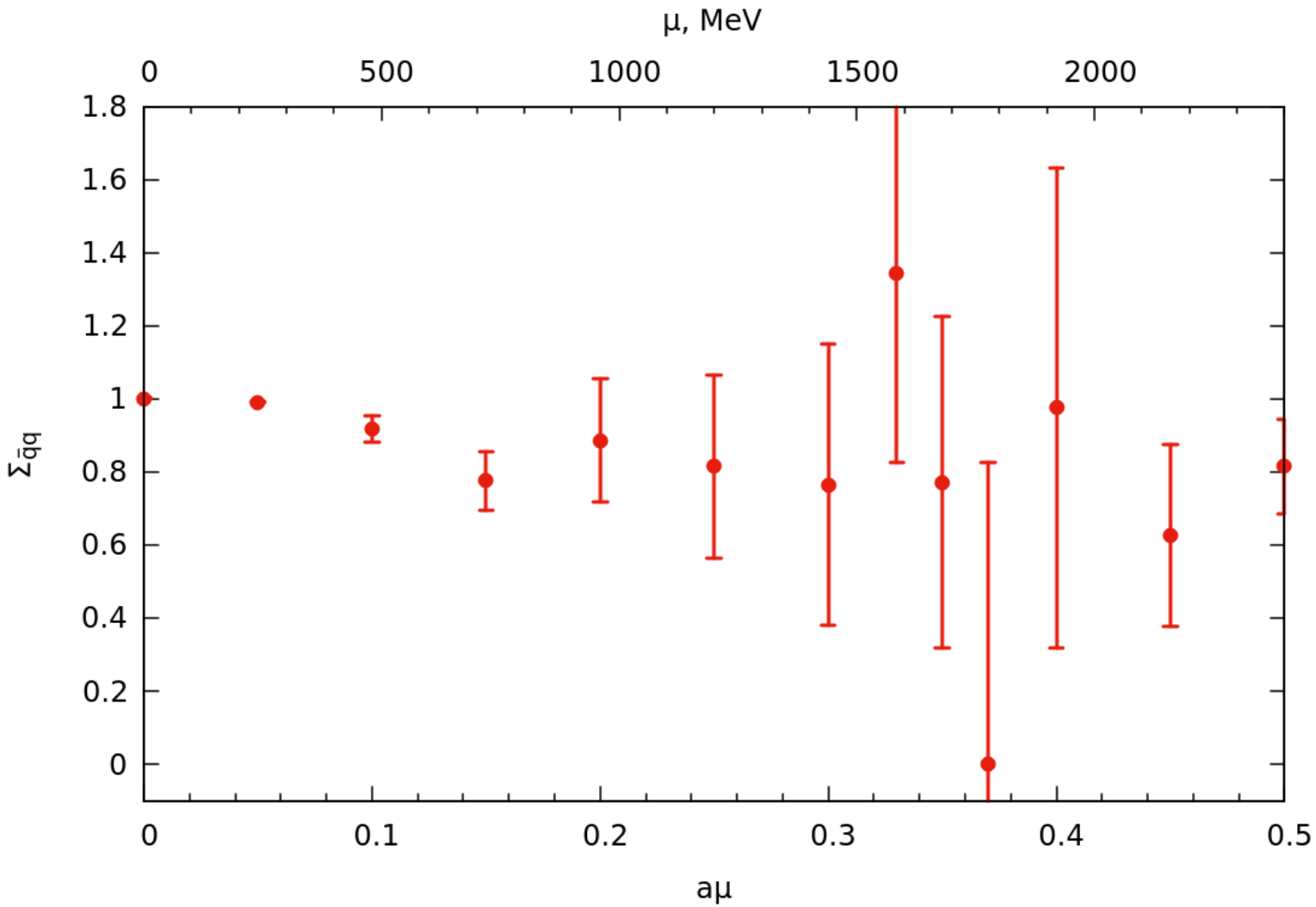
- Differences with the Wilson fermions results call for careful checks of lattice artefacts

- Results for the gluon propagator show nontrivial dependence on μ_q contrary to earlier claims

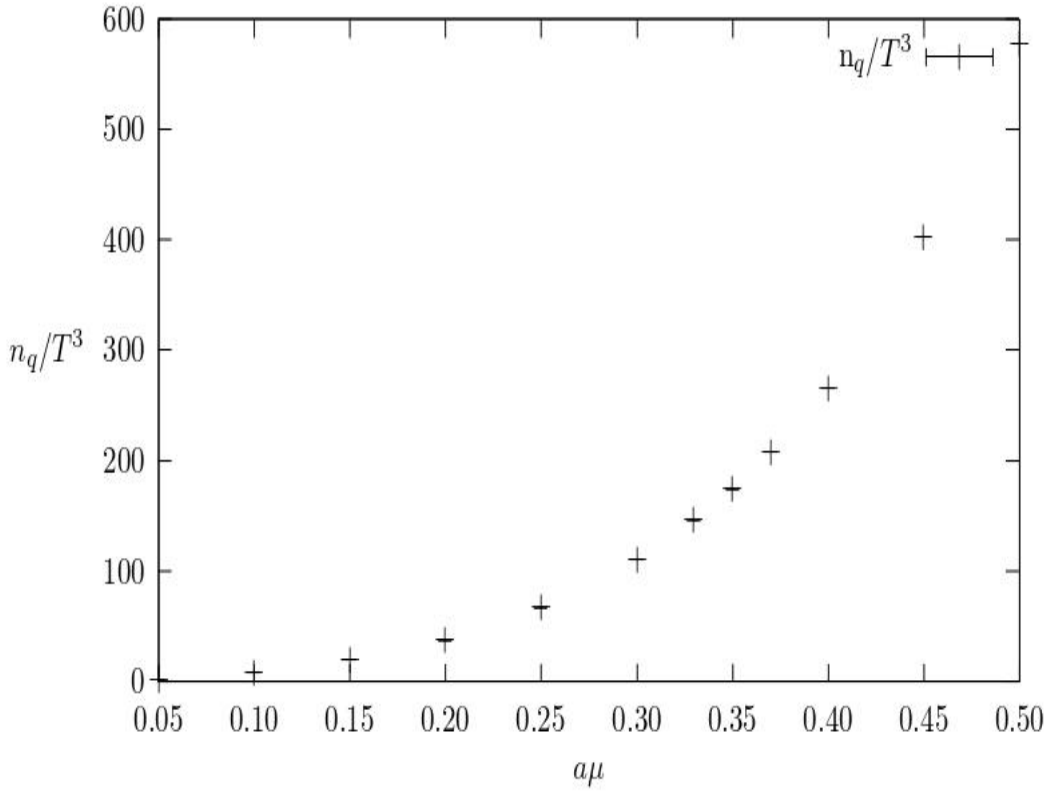
Diquark condensate



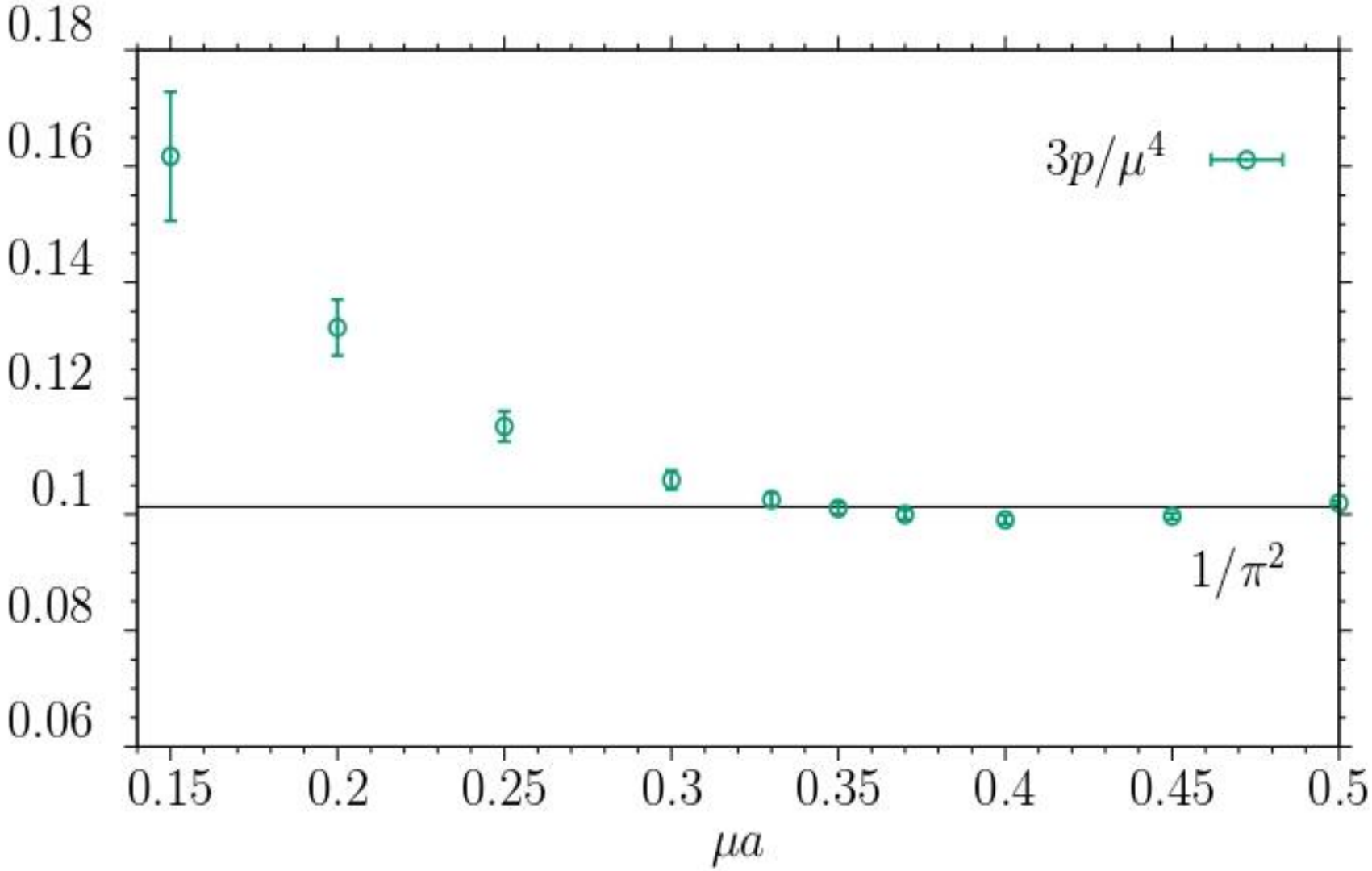
Chiral condensate



Quark number density



Pressure


0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
μa
$3p/\mu^4$
$1/\pi^2$