NEUTRINO OSCILLATIONS IN GRAVITATIONAL FIELDS

Maxim Dvornikov

IZMIRAN, Russia

Plan of the talk

- Introduction
- Flavor oscillations of SN neutrinos in stochastic gravitational waves (GWs)
- Neutrino spin oscillations in a scattering off a black hole (BH)
- Summary

Neutrino oscillations

Flavor oscillations

- Neutrinos interact with other leptons (e,μ,τ) as flavor eigenstates: $\nu = (\nu_e, \nu_\mu, \nu_\tau)$
- Flavor eigenstates do not have definite masses
- We introduce mass eigenstates $\psi = (\psi_1, \psi_2, \psi_3)$
- These bases are related by the unitary matrix transformation
$$\nu = U\psi$$
- These neutrino properties result in neutrino flavor oscillation, i.e. the change of a flavor content of the neutrino beam, which can happen even in vacuum

Spin oscillations

- Neutrinos are left-handed in the standard model, i.e. their spin is opposite to the neutrino momentum
- If the neutrino spin precesses in an external field, i.e. changes its direction with respect to the neutrino momentum, particles become right-handed
- Right-handed neutrinos are sterile in the standard model
- We will observe the effective reduction of the initial neutrino flux
- This process is called neutrino spin oscillations

Spin-flavor oscillations

- Both spin and flavor states are changed
- We observe the transitions like $\nu_{eL} \leftrightarrow \nu_{\mu L}$
- Unlike flavor oscillations, spin and spin-flavor ones happen only in the presence of external fields

Neutrino interaction with external fields

- Wolfenstein (1978); Mikheyev & Smirnov (1985) dicoveved that neutrino flavor oscillations are affected by neutrino interaction with background matter (electrons, protons, neutrons)
- MSW effect. It leads to a significant amplification of transition probability in background matter. Nowadays, it is the most plausible solution of the solar neutrino problem.
- Neutrinos are not fully electrically neutral particles. If neutrinos are massive, they can have nonzero magnetic moments (Lee & Shrock, 1977).
- The neutrino spin precession in an external electromagnetic field leads to neutrino spin oscillations; see, e.g., Voloshin et al. (1986).
- WHAT ABOUT GRAVITY?

Flavor oscillations in gravitational fields

- Ahluwalia & Burgard (1996); Fornengo et al (1997) established the evolution of neutrino mass eigenstates in a gravitational field

$$\psi_a(x,t) \sim \exp\left[-iS_a(x,t)\right] \qquad g_{\mu\nu}\frac{\partial S_a}{\partial x_\mu}\frac{\partial S_a}{\partial x_\nu} = m_a^2$$

- Neutrino oscillations in static gravitational fields, e.g., of a nonrotating black hole (Schwarzschild metric) were studied. Recently, Godunov & Pastukhov (2009) claimed that the effects of gravity are nonobservable in this case
- Visinelli (2015) studied neutrino flavor oscillations in Friedmann-Robertson-Walker metric of an expanding universe

Spinning particle in curved spacetime

- Papapetrou (1951) studied the motion of a spinning body in curved spacetime
- Wald (1972) derived equations for a spinning particle in a gravitational field
- The motion of a spinning body deviates from geodesics

$$\frac{DS^{\mu\nu}}{D\tau} = p^{\mu}v^{\nu} - p^{\nu}v^{\mu},$$

$$\frac{Dp^{\mu}}{D\tau} = -\frac{1}{2}R^{\mu}{}_{\nu\rho\sigma}v^{\nu}S^{\rho\sigma},$$

$$S_{\rho} = \frac{1}{2m}\sqrt{-g}\,\varepsilon_{\mu\nu\lambda\rho}p^{\mu}S^{\nu\lambda}$$

- Rietdijk and van Holten (1993) showed that for point like particles this deviation is negligible
- Pomeranskii & Khriplovich (1998) proposed the quasiclassical approach for a motion of an elementary spinning particle in curved spacetime
- The most general quantum description of a spinning fermion in a gravitational field was made by Obukhov et al. (2017) on the basis of the Dirac equation in curved spacetime
- The review of spin physics in noninertial frames is given by Vergeles et al. (2023)

Neutrino spin oscillations in gravitational fields

- Using the results of Pomeranskii & Khriplovich (1998), MD (2006,2013) studied neutrino spin oscillations in curved spacetime, in frames of the General Relativity, under the influence of external fields
- The neutrino invariant 3-spin vector, defined in the particle rest frame obeys

$$\frac{d\vec{\zeta}}{dt} = 2(\vec{\Omega} \times \vec{\zeta})$$

- One should take into account that both spin and neutrino velocity change in a gravitational field
- Alavi & Nodeh (2015); Chakraborty (2015); Mastrototaro & Lambiase (2021); Pantig et al. (2022) applied this approach in various extensions of GR

Spin-flavor oscillations in gravitational fields

- Piriz et al. (1996) studied spin-flavor oscillations near a rotating BH
- Quasiclassical expansion of the Dirac equation in curved spacetime is performed
- Generally, the approach is similar to flavor oscillations in gravitational fields

FLAVOR OSCILLATIONS OF SN NEUTRINOS IN RANDOM GWS

Motivation

- LIGO-Virgo collaborations in 2016 directly detected GWs from merging black holes (BHs)
- Recently, the North American Nanohertz Observatory for Gravitational Waves (NANOGrav), the European Pulsar Timing Array (EPTA), the Parkes Pulsar Timing Array (PPTA), and the Chinese Pulsar Timing Array (CPTA) collaborations independently reported on the evidence for a stochastic GW signal with a typical frequency $f = 1 yr^{-1} = 31.7\ nHz$
- Previously, neutrino oscillations in mainly static gravitational fields in the vicinity of BH were studied
- MD (2019); Koutsoumbas & Metaxas (2020) considered neutrino oscillations in a plane GW
- How a GW will influence flavor oscillations of neutrinos?

Evolution of a mass eigenstate in GW

Metric of a plane GW propagating along z-axis and having two independent polarizations h_+ and $h_\times$

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu$$
$$= dt^2 - (1 - h_+ \cos\phi)dx^2 - (1 + h_+ \cos\phi)dy^2 + 2dxdyh_\times \sin\phi - dz^2$$

Dvornikov (2021) found the perturbative solution of the Hamilton-Jacobi equation in a plane GW

Schrodinger equation and the effective Hamiltonian for neutrino flavor eigenstates

$$\left(H_m^{(+)}\right)_{aa} = -\frac{p^2 h_+}{2\sqrt{p^2 + m_a^2}} \sin^2\vartheta \cos(2\varphi)$$
$$\times \cos(\omega t[1 - v_a \cos\vartheta])$$

$$i\dot{\nu} = H_f \nu \qquad H_f = U H_m U^+$$

GW does not contribute to neutrino oscillations if neutrino beam propagates along GW ($\vartheta = 0$)

Stochastic GWs

- Neutrino interacts with randomly emitted GWs
- Density matrix (Loreti & Balantekin, 1994)
- Averaging over angles
- Gaussian distribution of strain with arbitrary correlator: $\langle h_{+,\times}(t_1) h_{+,\times}(t_2) \rangle = f_{+,\times}(|t_1 - t_2|)$
- We can find the correction to the probabilities of vacuum oscillations caused by GWs

$$\Delta P_\lambda(x) = 2 \sum_\sigma P_\sigma(0) \sum_{a>b} \left\{ \mathrm{Re}[U_{\lambda a} U^*_{\lambda b} U^*_{\sigma a} U_{\sigma b}] \cos\left(2\pi \frac{x}{L_{ab}}\right) + \mathrm{Im}[U_{\lambda a} U^*_{\lambda b} U^*_{\sigma a} U_{\sigma b}] \sin\left(2\pi \frac{x}{L_{ab}}\right) \right\} \left\{ 1 - \exp\left[-\frac{4\pi^2}{L_{ab}^2} \int_0^x \tilde{g}(t) dt \right] \right\}$$

Initial condition

We study neutrinos emitted in ν_e-burst in core-collapsing SN
SN is almost point-like source
The size of neutrinosphere is ~ 100 km
The contribution of solar oscillations channel is not smeared

Fluxes at a source are $(F_e, F_\mu, F_\tau)_S = (1:0:0)$

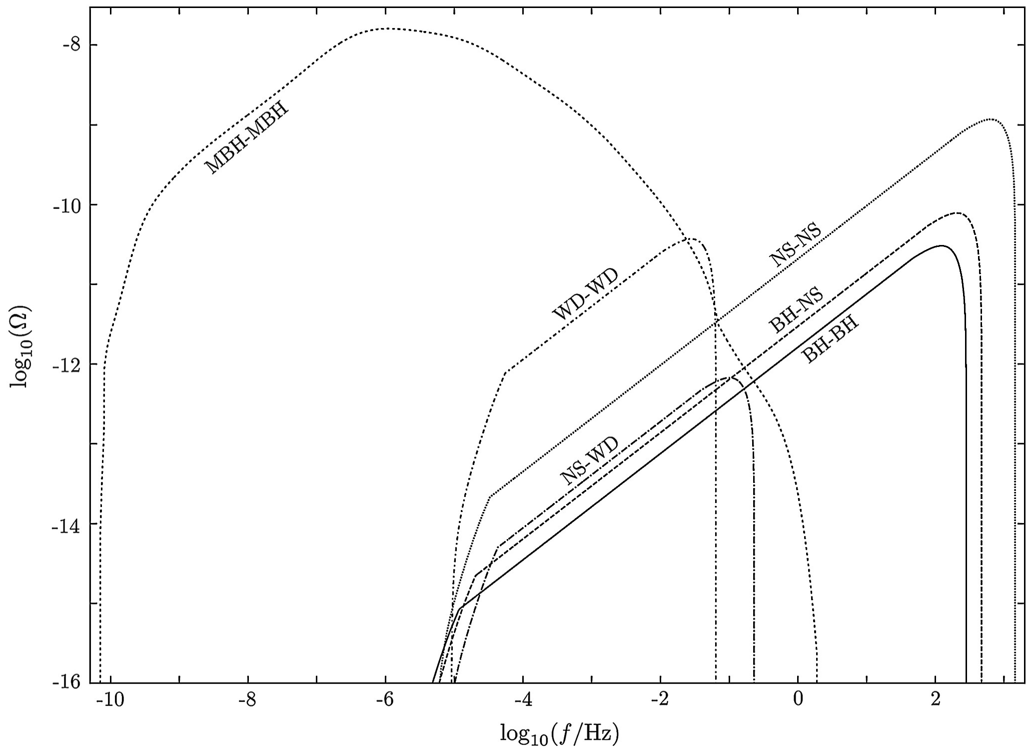
Initial condition $\rho_{11}(0) = 1, \rho_{22}(0) = 0, \rho_{33}(0) = 0$

GW emitted by randomly coalescencing supermassive BHs (SMBH)

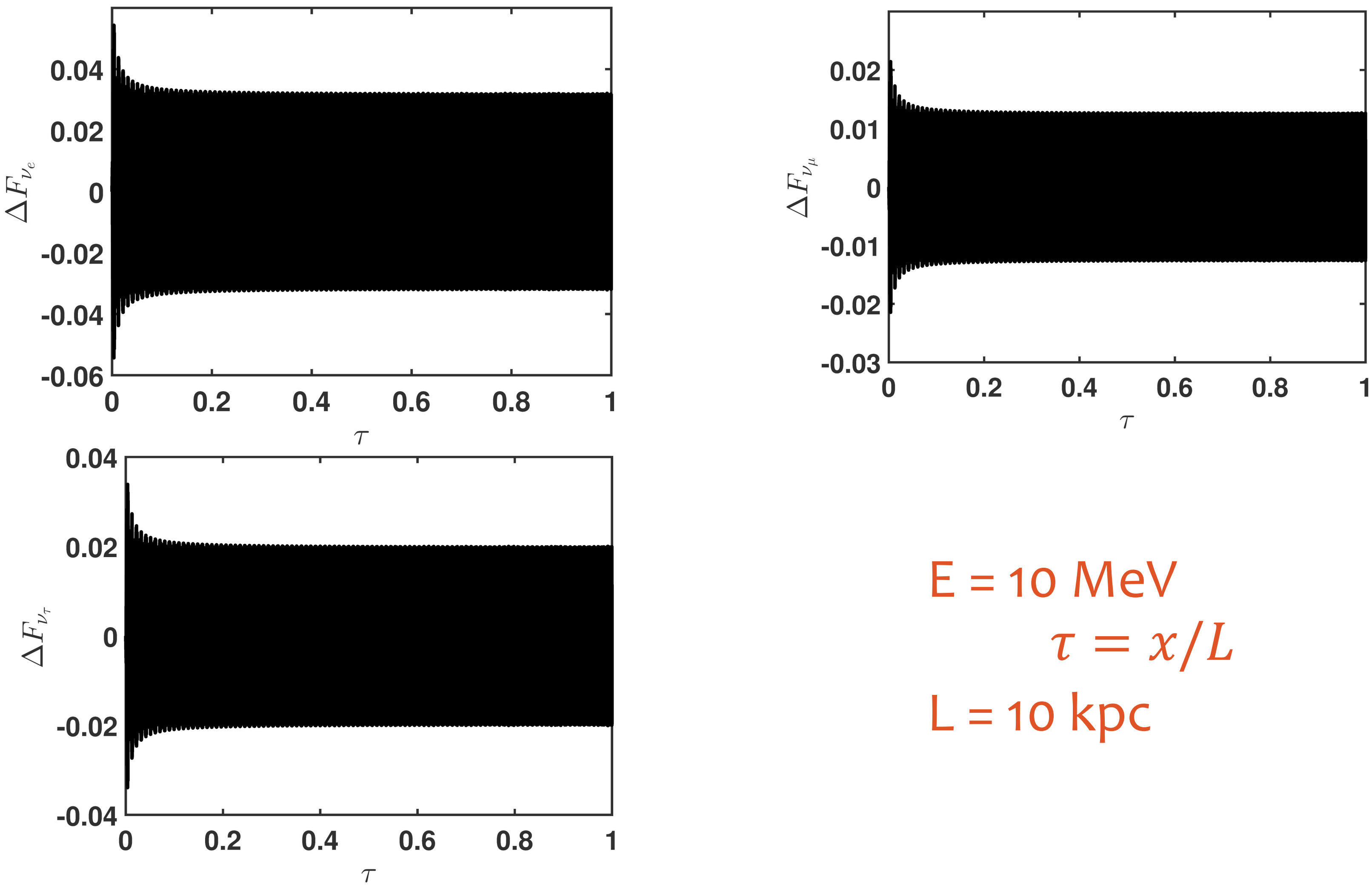
- Spectral function for GW from different types of merging BHs is calculated by Rosado (2011)
- Ω is the energy density of stochastic GWs per logarithmic frequency interval with respect to the closure density of the universe

$$\Omega \propto f^{\alpha}$$

- We will study the case of SMBH since they produce stochastic GWs with the major effect on neutrino oscillations



Fluxes of flavor neutrinos

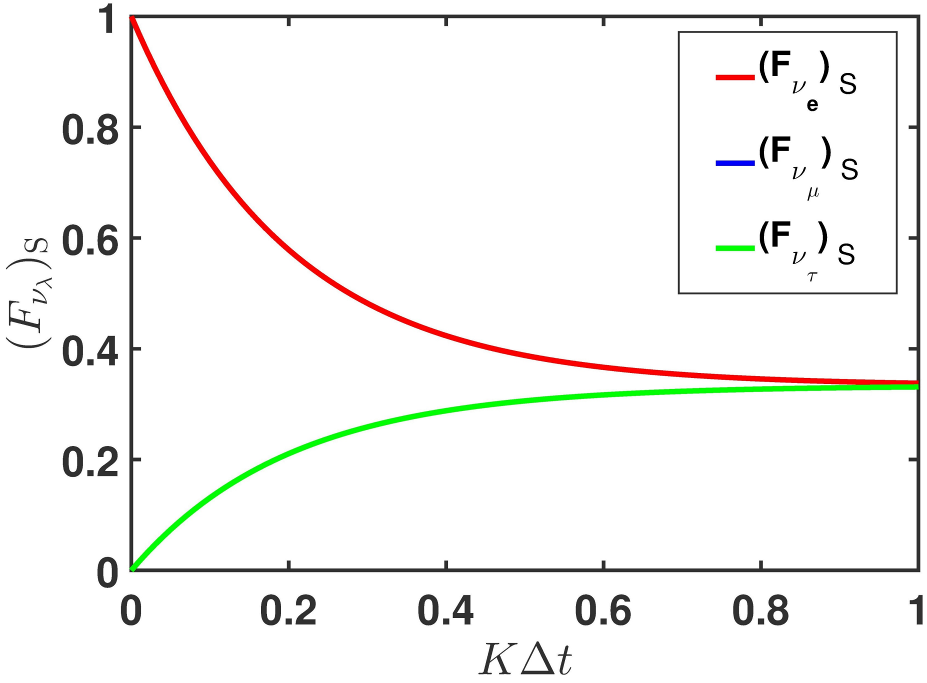


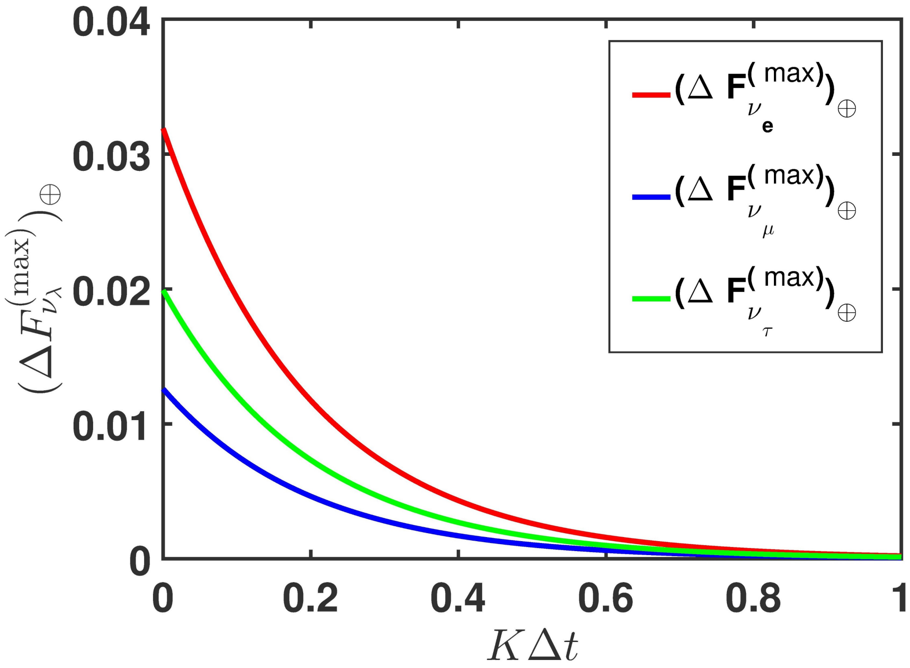
E = 10 MeV

$\tau = x/L$

L = 10 kpc

Neutrino fluxes after ν_e-burst





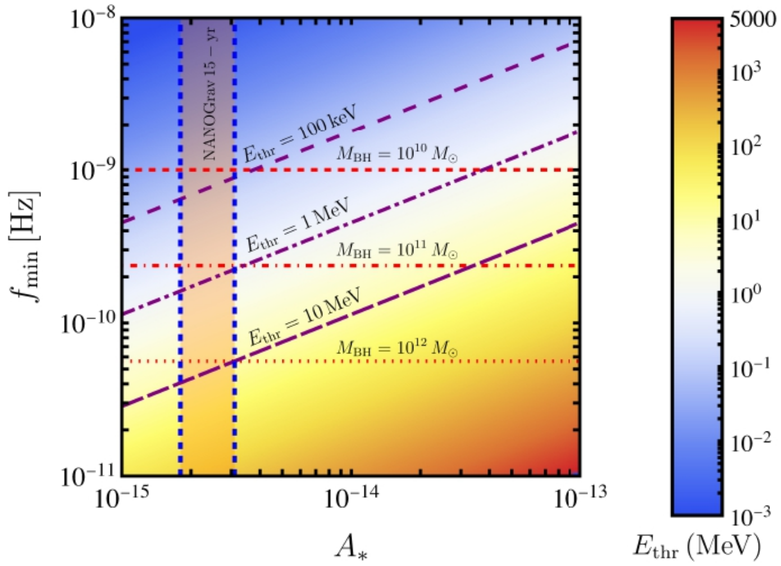
- Significant number of neutrinos of different flavors is emitted after ν_e-burst
- We study oscillations of such neutrinos in stochastic GWs
- $0 < \Delta t < 0.1\ s$ is the time after ν_e-burst. We approximate the fluxes at a source (SN) by exponents
- The contributions to the fluxes from stochastic GWs in a detector are vanishing at $\Delta t \sim 0.1\ s$

The decoherence of oscillations in GW background detected

- GW strain detected by NANOGrav, EPTA, PPTA, and CPTA

$$h(f) = A_* \left(\frac{f}{f_{yr}}\right)^{\frac{3-\gamma}{2}}$$

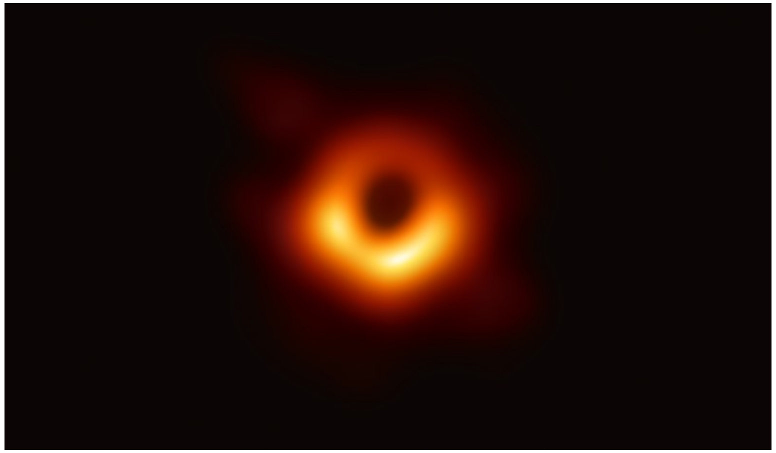
- Lambiase et al. (2023) studied the decoherence in flavor oscillations of SN neutrinos
- Assuming that GW background is by coalescing SMBHs, $\gamma = 13/3$ and $A_* \sim 10^{-15}$, oscillations are suppressed if $E < E_{thr}$
- JUNO: $E_{thr} = 100keV$



SPIN OSCILLATIONS IN GRAVITATIONAL SCATTERING

Motivation

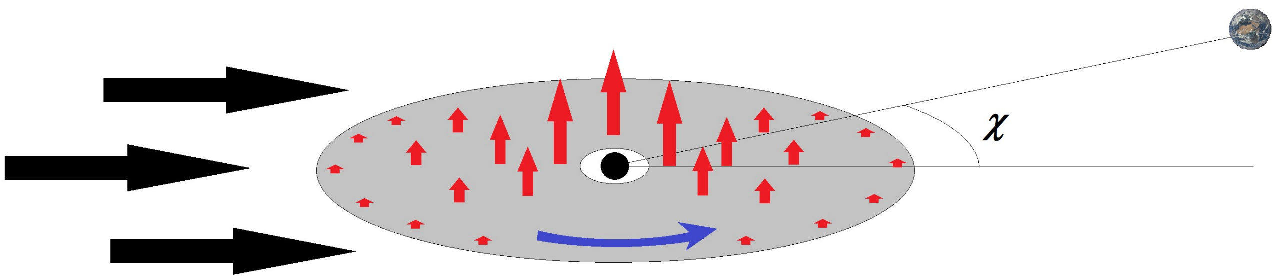
- The Event Horizon Telescope observed shadows of the supermassive BHs in the centers M87 and Milky Way
- Accretion disks in some galaxies can be sources of both photons and high energy neutrinos (Berezinsky & Ginzburg, 1981)



- These neutrinos are strongly gravitationally lensed and their spin precesses in external fields near BH, leading to spin oscillations
- What shall we see if we look at the center of a galaxy in a neutrino telescope?

Formulation of the problem

- Uniform flux of left-polarized neutrinos is parallel to the equatorial plane
- Neutrino motion in a gravitational field of BH is described exactly (Gralla et al., 2018)
- BH is surrounded by a thick realistic accretion disk, a Polish doughnut (see below)
- Toroidal and poloidal magnetic fields in the accretion disk is generated by the plasma motion

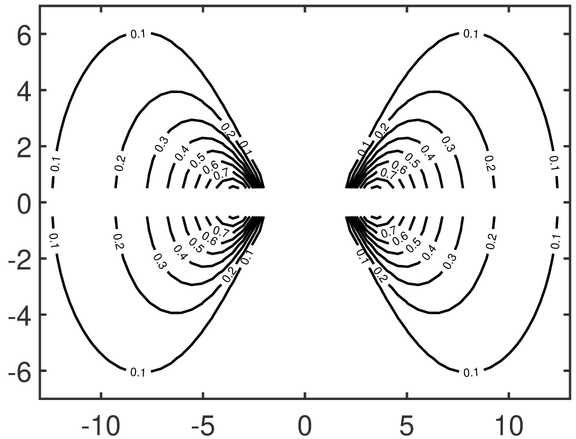


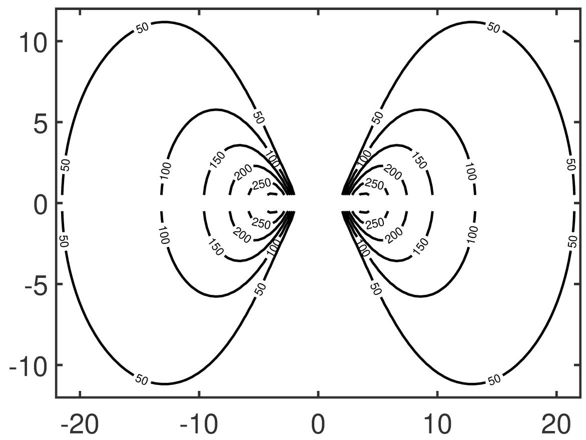
Neutrino helicity evolution in the locally Minkowskian frame under the influence of external fields

- The vector $\vec{\Omega}$ in the precession equation account for the neutrino interaction with gravity, the magnetic interaction and electroweak interaction with matter: $\vec{\Omega} = \vec{\Omega}_g + \vec{\Omega}_B + \vec{\Omega}_{matt}$. It is given in the explicit for in arXiv:2307.10126
- One should take into account that neutrino spin states depend on the helicity $h \propto (\vec{\zeta}\vec{u})$. The neutrino velocity also changes in gravitational scattering

Magnetized Polish doughnut

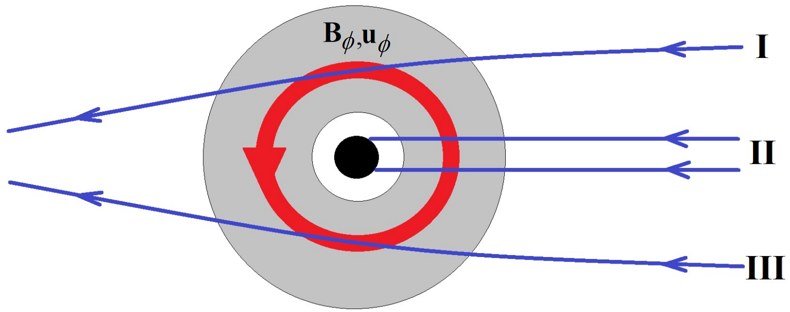
- Abramowicz et al. (1978) put forward the analytical model for the accretion disk around a rotating BH, called the Polish doughnut
- Komissarov (2006) generalized the Polish doughnut to include the magnetic field. Improtant: this magnetic field is toroidal
- Review: Abramowicz & Fragile (2013)





Factors not affecting spin oscillations

- Gravitational interaction only does not contribute to spin oscillations in scattering of ultrarelativistic neutrinos off a rotating BH. Previously, it was mentioned by Lambiase et al. (2005) for a weak gravitational lensing.
- Toroidal magnetic field with a reasonable strength (see below), inherent in the Polish doughnut model, does not change the neutrino helicity.
- Background matter + toroidal magnetic field = no spin oscillations (Voloshin et al., 1986).
- A poloidal magnetic field is needed
- One needs a sizable transverse component to have a significant spin-flip



Models for poloidal magnetic field

Tayler (1973) found that only toroidal and only poloidal magnetic fields are unstable

Wald (1974): Model 1

$$A_t = Ba\left[1 - \frac{rr_g}{2\Sigma}(1 - \cos^2\theta)\right]$$

$$A_\phi = -\frac{B}{2}\left[r^2 + a^2 - \frac{a^2 r r_g}{\Sigma}(1 - \cos^2\theta)\right]\sin^2\theta$$

- $B \propto r^{-5/4}$ (Blandford & Payne, 1982)
- Neronov et al. (2009) used analogous magnetic field explain high energy cosmic rays creation in the vicinity of BH

Fragile & Meier (2009): Model 2

$$A_\phi = b\rho$$

In both cases, we suppose that a poloidal field exists only inside the disk

Parameters of an accretion disk and a neutrino

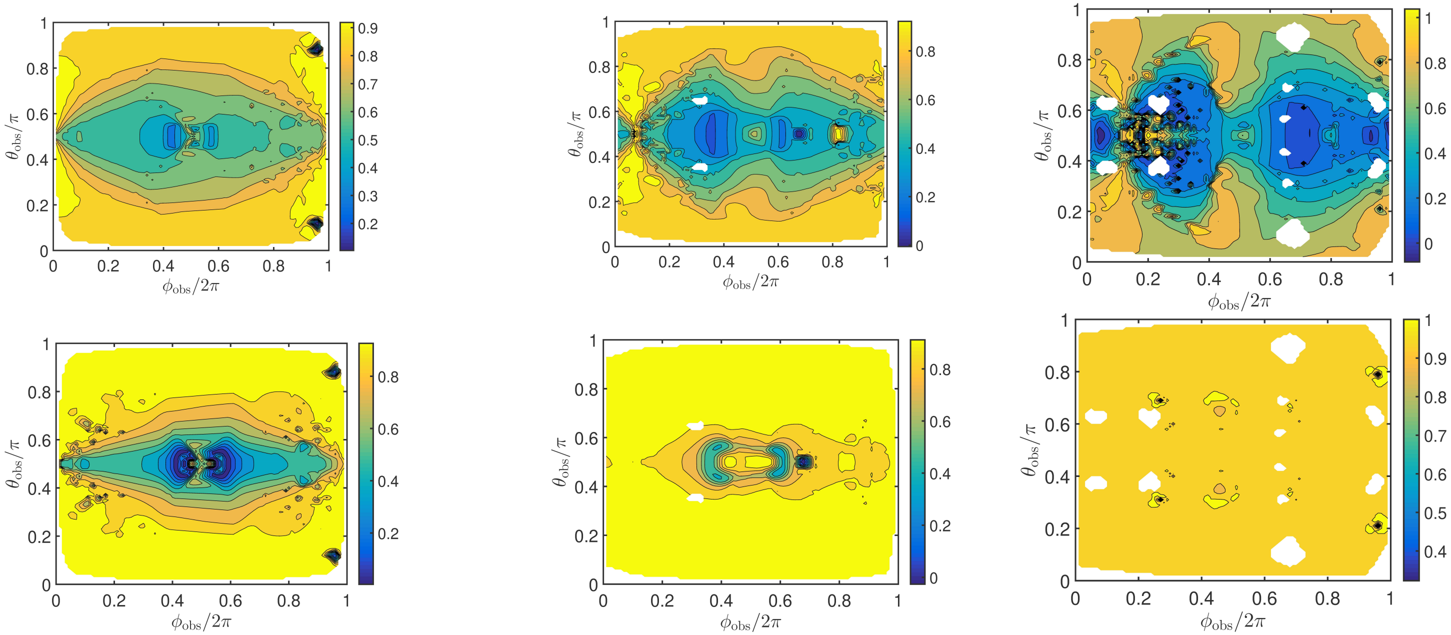
- The mass of SMBH is $10^8 M_\odot$. The BH spin is $0 < a < 0.9M$.
- The maximal strength of both poloidal and toroidal fields is $320G$. It is 1% of the Eddington limit for this BH mass (Beskin, 2010).
- The maximal matter density of hydrogen plasma is $10^{18}\ cm^{-3}$. Such density can be found in some AGN (Jiang et al., 2019).
- Neutrinos are Dirac particles having the magnetic moment $\mu = 10^{-13}\mu_B$. It is below the best astrophysical constraint established by Viaux et al. (2013).

Results

- Only left-polarized neutrinos interact with a detector. The measured flux $\propto P_{LL}F_0$, where F_0 is the flux of scalar particles
- We show the ratio F_ν/F_0 for BHs with different spins

Model 1:

Model 2:



a=0.02M

a=0.5M

a=0.9M

Publications

- **M. Dvornikov**, *Neutrino spin oscillations in a magnetized Polish doughnut*, arXiv:2307.10126.
- **M. Dvornikov**, *Scattering of neutrinos by a rotating black hole accounting for the electroweak interaction with an accretion disk*, Int. J. Mod. Phys. D (2023), arXiv:2212.03479.
- **M. Dvornikov**, *Gravitational scattering of spinning neutrinos by a rotating black hole with a slim magnetized accretion disk*, Class. Quantum Gravity **40**, 015002 (2023), arXiv:2206.00042.
- **M. Dvornikov**, *Interaction of supernova neutrinos with stochastic gravitational waves*, Phys. Rev. D **104**, 043018 (2021), arXiv:2103.15464.
- **M. Dvornikov**, *Flavor ratios of astrophysical neutrinos interacting with stochastic gravitational waves having arbitrary spectra*, JCAP 12 (2020) 022, arXiv:2009.02195.
- **M. Dvornikov**, *Neutrino flavor oscillations in stochastic gravitational waves*, Phys. Rev. D **100**, 096014 (2019), arXiv:1906.06167.

Summary

- Fluxes of neutrinos from SN in our Galaxy are changed owing to flavor oscillations in stochastic GWs
- The decoherence effects can be measurable in JUNO
- We have considered spin oscillations of high energy neutrinos in the particles scattering off SMBH surrounded by a Polish doughnut
- Observed fluxes of outgoing neutrinos are significantly modified by spin effects
- Neutrino tomography of magnetic fields distributions in accretion disks is possible