Hadronic light by light contribution to the fine and hyperfine structure of mounic atoms

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Actuality

Precise study of muonic bound states energy levels is a direction of fundamental research, that can shed light on manifestations of new interactions of particles. Muonium represents such systems.

Theoretical prediction of ground state hyperfine structure of muonium is:
(*M. Eides, Phys. Lett B 795, 113 (2019)*)

$$\nu_{hfs}^{theor}(1S) = 4463302872(515)\ Hz, \quad \delta = 1.3 \times 10^{-7}$$

Most accurat experimental value of ground state hyperfine structure:
(*W. lin, M. G. Boshier, S. Dhawan, et al., Phys. Rev. Lett., 82, 711 (1999)*)

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New experiment

MuSEUM experiment (J-PARK): new ground state hyperfine structure measurement with an accuracy of 1 ppb ($\sim 1Hz$)

R. Iwai, M. Abe, S. Fukumura, et al., Jour. of Phys.: Conf. Ser 2462, 012019 (2023)

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High precision measurements of HFS in muonium for a long time were considered as a test of the high precision QED and a source for precise values of the fine structure constant α and the muon-electron mass ratio m_μ/m_e.

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Our previous researches show that hadronic light by light contribution to the muonic hydrogen energy spectrum is significant:

- *Dorokhov A. E. et al. The contribution of pseudoscalar mesons to hyperfine structure of muonic hydrogen //Physics of Particles and Nuclei Letters. – 2017. – V. 14. – P. 857-864.*
- *Dorokhov A. E. et al. The contribution of axial-vector mesons to hyperfine structure of muonic hydrogen //Physics Letters B. – 2018. – V. 776. – P. 105-110.*
- *Dorokhov A. E. et al. The sigma-meson exchange contribution to the muonic hydrogen Lamb shift //EPJ Web of Conferences. – EDP Sciences, 2019. – V. 212. – P. 07003.*
- *Dorokhov A. E. et al. Tensor meson contribution to the Lamb shift and hyperfine splitting in muonic hydrogen //Journal of Physics: Conference Series. – IOP Publishing, 2020. – V. 1435. – N. 1. – P. 012004.*

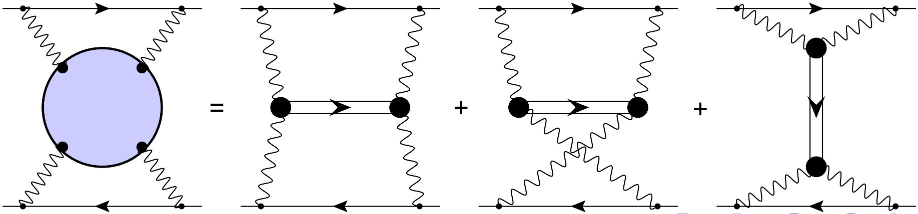
One meson interaction

There are a lot of QED corrections of higher order in α, that can contribute the value $\sim 1\ Hz$. In this work we study one of such contributions, connected with strong interaction.

Preliminary estimation of the contribution order

$$\Delta E \sim \alpha^3 E_F \sim 1700\ Hz, \quad E_F = \frac{8\alpha^4\mu^3}{3m_e m_\mu}$$

Figure: Hadronic light-by-light scattering amplitudes with horizontal (a, b) and vertical (c) exchanges. Top and bottom lines corresponds to electron and muon respectively. Wavy line corresponds to the virtual photon. The bold dot denotes the form factor of the transition of two photons into a meson.



$\gamma^* + \gamma^* \to M$ vertex parametrization

Axial-vector meson (*R. N. Cahn, Phys. Rev. D* **35**, *3342 (1987)*):

$$T^{\mu\nu\beta}_{AV}(k_1,k_2) = 4\pi i\alpha\varepsilon_{\mu\nu\alpha\beta}(k_1^\alpha k_2^2 - k_2^\alpha k_1^2)A(t^2,k_1^2,k_2^2),$$

Scalar meson (*M. K. Volkov, et al., Phys. Atom. Nucl.* **73**, *443 (2010)*):

$$T^{\mu\nu}_{S}(t,k_1,k_2) = 4\pi\alpha\Big\{A(t^2,k_1^2,k_2^2)(g^{\mu\nu}(k_1\cdot k_2) - k_1^\nu k_2^\mu) +$$
$$+B(t^2,k_1^2,k_2^2)(k_2^\mu k_1^2 - k_1^\mu(k_1\cdot k_2))(k_1^\nu k_2^2 - k_2^\nu(k_1\cdot k_2))\Big\},$$

Pseudoscalar meson (*V. Pauk and M. Vanderhaeghen, EPJ C* **74**, *3008 (2014)*):

$$T^{\mu\nu}_{PS}(k_1,k_2) = i\varepsilon^{\mu\nu\alpha\beta}k_{1\alpha}k_{2\beta}\frac{\alpha}{\pi F_\pi}F_{\pi^0\gamma^*\gamma^*}(k_1^2,k_2^2),$$

Tensor meson (*V. Pauk and M. Vanderhaeghen, EPJ C* **74**, *3008 (2014)*):

$$T^{\mu\nu\alpha\beta}_{T}(k_1,k_2) = 4\pi\alpha\frac{k_1k_2}{M_T}\mathcal{M}_{\mu\nu\alpha\beta}(k_1,k_2)\mathcal{F}_{T\gamma^*\gamma^*}(k_1^2,k_2^2),$$

$$\mathcal{M}_{\mu\nu\alpha\beta}(k_1,k_2) = \Big\{R_{\mu\alpha}(k_1,k_2)R_{\nu\beta}(k_1,k_2) + \frac{1}{8(k_1+k_2)^2\left[(k_1k_2)^2 - k_1^2k_2^2\right]}R_{\mu\nu}(k_1,k_2)\times$$
$$\left[(k_1+k_2)^2(k_1-k_2)_\alpha - (k_1^2-k_2^2)(k_1+k_2)_\alpha\right]\times\left[(k_1+k_2)^2(k_1-k_2)_\beta - (k_1^2-k_2^2)(k_1+k_2)_\beta\right]\Big\},$$

Axial vector exchange. Vertical diagram

- Interaction amplitude:

$$i\mathcal{M}_{AV}^{vert}=\frac{\alpha^2(Z\alpha)^2}{16m_1^2m_2^2}\int\frac{d^4k}{\pi^2}\frac{A(t^2,k_1^2,k_2^2)}{(k^2)^2}\int\frac{d^4r}{\pi^2}\frac{A(t^2,r_1^2,r_2^2)}{(r^2)^2}\frac{\varepsilon_{\mu\nu\alpha\beta}(k_1^\alpha k_2^2-k_2^\alpha k_1^2)}{(k^2-2k_0m_1)}\times$$
$$\frac{\varepsilon_{\sigma\lambda\rho\omega}(r_1^\rho r_2^2-r_2^\rho r_1^2)}{(r^2-2r_0m_2)}\left[\bar u(0)(\hat q_1+m_1)\gamma^\nu(\hat p_1-\hat k+m_1)\gamma^\mu(\hat p_1+m_1)u(0)\right]\left[\bar v(0)(\hat p_2-m_2)\times\right.$$
$$\left.\gamma^\sigma(\hat r_1-p_2+m_2)\gamma^\lambda(\hat q_2-m_2)v(0)\right]D^{\beta\omega}(t),$$

- The projection operators are constructed from the wave functions of the particles in their rest frame:

$$\hat\Pi_{S=0}=[u(0)\bar v(0)]_{S=0}=\frac{(1+\gamma^0)}{2\sqrt2}\gamma_5,\quad \hat\Pi_{S=1}=[u(0)\bar v(0)]_{S=1}=\frac{(1+\gamma^0)}{2\sqrt2}\hat\varepsilon.$$

- Trace calculation in package FORM in leading order gives:

$$N_{AV}^{vert}=\frac{1}{3}k^2r^2\mathbf{k}^2\mathbf{r}^2.$$

- We use the dipole parametrization for transition form factor:

$$A(t^2,k_1^2,k_2^2)=A(t^2,0,0)\frac{1}{(1-\frac{k_1^2}{\Lambda})(1-\frac{k_2^2}{\Lambda})},\ A(t^2,0,0)=A(M^2,0,0)e^{(t^2-M^2)/M^2}.$$

Axial vector exchange. Vertical diagram

- Contribution to the interaction operator in momentum representation takes the form:

$$\Delta V^{hfs}_{AV,vert} = -\frac{64}{9}\frac{\alpha^2(Z\alpha)^2}{\mathbf{t}^2+M_A^2}\int\frac{d^4k}{\pi^2}\frac{A(t^2,k^2,k^2)(2k^2+k_0^2)}{k^2(k^2-2m_1k_0)}\int\frac{d^4r}{\pi^2}\frac{A(t^2,r^2,r^2)(2r^2+r_0^2)}{r^2(r^2-2m_2r_0)}$$

- For the purpose of further integration over loop momenta, we pass to the Euclidean space:

$$k^2\to -k^2,\quad r^2\to -r^2,\quad k_0^2\to -k_0^2=-k^2\cos^2\psi_1,\quad r_0^2\to -r_0^2=-r^2\cos^2\psi_2.$$

- Momentum integrals can be calculated analytically:

$$I_e=\int d^4k\frac{(2k^2+k_0^2)}{k^2(k^2-2k_0m_1)}\frac{\Lambda^4}{(k^2-\Lambda^2)^2}=$$

$$-\frac{\pi^2\Lambda_A^2}{4(1-a_e^2)^{5/2}}\left[3\sqrt{1-a_e^2}-a_e^2(5-2a_e^2)\ln\frac{1+\sqrt{1-a_e^2}}{a_e}\right],\quad a_e=\frac{2m_1}{\Lambda}.$$

- After all transformation the contribution to the muonium hyperfine structure of 1S state:

$$\Delta E^{hfs}_{AV,vert}(1S)=-\frac{64\alpha^2(Z\alpha)^5\mu^3A(0,0,0)^2}{9\pi M_A^2\left(1+\frac{2W}{M_A}\right)^2}I_eI_\mu.$$

Axial vector exchange. Horizontal diagrams

In the case of horizontal exchanges, sum of direct and crossed diagrams gives:

$$N_{AV}^{hor} = (k_1^2 k_2^4 + k_2^2 k_1^4)(2\cos\Omega + \cos\psi_1\cos\psi_2) - k_1^3 k_2^3(1 + 3\cos^2\Omega + \cos^2\psi_1 + \cos^2\psi_2),$$

$$\cos\Omega = \cos\psi_1\cos\psi_2 + \sin\psi_1\sin\psi_2\cos\theta.$$

The contribution to the muonium hyperfine structure of 1S state takes the integral form:

$$\Delta E_{AV,hor}^{hfs}(1S) = \frac{16\alpha^2(Z\alpha)^5\mu^3\Lambda^2}{3\pi}\int_0^\infty dk_1 \int \frac{d\Omega_1}{\pi^2}\int_0^\infty dk_2 \int \frac{d\Omega_2}{\pi^2}\frac{A(M_A^2, k_1^2, k_2^2)}{(k_1^2 + a_e^2\cos^2\psi_1)}\times$$
$$\frac{A(M_A^2, k_1^2, k_2^2)}{(k_2^2 + a_\mu^2\cos^2\psi_2)}\frac{N_{AV}^{hor}}{(k_1^2 + k_2^2 + 2k_1k_2\cos\Omega + \frac{M_A^2}{\Lambda^2})},$$

For the numerical estimation all integrals are calculated numerically.

PS, S, T mesons. Horizontal diagrams

Vertical diagram in the case of scalar, pseudoscalar and tensor mesons in the leading order leads to contribution equal to zero. In the case of horizontal diagrams contributions can presented in integral form.

- Scalar meson:

$$\Delta E^{hfs}_{S,hor} = \frac{16\alpha^2(Z\alpha)^5\mu^3}{3\pi}\int_0^\infty dk_1 \int \frac{d\Omega_1}{\pi^2}\int_0^\infty dk_2 \int \frac{d\Omega_2}{\pi^2}\frac{A(M_S^2,k_1^2,k_2^2)}{(k_1^2+a_e^2\cos^2\psi_1)}\times$$
$$\frac{A(M_S^2,k_1^2,k_2^2)}{(k_2^2+a_\mu^2\cos^2\psi_2)}\frac{k_1^2k_2^2\cos\Omega(\cos\Omega\cos\psi_1\cos\psi_2-1-\cos^2\psi_1-\cos^2\psi_2-\cos^2\Omega)}{(k_1^2+k_2^2+2k_1k_2\cos\Omega+\frac{M_S^2}{\Lambda^2})},$$

- Pseudoscalar meson:

$$\Delta E^{hfs}_{PS,hor} = -\frac{\alpha^2(Z\alpha)^5\mu^3}{3\pi F_P^2}\int\frac{d^4k_1}{k_1\pi^4}\int\frac{d^4k_2}{k_2\pi^4}[F_{\pi^0\gamma^*\gamma^*}(k_1^2,k_2^2)]^2\times$$
$$\frac{(\cos\Omega+\cos^2\Omega-\cos^3\Omega+\cos\psi_1\cos\psi_2-\cos\Omega\cos^2\psi_1-\cos\Omega\cos^2\psi_2)}{(k_1^2+a_e^2\cos\psi_1^2)(k_2^2+a_\mu^2\cos\psi_2^2)((k_1+k_2)^2+\frac{M_P^2}{\Lambda^2})},$$

- Tensor meson:

$$\Delta E^{hfs}_{T,hor} = \frac{128\pi\alpha^2(Z\alpha)^5\mu^3}{3M_T^2}\int_0^\infty k_1^2dk_1\int_0^\pi\frac{\sin^2\psi_1}{\pi^3}\int_0^\infty k_2^2dk_2\int_0^\pi\frac{\sin^2\psi_2}{\pi^3}\int_0^\pi\sin\theta d\theta$$
$$\frac{A^2_{T\gamma^*\gamma^*}(M_T^2,0,0)(k_1^0k_2^0-k_1k_2+\frac{1}{k_1^2k_2^2-(k_1k_2)^2}[k_1^2(k_2^0)^2(k_1k_2)+k_2^2(k_1^0)^2(k_1k_2)-2k_1^2k_2^2k_1^0k_2^0])}{(k_1^2+1)^2(k_2^2+1)^2(k_1^2+a_e^2\cos^2\psi_1)(k_2^2+a_\mu^2\cos^2\psi_2)[(k_1+k_2)^2+\frac{M_T^2}{\Lambda^2}]}.$$

Meson parameters

Main parameters of mesons, such masses and decay width $\Gamma_{M\to\gamma\gamma}$ are presented in summary table of Particle Data Group collaboration (*R.L. Workman et al.(Particle Data Group), Prog.Theor.Exp.Phys.2022, 083C01 (2022)*).

- $f_1(1^{++}):\ \ M = 1281.5\ MeV,\ \ \Lambda = 1040\ MeV,\ \ A(M^2,0,0) = 0.226\ GeV^{-2}$
- $a_1(1^{++}):\ \ M = 1260\ MeV,\ \ \Lambda = 1040\ MeV,\ \ A(M^2,0,0) = 0.160\ GeV^{-2}$
- $f_1(1^{++}):\ \ M = 1426.3\ MeV,\ \ \Lambda = 926\ MeV,\ \ A(M^2,0,0) = 0.193\ GeV^{-2}$
- $\sigma(0^{++}):\ \ M = 550\ MeV,\ \ \Lambda = 2000\ MeV,\ \ A(M^2,0,0) = -0.596\ GeV^{-2}$
- $f_0(0^{++}):\ \ M = 980\ MeV,\ \ \Lambda = 2000\ MeV,\ \ A(M^2,0,0) = -0.085\ GeV^{-2}$
- $a_0(0^{++}):\ \ M = 980\ MeV,\ \ \Lambda = 2000\ MeV,\ \ A(M^2,0,0) = -0.086\ GeV^{-2}$
- $f_0(0^{++}):\ \ M = 1370\ MeV,\ \ \Lambda = 2000\ MeV,\ \ A(M^2,0,0) = -0.036\ GeV^{-2}$
- $\pi^0(0^{-+}):\ \ M = 135.9768\ MeV,\ \ \Lambda = 770\ MeV,\ \ A(M^2,0,0) = 0.025\ GeV^{-2}$
- $\eta(0^{-+}):\ \ M = 547.862\ MeV,\ \ \Lambda = 774\ MeV,\ \ A(M^2,0,0) = 0.024\ GeV^{-2}$
- $\eta'(0^{-+}):\ \ M = 957.78\ MeV,\ \ \Lambda = 859\ MeV,\ \ A(M^2,0,0) = 0.031\ GeV^{-2}$
- $f_2(2^{++}):\ \ M = 1275.4\ MeV,\ \ \Lambda = 2000\ MeV,\ \ A(M^2,0,0) = 0.498\ GeV^{-2}$

Values of transition form factor $A_{AV\gamma\gamma}(M^2,0,0)$ can be fixed from experimental data of L3 collaboration (*A. E. Dorokhov, et al., Phys. Lett. B, 776, 105 (2018)*). In the case of other mesons it can be expressed in term of decay width $\Gamma_{M\to\gamma\gamma}$:

$$A_{PS\gamma\gamma} = \sqrt{\frac{64\pi^3\Gamma_{PS\to\gamma\gamma}}{\alpha^2 M_{PS}^3}},\ A_{S\gamma\gamma} = \sqrt{\frac{4\Gamma_{S\to\gamma\gamma}}{\pi\alpha^2 M_S^3}},\ A_{T\gamma\gamma} = \sqrt{\frac{20\Gamma_{T\to\gamma\gamma}}{\pi\alpha^2 M_T}}.$$

Numerical results

- Axial vector meson:

$$\Delta E^{hfs,v}_{f_1(1285)}(1S) = -0.00028 \text{ Hz}, \quad \Delta E^{hfs,h}_{f_1(1285)}(1S) = -0.00311 \text{ Hz}.$$

$$\Delta E^{hfs,v}_{a_1(1260)}(1S) = -0.00011 \text{ Hz}, \quad \Delta E^{hfs,h}_{a_1(1260)}(1S) = -0.00115 \text{ Hz}.$$

$$\Delta E^{hfs,v}_{f_1(1420)}(1S) = -0.00007 \text{ Hz}, \quad \Delta E^{hfs,h}_{f_1(1420)}(1S) = -0.00096 \text{ Hz}.$$

- Scalar meson:

$$\Delta E^{hfs,v}_{\sigma(550)}(1S) = 0 \text{ Hz}, \quad \Delta E^{hfs,h}_{\sigma(550)}(1S) = 0.02701 \text{ Hz}.$$

$$\Delta E^{hfs,v}_{f_0(980)}(1S) = 0 \text{ Hz}, \quad \Delta E^{hfs,h}_{f_0(980)}(1S) = 0.00023 \text{ Hz}.$$

$$\Delta E^{hfs,v}_{a_0(980)}(1S) = 0 \text{ Hz}, \quad \Delta E^{hfs,h}_{a_0(980)}(1S) = 0.00023 \text{ Hz}.$$

$$\Delta E^{hfs,v}_{f_0(1370)}(1S) = 0 \text{ Hz}, \quad \Delta E^{hfs,h}_{f_0(1370)}(1S) = 0.00002 \text{ Hz}.$$

- Pseudoscalar meson:

$$\Delta E^{hfs,v}_{\pi^0(135)}(1S) = 0 \text{ Hz}, \quad \Delta E^{hfs,h}_{\pi^0(135)}(1S) = -0.00650 \text{ Hz}.$$

$$\Delta E^{hfs,v}_{\eta(550)}(1S) = 0 \text{ Hz}, \quad \Delta E^{hfs,h}_{\eta(550)}(1S) = -0.00170 \text{ Hz}.$$

$$\Delta E^{hfs,v}_{\eta'(960)}(1S) = 0 \text{ Hz}, \quad \Delta E^{hfs,h}_{\eta'(960)}(1S) = -0.00165 \text{ Hz}.$$

- Tensor meson:

$$\Delta E^{hfs,v}_{f_2(1275)}(1S) = 0 \text{ Hz}, \quad \Delta E^{hfs,h}_{f_2(1275)}(1S) = 0.00006 \text{ Hz}.$$

Conclusion

In comparison with previous results

- *R. N. Faustov, A. P. Martynenko, Phys. Lett. B 541, 135 (2002)*
- *S. G. Karshenboim, V. A. Shelyuto, A. I. Vainstein, Phys. Rev. D 78, 065036 (2008)*

we consider in our calculation

- interaction amplitudes of horizontal and vertical exchange,
- scalar, pseudoscalar, axial vector and tensor meson exchanges.

Total contribution:

$$\Delta E^{hfs}(1S) = 0.012 \text{ Hz}$$

The uncertainty of our theoretical result remains large because of large experimental errors in determination of transition formfactors parameters. We estimate the uncertainty is 50% for scalar meson exchange.

Thank You!

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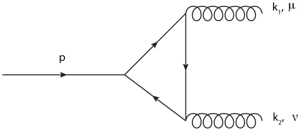
Transition form factor $\gamma^* + \gamma^* \to S$



Local quark model gives for $A(t^2, k_1^2, k_2^2)$:

$$A(t^2, k_1^2, k_2^2) = g_{S\gamma\gamma} \frac{N_c}{2\pi^2} Tr[\tau_M QQ] I_{S\gamma\gamma}.$$

For kinematics: $t^2 = 0,\ k_1^2 = -k^2,\ k_2^2 = -k^2$

$$I_{S\gamma\gamma}(0, -k^2, -k^2) = \frac{m_q}{k^2}\left(-2 + \frac{4m_q^2 ln\left(\frac{k\sqrt{4m_q^2+k^2}+2m_q^2+k^2}{2m_q^2}\right)}{\sqrt{k^2(4m_q^2+k^2)}}\right)$$

Expression for the integral at small momenta k allows to estimate transition form factor at zero:

$$A_S^{I=0} = -\frac{5}{18\pi^2 f_\pi}, \quad A_S^{I=1} = -\frac{1}{6\pi^2 f_\pi}$$