Muon pair production at the LHC with one proton tagging via $\gamma \gamma$ fusion and $\gamma Z$ fusion.

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## Introduction

- Searching for New Physics in muon pairs production at very high energies at the LHC is of a great interest.
- The ATLAS collaboration managed to measure the cross sections of dilepton production with one proton hitting the forward detector. (see Phys. Rev. Lett. 125, 261801 (2020))
- The analytical formulas describing fiducial cross section of the proton-proton scattering will be provided.
- The correction due to $Z$ boson exchange to the leading process of the pp scattering via $\gamma \gamma$ fusion will be investigated.


## Anomalous muon magnetic moment

Measurement of the Positive Muon Anomalous Magnetic Moment to 0.20 ppm (2023).

$$
\vec{\mu}=g_{\mu}\left(\frac{e}{2 m_{\mu}}\right) \vec{s}, \quad \text { where } \quad g_{\mu}=2\left(1+a_{\mu}\right)
$$



$$
a_{\mu}(\exp .)-a_{\mu}(S M)=(249 \pm 48) \times 10^{-11} \quad \text { with significance } 5 \sigma .
$$

## Experimental cuts

The ATLAS experimental constraints on the phase space volume are:

- $p_{i, T}>\hat{p}_{T}=15 \mathrm{GeV}$, where $p_{i, T}$ is a transversal momentum of a muon.
- $\left|\eta_{i}\right|<\hat{\eta}=2.4$, where $\eta_{i}$ is a pseudorapidity of a muon.
- $p_{T}^{\mu \mu}<\hat{p}_{T}^{\mu \mu}=5 \mathrm{GeV}$, where $p_{T}^{\mu \mu}$ is a transversal momentum of a muon pair.
- $20 \mathrm{GeV}<W<70 \mathrm{GeV}$ and $W>105 \mathrm{GeV}$, where $W$ is an invariant mass of a muon pair.
- $0.035<\xi<0.08 \rightarrow 227 \mathrm{GeV}=\omega_{\min }<\omega<\omega_{\max }=520 \mathrm{GeV}$, where $\xi$ is a fraction of the energy that the proton loses.


## Elastic case: $p p \rightarrow p p(\gamma \gamma) \rightarrow p \mu \mu p$



- Here both protons don't disintegrate $\rightarrow$ the EPA can be used.

$$
\sigma\left(p p \rightarrow p \mu^{+} \mu^{-} p\right)=\int_{0}^{\infty} \int_{0}^{\infty} \sigma\left(\gamma \gamma \rightarrow \mu^{+} \mu^{-}\right) n_{p}\left(\omega_{1}\right) n_{p}\left(\omega_{2}\right) \mathrm{d} \omega_{1} \mathrm{~d} \omega_{2}
$$

## Elastic case: $p p \rightarrow p p(\gamma \gamma) \rightarrow p \mu \mu p$

- The equivalent photon spectrum is:

$$
n_{p}(\omega)=\frac{2 \alpha}{\pi \omega} \int_{0}^{\infty} \frac{D\left(Q^{2}\right)}{Q^{4}} q_{\perp}^{3} \mathrm{~d} q_{\perp}
$$

where $Q^{2}=q_{\perp}^{2}+\omega^{2} / \gamma^{2}$.

- The value $D\left(Q^{2}\right)$ is a combination of form-factors:

$$
D\left(Q^{2}\right)=\frac{G_{E}^{2}\left(Q^{2}\right)+\frac{Q^{2}}{4 m_{P}^{2}} G_{M}^{2}\left(Q^{2}\right)}{1+\frac{Q^{2}}{4 m_{P}^{2}}},
$$

where $G_{E}\left(Q^{2}\right)$ and $G_{M}\left(Q^{2}\right)$ are the Sachs electric and magnetic form factors:

$$
G_{E}\left(Q^{2}\right)=\frac{1}{\left(1+Q^{2} / \Lambda^{2}\right)^{2}}, \quad G_{M}\left(Q^{2}\right)=\frac{\mu_{p}}{\left(1+Q^{2} / \Lambda^{2}\right)^{2}}
$$

## Elastic case: $p p \rightarrow p p(\gamma \gamma) \rightarrow p \mu \mu p$

- $W=4 \omega_{1} \omega_{2}$.
- $y=\frac{1}{2} \ln \frac{\omega_{1}}{\omega_{2}}$, where $y=\eta+\frac{1}{2} \ln \frac{1-\sqrt{1-4 p_{T}^{2} / W^{2}}}{1+\sqrt{1-4 p_{T}^{2} / W^{2}}}$.
- After imposing ATLAS experimental constraints one obtains:

$$
\sigma_{\text {fid. }}\left(p p \rightarrow p \mu^{+} \mu^{-} p\right)=8.6 \mathrm{fb}
$$

## Inelastic case: $p p \rightarrow p(\gamma \gamma) p \rightarrow p \mu \mu X$



- Here one of the protons disintegrates $\rightarrow$ the EPA can't be used. The calculation can be performed within the parton model.

$$
\sigma\left(p p \rightarrow p \mu^{+} \mu^{-} X\right)=\sum_{q} \sigma\left(p q \rightarrow p \mu^{+} \mu^{-} q\right)
$$

## Inelastic case: $p p \rightarrow p(\gamma \gamma) p \rightarrow p \mu \mu X$

The cross-section for the reaction $p q \rightarrow p \mu \mu q$ is:

$$
\begin{aligned}
d \sigma_{p q \rightarrow p \mu^{+} \mu^{-} q}= & \frac{Q_{q}^{2}(4 \pi \alpha)^{2}}{q_{1}^{2} q_{2}^{2}} \rho_{\mu \nu}^{(1)} \rho_{\alpha \beta}^{(2)} M_{\mu \alpha} M_{\nu \beta}^{*} \times \\
& \times \frac{(2 \pi)^{4} \delta^{(4)}\left(q_{1}+q_{2}-k_{1}-k_{2}\right) d \Gamma}{4 \sqrt{\left(p_{1} p_{2}\right)^{2}-m_{p}^{4}}} \times \\
& \times \frac{d^{3} p_{1}^{\prime}}{(2 \pi)^{3} 2 E_{1}^{\prime}} \frac{d^{3} p_{2}^{\prime}}{(2 \pi)^{3} 2 E_{2}^{\prime}} \times f_{q}\left(x, Q_{2}^{2}\right) d x,
\end{aligned}
$$

where $Q_{q}$ is a quark charge, $\rho_{\mu \nu}^{i}$ is a photon density matrix, $M_{\mu \alpha}$ is the amplitude of $\gamma \gamma^{*} \rightarrow \mu \mu$ process, $d \Gamma$ is a phase volume of the muon pair and $f_{q}\left(x, Q_{2}^{2}\right)$ is a parton distribution function (PDF).

## Inelastic case: $p p \rightarrow p(\gamma \gamma) p \rightarrow p \mu \mu X$

- Changing to the photons helicity basis one obtains:

$$
\rho_{1}^{\mu \nu} \rho_{2}^{\alpha \beta} M_{\mu \alpha} M_{\nu \beta}^{*}=(-1)^{a+b+c+d} \rho_{1}^{a b} \rho_{2}^{c d} M_{a c} M_{b d}^{*}
$$

where $a, b, c, d \in\{ \pm 1,0\}$

- Non-diagonal terms $(a \neq b$ or $c \neq d)$ vanish after integration over phase space.
- For the amplitudes $\left|M_{ \pm 0}\right|^{2}$ contribution we have:

$$
\left|M_{ \pm 0}\right|^{2} \sim Q_{2}^{2} / W^{2} \leq\left(\hat{p}_{T}^{\mu \mu} / W\right)^{2} \ll 1
$$

- The transversal term reveals the chiral anomaly:

$$
\left|M_{++}\right|^{2} \sim \sin ^{2} \theta[\ldots]+\left\{\frac{1-v^{2}}{(1+v \cos \theta)^{2}}+\frac{1-v^{2}}{(1-v \cos \theta)^{2}}\right\}
$$

## Inelastic case: $p p \rightarrow p(\gamma \gamma) p \rightarrow p \mu \mu X$

- The cross section $\sigma(\gamma \gamma \rightarrow \mu \mu)$ is a sum over transversal polarizations of photons:

$$
\begin{aligned}
\sigma(\gamma \gamma \rightarrow \mu \mu) & =\int \frac{1}{4}\left[\left|M_{++}\right|^{2}+\left|M_{+-}\right|^{2}+\left|M_{-+}\right|^{2}+\left|M_{--}\right|^{2}\right] \times \\
& \times \frac{(2 \pi)^{4} \delta^{(4)}\left(q_{1}+q_{2}-k_{1}-k_{2}\right) d \Gamma}{4 p_{1} p_{2}}
\end{aligned}
$$

- Similarly to the elastic case the equivalent photon spectrum of quark can be introduced:

$$
n_{q}(\omega)=\frac{2 Q_{q}^{2} \alpha}{\pi \omega} \int_{\omega / E}^{1} d x \int_{0}^{p_{T}^{\ell \ell}} d q_{2 \perp} \frac{q_{2 \perp}^{3}}{Q_{2}^{4}} f_{q}\left(x, Q_{2}^{2}\right)
$$

## Inelastic case: $p p \rightarrow p(\gamma \gamma) p \rightarrow p \mu \mu X$

- Under the approximation $\omega_{1} \ll E, \omega_{2} \ll x E$ one obtains:

$$
\begin{gathered}
\rho_{1}^{++}=\rho_{2}^{--} \approx D\left(Q_{1}^{2}\right) \cdot \frac{2 E^{2} q_{1 \perp}^{2}}{\omega_{1}^{2} Q_{1}^{2}}, \\
\rho_{1}^{++}=\rho_{2}^{--} \approx \frac{2 E^{2} x^{2} q_{2 \perp}^{2}}{\omega_{2}^{2} Q_{2}^{2}} .
\end{gathered}
$$

- Using parton distribution functions MSHT20nnlo_as118 (Eur. Phys. J. C81, 341 (2021)) provided by LHAPDF (Eur. Phys. J. C75, 132 (2015)) we get the cross section:

$$
\sigma_{\text {fid }}\left(p p \rightarrow p \mu^{+} \mu^{-} X\right)=9.6 f b .
$$

## Numerical results

- Elastic case:

$$
\sigma_{\mathrm{fid}}\left(p p \rightarrow p \mu^{+} \mu^{-} p\right)=8.6 \mathrm{fb}
$$

- Inelastic case:

$$
\sigma_{\mathrm{fid}}\left(p p \rightarrow p \mu^{+} \mu^{-} X\right)=9.2 \mathrm{fb}
$$

- Total elastic-inelastic cross-section is

$$
\tilde{\sigma}_{\mu \mu+p}^{\text {fid. }}=18 \pm 2 \mathrm{fb}
$$

- ATLAS results:

$$
\sigma_{\mu \mu+p}^{\exp }=7.2 \pm 1.6 \text { (stat.) } \pm 0.9 \text { (syst.) } \pm 0.2 \text { (lumi.) fb. }
$$

## Survival factor

- The so-called survival factor $S(b)$ depending on the impact parameter $b$ must be taken into account.
- For elastic case this factor provides $10 \%$ diminishing of the cross section. (see JHEP 2021, 234 (2021))
- For inelastic case this factor provides $50 \%$ diminishing of the cross section. (see Phys. Rev. D 104, 074009 (2021))
- Thus the derived formulas are in agreement with the experimental data at the level of $2-3$ standard derivations.


## $\gamma Z$ correction

- Interference between processes due to vector and axial interaction is identically zero.
- The $\left|M_{\gamma Z}^{V}\right|^{2}=\left|M_{\gamma Z}^{A}\right|^{2}$ in the limit $W \gg m$.
- $\left|M_{\gamma \gamma+\gamma Z}\right|^{2} \equiv \varkappa\left|M_{\gamma \gamma}\right|^{2}$, where

$$
\begin{aligned}
& \quad \varkappa\left(Q_{2}^{2}\right)=1+2 \frac{g_{V}^{\mu}}{Q_{\mu}} \frac{g_{V}^{q}}{Q_{q}} \lambda+\frac{\left(g_{A}^{q}\right)^{2}+\left(g_{V}^{q}\right)^{2}}{Q_{\mu}^{2}} \frac{\left(g_{A}^{\mu}\right)^{2}+\left(g_{V}^{\mu}\right)^{2}}{Q_{q}^{2}} \lambda^{2} \\
& \text { and } \lambda=\frac{1}{\left(2 s_{W} c_{W}\right)^{2}\left(1+M_{Z}^{2} / Q_{2}^{2}\right)} .
\end{aligned}
$$

## $\gamma Z$ correction

- The ratio $\frac{(d \sigma / d W)_{\gamma \gamma+\gamma z}}{(d \sigma / d W)_{\gamma \gamma}}$ is presented in the graph:

- If the lower limit on $p_{T}^{\mu \mu}$ grows the correction becomes larger:



## Conclusion

- The analytical formulae for the cross section of the processes $p p \rightarrow p \mu \mu p$ and $p p \rightarrow p \mu \mu X$ were obtained.
- After implying experimental cuts on the theoretically obtained formulae the fiducial cross section was calculated.
- After taking into account the survival factor the derived formulae are in good agreement with the ATLAS results.
- The value of the $\gamma Z$ correction was calculated. It is shown that with the larger lower limit on $p_{T}^{\mu \mu}$ the contribution goes up to 20\%.
- The numerical calculations were performed with the help of the library libepa: https://github.com/jini-zh/libepa developed within our group.
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