The next-to-leading approximation of BFKL for dijet production with large rapidity separation between jets at the LHC

ANATOLII EGOROV (1,2) , V.T. KIM (1,2), V.A. Murzin (1), V.A. Oreshkin (1)

1 - NRC KI Petersburg Nuclear Physics Institute, Gatchina, Russia

2 - Peter the Great St. Petersburg Polytechnic University, St.Petersburg, Russia

DGLAP vs BFKL

DGLAP

Gribov—Lipatov—Altarelli—Parisi—Dokshitzer

Bjorken limit

$$\sqrt{s} \to \infty; p_T \to \infty; x \sim \frac{p_T}{\sqrt{s}} \sim 1$$

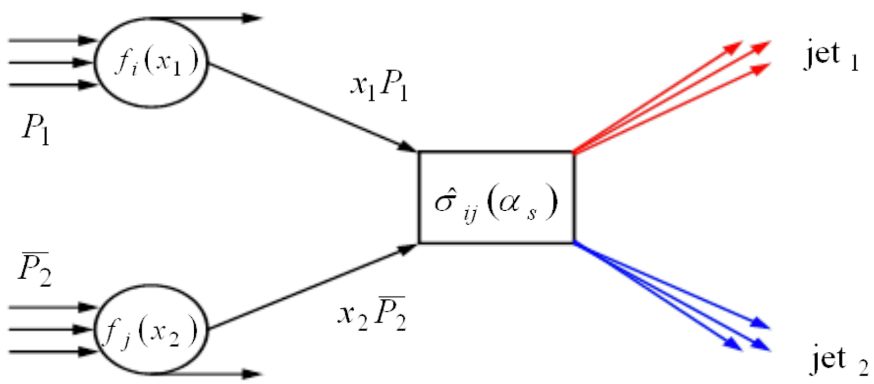
$$k_{Tn} \gg k_{Tn-1} \gg \ldots \gg k_{T2} \gg k_{T1}$$

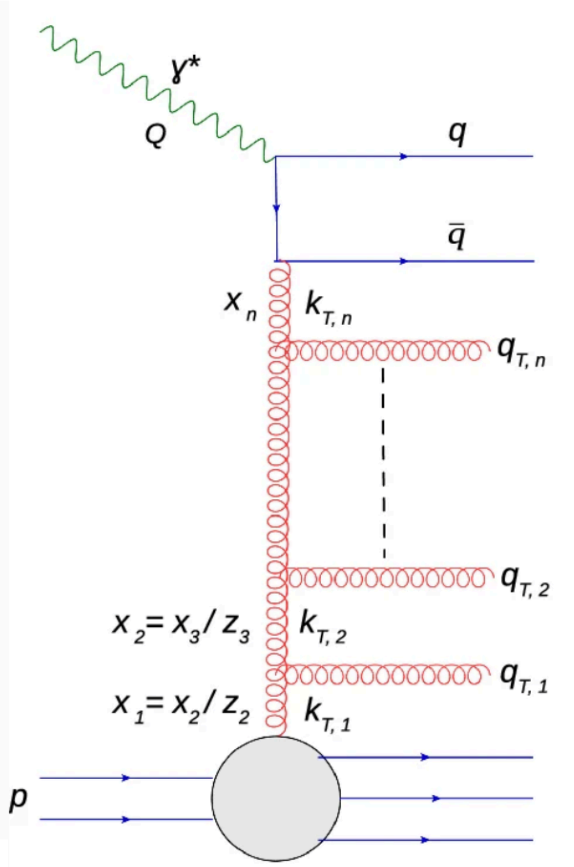
$$[\alpha_s \log Q^2]^n$$

DGLAP evolution

$$\frac{df_i}{d\log\mu^2} = \frac{\alpha_s}{2\pi}[P_{qq} \otimes f_i + P_{qg} \otimes f_g]$$

$$\frac{df_g}{d\log\mu^2} = \frac{\alpha_s}{2\pi}[P_{gq} \otimes \sum_i f_i + P_{gg} \otimes f_g]$$





BFKL

Balitsky—Fadin—Kuraev—Lipatov

BFKL kinematics (LLA): Regge-Gribov limit

$$\sqrt{s} \to \infty; p_T - finite; x \sim \frac{p_T}{\sqrt{s}} \to 0;$$

c $p_T \gg \Lambda_{QCD}$

$$x_n \gg x_{n-1} \gg \ldots \gg x_2 \gg x_1$$

$$k_{Tn} \sim k_{Tn-1} \sim \ldots \sim k_{T2} \sim k_{T1}$$

BFKL evolution

$$[\alpha_s \log(1/x)]^n$$

$$\frac{\partial f_g}{\partial \log 1/x} = K \otimes f_g = \omega f_g \quad \Rightarrow$$

$$f_g \sim \left(\frac{1}{x}\right)^{\omega} \sim \left(\frac{s}{s_0}\right)^{\omega} \sim e^{\omega \Delta y}$$

$$\omega_{\max} = \alpha_{I\!P}(0) - 1$$

LL BFKL provides too large intercept

$$\alpha_{I\!P}^{LL}(0) \approx 1.5$$

NLL BFKL: $\alpha_{I\!P}^{NLL} \approx 1.2$

S.J. Brodsky, V.S. Fadin, V.T. Kim, L.N. Lipatov, G.B. Pivovarov

[JETP Lett. 70 (1999) 155-160]

Obstacles in search of BFKL

- NLL BFKL methods of calculation are developed only for part of the measured observables
- Absence of PURE DGLAP based calculations. The colour coherence corrections are often included to Monte Carlo (MC) DGLAP-based generators such as PYTHIA8 and HERWIG

STRATEGY:

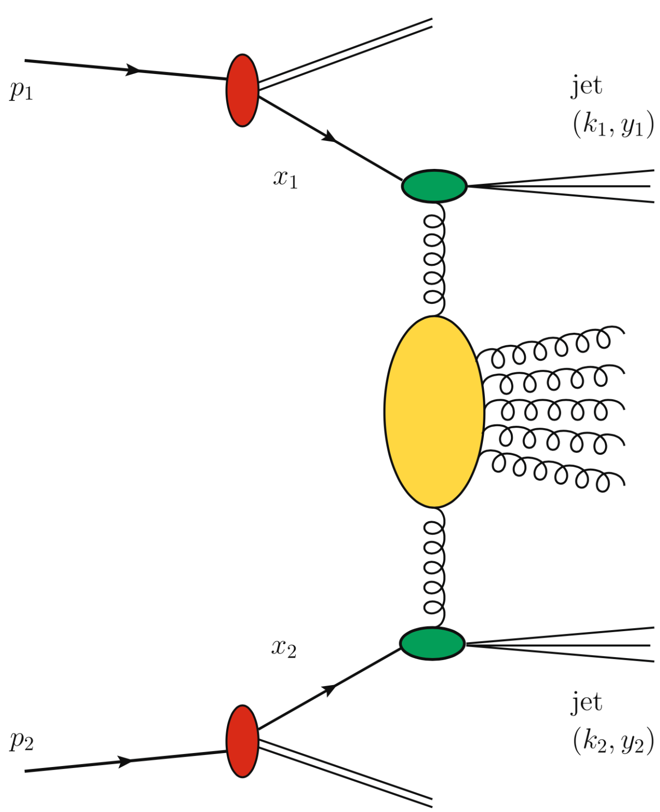
- Perform comparison of NLL BFKL calculations with measurements were it is possible
- Measure the existing observables at all available in experiments energies
- Develop calculations based on NLL BFKL for all existing observables
- Look for new observables which can be calculated at NLL BFKL and measured

Mueller-Navelet dijets

$\sqrt{s} = 2.76$ TeV

A. H. Mueller and H. Navelet [Nucl. Phys. B 282 (1987) 727]

σ^{MN}



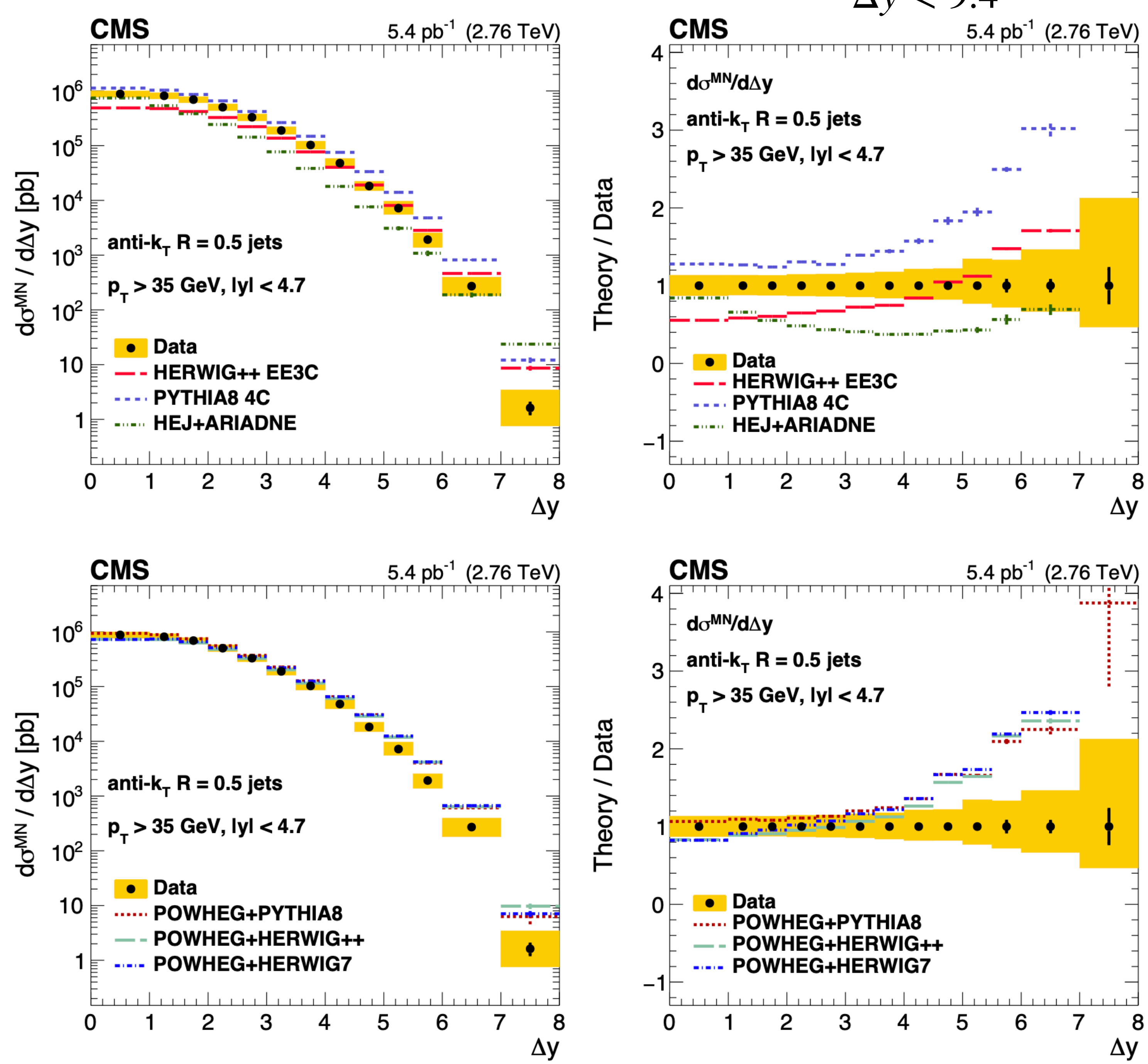
MN jet pair is a pair of jets with p_T above $p_{T\min}$ most separated in rapidity

$\Delta y = |y_1 - y_2|$

CMS [JHEP03(2022)189]

$p_{T\min} = 35$ GeV

$\Delta y < 9.4$



NLL BFKL for MN dijets (1)

$$\frac{d\sigma}{dy_1 dy_2 d^2\vec{p}_{T1} d^2\vec{p}_{T2}} = \sum_{ij} \int f_i(x_1,\mu_F) f_j(x_2,\mu_F) \frac{d\hat{\sigma}_{ij}(x_1 x_2 s, \mu_F, \mu_R)}{dy_1 dy_2 d^2\vec{p}_{T1} d^2\vec{p}_{T2}}$$

Large Δy: $$f^{\text{eff}}(x,\mu_F) = \frac{C_A}{C_F} f_g(x,\mu_F) + \sum_{i=q,\bar{q}} f_i(x,\mu_F),$$

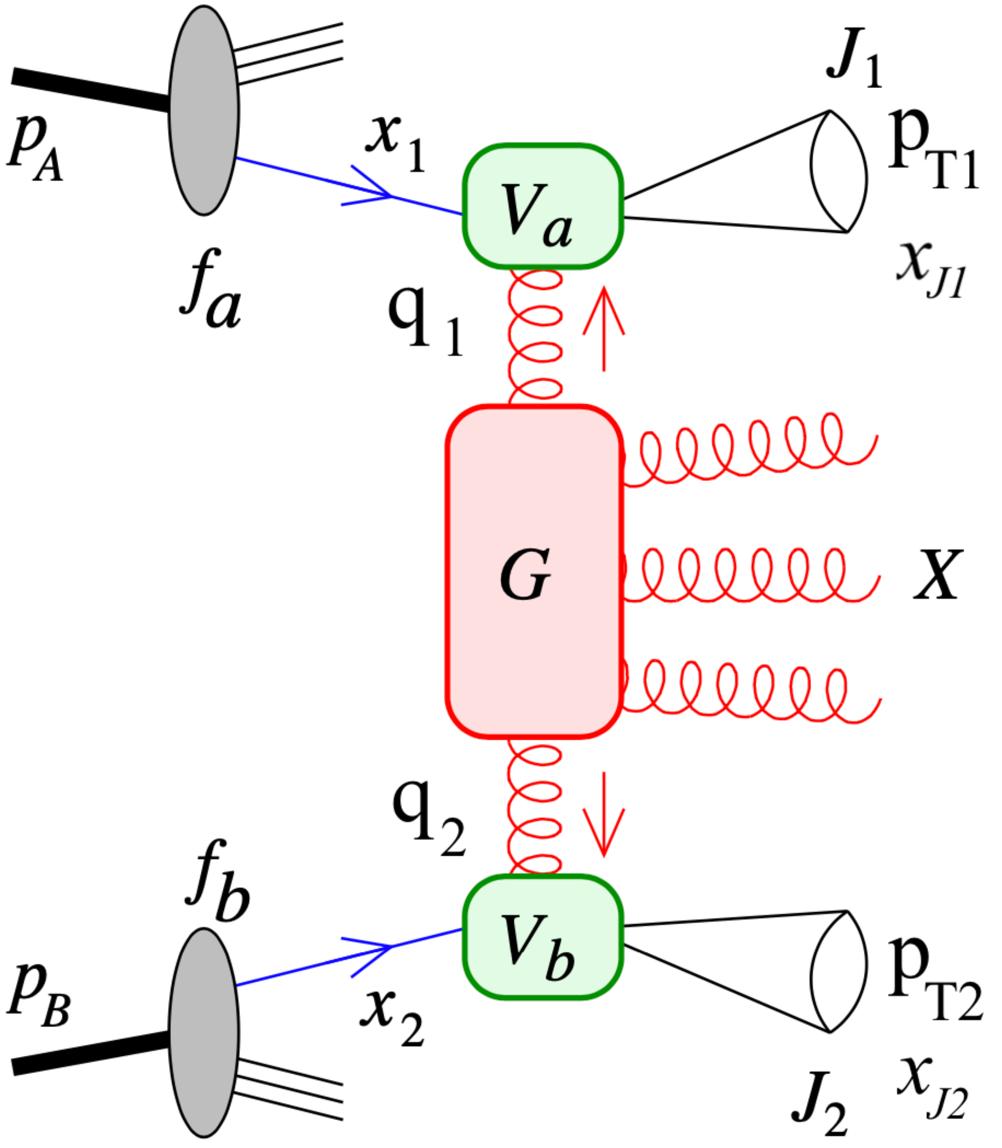
NLL BFKL

$$\frac{d\hat{\sigma}_{gg}}{dy_1 dy_2 d^2\vec{p}_{T1} d^2\vec{p}_{T2}} = \frac{x_{J1} x_{J2}}{(2\pi)^2} \int \frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}} V_1(\vec{q}_1, x_1, \vec{p}_{T1}, x_{J1}) \times \int \frac{d^2\vec{q}_2}{\vec{q}_2^{\,2}} V_2(-\vec{q}_2, x_2, \vec{p}_{T2}, x_{J2}) \times \int_C \frac{d\omega}{2\pi i} \left(\frac{x_1 x_2 s}{s_0}\right)^{\omega} G_\omega(\vec{q}_1, \vec{q}_2),$$

$$\Phi(\vec{q}, \vec{p}_T, x_J, \omega) \equiv \sum_i \int_0^1 dx f_i(x,\mu_F) \left(\frac{x}{x_J}\right)^{\omega} V_i(\vec{q}, x, \vec{p}_T, x_J),$$

$$\Phi_{1,2}(n,\nu,\vec{p}_{T1,2}, x_{J1,2}, \omega) = \alpha_s(\mu_R)[c_{1,2}(n,\nu) + \bar{\alpha}_s(\mu_R) c^{(1)}_{1,2}(n,\nu)]$$

$$\frac{d\sigma}{dy_1 dy_2 d|\vec{p}_{T1}| d|\vec{p}_{T2}| d\phi_1 d\phi_2} = \frac{1}{(2\pi)^2}\left[\mathscr{C}_0 + \sum_{n=1}^{\infty} 2\cos(n\phi)\mathscr{C}_n\right]$$



NLL BFKL for MN dijets

NLL BFKL

$$\frac{d\sigma}{dy_1 dy_2 d|\vec{p}_{T1}|d|\vec{p}_{T2}|d\phi_1 d\phi_2} = \frac{1}{(2\pi)^2}\left[\mathscr{C}_0 + \sum_{n=1}^{\infty} 2\cos(n\phi)\mathscr{C}_n\right]$$

$$\mathscr{C}_n = \frac{x_{J1}x_{J2}}{|\vec{p}_{T1}||\vec{p}_{T2}|}\int_{-\infty}^{+\infty} d\nu e^{(Y-Y_0)\bar{\alpha}_s(\mu_R)\chi(n,\nu)}\alpha_s^2(\mu_R)c_1(n,\nu)c_1(n,\nu)\Bigg[1 + \bar{\alpha}_s(\mu_R)\left(\frac{\bar{c}_1^{(1)}(n,\nu)}{c_1(n,\nu)} + \frac{\bar{c}_2^{(1)}(n,\nu)}{c_2(n,\nu)}\right.$$

$$\left. + \frac{\beta_0}{2N_c}\left(\frac{5}{3} + \ln\frac{\mu_R^2}{|\vec{p}_{T1}||\vec{p}_{T2}|}\right)\right) + \bar{\alpha}_s^2(\mu_r)\ln\frac{x_{J1}x_{J2}s}{s_0}\left\{\bar{\chi}(n,\nu) + \frac{\beta_0}{4N_c}\chi(n,\nu)\left(-\frac{\chi(n,\nu)}{2} + \frac{5}{3} + \ln\frac{\mu_R^2}{|\vec{p}_{T1}||\vec{p}_{T2}|}\right)\right\}\Bigg]$$

where

$$Y = y_1 - y_2 = \ln\frac{x_{J1}x_{J2}s}{|\vec{p}_{T1}||\vec{p}_{T2}|} \quad \text{and} \quad Y_0 = \ln\frac{s_0}{|\vec{p}_{T1}||\vec{p}_{T2}|}$$

$$\frac{d\sigma}{dy_1 dy_2 d|\vec{p}_{T1}|d|\vec{p}_{T2}|} = \mathscr{C}_0$$

- NLL BFKL contains both renormalization scheme and renormalization scale ambiguities

BFKLP prescription (1)

This is the generalisation of Brodsky-Lepage-Mackenzie (BLM) optimal scale setting procedure [Phys. Rev. D 28 (1983) 229].

Brodsky-Fadin-Kim-Lipatov-Pivovarov (BFKLP) [JETP Lett. 70 (1999) 155-160]

The ambiguity is related to large running coupling effects and non-Abelian nature of the QCD

$\Rightarrow$ one needs to use physical renormalization scheme, in which the non-Abelian contributions presented in the first order, for example physical momentum subtraction scheme (MOM).

MOM and $\overline{\mathrm{MS}}$ schemes are related:

$$\alpha_s^{\overline{\mathrm{MS}}} = \alpha_s^{\mathrm{MOM}}\left(1 + \frac{\alpha_s^{\mathrm{MOM}}}{\pi}(T^\beta + T^{\mathrm{conf}})\right),$$

$$T^\beta = -\frac{\beta_0}{2}\left(1 + \frac{2}{3}I\right),$$

$$T^{\mathrm{conf}} = \frac{C_A}{8}\left[\frac{17}{2}I + \frac{3}{2}(I-1)\xi + \left(1 - \frac{1}{3}I\right)\xi^2 - \frac{1}{6}\xi^3\right],$$

where $I \approx 2.3439$, ξ - gauge parameter

BFKLP prescription (2)

Transform to MOM scheme, and separate conformal (β_0 -independent) and non-conformal (β_0 -dependent) parts:

$$\mathscr{C}_n^{\mathrm{MOM}} = \frac{x_{J1}x_{J2}}{|\vec{p}_{T1}||\vec{p}_{T2}|}\int_{-\infty}^{+\infty} d\nu e^{(Y-Y_0)\bar{\alpha}_s^{\mathrm{MOM}}(\mu_R)\chi(n,\nu)}\left(\alpha_s^{\mathrm{MOM}}(\mu_R)\right)^2 c_1(n,\nu)c_2(n,\nu)$$
$$\times\left[1+\bar{\alpha}_s(\mu_R)\left(\frac{\bar{c}_1^{(1)}(n,\nu)}{c_1(n,\nu)}+\frac{\bar{c}_2^{(1)}(n,\nu)}{c_2(n,\nu)}+\frac{2T^{\mathrm{conf}}}{N_c}+\frac{\beta_0}{2N_c}\left(\frac{5}{3}+\ln\frac{\mu_R^2}{|\vec{p}_{T1}||\vec{p}_{T2}|}-2\left(1+\frac{2}{3}I\right)\right)\right)\right.$$
$$\left.+\left(\bar{\alpha}_s^{\mathrm{MOM}}(\mu_R)\right)^2\ln\frac{x_{J1}x_{J2}s}{s_0}\left\{\bar{\chi}(n,\nu)+\frac{T^{\mathrm{conf}}}{N_c}\chi(n,\nu)+\frac{\beta_0}{4N_c}\chi(n,\nu)\left(-\frac{\chi(n,\nu)}{2}+\frac{5}{3}+\ln\frac{\mu_R^2}{|\vec{p}_{T1}||\vec{p}_{T2}|}-2\left(1+\frac{2}{3}I\right)\right)\right\}\right],$$

choose μ_R scale so that the non-conformal part vanishes

$$\mathscr{C}_n^{\beta} = \frac{x_{J1}x_{J2}}{|\vec{p}_{T1}||\vec{p}_{T2}|}\int_{-\infty}^{+\infty} d\nu e^{(Y-Y_0)\bar{\alpha}_s^{\mathrm{MOM}}(\mu^{\mathrm{BFKLP}})\chi(n,\nu)}\left(\alpha_s^{\mathrm{MOM}}(\mu^{\mathrm{BFKLP}})\right)^3 c_1(n,\nu)c_2(n,\nu)$$
$$\times\frac{\beta_0}{2N_c}\left[\frac{5}{3}+\ln\frac{(\mu^{\mathrm{BFKLP}})^2}{|\vec{p}_{T1}||\vec{p}_{T2}|}-2\left(1+\frac{2}{3}I\right)+\bar{\alpha}_s^{\mathrm{MOM}}(\mu^{\mathrm{BFKLP}})\ln\frac{x_{J1}x_{J2}s}{s_0}\frac{\chi(n,\nu)}{2}\right.$$
$$\left.\times\left(-\frac{\chi(n,\nu)}{2}+\frac{5}{3}+\ln\frac{(\mu^{\mathrm{BFKLP}})^2}{|\vec{p}_{T1}||\vec{p}_{T2}|}-2\left(1+\frac{2}{3}I\right)\right)\right]=0,$$

Approximate BFKLP scale

Case a:

F. Caporale, D. Yu. Ivanov, B. Murdaca & A. Papa [Phys. Rev. D 91 (2015) 114009]

$$(\mu_a^{\rm BFKLP})^2 = |\vec{p}_{T1}||\vec{p}_{T2}|\exp\left[2\left(1+\frac{2}{3}I\right)-\frac{5}{3}\right],$$

$$\mathscr{C}_a^{\rm BFKLP} = \frac{x_{J1}x_{J2}}{|\vec{p}_{T1}||\vec{p}_{T2}|}\times\int_{-\infty}^{+\infty} d\nu\, e^{(Y-Y_0)\bar{\alpha}_s^{\rm MOM}(\mu_a^{\rm BFKLP})\left[\chi(n,\nu)+\bar{\alpha}_s^{\rm MOM}(\mu_a^{\rm BFKLP})\left(\bar{\chi}(n,\nu)+\frac{T^{\rm conf}}{N_c}\chi(n,\nu)-\frac{\beta_0}{8N_c}\chi^2(n,\nu)\right)\right]}$$

$$\times\,(\alpha_s^{\rm MOM}(\mu_a^{\rm BFKLP}))^2 c_1(n,\nu)c_2(n,\nu)\times\left[1+\bar{\alpha}_s^{\rm MOM}(\mu_a^{\rm BFKLP})\left\{\frac{\bar{c}_1^{(1)}(n,\nu)}{c_1(n,\nu)}+\frac{\bar{c}_2^{(1)}(n,\nu)}{c_2(n,\nu)}+\frac{2T^{\rm conf}}{N_c}\right\}\right]$$

Case b:

$$(\mu_b^{\rm BFKLP})^2 = |\vec{p}_{T1}||\vec{p}_{T2}|\exp\left[2\left(1+\frac{2}{3}I\right)-\frac{5}{3}+\frac{1}{2}\chi(n,\nu)\right],$$

$$\mathscr{C}_b^{\rm BFKLP} = \frac{x_{J1}x_{J2}}{|\vec{p}_{T1}||\vec{p}_{T2}|}\times\int_{-\infty}^{+\infty} d\nu\, e^{(Y-Y_0)\bar{\alpha}_s^{\rm MOM}(\mu_b^{\rm BFKLP})\left[\chi(n,\nu)+\bar{\alpha}_s^{\rm MOM}(\mu_b^{\rm BFKLP})\left(\bar{\chi}(n,\nu)+\frac{T^{\rm conf}}{N_c}\chi(n,\nu)\right)\right]}$$

$$\times\,(\alpha_s^{\rm MOM}(\mu_b^{\rm BFKLP}))^2 c_1(n,\nu)c_2(n,\nu)\times\left[1+\bar{\alpha}_s^{\rm MOM}(\mu_b^{\rm BFKLP})\left\{\frac{\bar{c}_1^{(1)}(n,\nu)}{c_1(n,\nu)}+\frac{\bar{c}_2^{(1)}(n,\nu)}{c_2(n,\nu)}+\frac{2T^{\rm conf}}{N_c}+\frac{\beta_0}{4N_c}\chi(n,\nu)\right\}\right],$$

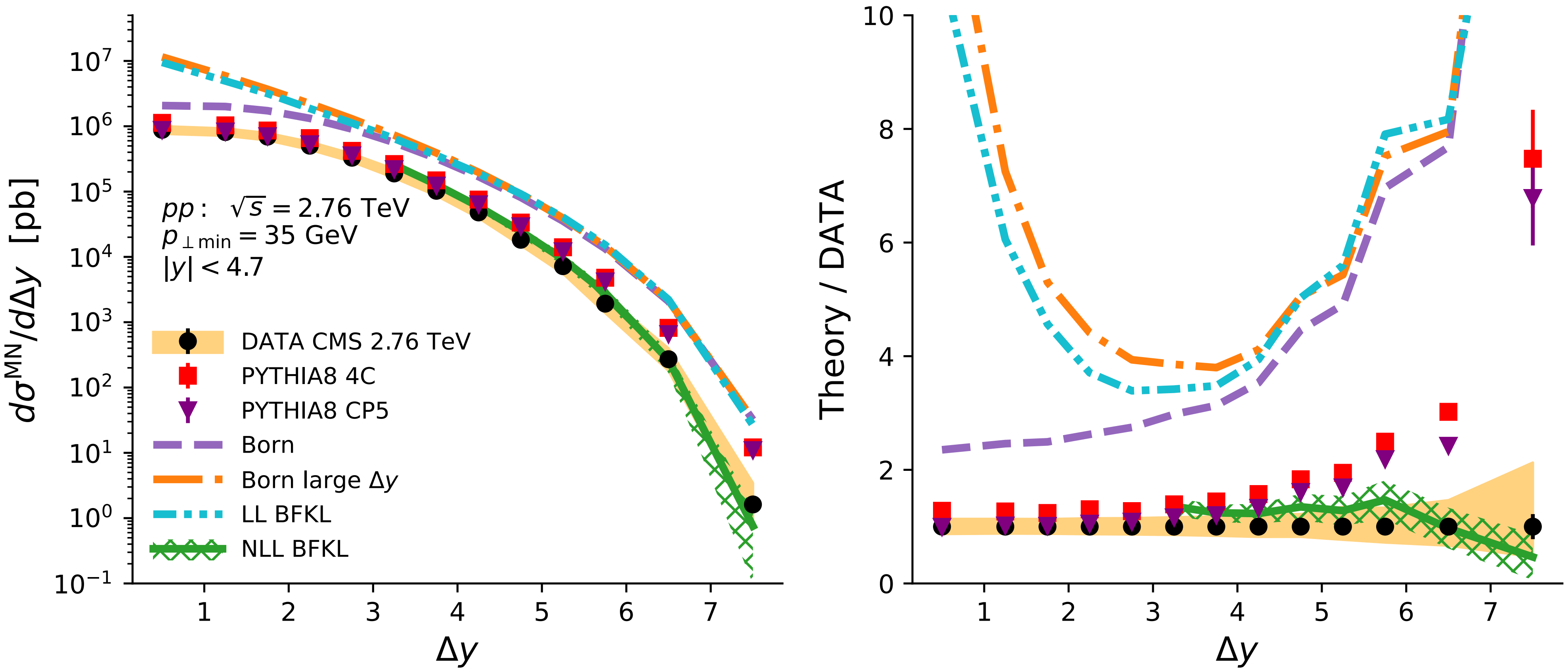
Case (a) better reproduce exact $\mu^{\rm BFKLP}$ settings for cross section $\mathscr{C}_0$

F. G. Celiberto, D. Yu. Ivanov, B. Murdaca & A. Papa [Phys. Rev. D 91 (2015) 114009]

Case (a) ⇒ estimate of MN x-section; Case (a) - Case(b) ⇒ estimate of theoretical uncertainty

First comparison: NLL BFKL for MN dijets @ 2.76 TeV

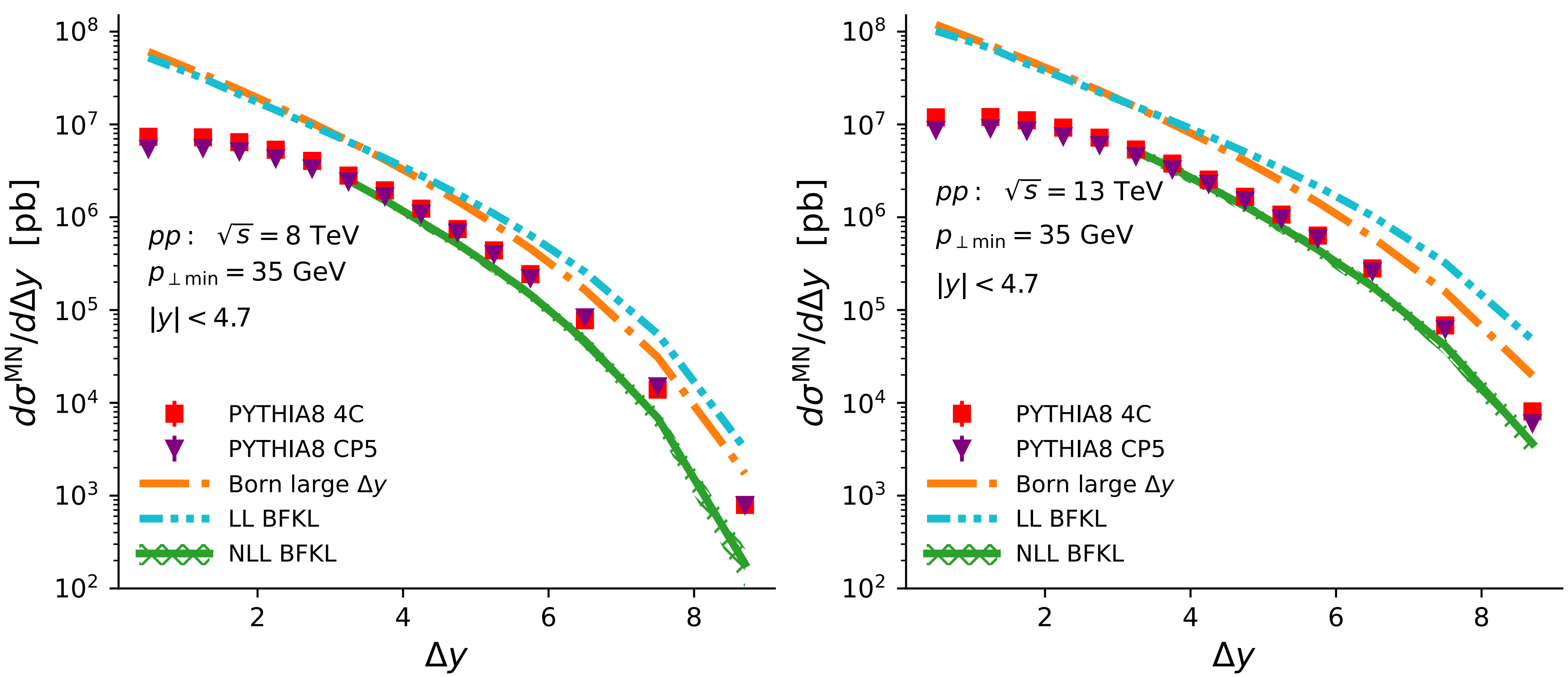
A. Iu. Egorov and V. T. Kim [Phys. Rev. D 108 (2023) 014010]



- NLL BFKL agrees with the CMS data at large Δy
- All other calculations based on LO+LL DGLAP overestimates the CMS data at large Δy (Born, PYTHIA8, HERWIG [JHEP03(2022)189])
- NLO+LL DGLAP POWHEG+PYTHIA8/HERWIG overestimates the CMS data at large Δy [JHEP03(2022)189].

NLL BFKL for MN dijets @ 8 and 13 TeV

A. Iu. Egorov and V. T. Kim [Phys. Rev. D 108 (2023) 014010]

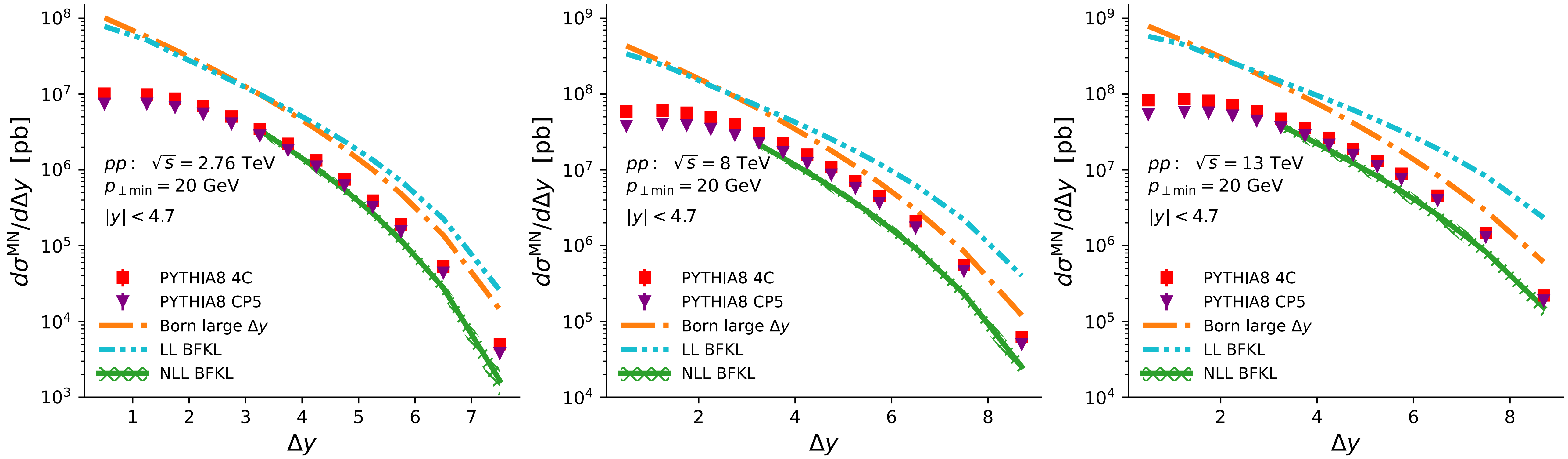


- NLL BFKL predicts lowest values of MN x-section at large Δy
- Difference between Born and LL BFKL rises with $\sqrt{s}$ and $\Delta y \Rightarrow$ expected BFKL effects become stronger with increasing $\sqrt{s}$ and Δy

NLL BFKL for MN dijets with lower $p_{\perp\min}$ = 20 GeV

@ 2.76, 8 and 13 TeV

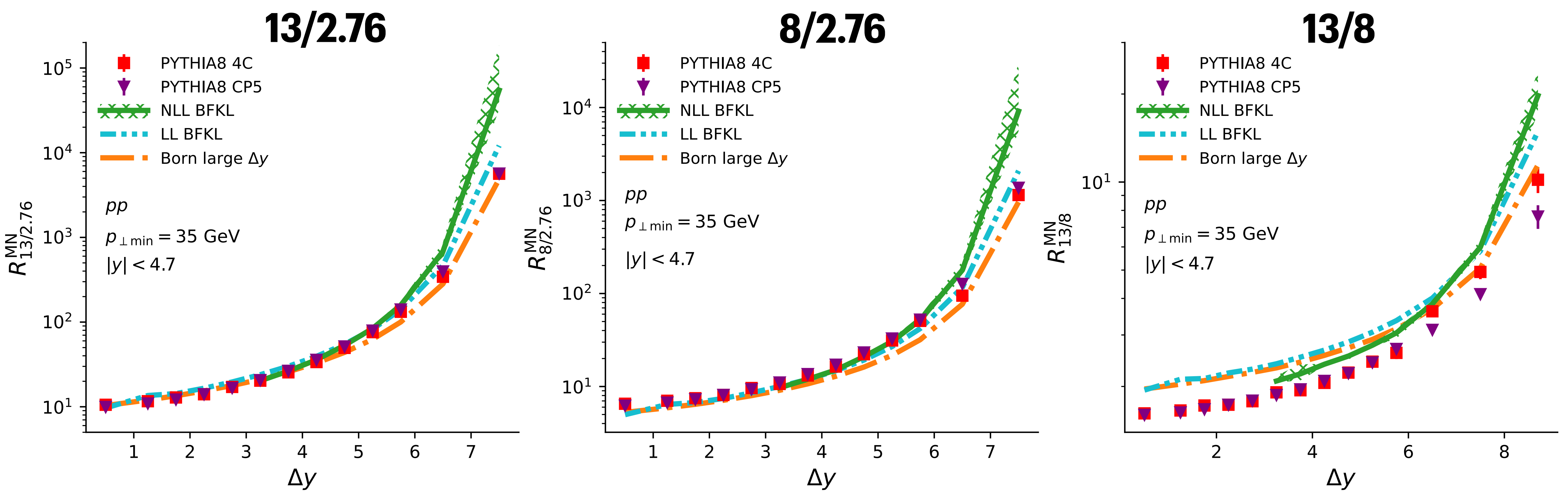
A. Iu. Egorov and V. T. Kim [Phys. Rev. D 108 (2023) 014010]



- NLL BFKL predicts lowest values of MN x-section at large Δy
- Lowering $p_{\perp\min} \Rightarrow$ increasing the sensitivity to expected BFKL effects

NLL BFKL for Ratios of MN cross sections at different energies, $p_{\perp\min} = 35$ GeV

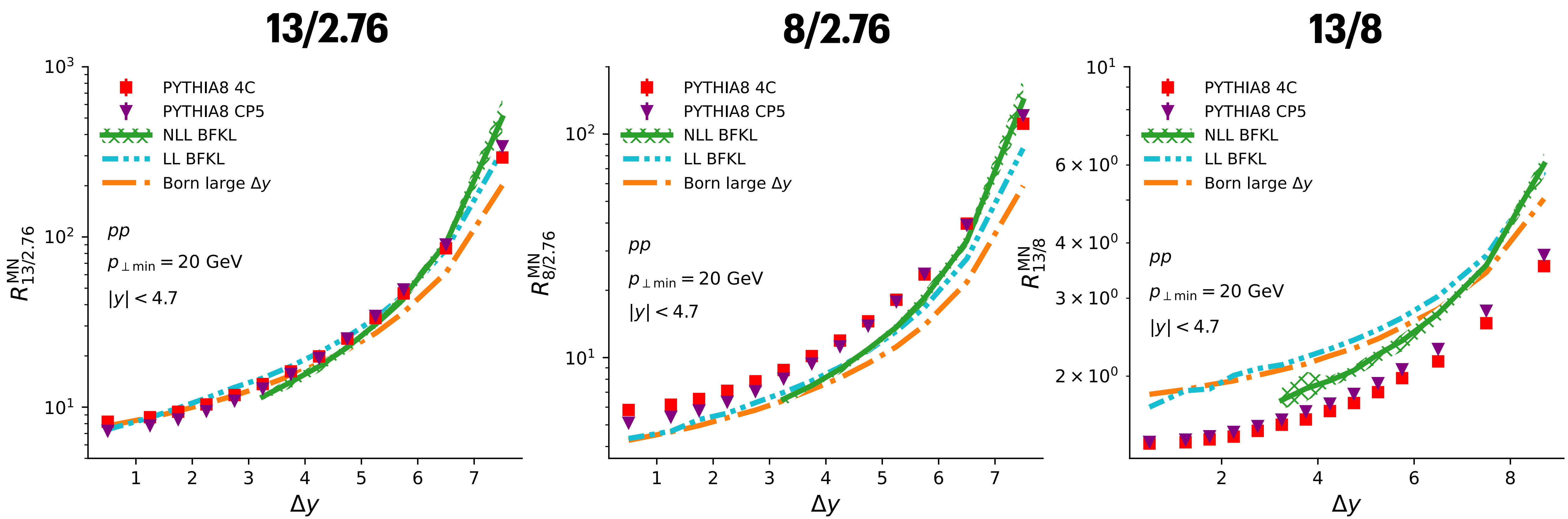
A. Iu. Egorov and V. T. Kim [Phys. Rev. D 108 (2023) 014010]



- NLL BFKL predicts fastest rise with Δy
- Predictions of DGLAP and BFKL-based calculations are well separated at large Δy
- These ratios are sensitive to BFKL

NLL BFKL for Ratios of MN cross sections at different energies, $p_{\perp\min} = 20$ GeV

A. Iu. Egorov and V. T. Kim [Phys. Rev. D 108 (2023) 014010]



- NLL BFKL predicts fastest rise with Δy
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Summary

- The first comparison of the NLL BFKL calculations for MN x-section with experimental data at $\sqrt{s} = 2.76$ TeV is presented.
- The NLL BFKL calculation with BFKLP scale setting agrees with MN x-section measurements of CMS at $\sqrt{s} = 2.76$ TeV. All DGLAP-based calculations are fail at largeΔy. These observations argue in favour of manifestation of the BFKL evolution
- Ratios of MN cross sections at different energies can be a sensitive probe for search of the BFKL evolution
- The predictions which can be tested at LHC are given

THANK YOU!