TWENTY-FIRST LOMONOSOV CONFERENCE August, 24-30, 2023 ON ELEMENTARY PARTICLE PHYSICS
MOSCOW STATE UNIVERSITY

Statistical properties of fractal entropy of K_S^0 meson production in Au+Au collisions at RHIC

Mikhail Tokarev* & Imrich Zborovský**

*JINR, Dubna, Russia
**NPI, Řež, Czech Republic

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Motivation & Goals

Search for new symmetries in Nature

Systematic analysis of inclusive cross sections of particle production in p+p, p+A and A+A collisions to search for general features of hadron and nucleus structure, constituent interaction and fragmentation process over a wide scale range

z-Scaling is a tool in high energy physics

Development of z-scaling approach for description of processes with multiparticle production in inclusive reactions and verification of fundamental physics principles of self-similarity, locality, fractality, maximal entropy, etc.

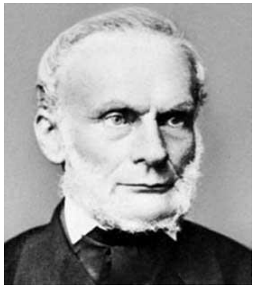
Concept of new entropy and statistical properties of nuclear system with fractal objects and processes probed by K_S^0 mesons produced in Au+Au collisions at RHIC

Entropy – basis notion of thermodynamics and stat. physics

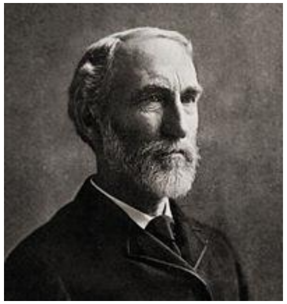
Thermodynamics

ἐντροπία

Statistical physics

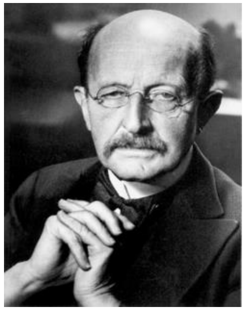


Rudolf Julius Emanuel Clausius



Josiah Willard Gibbs

Ludwig Eduard Boltzmann



Max Karl Ernst Ludwig Planck

Entropy is a function of state

$$dS = dU/T + pdV/T - \mu dN/T$$

$$S = S(U,V,N), \quad T=T(U,V,N)$$

Thermodynamic quantities and potentials are expressed via entropy

$$U=TS-pV+\mu N$$
$$H=U+pV$$
$$F=U-TS$$
$$G=U+pV-TS$$
$$\Omega=U-TS-\mu N$$

$$1/T = \partial S/\partial U|_{V,N}$$
$$p/T = \partial S/\partial V|_{U,N}$$
$$c_V = T\partial S/\partial T|_V$$
$$\partial p/\partial T|_{V,N} = \partial S/\partial V|_{T,N}$$
$$\partial V/\partial T|_{p,N} = - \partial S/\partial p|_{T,N}$$

$$S = k \cdot lnW$$

k - Boltzmann constant

W - number of microstate

Various forms of entropy:

Clausius (1865) S_C
Gibbs (1876) S_G
Boltzmann (1872) S_B
Sharma, Mittal (1975) S_{SM}
Tsallis (1988) S_q
Kaniadakis (2001) S_k
………

von Neumann (1932)
Shannon (1948) S_S
Kolmogorov (1954)
Khinchin (1957)
Rényi (1961) S_R
……….

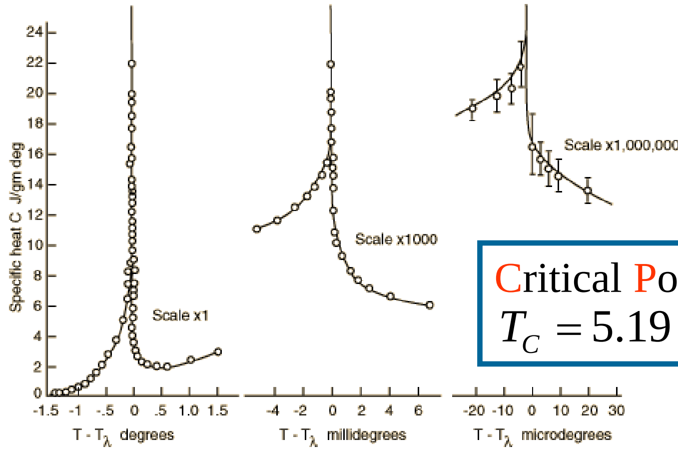
Properties of entropy

- Entropy is function of state of a (thermodynamic) system.
- Entropy is smooth function of thermodynamic parameters.
- For reversible processes $\oint dS = 0$.
- Basic concept in 2nd and 3d laws of thermodynamics.
- Entropy of phase transition.
- Entropy of extensive and non-extensive systems.
- Fractal entropy - entropy of systems with fractal objects.
- Entropy in quantum statistical mechanics.
- Entropy of Entanglement, Black Hole, Big Band, Universe, …
- Entropy and information content of the human genome, …
- ………

Singularity of specific heat near a Critical Point

Superfluid transition

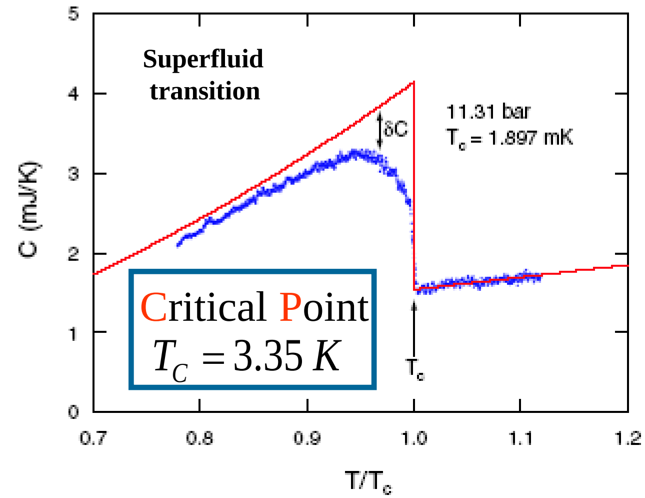
Specific heat of liquid ^4He



Critical Point
$T_C = 5.19\,K$

M. J. Buckingham and W. M. Fairbank, 1961
H.E. Stanley, 1971

Heat capacity of liquid ^3He



Critical Point
$T_C = 3.35\,K$

H. Choi et al., PRL 96, 125301 (2006)

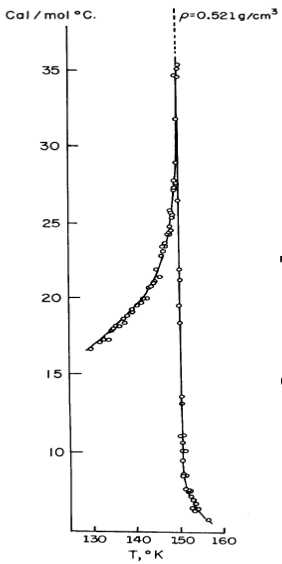
- Near a critical point the singular part of thermodynamic potentials is a Generalized Homogeneous Function (GHF).
- The Helmholtz potential $F(\lambda^{a_\varepsilon}\varepsilon, \lambda^{a_V}V) = \lambda F(\varepsilon, V)$ is GHF of (ε, V).

$$c_V \sim |\varepsilon|^{-\alpha} \qquad \varepsilon \equiv (T-T_c)/T_c \qquad c_V = -T(\partial^2 F/\partial T^2)|_V \qquad c_V = T(\partial S/\partial T)|_V$$

Critical exponents define the behavior
of thermodynamic quantities nearby the Critical Point.

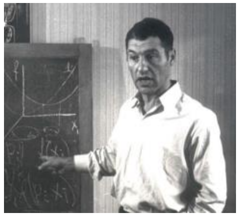
Singularity of heat capacity and thermal conductivity near a CP

Heat capacity of Ar



Critical Point

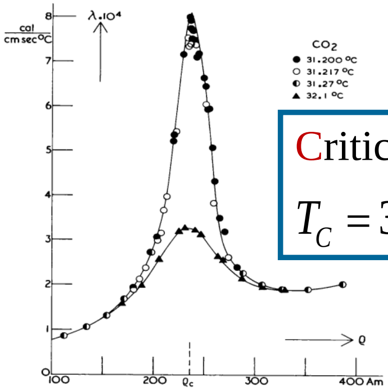
$T_C = 150.8\,K$



The isochoric heat capacity C_v of argon becomes infinite at the vapor-liquid critical point.

A.V. Voronel’ et al.
Zh. Exp. Teor. Fiz. 43, 728 (1962).

Thermal conductivity of CO_2



Critical Point

$T_C = 304.15\,K$

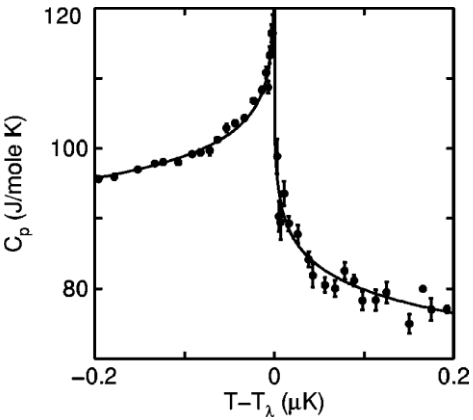
Maximum of thermal conductivity increased upon a gradual approach to the critical isotherm suggesting that it could tend to infinity at the critical point.

Owing to the discoveries made by A.Voronel and J.Sengers more 60 years ago, critical phenomena in fluids and fluids mixtures have become an integral part of condensed-mater physics.

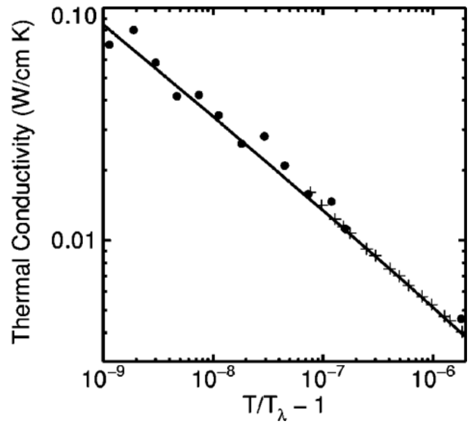
J.V. Sengers,
Thermal Conductivity Measurements at Elevated Gas Densities Including the Critical Region,
Thesis (Universiteit van Amsterdam, 1962).

Singularity of specific heat c_p of liquid ^4He in cosmic space

Specific heat



Thermal conductivity



Specific heat and thermal conductivity vs. reduced temperature near the lambda point

$$t = \frac{T - T_\lambda}{T_\lambda}$$

- Density gradients cause substantial distortion of the singularity for reduced temperatures.
- Transition broadening associated with gravity and relaxation phenomena.

The experiment was performed in Earth orbit to reduce the rounding of the transition caused by gravitationally induced pressure gradients on Earth.

Critical exponent describing the specific-heat singularity was found to be α = -0.01276 ±0.0003.

$$c_p = \frac{A^\pm}{\alpha} | t |^{-\alpha} + B^\pm$$

Expt. in space $|t|<10^{-10}$ (^{4}He , Lipa).
Expt. on Earth $|t|<10^{-7}$ (^{4}He , Fairbank).
Expt. on Earth $|t|<10^{-4}$ (Xe, Sengers,).

In space, the lambda transition is expected to be sharp to $|t|<10^{-12}$ in ideal conditions.

J. A. Lipa et al.,
"Specific heat of liquid helium in zero gravity very near the lambda point"
Phys. Rev. B **68**, 174518, (2003)

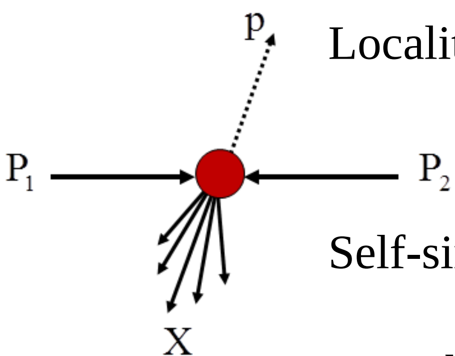
z-Scaling:
hypothesis, ideas, definitions, …

Basic principles:
locality, self-similarity, fractality,…

Phys.Rev. D 75 (2007) 094008
Int.J.Mod.Phys. A 24 (2009) 1417
J. Phys.G: Nucl.Part.Phys.
37 (2010) 085008
Int.J.Mod.Phys. A 27 (2012) 1250115
J.Mod.Phys. 3 (2012) 815
Int.J.Mod.Phys. A 32 (2017) 750029
Nucl.Phys. A 993 (2020) 121646
Nucl.Phys. A 1025 (2022) 122492

z-Scaling

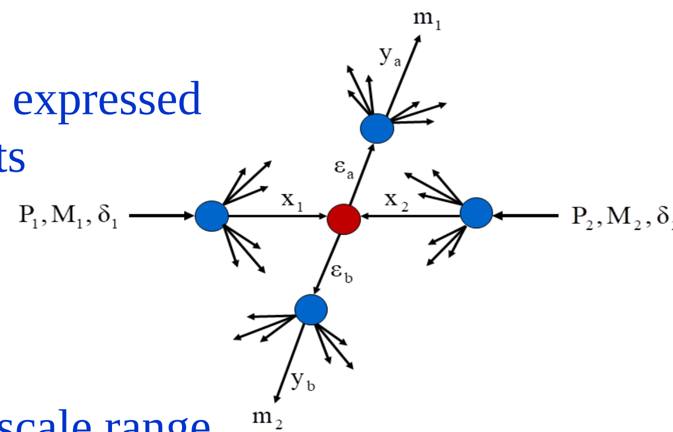
Principles: locality, self-similarity, fractality



Locality: collisions of hadrons and nuclei are expressed via interactions of their constituents (partons, quarks and gluons,...).

Self-similarity: interactions of constituents are mutually similar.

Fractality: self-similarity is valid over a wide scale range.



Hypothesis of z-scaling :

$s^{1/2}$, p_T, θ_{cms}

Inclusive particle distributions can be described in terms of constituent sub-processes and parameters characterizing bulk properties of the system.

x_1, x_2, y_a, y_b

$\delta_1, \delta_2, \varepsilon_a, \varepsilon_b, c$

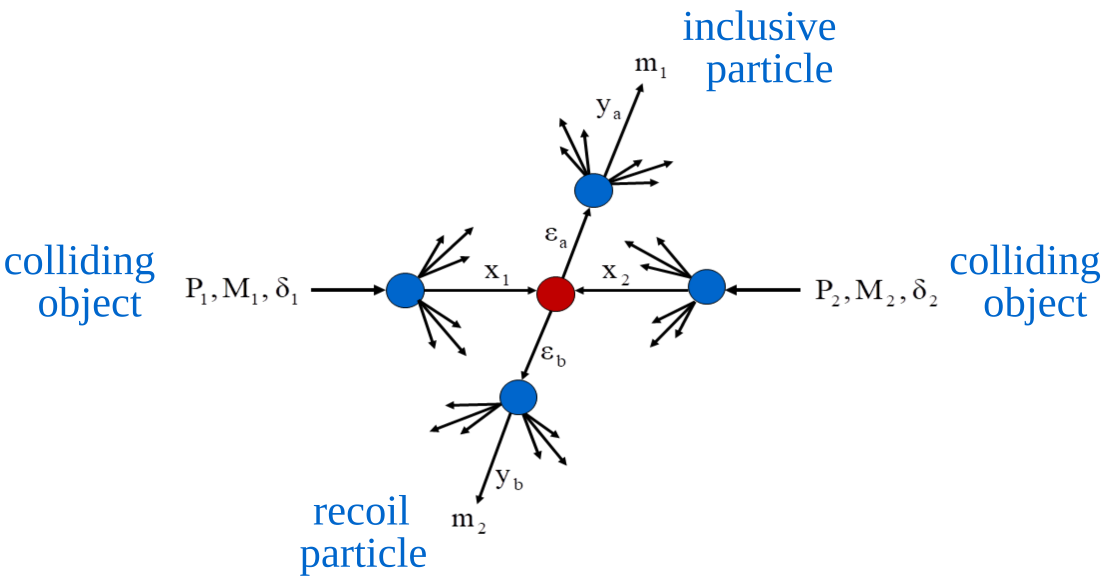
$Ed^3\sigma/dp^3$

Scaled inclusive cross section of particles depends in a self-similar way on a single scaling variable z.

$\Psi(z)$

Locality

Collisions of colliding objects are expressed via interactions of their constituents



P_1, P_2, p – momenta of colliding and produced particles

M_1, M_2, m_1 – masses of colliding and produced particles

x_1, x_2 – momentum fractions of colliding particles carried by constituents

y_a, y_b – momentum fractions of scattered constituents carried by inclusive particle and its recoil

δ_1, δ_2 – fractal dimensions of colliding particles

ε_a, ε_b – fractal dimensions of scattered constituents (fragmentation dimensions)

m_2 – mass of recoil particle

Elementary sub-process:

$$(x_1M_1) + (x_2M_2) \rightarrow (m_1/y_a) + (x_1M_1+x_2M_2+m_2/y_b)$$

Momentum conservation law for sub-process

$$(x_1P_1+x_2P_2-p/y_a)^2 = M_X^2$$

Mass of recoil system

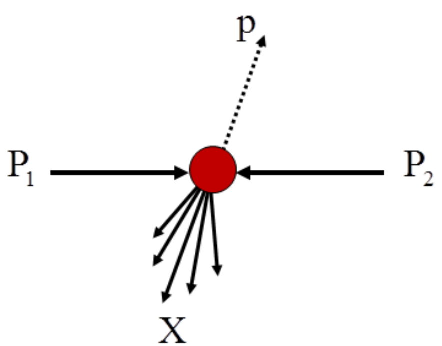
$$M_X = x_1M_1+x_2M_2+m_2/y_b$$

M.T., I.Zborovský
Yu.Panebratsev, G.Skoro
Phys.Rev.D54 5548 (1996)
Int.J.Mod.Phys.A16 1281 (2001)

Self-similarity

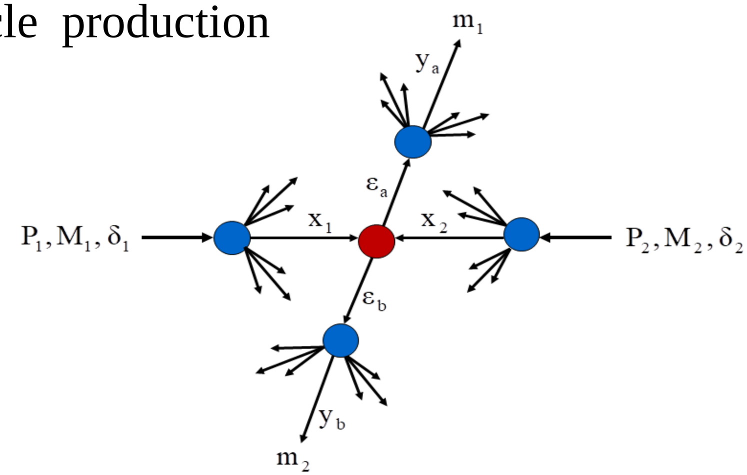
Interactions of constituents are mutually similar

The self-similarity parameter z is a dimensionless variable, expressed through the dimensional quantities P_1, P_2, p, M_1, M_2, m_1, m_2, characterizing the process of inclusive particle production



$$z = z_0 \cdot \Omega^{-1}$$

$$z_0 = \frac{\sqrt{s_\perp}}{(dN_{ch}/d\eta|_0)^c m_N}$$



- Ω^{-1} – the minimal resolution at which a constituent sub-process can be singled out of the inclusive reaction
- $\sqrt{s_\perp}$ – the transverse kinetic energy of the sub-process consumed on production of m_1 & m_2
- $dN_{ch}/d\eta|_0$ – the multiplicity density of charged particles at $\eta = 0$
- c – a parameter interpreted as a “specific heat” of created medium
- m_N – an arbitrary constant (fixed at the value of nucleon mass)

Fractality

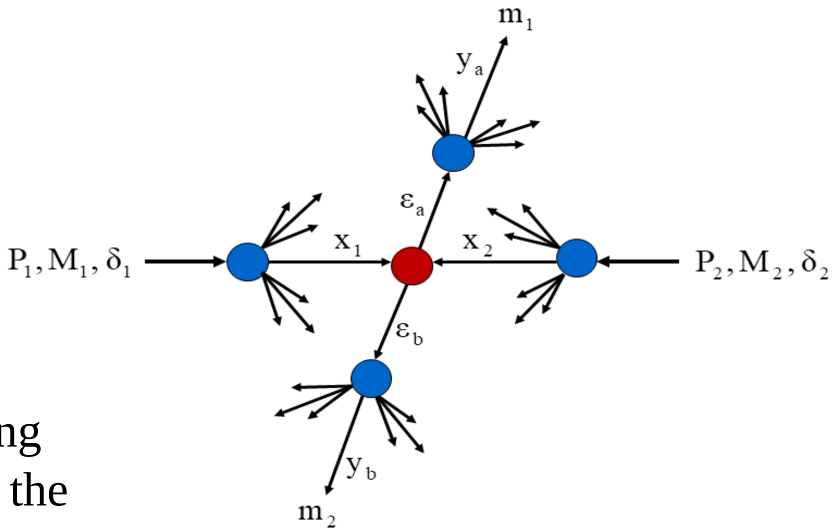
Self-similarity over a wide scale range

Fractal measure

$$z = z_0 \cdot \Omega^{-1}$$

$$\Omega = (1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon_a}(1-y_b)^{\varepsilon_b}$$

$$0 < x_1, x_2 < 1 \qquad 0 < y_a, y_b < 1$$



Ω – relative number of configurations containing a sub-process with fractions x_1, x_2, y_a, y_b of the corresponding 4-momenta

δ_1, δ_2, ε_a, ε_b – parameters characterizing structure of the colliding objects and fragmentation process, respectively

$\Omega^{-1}(x_1, x_2, y_a, y_b)$ characterizes resolution at which a constituent sub-process can be singled out of the inclusive reaction

The fractal measure z diverges as the resolution Ω^{-1} increases.

$$z(\Omega)|_{\Omega^{-1}\to\infty} \to \infty$$

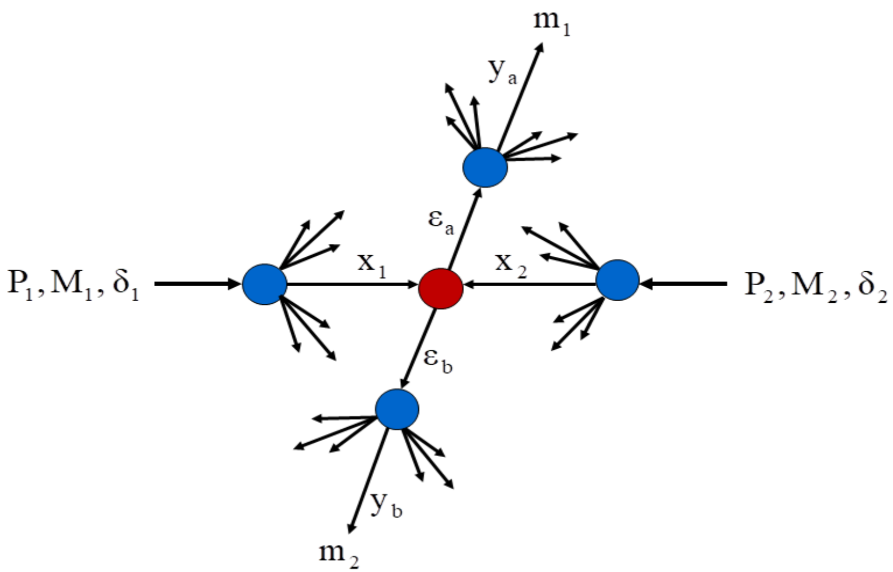
Minimal resolution of sub-process

Principle of minimal resolution: The momentum fractions x_1, x_2 and y_a, y_b are determined in a way to minimize the resolution Ω^{-1} of the fractal measure z with respect to all constituent sub-processes taking into account 4-momentum conservation law.

Momentum conservation law

$$(x_1P_1+x_2P_2-p/y_a)^2 = M_X^2$$

$$\begin{cases} \partial\Omega/\partial x_1 \big|_{y_a=y_a(x_1,x_2,y_b)} = 0 \\ \partial\Omega/\partial x_2 \big|_{y_a=y_a(x_1,x_2,y_b)} = 0 \\ \partial\Omega/\partial y_b \big|_{y_a=y_a(x_1,x_2,y_b)} = 0 \end{cases}$$



Resolution of sub-process

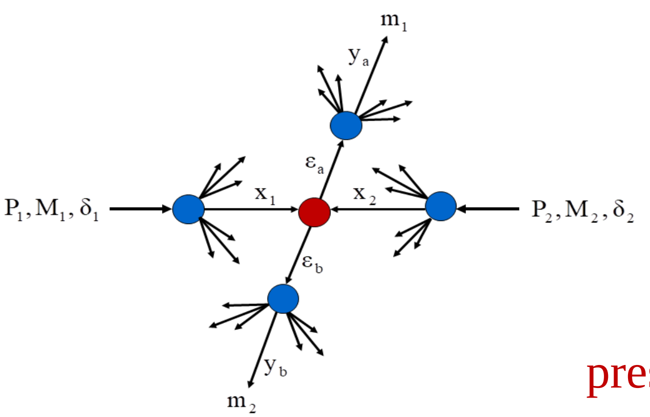
$$\Omega^{-1} = (1-x_1)^{-\delta_1}(1-x_2)^{-\delta_2}(1-y_a)^{-\varepsilon_a}(1-y_b)^{-\varepsilon_b}$$

Fractions x_1, x_2, y_a, y_b are expressed via Lorentz invariants – scalar products of 4-D momenta and particle masses.

Mass of the recoil system

$$M_X = x_1M_1+x_2M_2+m_2/y_b$$

Scaling function $\Psi(z)$



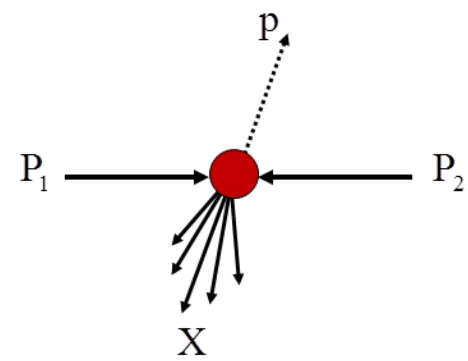
Normalization condition

$$\int_0^\infty \Psi(z)dz = 1$$

Scale transformation

$$z \to \alpha_F \cdot z \qquad \Psi \to \alpha_F^{-1} \cdot \Psi$$

preserves the normalization condition



$$\Psi(z) = \frac{\pi}{(dN/d\eta)\cdot\sigma_{inel}} \cdot J^{-1} \cdot E\frac{d^3\sigma}{dp^3} \quad \Longleftrightarrow \quad \int E\frac{d^3\sigma}{dp^3} dy d^2p_\perp = \sigma_{inel} \cdot \langle N \rangle$$

- σ_{inel} – the inelastic cross section
- $\langle N \rangle$ – the average multiplicity
- $dN/d\eta$ – the multiplicity density
- $J(z,\eta;p_T^2,y)$ – the Jacobian
- $Ed^3\sigma/dp^3$ – the inclusive cross section

The scaling function $\Psi(z)$ is a probability density to produce the inclusive particle with the corresponding value of self-similarity variable z.

Fractal entropy of nuclear system
&
self-similarity variable z

Physics 5 (2023) 537
Phys. Part. Nucl. 54 (2023) 640
Nucl. Phys. A1025 (2022) 122492
Nucl. Phys. A993 (2020) 121646

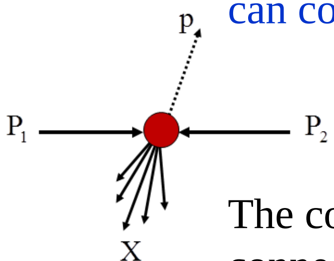
Entropy of nuclear system produced in A+A → h+X

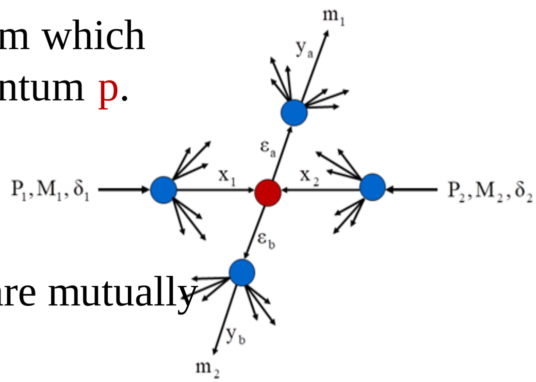
According to statistical physics, entropy of a system is given by a number Ws of its statistical states:

$$S = \ln W_S$$

The most likely configuration of the system is given by the maximal value of S.

For inclusive reactions, the quantity Ws is the number of all parton and hadron configurations in the initial and final states of the colliding system which can contribute to the production of inclusive particle with momentum p.





The configurations comprise all constituent configurations that are mutually connected by independent binary subprocesses:

$$(x_1M_1)+(x_2M_2) \rightarrow (m_a/y_a)+(x_1M_1+x_2M_2+m_b/y_b)$$

The subprocesses corresponding to the production of the inclusive particle with the 4-momentum p are subject to the momentum conservation law:

$$(x_1P_1+x_2P_2-p/y_a)^2=(x_1M_1+x_2M_2+m_b/y_b)^2$$

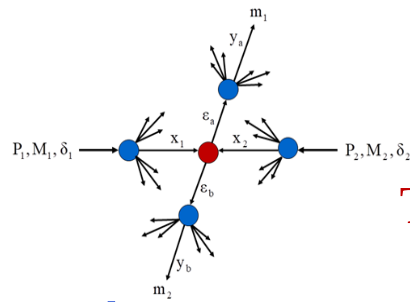
The underlying subprocess, which defines the variable z, is singled out from the corresponding subprocesses by the principle of maximal entropy S.

Self-similarity variable z & Fractal entropy $S_{\delta,\varepsilon}$

$$z = z_0 \Omega^{-1}$$

$$z_0 = \frac{\sqrt{s_\perp}}{(dN_{ch}/d\eta|_0)^c m_N}$$

$$\Omega = (1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon_a}(1-y_b)^{\varepsilon_b}$$



Statistical entropy

$$S = \ln W_S$$

Thermodynamical entropy for ideal gas

$$S = c_V \ln T + R \ln V + S_0$$

The quantity W_S is the number of all parton and hadron configurations in the initial and final states of the colliding system which can contribute to the production of inclusive particle with momentum p

Fractal entropy for independent processes

$$z = \frac{\sqrt{s_\perp}}{W}$$

$$W_S = W \cdot W_0 = (dN_{ch}/d\eta|_0)^c \cdot \Omega \cdot W_0$$

$$S_{\delta,\varepsilon} = c \cdot \ln(dN_{ch}/d\eta|_0) + \ln(V_{\delta,\varepsilon}) + \ln W_0$$

Entropy $S_{\delta,\varepsilon}$ for systems with structural constituents

$$S_{\delta,\varepsilon} = c \cdot \ln(dN_{ch}/d\eta|_0) + \ln[(1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon_a}(1-y_b)^{\varepsilon_b}] + \ln W_0$$

- $dN_{ch}/d\eta|_0$ characterizes "temperature" of the colliding system.
- c has meaning of a "specific heat" of the produced medium.
- Fractional exponents $\delta_1, \delta_2, \varepsilon_a, \varepsilon_b$ are fractal dimensions in the space of $\{x_1, x_2, y_a, y_b\}$.
- $V_{\delta,\varepsilon} = \Omega$ is fractal volume in the space of momentum fraction.

Maximum entropy principle & New conservation law

Principle of maximal entropy:

The momentum fractions x_1, x_2, y_a, y_b are determined in a way to maximize the entropy $S_{\delta,\varepsilon}$ with a kinematic constraint (momentum conservation law).

Maximum of $S_{\delta,\varepsilon}$

$$\begin{cases} \partial\Omega/\partial x_1 = 0 & \partial\Omega/\partial y_a = 0 \\ \partial\Omega/\partial x_2 = 0 & \partial\Omega/\partial y_b = 0 \end{cases}$$

Momentum conservation law

$$(x_1P_1 + x_2P_2 - p/y_a)^2 = M_X^2$$

Mass of recoil system

$$M_X = x_1M_1 + x_2M_2 + m_2/y_b$$

Resolution of sub-processes

$$\Omega^{-1} = (1-x_1)^{-\delta_1}(1-x_2)^{-\delta_2}(1-y_a)^{-\varepsilon_a}(1-y_b)^{-\varepsilon_b}$$

Equivalence of maximal entropy principle and minimal resolution principle

Conservation law

$$\delta_1\frac{x_1}{1-x_1} + \delta_2\frac{x_2}{1-x_2} = \varepsilon_a\frac{y_a}{1-y_a} + \varepsilon_b\frac{y_b}{1-y_b}$$

for arbitrary $P_1, P_2, p, \delta_1, \delta_2, \varepsilon_a, \varepsilon_b$!!!

The conservation law corresponds to maximum of fractal entropy $S_{\delta,\varepsilon}$

I.Zborovsky,
Int. J. Mod. Phys. A 33, 1850057 (2018)

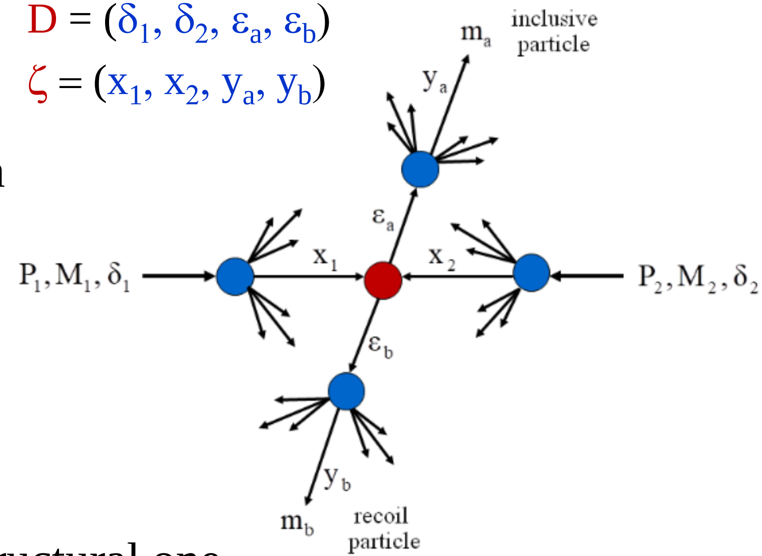
Conservation law for fractal cumulativity

"Fractal cumulativity"

$$C(D,\zeta) = D \cdot \frac{\zeta}{1-\zeta}$$

$D = (\delta_1, \delta_2, \varepsilon_a, \varepsilon_b)$

$\zeta = (x_1, x_2, y_a, y_b)$



The fractal cumulativity before a constituent interaction is equal to the fractal cumulativity after a constituent interaction for any binary constituent sub-process

$$\sum_i^{in} C(D_i,\zeta_i) = \sum_j^{out} C(D_j,\zeta_j)$$

We assume that

- every physical particle is a structural one
- particle's constituents possess a fractal-like structure
- fragmentation is a fractal-like process
- compactness of the fractal structures is governed by the Heisenberg uncertainty principle

Fractal cumulativity is a property of a fractal-like object (or fractal-like process) named a FRACTALON with fractal dimension D to form a "structural aggregate" with certain degree of local compactness which carries the momentum fraction ζ.

Fractal entropy $S_{\delta,\varepsilon}$ near a fractal limit $\Omega^{-1} \to \infty$

Entropy decomposition

$$S_{\delta,\varepsilon} = S_{\Upsilon} - S_{\Gamma} + S_0$$

S_{Υ} depends on momenta and masses of the colliding and inclusive particles

S_0 is some constant guaranteeing positivity of $S_{\delta,\varepsilon}$

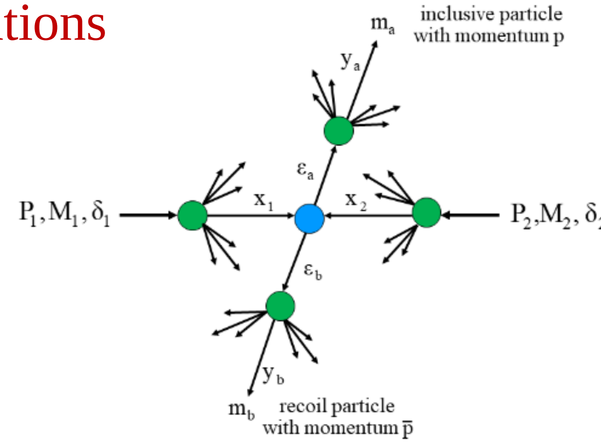
S_{Γ} depends *solely* on fractal dimensions

$$S_{\Gamma} = (\delta_1+\delta_2+\varepsilon_a+\varepsilon_b)\ln(\delta_1+\delta_2+\varepsilon_a+\varepsilon_b) - \delta_1\ln\delta_1 - \delta_2\ln\delta_2 - \varepsilon_a\ln\varepsilon_a - \varepsilon_b\ln\varepsilon_b$$

S_{Γ} enters with minus sign in decomposition of $S_{\delta,\varepsilon}$ and diminishes the fractal entropy

Entropy S_Γ of a statistical ensemble

Statistical ensemble of interacting fractal configurations



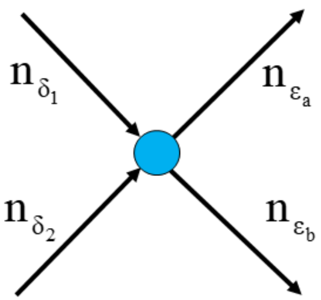
Large collection of the interacting fractals

– with random configurations $\{x_1, x_2, y_a, y_b, ...\}$
– with the same fractal dimensions $\{\delta_1, \delta_2, \varepsilon_a, \varepsilon_b\}$

Number of configurations

n_{δ_1} – internal structure of M_1
n_{δ_2} – internal structure of M_2
n_{ε_a} – fragmentation process to m_a
n_{ε_b} – fragmentation process to m_b

The statistical ensemble is considered as a collection of n_{δ_1} fractals with random configurations but with the same fractal dimension δ_1, together with an analogous set of n_{δ_2} interacting fractals with the fractal dimension δ_2, which recombined via binary sub-processes with the collection of n_{ε_a} fractals with random configurations but with the same fractal dimension ε_a in the final state, and the corresponding set of n_{ε_b} fractals with the fractal dimension ε_b.



Statistical weight

$$\Gamma_{\delta_1,\delta_2,\varepsilon_a,\varepsilon_b} \equiv \frac{\left(n_{\delta_1}+n_{\delta_2}+n_{\varepsilon_a}+n_{\varepsilon_b}\right)!}{n_{\delta_1}!\,n_{\delta_2}!\,n_{\varepsilon_a}!\,n_{\varepsilon_b}!}$$

Entropy of the whole statistical ensemble

$$S_\Gamma = d \cdot \ln\left(\Gamma_{\delta_1,\delta_2,\varepsilon_a,\varepsilon_b}\right)$$

Statistical interpretation of entropy S_Γ

The entropy S_Γ can be presented as logarithm of the number of different ways to share identical dimensional quanta d among fractal dimensions of the interacting fractal structures.

$$S_\Gamma = d \cdot \ln\left(\Gamma_{\delta_1,\delta_2,\varepsilon_a,\varepsilon_b}\right)$$

$$\Gamma_{\delta_1,\delta_2,\varepsilon_a,\varepsilon_b} \equiv \frac{\left(n_{\delta_1}+n_{\delta_2}+n_{\varepsilon_a}+n_{\varepsilon_b}\right)!}{n_{\delta_1}!n_{\delta_2}!n_{\varepsilon_a}!n_{\varepsilon_b}!} = \Gamma_{\delta_1,\delta_2;\varepsilon_a,\varepsilon_b} \cdot \Gamma_{\delta_1,\delta_2} \cdot \Gamma_{\varepsilon_a,\varepsilon_b}$$

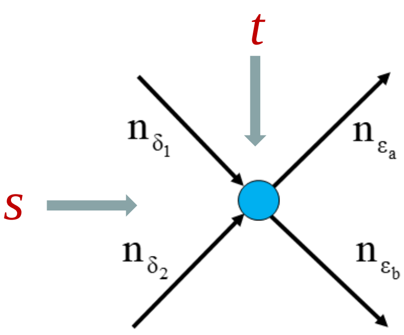
$$\Gamma_{\delta_1,\delta_2;\varepsilon_a,\varepsilon_b} = \frac{(n_{\delta_1}+n_{\delta_2}+n_{\varepsilon_a}+n_{\varepsilon_b})!}{(n_{\delta_1}+n_{\delta_2})!(n_{\varepsilon_a}+n_{\varepsilon_b})!} \qquad \Gamma_{\delta_1,\delta_2} = \frac{\left(n_{\delta_1}+n_{\delta_2}\right)!}{n_{\delta_2}!\; n_{\delta_3}!} \qquad \Gamma_{\varepsilon_a,\varepsilon_b} = \frac{\left(n_{\varepsilon_a}+n_{\varepsilon_b}\right)!}{n_{\varepsilon_a}!\; n_{\varepsilon_b}!}$$

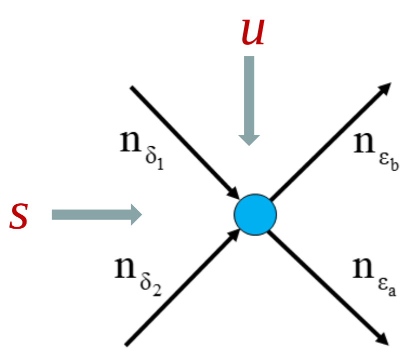
Such interpretation of the entropy S_Γ within statistical ensemble of fractal configurations of the internal structures of the colliding hadrons (or nuclei) and fractal configurations corresponding to the fragmentation processes in the final state is only possible if quantization of fractal dimensions takes place:

$$\delta_1 = d \cdot n_{\delta_1}\ ,\ \ \delta_2 = d \cdot n_{\delta_2}\ ,\ \ \varepsilon_a = d \cdot n_{\varepsilon_a}\ ,\ \ \varepsilon_b = d \cdot n_{\varepsilon_b}$$

d – quant of fractal dimension, $n_{\delta_1}, n_{\delta_2}, n_{\varepsilon_a}, n_{\varepsilon_b}$ – quantum numbers of fractal dimension

Crossing symmetry for entropy S_Γ





Statistical weight

$$S_\Gamma = d \cdot \ln\left(\Gamma_{\delta_1,\delta_2,\varepsilon_a,\varepsilon_b}\right) \qquad \Gamma_{\delta_1,\delta_2,\varepsilon_a,\varepsilon_b} \equiv \frac{\left(n_{\delta_1}+n_{\delta_2}+n_{\varepsilon_a}+n_{\varepsilon_b}\right)!}{n_{\delta_1}!n_{\delta_2}!n_{\varepsilon_a}!n_{\varepsilon_b}!}$$

s - channel

$$\boxed{\Gamma_{\delta_1,\delta_2,\varepsilon_a,\varepsilon_b} = \Gamma_{\delta_1,\delta_2;\varepsilon_a,\varepsilon_b} \cdot \Gamma_{\delta_1,\delta_2} \cdot \Gamma_{\varepsilon_a,\varepsilon_b}}$$

$$\Gamma_{\delta_1,\delta_2;\varepsilon_a,\varepsilon_b} = \frac{(n_{\delta_1}+n_{\delta_2}+n_{\varepsilon_a}+n_{\varepsilon_b})!}{(n_{\delta_1}+n_{\delta_2})!(n_{\varepsilon_a}+n_{\varepsilon_b})!}$$

$$\Gamma_{\delta_1,\delta_2} = \frac{\left(n_{\delta_1}+n_{\delta_2}\right)!}{n_{\delta_1}!\ n_{\delta_2}!}$$

$$\Gamma_{\varepsilon_a,\varepsilon_b} = \frac{\left(n_{\varepsilon_a}+n_{\varepsilon_b}\right)!}{n_{\varepsilon_a}!\ n_{\varepsilon_b}!}$$

t - channel

$$\boxed{\Gamma_{\delta_1,\delta_2,\varepsilon_a,\varepsilon_b} = \Gamma_{\delta_1,\varepsilon_a;\delta_2,\varepsilon_b} \cdot \Gamma_{\delta_1,\varepsilon_a} \cdot \Gamma_{\delta_2,\varepsilon_b}}$$

$$\Gamma_{\delta_1,\varepsilon_a;\delta_2,\varepsilon_b} = \frac{(n_{\delta_1}+n_{\varepsilon_a}+n_{\delta_2}+n_{\varepsilon_b})!}{(n_{\delta_1}+n_{\varepsilon_a})!(n_{\delta_2}+n_{\varepsilon_b})!}$$

$$\Gamma_{\delta_1,\varepsilon_a} = \frac{\left(n_{\delta_1}+n_{\varepsilon_a}\right)!}{n_{\delta_1}!\ n_{\varepsilon_a}!}$$

$$\Gamma_{\delta_2,\varepsilon_b} = \frac{\left(n_{\delta_2}+n_{\varepsilon_b}\right)!}{n_{\delta_2}!\ n_{\varepsilon_b}!}$$

u - channel

$$\boxed{\Gamma_{\delta_1,\delta_2,\varepsilon_a,\varepsilon_b} = \Gamma_{\delta_1,\varepsilon_b;\delta_2,\varepsilon_a} \cdot \Gamma_{\delta_1,\varepsilon_b} \cdot \Gamma_{\delta_2,\varepsilon_a}}$$

$$\Gamma_{\delta_1,\varepsilon_b;\delta_2,\varepsilon_a} = \frac{(n_{\delta_1}+n_{\varepsilon_b}+n_{\delta_2}+n_{\varepsilon_a})!}{(n_{\delta_1}+n_{\varepsilon_b})!(n_{\delta_2}+n_{\varepsilon_a})!}$$

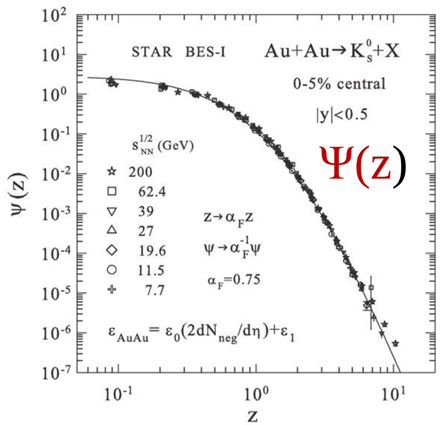
$$\Gamma_{\delta_1,\varepsilon_b} = \frac{\left(n_{\delta_1}+n_{\varepsilon_b}\right)!}{n_{\delta_1}!\ n_{\varepsilon_b}!}$$

$$\Gamma_{\delta_2,\varepsilon_a} = \frac{\left(n_{\delta_2}+n_{\varepsilon_a}\right)!}{n_{\delta_2}!\ n_{\varepsilon_a}!}$$

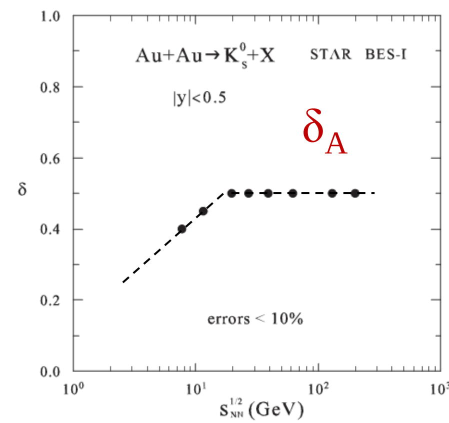
Crossing symmetry for entropy S_Γ in terms of quantum numbers n_i

Features of K_S^0 meson production in Au+Au at RHIC

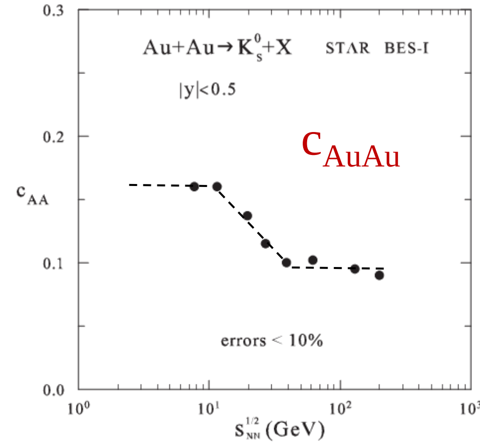
"Collapse" of data points onto a single curve



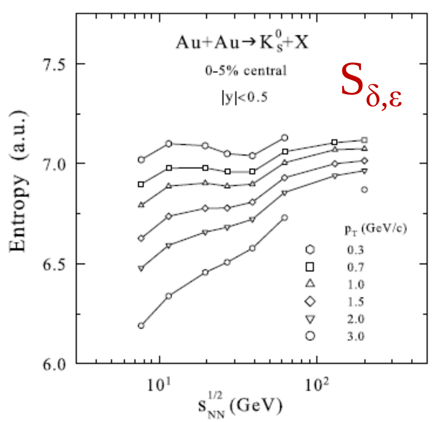
Additivity of nucleus fractal dimension



Anomalous behavior of specific heat



Anomaly of fractal entropy in central Au+Au collsions



Results of analysis:

- Self-similarity and fractality over a wide range of energy, centrality, p_T were established.
- Fractal entropy vs. energy, centrality, p_T was studied.
- Anomalous behavior of $S_{\delta,\varepsilon}$ in central Au+Au collisions at small p_T was found.
- Constancy of fractal dimension δ_A at high energy was found.
- Abrupt decrease of specific heat c_{AuAu} in the range $\sqrt{s_{NN}}$ = 11.5-39 GeV was observed.

Summary

- Fractal entropy introduced in z-scaling approach was discussed.
- Principles of self-similarity, locality, and fractality were verified.
- Equivalence of principle of minimal resolution and principle of maximal fractal entropy was shown.
- Conservation law of fractal cumulativity was formulated.
- Quantization of structural and fragmentation fractal dimensions was suggested.
- Statistical properties of fractal entropy were described.
- Self-similarity of K_S^0 meson production in Au+Au collisions at RHIC was confirmed.
- Anomalous behavior of specific heat of produced medium as a parameter of fractal entropy was found.

Outlook

Verification with high statistical accuracy:

- Scaling behavior and the shape of $\Psi(z)$ over a wider range of z .
- Energy dependence of the fractal dimension δ_A .
- Energy dependence of the specific heat c_{AuAu} at $\sqrt{s_{NN}} > 7.7$ GeV
- Anomalous dependence of the fractal entropy $S_{\delta,\varepsilon}$ at low $p_T < 300$ MeV/c

Thank You Very Much for Your Attention !